

1 **A critique of general allometry-inspired models for estimating**
2 **forest carbon density from airborne LiDAR**

3 Short title: LiDAR estimation of forest carbon density

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18 **Abstract**

19 There is currently much interest in developing general approaches for mapping forest
20 aboveground carbon density using structural information contained in airborne LiDAR data. The
21 most widely utilized model in tropical forests assumes that aboveground carbon density is a
22 compound power function of top of canopy height (a metric easily derived from LiDAR), basal
23 area and wood density. Here we derive the model in terms of the geometry of individual tree
24 crowns within forest stands, showing how scaling exponents in the aboveground carbon density
25 model arise from the height–diameter (H–D) and projected crown area–diameter (C–D)
26 allometries of individual trees. We show that a power function relationship emerges when the
27 C–D scaling exponent is close to 2, or when tree diameters follow a Weibull distribution (or
28 other specific distributions) and are invariant across the landscape. In addition, basal area must
29 be closely correlated with canopy height for the approach to work. The efficacy of the model was
30 explored for a managed uneven–aged temperate forest in Ontario, Canada within which stands
31 dominated by sugar maple (*Acer saccharum* Marsh.) and mixed stands were identified. A much
32 poorer goodness–of–fit was obtained than previously reported for tropical forests ($R^2 = 0.29$ vs.
33 about 0.83). Explanations for the poor predictive power on the model include: (1) basal area was
34 only weakly correlated with top canopy height; (2) tree size distributions varied considerably
35 across the landscape; (3) the allometry exponents are affected by variation in species
36 composition arising from timber management and soil conditions; and (4) the C–D allometric
37 power function was far from 2 (1.28). We conclude that landscape heterogeneity in forest
38 structure and tree allometry reduces the accuracy of general power-function models for
39 predicting aboveground carbon density in managed forests. More studies in different forest types

40 are needed to understand the situations in which power functions of LiDAR height are
41 appropriate for modelling forest carbon stocks.

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43 *Keywords:* biomass, temperate forest carbon, airborne LiDAR, scaling relationships, crown
44 area–diameter allometry, heterogeneity

45 **Introduction**

46 Aboveground carbon density (ACD) is an important forest property to map in the context of the
47 global carbon cycle [1-3]. Classically, ACD has been estimated using tree size measurements
48 recorded from networks of forest plots, with generalised or species–specific allometries used to
49 convert field measures of diameter and height into tree biomass estimates, and then into ACD
50 estimates [4, 5]. More recently, methods using remote sensing technologies have been developed
51 to complement these plot networks: airborne or spaceborne LiDAR sensors have proven to be
52 particularly effective for estimating ACD because they provide detailed information about forest
53 structure, which is in turn closely related to ACD [6].

54 There is currently much interest in developing a general method for predicting ACD from
55 LiDAR [7, 8]. A common approach has been to estimate ACD in field plots and then use
56 regression to relate these measurements to various LiDAR metrics [9]. This approach can deliver
57 accurate estimation models within sampling regions, but the models lack physical underpinnings
58 because they are purely empirical. Consequently, they either need to be re-parameterized for
59 each new site, or generalised by estimating how parameters vary geographically. Asner and
60 Mascaro [8] have developed a General Model (henceforth AM–GM) for predicting ACD, which

61 uses measures of the top canopy height derived from LiDAR (H_L), along with local relationships
62 predicting basal area (B_P) and basal–area–weighted mean wood density ($\bar{\rho}_P$):

$$ACD = aH_L^{b_1}B_P^{b_2}\bar{\rho}_P^{b_3} \quad (1)$$

63 where a , b_1 , b_2 and b_3 are parameters estimated by regression using the log–transformed
64 function. Note that subscript L denotes a LiDAR–based measurement, and subscript P a
65 plot–based measurement. Asner and Mascaro [8] argue that this model is analogous to the
66 allometric formula used to calculate an individual tree’s biomass from its height H_i , diameter D_i
67 and wood density ρ_i measurements, namely $aH_i^bD_i^c\rho_i^d$ where a , b , c and d vary with forest type
68 [10] and i denotes measurements on an individual tree. Fitting the AM–GM to data from four
69 contrasting tropical forests, Asner et al. [7] found that a single, universally fitted relationship
70 reduced model accuracy by no more than 1% relative to regional–specific models. Furthermore,
71 the accuracy was only slightly diminished by replacing plot–level measurements of B_P and $\bar{\rho}_P$
72 with regional averages and, as a result, the major benefit of their approach is that it requires less
73 additional field data to calibrate than traditional regressions [11].

74 A key reason why the AM–GM has worked well, where it has, is that basal area and top-
75 of-canopy height were closely correlated in the forests investigated. Asner and Mascaro [8]
76 showed that – for the four tropical forests studied – the AM–GM could be calibrated simply by
77 generating a local relationship estimating B_P from LiDAR and finding a regional $\bar{\rho}_P$ estimate.
78 Others have questioned the generality of the approach [12,13]. In some forest types the
79 correlation between forest height and basal area is weak, especially for mature stands. In these
80 situations two stands can have the same top-of-canopy height, but quite different basal area
81 [14,15].

82 The problem is that the carbon density of a plot is obtained by summing the biomass of
83 individual trees, but because a tree's biomass is non-linearly related to its dimensions (height,
84 stem diameter), this summation is only exact under certain conditions that we explain below.
85 Although Asner et al. [7] did not claim that the AM-GM could be applied outside the tropics,
86 testing the accuracy of the model across different forest types is important to understanding the
87 applicability and limitations of the general model. For example, tropical and temperate forests
88 have contrasting size structures: rain forests contain shade-tolerant species that develop a dense
89 understory beneath the upper canopy (i.e. stands contain many small trees and few large trees),
90 while temperate forests often lack dense understories and can have unimodal size-frequency
91 distributions [16]. Perhaps for this reason the AM-GM had low goodness-of-fit when applied to
92 broadleaf and coniferous forests in the USA [13], but this has yet to be evaluated critically.
93 Vincent et al. [12] suggest that forests should first be delineated into homogenous regions with
94 respect to the relationships between forest structure and LiDAR data to improve model
95 performance. Unfortunately, this requirement would severely limit the generality of the model.

96 The aim of this study is to derive the AM-GM from first principles using the geometry of
97 individual trees and, by doing so, to improve understanding of when the AM-GM is likely to
98 yield accurate predictions (i.e., have high goodness-of-fit when applied to data from the field
99 and from LiDAR scanners). Our individual-tree-based general model (ITB-GM) has the same
100 functional form as the AM-GM (1), but its parameters are derived from individual tree
101 allometries and other assumed scaling relationships. We fit the AM-GM to data from an
102 uneven-aged forest in central Ontario, Canada and compare the parameter estimates with those
103 obtained from tree-based measurements using the ITB-GM. By doing so, we explore why the
104 AM-GM has poor predictive ability in this temperate forest. We then examine whether fitting

105 separate models for two forest types within the Canadian dataset leads to significant
 106 improvements in goodness-of-fit. Finally, we outline forest conditions that determine the
 107 accuracy of the AM-GM.

108 **Theory: An individual-tree-based general model**

109 Consider a tree with stem diameter D_i (in cm), height H_i (in m), vertically projected crown area
 110 C_i (in m²) and wood density ρ_i (in g/cm³) growing in a plot with an area A_p (in ha). The tree's
 111 aboveground biomass can be modelled as $a_1\pi D_i^2 H_i \rho_i$ where a_1 is a species-specific coefficient
 112 that depends on crown and stem form. The total aboveground biomass of the plot is found by
 113 summing the biomasses of all N_p trees in the plot. ACD is calculated by dividing this biomass
 114 value by A_p and multiplying by carbon content a_0 (typically 0.5):

$$ACD = a_2 \sum_{i=1}^{N_p} a_0 a_1 \rho_i D_i^2 H_i \quad (2)$$

115 where $a_2 = \pi/A_p$. For ease of presentation, the limits of summations are dropped in subsequent
 116 equations, but remain the same throughout.

117 Assuming that a tree's height is related to its diameter by a power function ($H_i =$
 118 $a_H D_i^{k_H}$), we get:

$$ACD = a_2 \sum a_0 a_1 a_H \rho_i D_i^{2+k_H} \quad (3)$$

119 We can use individual tree heights and crown areas to estimate the average top canopy height
 120 H_p : this is calculated by summing the crown top height of all trees in the plot, weighted by their
 121 crown areas, $H_p = (\sum a_3 C_i H_i) / C_p$ where the canopy area of the plot is $C_p = \sum_{j=1}^{N_p} C_j$ and a_3 is a
 122 multiplier that takes into account that the average height of each tree's crown is some fraction of

123 that tree's maximum height [15]. Assuming that crown area is also a power function of stem
 124 diameter ($C_i = a_C D_i^{k_C}$), and that $H_i = a_H D_i^{k_H}$ as before, we get:

$$H_P = \frac{1}{C_P} \sum a_3 a_H a_C D_i^{k_C + k_H} \quad (4)$$

125 Our aim is to substitute (4) into (3) to remove the D_i terms, so that ACD is expressed in terms of
 126 H_P , B_P and $\bar{\rho}_P$. However, this is not straightforward for two reasons. The first problem is that
 127 a_0, a_1, a_3, a_H, a_C and ρ are inside the summations, but cannot necessarily be moved outside the
 128 summations because they are species-specific variables. As an approximation, we represent
 129 them by tree-volume-weighted mean values and take them outside of the summation [12] to
 130 give $ACD \approx a_2 \bar{a}_0 \bar{a}_1 \bar{a}_H \bar{\rho}_P \sum D_i^{2+k_H}$ and $H_P \approx \bar{a}_3 \bar{a}_H \bar{a}_C C_P^{-1} \sum D_i^{k_C+k_H}$. The second problem is
 131 that D_i is raised to different exponents inside the two summations (except when $k_C=2$). In order
 132 to progress, we need to assume that the two summations are themselves related by a scaling
 133 function: $\sum D_i^{2+k_H} \approx a_D (\sum D_i^{k_C+k_H})^{k_D}$; we call this the volume summation scaling relationship.
 134 The canopy area can be substituted with basal area by assuming a second scaling function: $C_P \approx$
 135 $a_B B_P^{k_B}$; we call this the canopy area scaling relationship. Making these substitutions, we obtain
 136 an individual-tree-based general model (ITB-GM):

$$ACD \approx a_4 (H_P)^{k_D} (B_P)^{k_D k_B} \bar{\rho}_P \text{ where } a_4 \approx \bar{a}_0 \bar{a}_1 \bar{a}_H a_2 a_D \left(\frac{a_B}{\bar{a}_3 \bar{a}_H \bar{a}_C} \right)^{k_D} \quad (5)$$

137 This equation is analogous to the AM-GM, given in (1), with $a = a_4$, $b_1 = k_D$, $b_2 = k_D k_B$ and
 138 $b_3 = 1$, but it has more parameters and so is less powerful for predictions.

139 Our derivation based on tree allometries shows that certain parameters in the AM-GM
 140 depend on the exponents of the volume scaling relationship and canopy area scaling relationship.
 141 It is important to realise that it would be impossible to derive a function having the form of the
 142 AM-GM unless these scaling relationships are valid. In the Supporting Information (S1 Text)

143 we show that these relationships are mathematically valid when tree sizes are precisely power-
 144 law or Weibull distributed. If the tree size distributions of all stands across a forest follow one of
 145 these functions (with identical parameters), the summation can be replaced by an integral that
 146 has an analytical solution. Specifically, if a large number of diameters (D_1, \dots, D_N) are drawn from
 147 $p(D) = \alpha D^{-\beta}$ (where α is a normalising constant), then a given power function summation can be
 148 approximated by:

$$\sum_{i=1}^N D_i^\gamma \approx N \int_{D_{min}}^{D_{max}} D^\gamma p(D) dD = \alpha N \int_{D_{min}}^{D_{max}} D^{\gamma-\beta} dD \quad (6)$$

149 which can in turn be solved to give:

$$\sum_{i=1}^N D_i^\gamma \approx \frac{\alpha N}{\gamma - \beta + 1} [D_{max}^{\gamma-\beta+1} - D_{min}^{\gamma-\beta+1}] \quad (7)$$

150

151 A similar property holds for a Weibull distribution of tree diameters (see S1 Text). If the power
 152 or Weibull distribution is identical across stands, it can be shown that $k_D = k_B = 1$ and a_D and
 153 a_B are both predictable.

154 We now compare the performance of the AM–GM and ITB–GM using data from a
 155 temperate forest, to gain a better understanding of when these models are appropriate for
 156 estimating ACD from LiDAR data.

157 **Materials and methods**

158 *Study area and inventory dataset*

159 We used datasets from Haliburton Forest and Wildlife Reserve in central Ontario, Canada
 160 (45°13'N, 78°35'W). The forest is managed using selection silviculture and consists mostly of
 161 uneven-aged stands [17]. Sugar maple (*Acer saccharum* Marsh.) is the most prevalent species,

162 but a number of other species are common, including eastern hemlock (*Tsuga canadensis* (L.)
163 Carrière), balsam fir (*Abies balsamea* (L.) Mill.) and American beech (*Fagus grandifolia* Ehrh.).
164 There were 154 circular plots inventoried across the forest each with an area of 2500 m². The
165 plot locations were chosen to stratify the variation across the forest. The stem diameters of all
166 trees with a stem diameter equal to or greater than 8 cm were recorded along with their species
167 identity. The plots were randomly split into a calibration (114 plots) and a validation dataset (40
168 plots). The calibration dataset was used for fitting the models and relationships, whilst the
169 validation dataset was reserved for assessing model performance.

170 ACD was estimated for each plot using species-specific allometric equations developed
171 for Canadian inventories, which relate stem diameter to aboveground tree biomass [18, 19].
172 Species-specific equations were used for the seven most prevalent species and then generic
173 conifer and broadleaf equations were used for all remaining species (~ 17% of total trees). The
174 individual tree aboveground biomasses were summed for each plot and converted to a per
175 hectare estimate; this aboveground biomass estimate was then multiplied by the carbon content
176 of wood (0.5; [20]) to estimate ACD. Wood density estimates were extracted from [21] and
177 represent the oven dry mass divided by green volume. To parameterise the LiDAR models
178 (AM-GM and ITB-GM), wood density was summarised as a volume-weighted average for
179 each plot ($\bar{\rho}_p$). Finally, we succinctly described the tree size distribution of each plot by
180 calculating the quadratic mean diameter (QMD) as $200\sqrt{A_p B_p / (\pi N_p)}$, and by fitting a Weibull
181 distribution to the list of stem diameters.

182 *Airborne LiDAR*

183 The LiDAR data were collected using an Optech ALTM 3100 four-pass system flown in August
184 2009 (altitude = 1500 m; pass overlap = 30%; pulse density = 2 pulses/m²). The dataset consisted
185 of x, y and z coordinates (converted to the height above the ground by subtracting the digital
186 elevation model) with up to four returns recorded from a single pulse. We used discrete-return
187 airborne LiDAR data clipped in ArcGIS 10 to overlay the inventoried plots, which had been
188 georeferenced to sub-metre accuracy using a Trimble Geo XH 6000. The LiDAR metrics used
189 in the analyses were H_L and gap fraction (G_L) (

190 Table 1). We split each plot into 1 m by 1 m tiles and extracted the maximum recorded height of
191 pulses in each of those tiles. H_L was calculated as the mean of the tile heights that were recorded
192 at 2 m and above, which excluded the tiles where LiDAR pulses were not intercepted by the
193 canopy. G_L was calculated as the proportion of first returns recorded at a height less than 2 m
194 above the ground.

195

196

Table 1 Definitions of all terms and parameters in the AM–GM and ITB–GM.

Term	Definition	Units
Lidar metrics		
H_L	Top canopy height	m
G_L	Gap fraction	No units
Tree level measurements		
ρ_i	Wood density	$Mg\ m^{-3} \equiv g\ cm^{-3}$
D_i	Diameter	cm
B_i	Basal area	m^2
H_i	Stem height	m
C_i	Crown area	m^2
Plot based measurements		
ACD	Aboveground carbon density	$Mg\ C\ ha^{-1}$
$\bar{\rho}_P$	Mean wood density (weight by relative abundances of species)	$Mg\ m^{-3} \equiv g\ cm^{-3}$
C_P	Canopy area ($C_P = \sum_{j=1}^N C_j$)	m^2
A_P	Plot area	ha
N_P	Total number of stems in a plot	No units
H_P	Average top canopy height	m
B_P	Basal area	$m^2\ ha^{-1}$
QMD	Quadratic mean diameter	cm
Model parameters		
a_0	Carbon content of trees	
a_1	Coefficient related to crown and stem form	
a_2	Factor scaling stem diameter to plot level basal area	
a_3	Average crown height as a proportion of tree height	
a_4	Coefficient in final ACD equation which amalgamates other coefficients	
$\bar{a}_0, \bar{a}_1, \bar{a}_3$	Means of a_0 , a_1 , and a_3 , weighted by tree volumes	
a_H, k_H	Coefficient and exponent of scaling relationship between stem diameter and height (H–D)	
a_C, k_C	Coefficient and exponent of scaling relationship between stem diameter and crown area (C–D)	
a_D, k_D	Coefficient and exponent of scaling relationship between two summations of stem diameter raised to different powers (volume scaling relationship)	
a_B, k_B	Coefficient and exponent of scaling relationship between canopy area and basal area (canopy area scaling relationship)	

199 *Forest types from aerial photography*

200 The study area was classified into two forest types using aerial photographs (captured by an
201 ADS52 Leica camera). The photographs were manually delineated into 42 forest types using
202 standard methods developed by Ontario's Forest Resources Inventory programme [22]. We
203 reduced the number of forest types to just two according to estimated species composition:
204 stands dominated by sugar maple, and mixed stands that contained a significant coniferous
205 component alongside sugar maple (see [23] for further details on the method used).

206 *Fitting the AM–GM to the Canadian data*

207 The log–transformed AM–GM was fitted using least squares regression to ACD measured in the
208 calibration plots:

$$\ln ACD = \ln a + b_1 \ln H_L + b_2 \ln B_P + b_3 \ln \bar{\rho}_P \quad (8)$$

209
210 Predicted ACD values included a $e^{MSE/2}$ multiplier (where MSE is the mean square error of the
211 regression) to correct for a bias introduced by the log transformation [24]. B_P and $\bar{\rho}_P$ were
212 estimated from relationships with LiDAR so that the model could be used to predict ACD
213 outside of the measured plots. We compared the accuracy of models based on LiDAR estimates
214 of B_P and $\bar{\rho}_P$ against models where B_P and $\bar{\rho}_P$ were ground measurements, to quantify the loss in
215 accuracy as a result of this estimation approach.

216 We measured the accuracy of the 40 validation plot predictions of the ACD model and
217 the B_P and $\bar{\rho}_P$ equations using the coefficient of determination (R^2):

$$R^2 = 1 - \frac{\sum_{j=1}^{40} (P_j - O_j)^2}{\sum_{j=1}^{40} (O_j - \bar{O})^2} \quad (9)$$

218 where the observed and predicted value for each plot is denoted by O_j and P_j , respectively, and
 219 the overall mean observed value is denoted by \bar{O} . We compared model support using the Akaike
 220 information criterion (AIC) where k is the number of estimated parameters and L is the
 221 maximised likelihood function:

$$AIC = 2k - 2\ln(L) \quad (10)$$

222

223 We also calculated the percentage root mean square error (% RMSE) which is normalised
 224 using the mean of the observed values:

$$\% RMSE = \frac{100}{\bar{O}} \sqrt{\frac{\sum_{j=1}^{40} (P_j - O_j)^2}{40}} \quad (11)$$

225 *Estimating the parameter values of the ITB–GM from tree level information*

226 Exponents k_B and k_D of the ITB–GM equation ($ACD \approx a_4 H_L^{k_D} B_P^{k_D k_B} \bar{\rho}_P$) are derived from the
 227 volume summation and canopy area scaling relationships. To estimate these, we first estimated
 228 allometric scaling exponents k_H and k_C from dimensional measurements of 5436 trees at a site
 229 230 km from the study area [25]. We calculated the relative abundances of species within the
 230 114 calibration plots (Table S1), then drew 500 trees at random from the height and crown radius
 231 dataset such that the species composition of the sample was the same as observed in the plots.
 232 Power functions were then fitted to the height vs. diameter and crown area vs. diameter
 233 relationships for these 500 trees. The fitted power functions gave values for k_H and k_C that were
 234 representative of the species composition in our study area. Exponent k_D (of the volume scaling
 235 relationship) was estimated by calculating $\log(\sum D_i^{2+k_H})$ and $\log(\sum D_i^{k_C+k_H})$ for each of the 114
 236 calibration plots, and then fitting a power function through these data. Similarly, exponent k_B of
 237 the canopy area scaling relationship was estimated by calculating $\log(C_P)$ and $\log(B_P)$ for each

238 of the 114 calibration plots, and then fitting a power function through these data. Theoretically,
239 a_4 in the ITB–GM could be calculated as $\bar{a}_0 \bar{a}_1 \bar{a}_H a_2 a_D a_B^{k_D} (\bar{a}_3 \bar{a}_H \bar{a}_C)^{-k_D}$ but in practice several
240 of these variables are hard to determine. For this reason, a_4 was estimated by linear regression:
241 we fit $\log(\text{ACD})$ as a linear function of $\log H_L$, $\log B_P$ and $\log \bar{\rho}_P$ with the coefficients associated
242 with these explanatory variables fixed at the values calculated from individual–tree–based
243 information, such that only a_4 was estimated.

244 *Testing whether forest type information improves model accuracy*

245 To explore whether incorporating forest type information improved the predictive power of the
246 estimation model, we split the plots into sugar maple and mixed stands using the aerial
247 photographs and repeated the same procedures as above for fitting AM–GM and ITB–GM.
248 Forest type was incorporated into both of these models and into the equations estimating B_P and
249 $\bar{\rho}_P$ from H_L and G_L .

250 **Results**

251 *Predicting temperate forest biomass using general power-law models*

252 A summary of the coefficients and goodness–of–fit estimates of the AM–GM (1) fitted to the
253 Canadian temperate forest dataset are provided in Table 2. The coefficient of the $\log(\bar{\rho}_P)$ term
254 was not significantly different from zero, so we set the power (b_3) to 1 to match the ITB–GM.
255 The resulting model performed relatively poorly, as the R^2 of the fit to the validation plots was
256 only 0.18. Fitting the model with ground–measured B_P and $\bar{\rho}_P$ increased the R^2 to 0.41, but
257 unfortunately B_P was poorly predicted from LiDAR estimates of H_L and G_L ($R^2 = 0.09$; Table 3),

258 and $\bar{\rho}_P$ was unrelated to the LiDAR metrics (Figs 1, 2). As a result, we found that ACD could be
 259 estimated using the AM–GM with relatively low accuracy (22.5% RMSE; equivalent to a RMSE
 260 of 15.7 Mg C ha⁻¹; Fig 3).

261

262 **Table 2. Aboveground carbon density (ACD) estimation models fit to a Canadian temperate**
 263 **forest dataset containing sugar maple and mixed broadleaf-conifer stands.** Parameters
 264 shown in bold were estimated from individual tree data, while all other parameters were
 265 estimated using least-squares regression of calibration plot data. The AIC gives the relative
 266 performance of the models and the R² denotes the fit to the validation plots: 1) using ground
 267 measured B_P and $\bar{\rho}_P$ and 2) using LiDAR estimated B_P and $\bar{\rho}_P$.
 268

Model type	ACD estimation equation	AIC	1) ground	2) LiDAR
			<i>B_P and $\bar{\rho}_P$</i>	<i>B_P and $\bar{\rho}_P$</i>
			R ²	R ²
<i>Asner and Mascaro's General Model (AM–GM)</i>				
All stands	$5.11H_L^{0.271}B_P^{0.808}\bar{\rho}_P$	947.8	0.405	0.179
Sugar maple stands	$2.99H_L^{0.258}B_P^{0.991}\bar{\rho}_P$	944.4	0.453	0.292
Mixed stands	$10.1H_L^{0.258}B_P^{0.616}\bar{\rho}_P$			
<i>Individual Tree Based General Model (ITB–GM)</i>				
All stands	$0.285H_P^{1.24}B_P^{0.870}\bar{\rho}_P$	1009.6	-0.111	-0.213
Sugar maple stands	$0.552H_P^{1.15}B_P^{0.729}\bar{\rho}_P$	1002.5	-0.088	-0.330
Mixed stands	$0.314H_P^{1.22}B_P^{0.867}\bar{\rho}_P$			

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272 **Table 3. Basal area and wood density estimation equations obtained by least squares**
 273 **regression.** Explanatory variables were LiDAR metrics top canopy height (H_L) and gap fraction
 274 (G_L) and forest type derived from aerial photographs in the sugar maple and mixed stand specific
 275 equations. The AIC gives the relative performance of the models and the R^2 denotes the fit to the
 276 validation plots.
 277

Response variable	Estimation equations	AIC	R^2
<i>Basal area</i>			
B_P (all stands)	$14.2 + 0.871 H_L - 29.4 G_L$	728.5	0.093
B_P (sugar maple stands)	$4.83 + 1.21 H_L - 20.3 G_L$	666.2	0.286
B_P (mixed stands)	$12.5 + 1.21 H_L - 20.3 G_L$		
<i>Volume-weighted mean wood density</i>			
$\bar{\rho}_P$ (all stands)	0.533	-307.0	-0.022
$\bar{\rho}_P$ (sugar maple stands)	0.576	-364.3	0.188
$\bar{\rho}_P$ (mixed stands)	0.497		

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 279
 280
 281 **Fig 1. Relationships between field-measured biophysical properties (basal area B_P and**
 282 **wood density $\bar{\rho}_P$) and LIDAR metrics (top-of-canopy height H_L and gap fraction G_L).** The
 283 lines are predictions from multiple regression analyses of data from all sites (solid), mixed stands
 284 (dashed) and sugar maple (dotted). For panels (a) and (c), the predicted lines are obtained by
 285 holding G_L constant at its mean value, whilst for panels (b) and (d) the value of H_L was held at its
 286 mean value.
 287

288 **Fig 2. Predictions made for the validation plots by multiple-regression models for basal**
 289 **area (left column) and volume weighted wood density (right column) with: a) no species**
 290 **information and b) forest types.**
 291

292 **Fig 3. LiDAR vs ground estimated ACD in 40 validation plots, where LiDAR estimates are**
 293 **based on Asner and Mascaro's general model (AM-GM; first column) and the individual**
 294 **tree based general model (ITB-GM; second column).** The first row gives the fit of the
 295 AM-GM and ITB-GM to the 40 validation plots (AM-GMa and ITB-GMa) and the second
 296 row gives the fit of the models fitted separately to 24 sugar maple and 16 mixed-species stands
 297 (AM-GMb and ITB-GMb). The overall RMSE value for each model version is given in the
 298 bottom right corner of the plot and the individual RMSE for the sugar maple (Mh) plots and
 299 mixture plots (Mix) are given above the plot.
 300

301

302 Including forest type into the B_P and $\bar{\rho}_P$ estimation models led to increased
303 goodness-of-fit (R^2 rose from 0.09 to 0.29 in the B_P models and from -0.02 to 0.19 in the $\bar{\rho}_P$
304 equations; Table 3) and was strongly supported by AIC (B_P : $\Delta = 62.3$; $\bar{\rho}_P$: $\Delta = 57.3$). The %
305 RMSE of the B_P estimator fell from 23.3 to 20.7% and that of $\bar{\rho}_P$ from 11.7 to 10.4% (Fig. 2b).
306 The mixed-forest plots had higher basal area and lower wood density than the sugar maple plots
307 (Fig 1). Incorporating forest type improved overall performance of the AM-GM with the R^2
308 rising from 0.18 to 0.29 (RMSE: 20.9 vs. 22.5%), with moderate AIC support ($\Delta = 3.4$).

309 *Estimating the exponents of individual-tree-based generalised model (ITB-GM)*

310 The ITB-GM model, which fixed the values of model parameters based on the field-
311 measured allometries of individual trees, performed less well than the Asner-Mascaro model in
312 which the parameters were estimated by regression. The exponents of ITB-GM estimated from
313 the fitted allometric powers of the H-D and C-D relationships are presented in Table 4 and the
314 fitted relationships are presented in Fig 4. For all stands, height and crown area were fitted as
315 power functions of diameter, with exponents of 0.521 and 1.28 respectively. The log-log
316 regression relationship between summed stem volume ($\sum D_i^{2+k_H}$) and the maximum canopy
317 volume ($\sum D_i^{k_C+k_H}$) had a higher goodness-of-fit ($R^2 = 0.814$) than the log-log regression
318 relationship between canopy area (C_P) and basal area (B_P) ($R^2 = 0.654$) indicating that the
319 volume scaling relationship was better supported than the canopy area scaling relationship.

320

321

322 **Table 4. Estimates of power function parameters of relationships between (a) height vs**
 323 **diameter; (b) crown area vs diameter; (c) summed diameters raised to 2 different powers**
 324 **(see text; crown volume scaling relationship); (d) basal area vs canopy area (canopy area**
 325 **scaling relationship).**

Model version	(a) H_i vs D_i			(b) C_i vs D_i			(c) $\sum D_i^{2+k_H}$ vs $\sum D_i^{k_C+k_H}$		(d) C_P vs B_P	
	a_H	k_H	R^2	a_C	k_C	R^2	k_D	R^2	k_B	R^2
All stands	3.26	0.521	0.593	0.465	1.28	0.419	1.24	0.814	0.701	0.654
Sugar maple stands	3.89	0.476	0.634	0.898	1.10	0.431	1.15	0.659	0.632	0.503
Mixed stands	3.73	0.466	0.503	0.397	1.29	0.378	1.22	0.813	0.711	0.676

326

327

328 **Fig 4. Height–diameter power relationships are given in the left panel whilst the crown**
 329 **area–diameter power relationships are given in the right panel.** The exponents from these
 330 fitted power functions are used to estimate the powers in the ITB–GM model (Table 4): top row
 331 for all stands, middle row for sugar maple stands and the bottom row for mixed stands.
 332

333 Both scaling relationships contained residual error and had exponent values different
 334 from 1 because our set of plots did not follow a single diameter distribution (Fig 5). Although the
 335 Weibull distributions that we fit showed that stem diameters were monotonically decreasing in
 336 most calibration plots, quadratic mean diameter ranged from 13 to 33 cm across the plots. Plots
 337 with a higher QMD generally had a higher top canopy height as measured by LiDAR. In the
 338 Supporting Information (S1 Text, Figs S1-S3), we provide a comprehensive analysis of how
 339 variation in tree diameter distributions affects model fit for a range of different H–D and C–D
 340 scaling relationships.

341

342 **Fig 5. Weibull distributions of tree diameters in each calibration plot.** The rug plot along the
 343 x-axis shows the quadratic mean diameter of each plot, coloured according to top canopy height.
 344 The left panel represents sugar maple stands; the right panel represents mixed stands.
 345

346 The ITB–GM, with exponents fixed at their theoretical values and a_4 fitted by linear
347 regression is given in Table 2. The exponent associated with B_P was similar in the two models
348 (AM–GM: 0.81 vs ITB–GM: 0.87), but the exponent associated with H_L differed greatly (0.271
349 vs 1.24). The ITB–GM model had a RMSE of 27.3%, indicating it is less able to explain
350 variance in biomass than the AM–GM (22.5%).

351 The best predictions were obtained by using the AM–GM and including forest type
352 information (RMSE: sugar maple: 15.8%; mixture: 25.5%). The exponent of the H_L term in the
353 AM–GM was unaffected by forest type, but the B_P exponent of mixed stands was much lower
354 than the sugar maple exponent (0.616 vs 0.991; Table 2). Including forest type led to greater
355 improvements in the fit of the ITB–GM than that of the AM–GM ($\Delta AIC = 7.1$ vs. $\Delta AIC = 3.4$).
356 However, the predictions to the validation plots of the ITB–GM were slightly less accurate
357 (RMSE: no forest types = 27.3%; forest type = 28.6%). In all versions of the model, the sugar
358 maple plots were predicted more accurately than the mixture plots.

359 **Discussion**

360 Deriving the AM–GM from individual tree measurements has revealed the origins of its
361 parameters, the assumptions behind the power function formula, and the situations in which it is
362 unlikely to make accurate predictions. Below, we explore specific explanations for low
363 goodness-of-fit, including that (1) the basal area and wood density of plots are not closely
364 correlated with top canopy height or gap fraction as measured by LiDAR; (2) tree size
365 distributions are not conserved across the landscape; and (3) the exponents of the allometries are
366 affected by systematic changes in species composition, and the exponent of the crown area

367 allometry deviates from 2. Our findings suggest that among-stand variability in structure and
368 composition are key factors in determining the accuracy of the AM-GM.

369 *Basal area is weakly correlated with height*

370 Basal area is a key element of allometry-inspired models for estimating forest carbon. It is not
371 directly measured from LiDAR, but instead is inferred indirectly from other height metrics [23].
372 The goodness-of-fit of the AM-GM for this Canadian forest was substantially reduced when
373 ground-measured B_p and $\bar{\rho}_p$ were replaced with LiDAR estimates ($R^2 = 0.41$ vs 0.18) and
374 therefore LiDAR offered a poor substitute for ground data on these quantities. Predictions of B_p
375 from LiDAR metrics were weak in our study area ($R^2 = 0.09$; Fig 1) compared with that reported
376 by Asner et al. [7] for tropical forests ($R^2 \geq 0.55$), although the fit was improved by splitting the
377 plots into two forest types ($R^2 = 0.29$). Improving the accuracy of the LiDAR-based models of
378 B_p may therefore require other metrics than H_L and G_L to be included in regression relationships
379 [9, 12] or the application of individual-based approaches [14].

380 *Tree size distributions vary across the landscape*

381 Basal area is fundamentally linked with the stem diameter distribution, and variability in
382 this distribution weakens the correlation between canopy height and basal area. When the stem
383 diameter distribution follows either a power- or truncated-Weibull function and is conserved
384 across a landscape, then the volume summation and crown area scaling relationships are exact
385 and the exponents of the AM-GM all reduce to 1 (see S2 Fig.1 and derivation in S1 Text).
386 However, when the underlying diameter distributions vary among stands, the exponents relating
387 these quantities will deviate from 1 and the accuracy of the relationships will decrease (Fig S3).

388 The AM–GM is therefore likely to be less accurate in forests where there is large variability in
389 tree size distributions.

390 *Why are size distributions more variable in temperate forests than in natural tropical forests?*

391 Size distributions of forests are linked to size–dependent growth and mortality [26], and can be
392 similar across forested landscapes if these demographic functions remain constant over space and
393 time [27, 16]. This may be a reasonable assumption in old-growth tropical forests where size
394 distributions are often close to power functions with exponents of roughly -2 [28, but see 27].
395 Temperate forests are often managed and comprise a patchwork of stands at different stages of
396 recovery following disturbance (natural or human). Temperate forest size distributions tend to be
397 more variable [29] and are often modelled by a Weibull distribution with the flexibility to fit
398 both unimodal and power function–type distributions [16]. The selection–managed forests
399 considered here are uneven–aged, and exhibit varying tree size distributions as a legacy of their
400 management history. Our analyses suggest that assumptions of the AM–GM are compromised in
401 structurally heterogeneous forests, and that this model is not expected to produce high
402 goodness–of–fits in such areas. In our particular study area, changing management practices
403 over time have produced a wide range of diameter distributions, which in turn have weakened
404 the accuracy of the AM–GM.

405 *Wood density is very weakly correlated with LiDAR-measured height*

406 LiDAR and RADAR measure forest structure, but not wood density. Predictive models can give
407 rise to markedly different maps of ACD depending on the assumed spatial variation in wood
408 density [36]. Wood density ($\bar{\rho}_p$) was even less well predicted ($R^2 = -0.02$) from LiDAR than

409 basal area, but was improved by separating the landscape into forest types ($R^2 = 0.19$) because
410 conifer and broadleaf species vary in wood density. There is no evidence in our derivation, or
411 from previous work [10, 38], that $\bar{\rho}_P$ should have an associated power in the AM–GM, even
412 though the model has commonly been fitted with an $\bar{\rho}_P$ exponent included [7, 10]. Consistent
413 with theory, we found that including the $\bar{\rho}_P$ exponent (b_3) did not lead to significant
414 improvements in model fit in our temperate data.

415 *Influences of crown area allometry on goodness of fit*

416 The exponent of the C–D relationship, k_C , can also affect accuracy. When $k_C = 2$, the powers in
417 the ITB–GM all reduce to 1, total stem volume is directly proportional to the maximum canopy
418 volume and canopy area is directly proportional to basal area. The AM–GM is therefore most
419 accurate when $k_C = 2$; conversely, the further k_C departs from 2, the more inaccurate the volume
420 and crown area summation scaling relationships become (see S1 Text for a detailed exploration).
421 Even with variable size distributions, the goodness–of–fit of the total stem volume vs canopy
422 volume relationship is high ($R^2 > 0.8$) when k_C is greater than 1.3. There is a sharp drop off in
423 the accuracy of the volume scaling relationship if the C–D exponent is less than 1.3 (Fig S1), and
424 the AM–GM is expected to perform poorly in forests with variable size distributions when the
425 C–D exponent has a lower value. Since k_C was 1.28 for the Canadian temperate forest, the
426 crown area allometry also contributed to low model accuracy.

427 We lack a clear picture of how k_C varies globally, but there is some evidence that values
428 are lower for temperate forests. Classical self–thinning theory was based on an assumption of an
429 exponent of 2 [30, 31], whereas metabolic scaling theory predicts an exponent of 4/3 [32], both
430 above the threshold of 1.3 below which accuracy deteriorates. An average value of $k_C = 1.36$

431 was found for tropical forests [30], whereas a wide range of k_C values have been reported for
432 temperate forests (0.85 for Virginia, USA, [33]; 1.19 for European beech, [34]; 2.16 for New
433 Zealand mountain beech, [31]). Competition amongst the trees becomes an important feature
434 determining crown shape and the C-D exponent [31] and that too varies at different scales. The
435 goodness-of-fits of the C-D power functions in our analyses were low ($R^2 < 0.45$), suggesting
436 that uneven-aged stands may require a variable relationship between height and diameter, which
437 would consequently require an alternative formulation of the AM-GM. Dietze et al. [35] found
438 that the C-D scaling relationship was more variable than the H-D relationship for two managed
439 temperate forest sites in North Carolina, USA.

440 The H-D scaling exponent, k_H , has less influence on the ITB-GM than k_C , as it only
441 contributes to the volume scaling relationship and appears on both sides of this equation. The
442 magnitude of k_H affects the accuracy of the power function by influencing the relative
443 magnitude of the summations; increasing k_H would mitigate the effects of k_C deviating from 2
444 (Fig S1).

445 *Influences of forest composition on power-law exponents and goodness of fit*

446 Changes in forest composition within a landscape can have major effects on ACD estimates if
447 those changes are associated with systematic variation in crown geometry and wood density [12,
448 36]. In our study area, the model was not substantially improved when forest type was accounted
449 for (Fig 3), but an examination of its assumptions highlighted some combinations of H-D and
450 C-D exponents where forest type could influence the generality of the model (Fig S1). Given
451 that the AM-GM is based on scaling relationships of individual trees (H-D and C-D), it is clear
452 that species composition may be important if it results in changes to these allometric functions

453 across the landscape. Previous studies indicate that H–D and C–D power functions vary with site
454 and species, suggesting that AM–GM exponents will vary across heterogeneous landscapes. The
455 inclusion of forest type improved the ACD predictions of the sugar maple stands more than the
456 mixed stands. Delineation of the sugar maple forest type, which essentially represents a single
457 species, may therefore have been beneficial because there is expected to be more variation in
458 allometry between species than within species. Lines *et al.* [37] noted that the H–D relationships
459 of Spanish conifer species had exponents close to 2/3 (the value predicted by biomechanical
460 theory), but those of broadleaf species were much more variable and often less than 2/3 [34].
461 Such differences between conifers and broadleaves could result in different AM–GM exponents
462 across forests with shifting species dominance.

463 **Conclusion**

464 The allometry-inspired AM-GM model appears to predict forest carbon more reliably in tropical
465 forests than in temperate ones. Asner and Mascaro [8] achieved a goodness-of-fit of $R^2 = 0.83$
466 compared with $R^2 = 0.18$ in this study, even though the models were identical (Table 2). Their
467 RMSE was 9% of the mean ACD compares with 23% for our models (Fig 3). Duncanson et al.
468 [14] also observed poor model performance when testing the AM–GM in two out of three
469 temperate forest sites in the USA ($R^2 = 0.13, 0.18$ and 0.73).

470 A key issue is that stand basal area is weakly correlated with canopy height in temperate
471 landscapes comprised of patchworks of stands at various stages of succession/development after
472 disturbance. Selection management created a variety of structural conditions in the Canadian
473 forests studied here, whereas in natural temperate forests variation in stand structure is induced
474 by disturbance from wind, disease, fire and pests. Variability in regeneration, growth and

475 mortality among these stands leads to weak correlations between basal area and height – whereas
476 these are closely coupled in many tropical forests [7]. The allometry-inspired model is reliant on
477 predicting basal area from height, which is a particular problem in heterogeneous landscapes.

478 Deriving the AM–GM from individual tree information further underscores the importance
479 of variability in size distributions across landscapes. Given that a tree’s biomass is obtained by
480 multiplying its wood volume by its wood density (and assuming conical form), the values of b , c
481 and d in the individual biomass model function $aH_i^b D_i^c \rho_i^d$ should be close to 1, 2 and 1,
482 respectively [10, 38]. By analogy we would expect b_1 , b_2 and b_3 to all be approximately 1 in the
483 AM–GM if the summation had no effect on exponents; however, two of the exponents are far
484 from 1 for the tropical forests analysed by Asner and Mascaro [8] ($b_1 = 0.28$, $b_2 = 0.97$ and $b_3 =$
485 1.38). Non-linearities in the process of scaling from trees to stands are clearly influential in
486 determining these exponents. This also explains why our ITB-GM was ineffective.

487 This paper has described the theoretical basis of the AM-GM, demonstrating that the
488 reliability of the approach is dependent on having invariant size distributions across landscapes
489 and on the crown area-diameter power relationship of individual trees. Landscape heterogeneity
490 in these attributes resulted in the poor performance of the AM-GM in a managed temperate
491 system compared with species-rich tropical forests. Model performance is improved by
492 stratification into forest types, but this does not address the issue of varying size distributions.
493 More studies into the spatial variability of tree size distribution are needed to understand when
494 allometry-inspired general models can be reliably used to predict forest aboveground carbon
495 stocks.

496

497

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597

598 **Supporting information captions**

599 **S1 Text. Assessing the validity of the volume and canopy area scaling relationships.**

600

601 **S1 Table. Species compositions extracted from the 114 calibration plots.** Trees from the tree
602 height and crown area dataset were sampled to match these compositions.

603

604 **S1 Fig. Goodness-of-fit with different values of k_H and k_C when substituting top canopy**
605 **height (H_L) and basal area (B_P) into the ITB-GM (5) using the relationships modelled by**
606 **the volume and canopy area scaling relationships.** Parameter values for the modelled
607 relationships are also given. For particular values of k_H and k_C , each matrix cell represents a
608 relationship fitted to the 114 calibration plots. The square matrices give the power (k_D),
609 coefficient (a_D) and R^2 of the relationship in the volume scaling relationship, whilst the bars give
610 the equivalent (k_B , a_B and R^2) for the canopy area scaling relationship. In the square matrices,

611 both k_H and k_C vary, whilst only the latter affects the bars. Points represent the values of k_H and
612 k_C estimated from allometric data (Table 4).

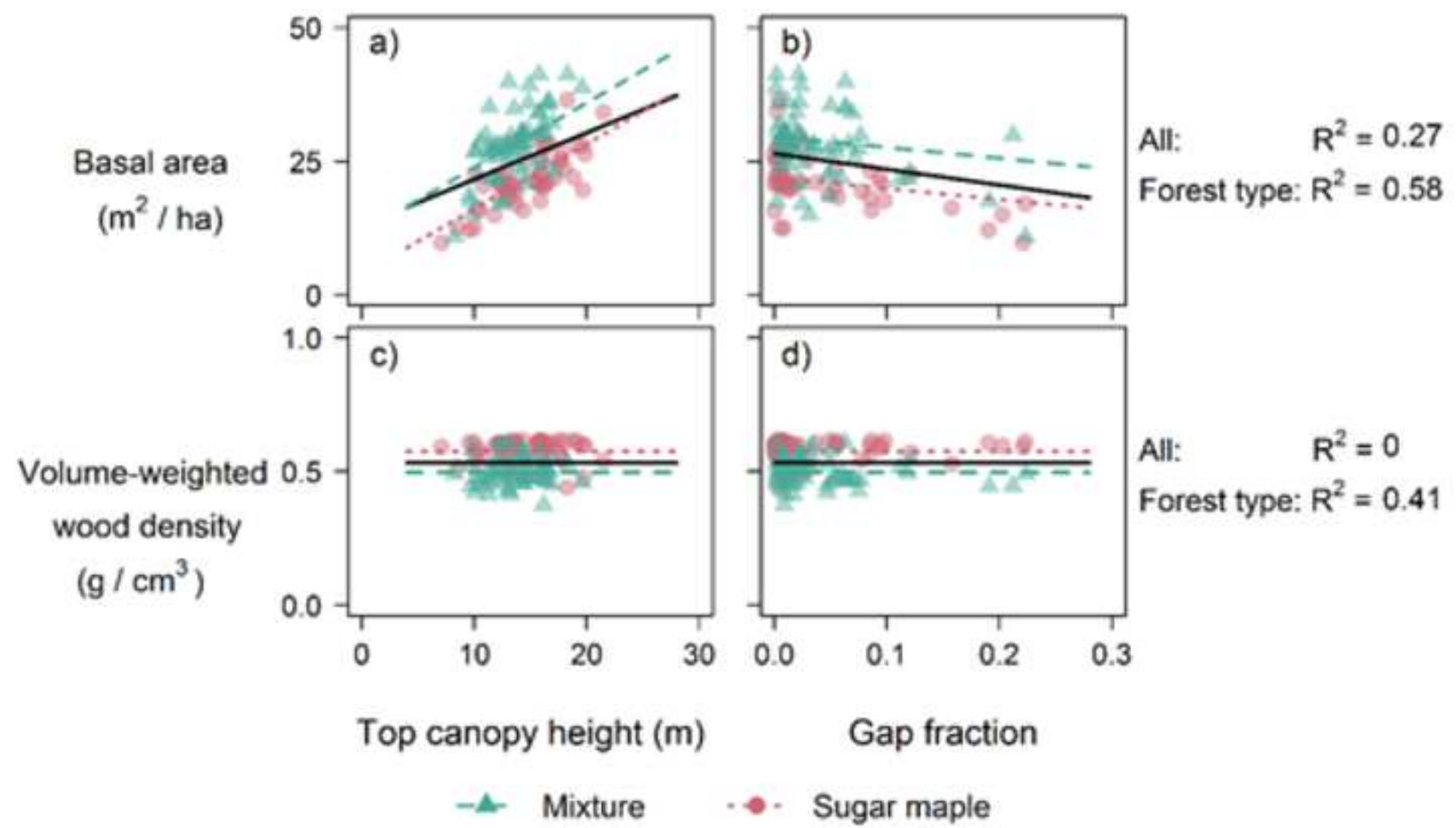
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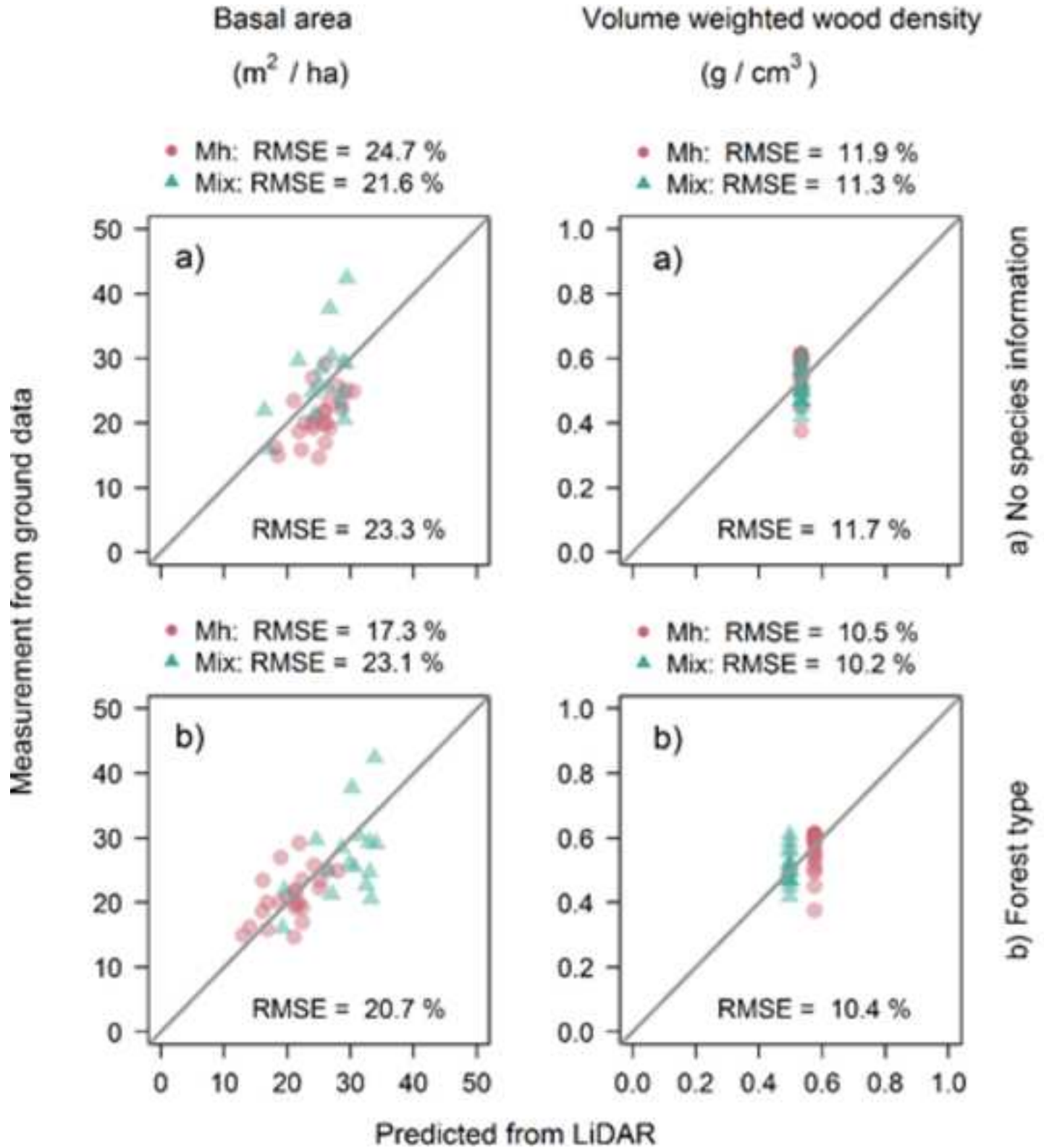
614 **S2 Fig. Goodness-of-fit of the scaling relationships when underlying size distributions**
615 **follow a power function or Weibull distribution.** The square matrices represent R^2 values for
616 the volume scaling relationship and the bars represent R^2 values for the canopy area scaling
617 relationship as the exponent parameters of H-D and C-D are varied. The leftmost and centre
618 panels represent pseudo-data plots that exhibit a power function and a Weibull distribution,
619 respectively. The rightmost panels show the difference in R^2 for each combination of k_H and k_C .

620 **S3 Fig. Exponent values and goodness-of-fit of the volume summation and crown area**
621 **assumptions as the H-D and C-D relationships are varied and as the Weibull stem**
622 **diameter distributions become more variable.** The exponent of the volume scaling
623 relationship when the Weibull parameters were changed to produce low and high variance is
624 given in the top row. The difference in R^2 between each of the variable Weibull datasets and the
625 fixed Weibull is given in the bottom matrices, where the square matrices correspond to the
626 volume scaling relationship and the bars correspond to the canopy area scaling relationship.

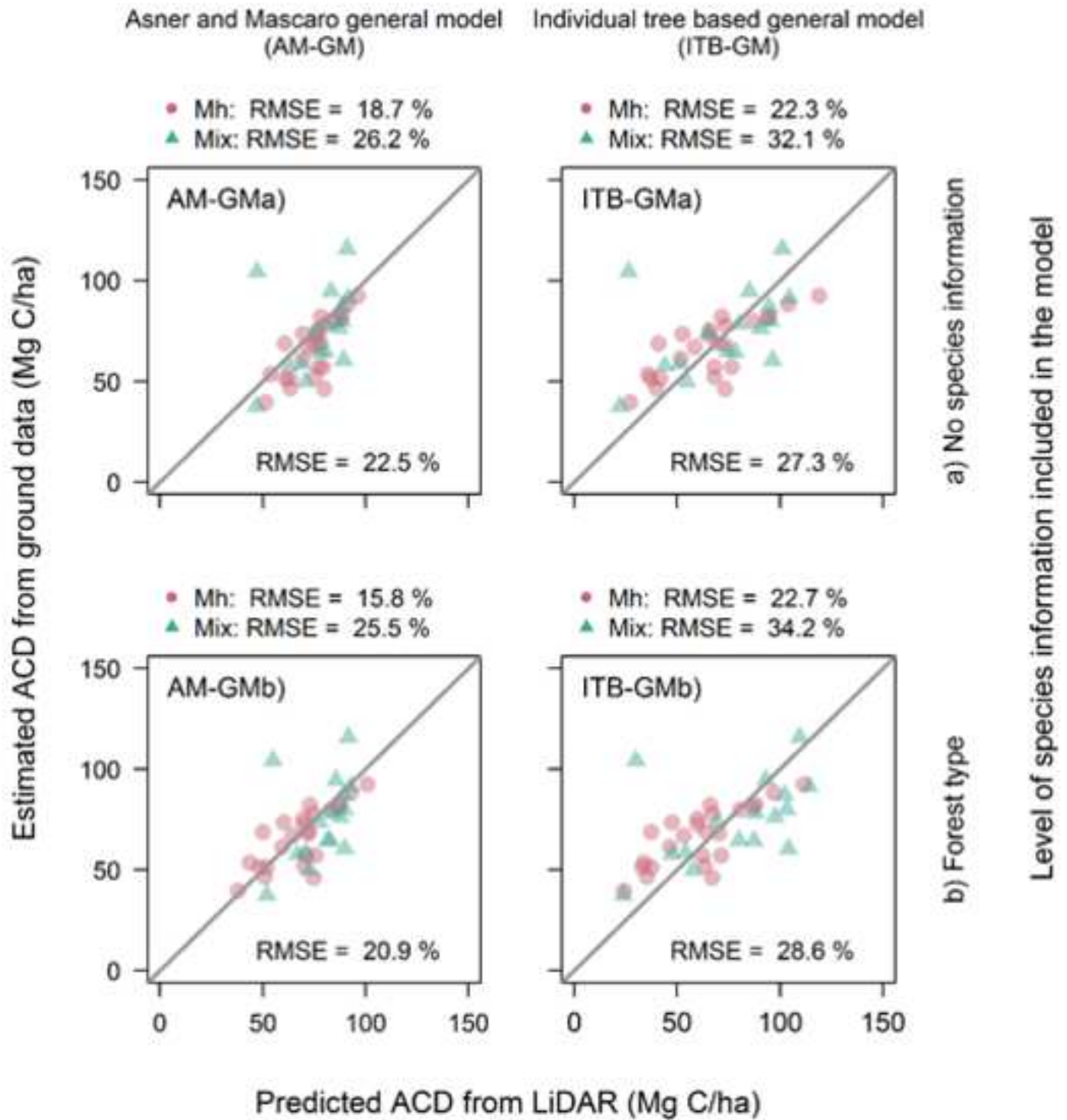
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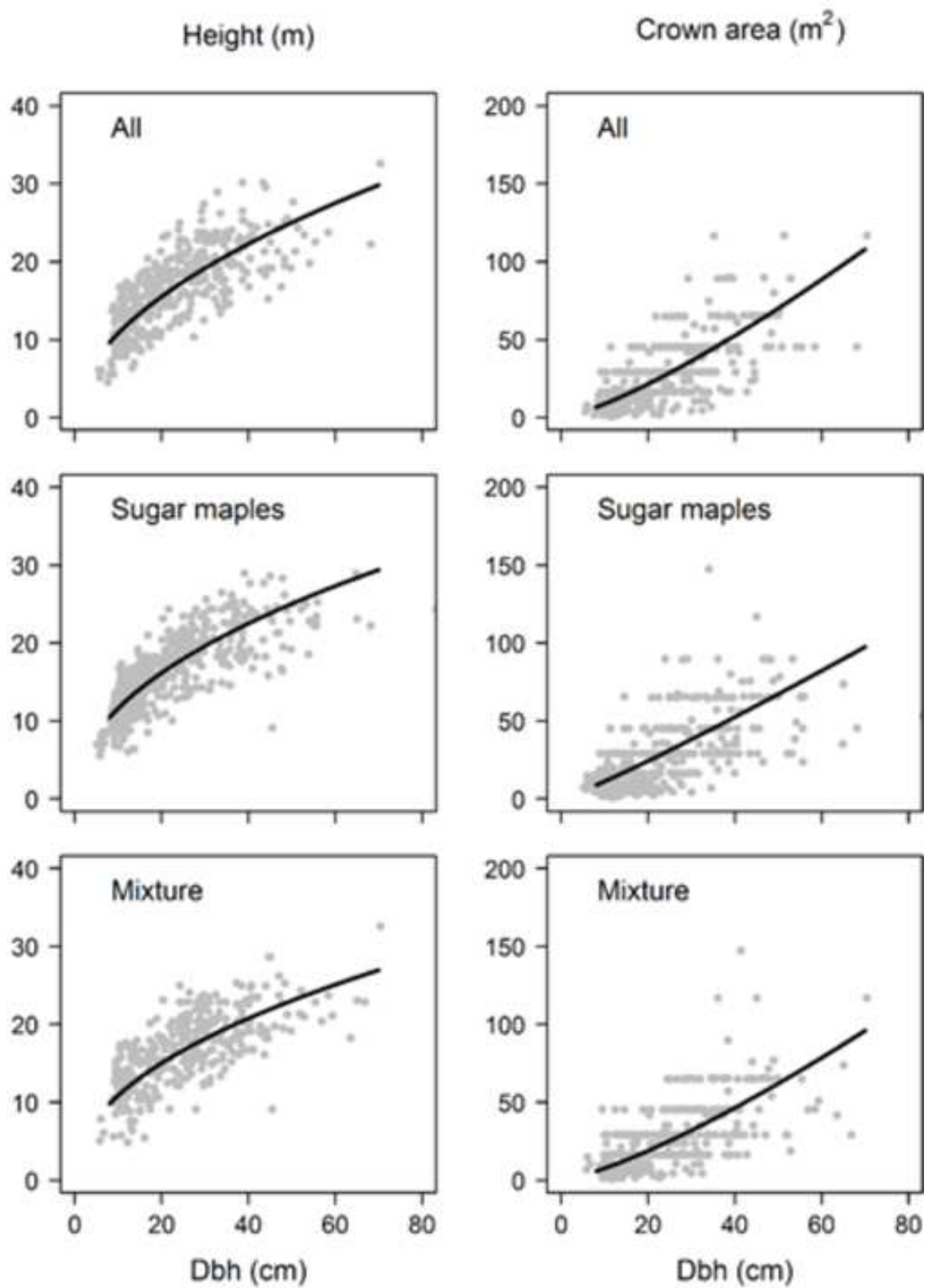
628 **S1 Dataset. Plot data used for main analyses.**

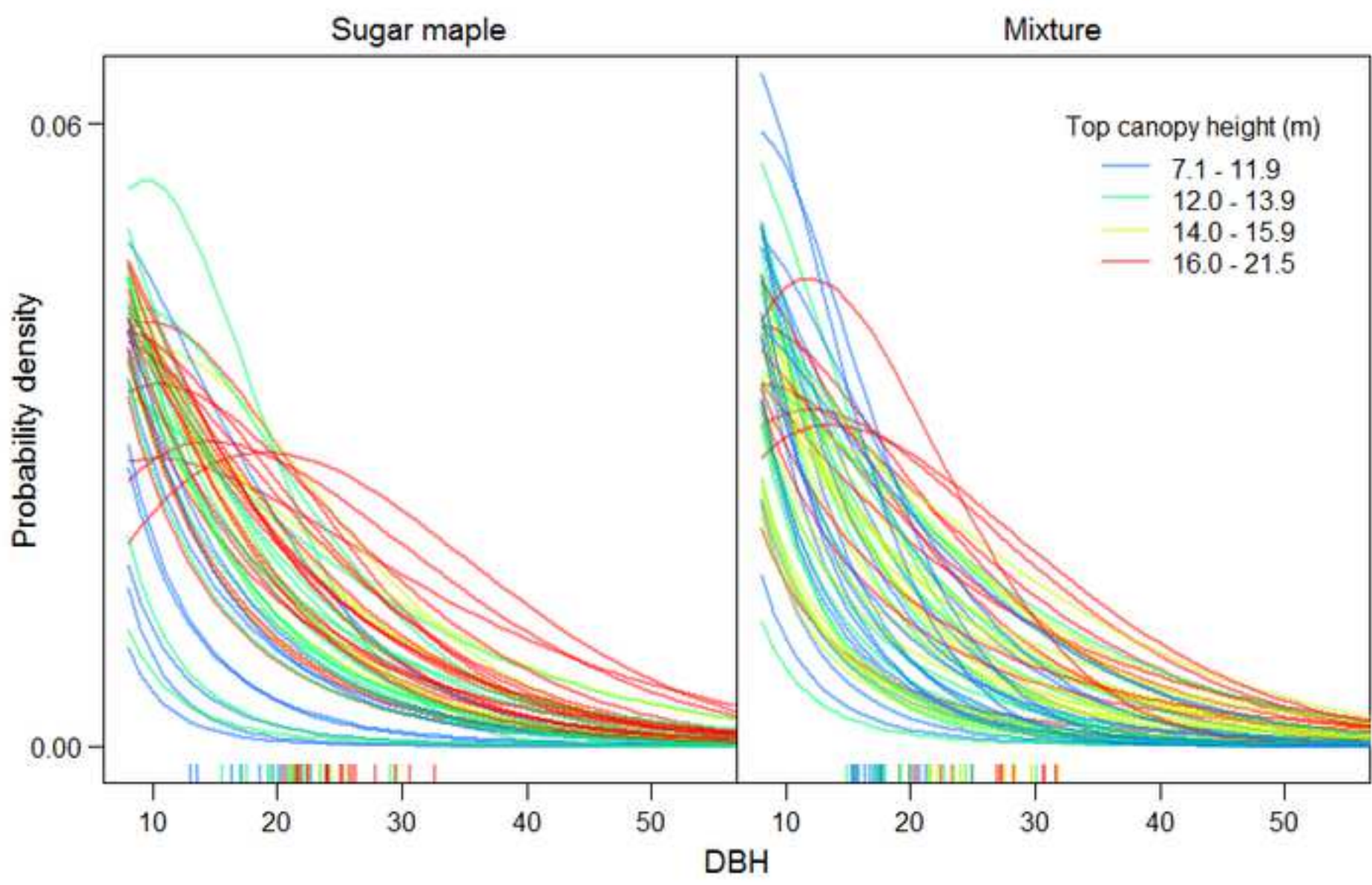




Version of the model





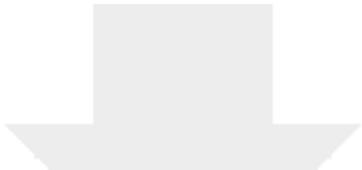






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Fig S2.png

