British Educational Research Journal
Vol. 44, No. 3, June 2018, pp. 496–514
DOI: 10.1002/berj.3444

Operational, interpersonal, discussional and ideational dimensions of classroom norms for dialogic practice in school mathematics

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While research suggests that interactive pedagogy drawing on students' ideas can improve learning outcomes, it has been found difficult to change mathematics classroom practice in this direction. The reasons for this difficulty remain poorly understood, hindering change at scale. This article focuses on the under-researched normative aspect of such practice which shapes participants' actions and expectations. Drawing on theories of social practice and interaction, we define norms as recurrent and socially obligating patterns of, and rationales for, behaviour in a particular social practice. We then examine empirically what and how (new) norms associated with this type of pedagogy are manifest in classroom discursive activity by examining talk across 21 school mathematics lessons by 12 teachers implementing a dialogic intervention. While there is a clear distinction between surface norms and underlying rationales, and a consistent set of surface norms relating to classroom talk can be identified, deeper analysis finds norms to be multi-dimensional. We illustrate how a surface norm, such as 'Respect others' ideas', can be enunciated in terms of multiple underlying rationales which we term operational, interpersonal, discussional and ideational. Our findings shed new light on why the dialogic intentions of such interventions are often realised in a superficial way. We further examine the ways in which teachers hold students and themselves accountable to the ideational dimension—the dimension that relates to taking students' ideas seriously in classroom dialogue.

Keywords: analysis of norms; classroom dialogue; interactive pedagogy; school mathematics

Introduction: Dialogic teaching and the difficulty of changing classroom practice

Syntheses of research on teaching suggest that effective pedagogy in mathematics and science is characterised by interactive whole-class teaching and collaborative small-group work, notably drawing on student thinking (Schroeder *et al.*, 2007; Slavin *et al.*, 2009; Ruthven, 2011b). These findings resonate with research on classroom pedagogy emphasising the benefits of *dialogic* teaching (Mercer *et al.*, 2004; Mercer & Sams, 2006; Scott *et al.*, 2006; Kazak *et al.*, 2015). Definitions of dialogic teaching characteristically invoke two broad aspects of classroom activity. One concerns the *distribution* of opportunities for talk (who gets to speak), emphasising the importance of opportunities for students to share their ideas and reasoning and

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take part in discussions (Mercer & Littleton, 2007; Kyriacou & Issitt, 2008; Mercer & Howe, 2012). The other set of features concerns the *ideas* involved, emphasising the importance of valuing and giving space to multiple viewpoints (e.g. Scott *et al.*, 2006; Ruthven & Hofmann, 2013; Kazak *et al.*, 2015). Our own research suggests that while these features are linked, they are not identical (Maine & Hofmann, 2016; Ruthven *et al.*, 2017).

Such dialogic pedagogy represents a significant shift from typical mathematics classroom practice (Henningsen & Stein, 1997; Chiu, 2004; Webb *et al.*, 2006). It is, therefore, not surprising that observational studies of dialogic teaching interventions have found that implementation is often superficial. While there is often evidence of some pedagogic elements of the intervention, these are interpreted so as to fit with existing practice (Wolf *et al.*, 2006; Mercer *et al.*, 2009; Maine & Hofmann, 2016; cf. Ruthven *et al.*, 2017). Targeted small-scale studies suggest that genuine dialogue is possible in mathematics and science (Mercer & Sams, 2006; Hennessy, 2011; Ruthven *et al.*, 2011; Kumpulainen & Rajala, 2017; van de Pol *et al.*, 2017), yet research conducted at scale, while sparse, suggests that it is difficult to implant widely (Webb *et al.*, 2006; Osborne *et al.*, 2013; Ruthven *et al.*, 2017). Interactive classroom teaching involving the elicitation of extended student responses can be focused on seeking a 'correct' or authoritative answer, or it can be oriented towards the elicitation and discussion of multiple different ideas. The findings from a recent large-scale UK field trial on dialogic teaching in secondary science and mathematics suggest that while teachers commonly succeed in increasing student contributions, actually making use of students' ideas is a more demanding development in classroom practice (Ruthven *et al.*, 2017). The reasons for these difficulties remain poorly understood, with many authors of a recent edited compilation on the topic calling for more research (Michaels & O'Connor, 2015; Stein *et al.*, 2015; Wilkinson *et al.*, 2015).

Our research contributes to the literature by examining the nature of this difficulty in the context of a large-scale pedagogic intervention aimed at increasing both of these dimensions of talk (Ruthven & Hofmann, 2013), and thereby addresses a further gap in the field: Howe and Abedin's (2013) systematic review found that one of the challenges of the field is its dependence on very small-scale studies. Many of the studies on the implementation of dialogue pedagogy cited here include between one and three teachers.[1] The study at hand contributes to the field by examining a broader range of teachers: it draws on observational data from our large-scale trial (Ruthven *et al.*, 2017) and includes 12 secondary mathematics teachers who were part of the intervention.

While it is widely acknowledged that extensive continuing professional development (CPD) is necessary to facilitate pedagogic change (Guskey & Yoon, 2009), Webb *et al.* (2006) found that such CPD does not guarantee change of classroom interaction towards more dialogic talk (cf. Michaels & O'Connor, 2015). They speak of an 'entrenched culture of low-level questions and explanations' in mathematics classrooms (Webb *et al.*, 2006, p. 109). Rather than construing this as a failure of individual teachers to enact new practice, cross-cultural and cultural–historical research shows that classroom cultures are not simply attributable to the teacher. The seminal work of Michaels, O'Connor and Resnick argues

that in order to change classroom culture and ground rules towards accountable discourse, research needs to attend to the sociocultural *norms* affecting classroom practice at the utterance-to-utterance level of interaction (Michaels *et al.*, 2008; Michaels & O'Connor, 2015).

Research in mathematics classrooms also illustrates how teachers' intentions to change their practice are often not realised if dominant sociocultural norms of classroom practice are not explicitly and successfully challenged (e.g. Keitel, 2006; Turner *et al.*, 2011). Culturally comparative research on teaching mathematics suggests that norms reaching well beyond an individual teacher play a central role in shaping classroom practice and its underlying assumptions about teaching and learning (Clarke & Xu, 2008). Research informed by cultural–historical theorising has argued that this similarity arises from a relatively enduring, historically accumulated, normative structure underlying classroom practice (Engeström, 1991, 1998). These collective norms both shape and give meaning to the surface-level actions of participants and hence cannot be ignored. This largely invisible, taken-for-granted normative dimension of classroom practice has received insufficient attention in attempts at school reform (Engeström, 2008; Michaels *et al.*, 2008; Hofmann, 2016). While the 'idealised' norms of classroom dialogue supportive of learning are well known in the literature, the gap between the 'idealised' and the 'realised' discourse norms in many classrooms remains 'daunting' and there are significant continuing challenges associated with bringing idealised norms into realisation (Michaels *et al.*, 2008; Michaels & O'Connor, 2015).

Thus, our article examines this normative dimension of classroom talk at the level of individual utterances. Focusing on dialogic teaching and learning in mathematics lessons, we will highlight aspects relating to classroom interaction and students' mathematical work. Students' work is regulated by mathematical norms pertaining in school to the curricular object of study (Chevallard, 1985), but it is also regulated by what Yackel and Cobb (1996) call socio-mathematical norms (pertaining to teaching mathematics in a classroom context). Yackel and Cobb distinguish these from social norms (that sustain inquiry-based discussion and argumentation but are not specific to a subject). Moreover, conversational norms frame aspects of classroom interaction: turn-taking, permissible pauses, who can ask questions and evaluate responses (McHoul, 1978; Ingram & Elliott, 2014).

Our study seeks to understand the processes and mechanisms of change from a perspective attentive to these norms. It addresses the gap that the literature identifies in understanding the reasons for, and the nature of, the difficulty of implementing dialogic pedagogy and enacting pedagogic change more generally, through analysing challenges at the level of classroom interaction. Our analysis extends the existing body of research on dialogic teaching, on the one hand, and on mathematics classroom norms, on the other, by offering an analysis which is specifically linked with an attempt to *change* classroom practice, particularly to foster dialogue. Studying classroom norms in the context of change affords us the opportunity to consider and compare both idealised and realised dialogue norms, but first we will attend to a conceptual definition of a norm.

The normative nature of classroom discursive practice

While the important role of shared norms is widely recognised in approaches which theorise shared social practice, various theoretical fields offer differing conceptualisations of 'norm'. After surveying the most important of these, we will develop a particular characterisation suited to the goals of our study.

Research drawing on a Bourdieusian perspective to theorise teachers' classroom actions as a 'practical rationality' (Herbst *et al.*, 2011; Herbst & Chazan, 2012) discusses norms as patterns of, and rationales for, behaviour. 'Norms' relate to those recurrent patterns of behaviour which the situation customarily requires from, and which are expected by, the participants. Herbst *et al.* (2011) describe teachers' logic of practice through what they call 'categories of perception and appreciation': teachers have a disposition to act in certain—observable—ways because they perceive as normative certain (teacher and student) actions. Such preferred ways of acting typically relate to some underlying rationale for why such actions are desirable. Even where accountability for norms is tacit and not authoritatively imposed (e.g. by policy or school regulations), norms remain binding: participants hold themselves accountable to fulfil them. Thus, central to our study is the notion that norms, as such, are expressed in patterns of behaviour that are typically unmarked and unremarked upon when they occur, but which call for explicit elaborations or repairs when those behaviours are absent or when established norms are being challenged.

Ethnomethodological and conversation analytic research has illustrated how interactions between people are governed by (often unspoken) *conversational* norms. Norms, in this perspective, influence both the course of specific interactions and their perceived *meaning* (Schegloff, 2007; Heritage, 2011). Conversation analysis illustrates how manifest patterns of behaviour are seen as being indicative of underlying rationales, and that these inferred rationales depend on the overall normative structure of the interactive context. Pauses in conversation are an example. In everyday conversation, pauses are seen to indicate that another person is free to self-select as the next speaker, whereas in formal classroom interaction (where the teacher controls turn-taking), a pause is taken as indicative of the underlying principle of giving 'thinking time' (Ingram & Elliott, 2014). Such analyses have illustrated how individual speakers in a social encounter cannot simply change or ignore these norms if the interaction is to run smoothly and engage the cooperation of participants (e.g. McHoul, 1978). If teachers deviate from established classroom norms, they can no longer draw on a shared understanding of what their spoken actions are trying to accomplish. The conversation analytic tradition also highlights how norms may become manifest in breakdowns which result from deviating from established norms, and the need to understand the overall normative structure of an activity that influences the rationales for, and meanings of, manifest surface behaviours.

However, we are interested in how classroom talk can be *changed* to better facilitate the development of understanding about mathematics. This requires the re-thinking of the activity of teaching and learning mathematics more broadly. We will draw on a further theoretical perspective focused on changing collective practice, cultural–historical activity theory (CHAT).

CHAT also refers to norms as the deep-seated explicit and implicit conventions that govern actions and interactions within a particular activity, and give rise to patterns of behaviour characteristic of that activity. Norms also relate to accepted ways of engaging with the shared object of that activity, the material and symbolic tools involved and the criteria for evaluating outcomes (Engeström, 1998). Competence in particular traditions of institutional practice requires successful acquisition of such symbolic artefacts (Hedegaard, 2007). Given their function of maintaining established ways of acting and thinking within an activity, addressing norms plays a central role in changing shared practice (Engeström, 2008; Edwards, 2011). CHAT argues that adopting new norms often leads to contradictions within the activity (Engeström, 2001). If such contradictions are resolved by simply adapting the new norms into existing forms of activity, these new ideas can become frozen. However, such contradictions can also give rise to innovative attempts to change the activity. For CHAT, such disruption, by individual participants, of established norms of practice has the potential to lead to deliberate change effort to embrace a wider '"horizon of possibilities" than in the previous mode of activity' (Engeström, 2001). To avoid chaos, change requires the mutual construction of new shared norms and their operationalisation as appropriate new routines (Ruthven, 2011a). This process may involve the new norm being first made, in itself, the object or outcome of the activity, before it can become the means of producing a new kind of outcome, and perhaps later a new, implicit and shared norm (Engeström, 1991).

Drawing on these theoretical literatures, we conceptualise norms as recurrent and socially obligating patterns of behaviour in a particular type of social encounter. These relate to interactions, but also to accepted ways of engaging with the shared object of the activity, including use of resources and criteria for evaluating outcomes. While often salient, norms for accepted ways of acting and interacting may go unarticulated, even unacknowledged, until broken. Norms in this understanding involve both a surface behavioural level and an underlying rationale for why those actions and interactions are deemed important. They regulate what actions participants consider appropriate, and how they understand those actions. The theoretical perspectives suggest that classroom norms may be manifest not only in recurrent patterns of behaviour, but also in teachers' evaluative appraisals of students' actions and work.

Moreover, challenging established norms and introducing new ones requires teachers to make explicit desirable patterns of behaviour and the rationales behind them. In examining how teachers attempt to change classroom norms regarding talk, Michaels and O'Connor (2015) propose an operationalisation of this conceptual approach through suggesting that teachers need to come to understand different kinds of talk moves as *tools*; in line with cultural–historical theorising, 'in order to understand a tool, you need to know what to use it for' (p. 351). Therefore, in order to understand the role of norms in implementing dialogic pedagogy, we need to attend both to patterns of behaviour and to teacher appraisals of these patterns, and to teachers' expressed rationales for these patterns: their explicit talk about *what* different norms for talk are intended to achieve.

We will study the normative dimension of classroom dialogue in the context of a pedagogic intervention developed and implemented as part of a larger research project (*epiSTEMe*) involving a previously reported large-scale trial of a dialogic teaching intervention in secondary science and mathematics (Ruthven *et al.*, 2017).

Research questions and methods

Our study sought to address the following research questions.

RQ1: What explicit norms for interactive mathematics classroom activity can be identified in the lessons observed?

RQ2: What rationales are expressed for these norms?

RQ3: What evidence is there that the teachers hold the students, and/or themselves, accountable to these norms?

The following subsections address these questions.

The epiSTEMe project and the dataset

The dataset for this study comes from lower-secondary mathematics classes implementing the Probability module of the *epiSTEMe* intervention. This intervention, described in detail elsewhere (Ruthven & Hofmann, 2013; Ruthven *et al.*, 2017), drew on research-based ideas of dialogic pedagogy; it involved whole-class and small-group discussions around rich topic-related tasks. The intervention was based on a short introductory module designed to prepare classes for dialogic teaching, and a topic module for this (and three further) topics, with associated professional development. Lessons were observed, both during the pilot (Phase 2) and scale-up (Phase 3) phases of the project, by the first author, video or audio recorded, and transcribed verbatim. The dataset contains 21 whole lessons (of duration c. 60 minutes each) by 12 different teachers teaching 14 different classes in 8 different schools across the East of England. Our analysis focuses on the whole-class interaction in these lessons. Data extracts are marked as a letter signifier for teacher/class[2] and a number signifier for the module lesson.

Although, in the first instance, it was schools which opted to join the project, the teachers that schools nominated to participate were all volunteers.[3] In general, these teachers were unfamiliar with the notion of 'dialogic teaching' prior to the project, but their volunteering and maintaining participation suggest that the notion resonated with their pedagogical orientations. At the same time, our interactions with teachers and observations of their teaching indicated that developing a dialogic approach represented a significant shift in practice for most. To assist this process, the project provided teachers with topic-specific lesson sequences and task materials specifically crafted to elicit student contributions capable of supporting dialogic exchanges, and with pedagogical guidance intended to support teachers in fostering such exchanges; for example, through establishing appropriate ground rules (Ruthven & Hofmann, 2013).

Data analysis

Considering our conceptualisation of norms, and drawing on empirical analyses of structures of classroom talk (Sinclair & Coulthard, 1975; McHoul, 1978; Yackel *et al.*, 1991), we developed an analytic framework for analysing the whole-class talk in our dataset.

Firstly, this involved paying particular attention to phases of lessons where prior research on patterns of classroom interactions indicates explicit talk about classroom norms and rules to be likely, such as talk at the start of lessons and tasks, when teachers frame and focus the lesson/task or direct and instruct students. We looked for explicit guidance for a lesson/task as well as justifications and explanations provided for the guidance. This allowed us to identify subsequent sequences of classroom interaction in which aspects of interaction/activity made salient in the instructions were manifest, to examine accountability. We also examined teacher feedback during class discussions, particularly during instances of norm-incongruent behaviour by students where a 'break-down' of norms might require them to be explicitly reiterated. Finally, we looked at overall feedback given by teachers at the end of activities and lessons, which provided a rich source of explicit norm talk.

The analysis took place in several stages. Informed by the literature, we attended both to talk whereby 'norms' were being made the focus of discussion and to manifestations of these aspects on subsequently activity, as well as the differences between these. We first identified teachers' explicit talk about the classroom norms throughout the lessons. We cross-sectionally indexed explicit norm talk (Mason, 2002), comparing each example with all other instances to establish similarities and differences (cf. Silverman, 2011). This enabled us to identify the core expectations put forward by the teachers in these lessons. We then re-examined the original data with these norms in mind to seek less explicit references to those norms throughout the lessons. We further examined the ways in, and extent to, which the participants offered explicit rationales for the norms referred to in this talk. Two later subsections illustrate this process. To understand the normative structure appealed to in this talk, we heeded the recent calls by various scholars to attend to the nuances of different dialogic norms (Michaels & O'Connor, 2015; Pauli & Reusser, 2015; Wilkinson *et al.*, 2015) as well as to the links between different norms (Michaels *et al.*, 2008).

We recognise that our cross-sectional analysis focuses primarily on talk about norms and patterns of behaviour as offered by the teachers, rather than patterns of behaviour themselves. Such analysis was necessary to identify new norms that the teachers in these lessons were attempting to establish, as has been discussed above. However, it is insufficient for examining the normative structure of these lessons. For teachers' proposed expectations to be considered part of the normative structure of the activity, teachers need to hold students, and themselves, accountable for those. Hence in the second stage we sought to identify pedagogic sequences (Silverman, 2011) in which the norms identified in the above manner were being implemented, and undertook contextualised analysis of these. A third later subsection illustrates this process.

Findings

Dimensionality of norms

Much of the explicit talk about expected patterns of behaviour, across classrooms, concerned a small number of clearly identifiable norms, such as 'Listening to others' and 'Respecting other people's ideas'. Five such distinct explicit norms for classroom talk were identified and are discussed below. Close analysis of teachers' explanations of these norms within classroom talk revealed, however, that beneath similar surface expressions often lay differing types of rationale. A distinction, then, needs to be drawn between surface expression and deeper norm structure. Moreover, the surface expression of a 'norm' may only be a partial one. Our analysis showed that, while often labelled in largely similar ways across occasions and classrooms, these 'surface' norms were not always understood in identical ways. Moreover, such differences originated not just in a norm being understood and applied in differing ways by a range of teachers, they also reflected variation in the terms in which the norm was sometimes enunciated by the same teacher. Our analysis of these terms indicated that the rationales for any norm could be understood as relating to (one or more of) four *dimensions of interaction*: what we will call the *operational*, *interpersonal*, *discussional* and *ideational* dimensions.

The *operational* dimension of classroom norms relates to what are deemed appropriate ways of carrying out classroom tasks. This may involve, for example, specifying the ways in which students' responses or ideas about the mathematical tasks should be presented so as to be acceptable, as in 'Let's try and show some kind of working' [A5]. This operational dimension is often prominent when reference is being made to received practice, and may not be associated with any explicit rationale for why or how those ways are intended to be supportive of students' learning of mathematics. Indeed, such norms may have become frozen in the sense that they have become detached from the rationale(s) that gave rise to them; what remains is the bald operational dimension of the norm prescribing particular actions.

Many of the norms identified by our analysis refer to what we have called an *interpersonal* dimension: 'Don't laugh at other people' [H2]. This dimension relates to expectations about how students should treat each other in the course of the classroom activities and discussions. As we will show, this dimension is rooted in principles of being 'kind' and 'fair'. We would expect to find the operational and interpersonal dimensions of norms expressed in any mathematics classroom, and in relation to many pedagogical approaches.

In this project, the intervention classes were specifically instructed to carry out (small-group and whole-class) discussions. Indeed, we identified a *discussional* dimension in the teachers' norm talk, in which norms were related to a rationale of promoting discussions: 'The whole point is about discussion' [I2]. This dimension focuses on the kind of discussion that should be taking place, asking students 'to discuss with yourselves, to explain, give your group a reason' [D4]. Discussing, in this dimension, becomes about asking questions, sharing ideas and backing them up with reasons.

Finally, in some lessons we identified talk which focuses on the content of such discussions and the role that should play in students' learning of mathematics. This norm talk promotes the significance of the various (mathematical) ideas produced in the discussions and encourages critical examination of them. We call this the *ideational* dimension. While the discussional dimension of norms can be generic and apply to any discussion, the ideational dimension evokes the idea that subsequent interactions should be contingent upon the content of those discussions: you 'might end up with a different opinion to what you started with' [G2].

Our analysis demonstrates that a surface-level norm can relate to *one or more* of these underlying dimensions, and do so differently on different occasions. We suggest that the most helpful way of thinking about this is through considering the *multi-dimensionality* of classroom norms.

The different norms and their dimensionality

The clearest, and most familiar, operational norms expressed in these mathematics lessons entail that:

-everyone has to contribute
-others should be listened to when they are speaking
-other people and their ideas should be treated with respect
-mere (numerical) answers are not sufficient

as well as, particularly linking with the *epiSTEMe* pedagogic approach:

-students need to seek group agreement during planned small-group work.

In this section we will examine these norms and the rationales provided for them in turn, discussing their multi-dimensionality. To illustrate findings from our cross-sectional analysis in a comprehensive yet efficient way, we integrate short extracts from across our data within the main body of text, with a longer extract at the end of this section illustrating how talk references multiple norms. Our longer examples focus on the discussional and ideational dimensions of the norms, as these are the ones more specifically associated with dialogic classroom pedagogy.

Contributing: From maintaining fairness to sharing ideas. A norm commonly expressed in the data is, expressed in the operational dimension, that 'everyone must contribute to the discussion' [F2; H2]. In the interpersonal dimension the rationale for this norm is about *fairness*. Students need to 'give everybody a fair chance' [N5], to 'let everyone speak' [H2]. And they need to 'contribute otherwise it's not fair' [G2]—they cannot just 'sit back and let everybody else do the work' [N5; F2]. On some occasions, this norm of contributing is also related to *sharing ideas*, so entering the discussional dimension: the underlying rationale here is that students need to 'practice making contributions, I know you all have something valuable to say' [B4]: 'carry on this kind of discussion that you're having' [J4]. In some of the norm talk the expectation for students to contribute is explicitly taken into the *ideational* dimension, emphasising the specific content of students' ideas and the role these play for others' learning. Students need to contribute to the discussion because other people's ideas help us 'think

through' and test our own ideas, which improves our understanding, rather than 'just kind of know[ing]' [A4].

Listening: From attending to others to changing one's mind. Another central and familiar norm, linked directly with making contributions, is expressed in the operational requirement to '*listen*' in the classroom, and similarly found to be multi-dimensional. In the interpersonal dimension, this norm is framed as 'tak[ing] it in turns to speak' [B4], being 'quiet when other people are speaking' [G2]. Listening, too, is about fairness: 'You were just talking and everybody was quiet for you, and now we've got another group talking but you're all chatting, and that's not fair, is it?' [G2]. 'Listening' is about smooth operation of classroom activity. On some occasions, this norm is linked with the discussional and ideational dimensions. Students are asked to 'listen very carefully because I'm going to ask you other groups to contribute to what [another] group says' [B4]. And as with making contributions, *listening* to the content of others' contributions is linked with opportunities for learning, as hearing others' ideas can make us 'change our minds' [B4] 'if you hear an argument that is convincing' [N5].

Respecting: From behaving kindly to exploring disagreement. It is made clear across the data that students need to 'respect each other's ideas' [e.g. H2; I2]. Beyond remaining quiet when others are speaking, in the interpersonal dimension 'respect' involves kindness, not 'laughing' or 'shouting' [H2] at others. As with contributing and listening, the norm of respecting others' ideas can remain within the interpersonal dimension, emphasising polite behaviour. Elsewhere, when the norm 'don't argue' [B4] is explicitly expressed, it is suggested that conducted respectfully, *mathematical* disagreements are acceptable in the context of classroom discussion: 'if someone doesn't agree with you, then talk it out with them' [D4]. This is not only about the discussional dimension. Respecting other people's ideas involves giving them serious consideration and re-evaluating our own ideas, as in the words of a teacher: 'We are going to respect what other people say. We are not going to say oh that's a bad idea. If we disagree, mathematically about something, we can discuss it and try to persuade the other person to change their mind' [A4]. Sometimes, the norm of being respectful is justified explicitly through linking it with the ideational dimension and the role of considering others' ideas for students' learning.

Demonstrating: From showing working to arguing persuasively. A norm expressed in basic operational terms in our data is that it is not sufficient for students to simply express an answer (typically in numerical form). A very familiar dimension of this norm is that students need to 'show some kind of working' [A5]. This demonstration can relate to the interpersonal dimension: 'Let's try and show some kind of working, so that if we ask you to come and explain it to everybody else, you've got a diagram or something that you could share' [A5]. The norm is also linked with the requirement to 'discuss your answers' in small groups [D4], as has been illustrated above. Furthermore, there are various instances of linking this norm with the ideational dimension as well as the discussional one. If a student thinks they know the answer, they need to 'think of something that you can tell us that will convince people of what's going on,

convince them of what you believe', because other students may 'have good reasons for not agreeing' [E5]. This links with the idea about changing one's mind. The aim is not to simply come up with answers, but try and persuade other people of your ideas.

Agreeing: From forming a majority to negotiating for consensus. A requirement in the *epiSTEMe* pedagogic approach, that students need to try to reach a consensus during planned small-group work (although the answer they propose may or may not be correct), is well visible in the directive talk during these lessons. Often, it is simply stated without further explanation or elaboration: 'What I'd like you to do is come up with a *group agreement* of how you would estimate the probabilities' [C4]. Nevertheless, where justifications are provided, there are differences in the understandings communicated about the nature of such agreement. On one occasion group consensus is described as a majority decision: 'not everyone might necessarily agree – but as long as we've got a majority, so the three of you come to a vote, come to an agreement' [F2]. In this definition, group consensus remains firmly within the *interpersonal* dimension, as a fair representation of the majority's view within the group. In other lessons, the norm is expressed in more *discussional* terms: it is suggested that consensus is to be achieved through discussion and persuasion. Here, achieving a group consensus is a matter not of a majority view but of negotiation in which group members 'might end up with a different opinion to what you started with, and that's absolutely fine' [G2]. In some of the lessons group consensus also applies at the level of the whole class: the entire class needs to be convinced about and understand the idea. As a teacher says: 'There is an answer to this and we need to work out what it is. And we all need to believe it. Because this is really quite important' [E5]. This appears to link consensus to the ideational dimension, although not very explicitly: consensus is taken to be about everyone coming to understand the accepted mathematics.

The extract below illustrates teacher guidance alluding to several of the above-discussed norms enunciated in the *discussional* dimension, focusing on sharing and talking through ideas as the rationales for contributing, listening, respecting, demonstrating and agreeing.

> T: *I want you to come to some sort of group agreement. If you decide that you agree with this, I want to see if you can convince your group in what you believe in, that this one is the one that you think is the right answer. Okay? Or 'What happens when you spin a coin, it can't be affected by other results'. If you think that's true, okay, then you try and convince the rest of your group that that's true. If they don't agree with you, then you need to have a discussion and ask them why, ask them to explain their answer, okay?* [D4]

Another extract illustrates teacher guidance alluding to several of the above-discussed norms enunciated in the ideational dimension, making salient the expectation to consider others' ideas, to think through your own ideas, to contribute to others' learning and the emphasis on understanding rather than solely correctness:

> T: *Now, fresh start, new lesson, I know you've got lots of really good ideas. I know a long time ago somebody said to me the reason I don't always give my ideas is because I feel that people are going to tease me, these lessons really rely on us respecting each other and giving our ideas because if somebody says something that you don't agree with, it helps you to think about what you are*

> *doing. I was teaching this actual lesson with [another class] yesterday and one of the kids said something that had just never occurred to me. And I really had to think, now if he hadn't said what he said, I wouldn't have thought about it. So we're going to respect each other's ideas. We're going to make sure that everyone is involved. We're going to ask that everybody gives their ideas. I've just explained that other people's ideas help you to think and it's not whether your idea is right, it's whether your idea helps other people to think about what their views are. And we need some wrong ideas because if everybody says the right idea everybody thinks 'oh well I know it' and they never think it through and they don't really know it for 'sure sure', they just kind of know it.* [A4]

Accountability for the expressed ideational norms

We have discussed the kinds of expected patterns of behaviour expressed in these lessons. For these to be considered as norms, they should be (at least socially) obliging for participants and their absence lead to disruption. We know from our earlier quantitative analyses that the *discussional* dimension of the norms is manifest in the observable patterns of behaviour in these lessons: teacher solicitation of students' ideas and extended pupil contributions to discussions occur consistently (Ruthven *et al.*, 2017). We therefore ask, in this subsection, what evidence there is in the data that teachers hold students, and themselves, accountable to the above-described patterns of behaviour with regard to the *ideational* dimension of a dialogic approach to teaching mathematics. To examine this question, we conducted a second type of analysis looking at the embedding of the expressed ideational norms in the wider interaction. Our quantitative analysis (Ruthven *et al.*, 2017) suggested that aspects of interaction relating to what we have identified here as the ideational dimension do not manifest themselves in patterns of behaviour in these lessons as consistently as the discussional dimension. We want to understand what accountability to such norms may look like when it occurs, given that it appears to be so difficult.

We have illustrated how, in the ideational dimension, the rationales for the surface norms of contributing, listening, respecting, agreeing and demonstrating focus on taking students' ideas seriously and critically considering them to test one's own ideas and understandings about mathematics. They further focus on using one's own understandings to persuade others. Such a comparison of ideas inevitably involves not only eliciting contributions from different people, but eliciting different contributions. And it involves examining the quality of those ideas.

There are instances in the data where the teacher holds the students and her/himself accountable to the norm of eliciting multiple ideas. Here in introducing the whole-class discussion after small-group work:

> T: *I'm going to pick some groups to see what their opinion was, and then we're going to discuss to see if anyone had any questions as to why that group thought it, or whether they disagree.* [F2]

And here in initiating an examination of such ideas:

> T: *So we've got two schools of thought here and I want you to just compare those two.* [C4]

This norm could also be manifest more indirectly. For example, one teacher discontinues the elicitation of responses from small-group work when no new/different

ideas are forthcoming, rather than following the more typical norms either of collecting each group's response or of stopping once a correct response is forthcoming

> T: *Okay I'm moving on a little bit now because everyone is saying more or less the same thing.* [G2]

In another episode, the teacher starts by asking one group: 'What do you think' about the question of whether a dice game of sixes is a game of pure chance? The group provides a correct answer, but the teacher does not offer an evaluative appraisal, asking instead: 'Does anybody disagree with that?'; and then invites the student (Maia), who said yes, to explain. Maia struggles to explain her idea, saying 'It's really confusing me', but the teacher guards her opportunity to keep talking as other students want to step in to correct her. The teacher then comments:

> T: *The really important thing about these lessons and what they're looking for, is are we talking about our maths, and Maia, you got it right, because you very clearly told us what you were thinking so that we were able to understand. You've listened to somebody else who made another comment. And that other comment helped you to change your mind. Now, what you did was right, because we're trying to think about what maths we're thinking. It's not about getting the right answer. It's about thinking about our maths and changing our ideas, yeah? So you got it right.* [A2]

We see here accountability to the ideational dimension for norms with regard to the *role* of ideas. Students, and teachers, are held accountable to listening to a range of ideas and to thinking through their own, and others', mathematical ideas, rather than foreclosing on a single approved one.

Elsewhere (Ruthven *et al.*, 2011) we have analysed a discussion episode which starts with the teacher eliciting different student answers; she then leads an extended classroom discussion in which the proponents of each are expected to 'sell' their idea to the other students [E5]. The role of students' ideas is highly salient in the episode. Throughout the class discussion, the students are expected to discuss and defend differing views; they are hereby held accountable to the norms of demonstrating your understanding through explaining your ideas in order to persuade others, as well as to considering others' mathematical ideas to test one's own mathematical thinking. As some of the students grow frustrated that the teacher is declining to confirm the answer they consider correct, the teacher insists on her position:

> T: *All those people who are getting restless think of something that you can tell us that will convince people of what's going on, convince them of what you believe, because the majority of you are saying that there's a fifty per cent chance they will have the same grouping as their parents. We have some people who don't agree, and they have good reasons for not agreeing, but, if you're sitting there fiddling, think of something you can say to help them understand.* [E5]

We witness here accountability to taking seriously others' ideas to test our own thinking and being responsible for others' learning, whereby students need not only to know the right answer but be able, and willing, to argue for it to convince others. At the same time, we also need to consider evidence of accountability to the norm of systematically comparing different ideas together. There is also some evidence in the data of students and teachers being held accountable to considering the *quality* of

those ideas. This may take the form of insisting that ideas need to be backed up by evidence:

> T: *What evidence can you give me that that's what's happening?* [A4]

Students are also, on occasion, reminded what counts as evidence, as below in the middle of a heated discussion:

> T: *If you think [the other group's dice] scores are unlikely, can I have reasons why you think that? And [claiming they were] cheating is not acceptable. – You need to think a lot harder, please, about what else might be going on, if you think that the results aren't reasonable.* [E2]

After the above discussion, the teacher returns to the norm that it is about 'the ideas and not the people', which she suggests was breached in the discussion.

> T: *A little bit of disappointment about yesterday, I'm not sure about you but certainly for me. Having spent time working out these rules, there seems to be a certain amount of disregard for them. – The ideas and not the people. I did feel that there was a certain amount of personal comments creeping in which actually detracted hugely from the discussion that followed. So can you ensure please that everything you say is about ideas. And not about any sort of feelings you have about other people in the group.* [E3]

However, as discussed previously, surface norms are not always developed to encompass the ideational dimension. Even when the ideational dimension is evoked in teacher instructions, teachers do not always hold the students and themselves accountable to those norms. What we have illustrated in this subsection are some of the forms that such accountability to an underlying rationale based on ideas can take in classroom discussion.

Discussion and conclusions

Our study has addressed the challenge that while dialogic pedagogy has been found to be effective for improving student learning, changing classroom practice at scale remains difficult and poorly understood. Based on a review of research on classroom practice, as well as theories of social interaction, we argued that to understand the processes and mechanisms of change towards more dialogic pedagogy, research needs to attend to the sociocultural norms affecting classroom practice. We have argued that changing classroom practice to incorporate dialogue which takes students' ideas seriously requires the explicit development, and mutual appropriation, of new interactional norms. In this study we characterised norms as involving both a surface level of patterns of behaviour that are recurrent and obligated in the particular social practice of the classroom, and an underlying rationale for such actions and interactions. Our analysis has demonstrated that norms relating to classroom talk and mathematical work combine a surface manifestation with an underlying layering of rationales. We identified a set of consistent normative injunctions for classroom interaction, present across classrooms: these involved the requirement to contribute to classroom discussion, to listen to others when they are speaking, to treat other people and their ideas respectfully, to elaborate on one's answers and to try to reach consensus. However, rather than each of these surface norms being linked with a single underlying

rationale, our analysis revealed that norms are in fact multi-dimensional. Surface-level actions and behaviours can relate to, and draw their meaning from, a range of underlying rationales that frame the nature of the mathematical activity in different ways. We termed these the *operational* (relating to ways of carrying out mathematical tasks), *interpersonal* (relating to ways of treating others), *discussional* (relating to promoting discussion) and *ideational* (relating to the content of the discussions and the mathematical ideas involved) dimensions.

Our findings are significant for better understanding the difficulty of fostering classroom discussion that genuinely takes seriously students' ideas about the mathematics being studied. Previous studies had found that dialogic interventions are often implemented in a way which may increase the number of extended student contributions but struggle to pay attention to and challenge students' ideas and ways of thinking (cf. Alexander *et al.*, 2017). However, research also suggests that simply increasing the amount of student talk/discussion fails to fully capitalise on the potential of talk to support learning (Murphy *et al.*, 2009), making this an important challenge.

Our analysis sheds light on this difficulty. Firstly, it theoretically illuminates the challenge in changing classroom interactive practice. Classroom norms are rooted not only in teacher practice, but also in wider educational practices and student expectations of those and changing them requires explicit efforts, time for consolidation and buy-in from all participants. Above we have illustrated how students may find new classroom norms unexpected, and even try to reinforce old established ones. This resonates with the suggestion that establishing new norms may require the new norm to be first made, in itself, the object of the lesson activity, to make possible a wider 'horizon of possibilities' for the activity.

Secondly, our finding of the multi-dimensionality of norms demonstrates that the behavioural norms required for an ideational dimension to be enacted in classroom talk are, on the surface, the *same* norms which can be expressed solely in terms of interpersonal and discussional dimensions. Implementing these norms in the discussional dimension in itself often represents a shift in mathematics classroom practice (as illustrated by previous research). Nevertheless, teachers, and researchers, may not have appreciated that a further extension of these norms into the ideational dimension is required to genuinely incorporate students' ideas and thinking into the teaching and learning process. Moreover, the second stage of our analysis on accountability provides insights into what such extension and re-interpretation may look like. It suggests that to consider accountability for the ideational dimension of these norms, teachers and researchers may want to attend to two key dimensions of classroom discussion: focusing on students' mathematical *ideas*, rather than simply encouraging students to make contributions, and emphasising joint examination of the *quality* of those ideas and establishing shared criteria for doing so, rather than making an immediate evaluation. Genuine incorporation of an ideational dimension of classroom talk requires consideration of what we have elsewhere (Ruthven & Hofmann, 2016) called epistemic initiative (who sets the ideas to be discussed in the classroom, and how). Moreover, joint evaluation of the quality of those ideas in the course of classroom discussion requires attention to what we have termed epistemic appraisal (who can and should judge the quality of ideas) and epistemic

framing (in what terms such judgements should proceed) (cf. Ruthven & Hofmann, 2016). What is taken as evidence of students' understanding may consequently change.

While our study has focused on mathematics classrooms, the broad normative dimensions that it has identified could be considered generic. Thus, whatever the discipline, indeed whatever the approach taken to a discipline, we would expect to find operational and interpersonal dimensions relating, respectively, to appropriate ways of carrying out classroom tasks and treating one other, even if particular norms—such as 'show your working'—may not be universal. Likewise, this study suggests that successful implementation of any dialogic approach calls for discussional and ideational dimensions to be salient for both teacher and students, even if the specific normative terms and forms in which these dimensions are expressed are open to variation. In that light, it is important to acknowledge that the ways in which the discussional and ideational dimensions were expressed in this study reflect, to some degree, a particular influence of the teaching materials and pedagogical guidance provided by the *epiSTEMe* project: for example, the norm that students should try to reach agreement during discussion; although, as we have noted, this norm could, in practice, be interpreted in terms of differing rationales leading to different operational forms.

Acknowledgements

The authors wish to thank the Economic and Social Research Council for funding the *epiSTEMe* project (RES-179-25-0003), fellow members of the research team (Christine Howe, Neil Mercer, Keith Taber, Stephanie Luthman, Paula Guardia and Fran Riga) for their support, and the participating students and teachers for their engagement in the study.

NOTES

[1] For example, of the studies we discuss, Maine and Hofmann (2016) includes three teachers, as does (per country) Clarke and Xu (2008). Mercer *et al.* (2009) and van de Pol *et al.* (2017) include two teachers, while Yackel and Cobb (1996), Ruthven *et al.* (2011) and Ruthven and Hofmann (2016) include one teacher.
[2] To protect the anonymity of the teachers, we are not identifying which teachers taught two different classes.
[3] Details of recruitment to the project can be found in Ruthven *et al.* (2017).

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