ON THE DETERMINATION OF EARTHQUAKE MAGNITUDES

by

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SUMMARY

The paper consists of two parts. In the first one the magnitude equation for Budapest is derived on the hand of 229 shallow-focus shocks. This equation is good for an epicentral distance of $10^\circ < \Delta^\circ < 180^\circ$. The mean error of magnitude determination is 0,34 M. In the second part a new approach to magnitude determination is treated. According to this new method there is a linear connection between the magnitude of the shock and the logarithm of the decay time $(F - eL)$ of the same. The mean error of magnitude determination on the hand of this new relation is 0,32 M.

The magnitude of a shallow shock is computed after Gutenberg [1] by

$$M = \log \frac{A_{20}}{B} + C = \log A_{20} - \log B + C \tag{1}$$

M being the magnitude of the shock, A_{20} the maximum soil amplitude of the surface waves of 20 sec period, C a constant depending on the location and characteristics of the instrument, and B the amplitude of the so-called zero-magnitude shock at a distance identical to that of the shock in question.

When considering the central part of (1) it is seen that the magnitude is, as a matter of fact, the logarithm of a ratio, namely the ratio of the amplitude of a shock of unknown magnitude to that of a zero magnitude shock, the distance of the epicenters being the same.

Gutenberg has given a formula [1] for $-\log B$, namely

$$-\log B = 5{,}04 + \frac{1}{2}\left[48{,}25\,\varkappa\,(\Delta^\circ - 90^\circ) + \log \sin \Delta^\circ + \frac{1}{3}(\log \Delta^\circ - 1{,}954)\right] \tag{2}$$

where $\varkappa$ is the absorption coefficient of the surface wave and Δ° the epicentral distance.

As in this equation $-\log B$ is, within a broad interval, an approximately linear function of $\log \Delta^\circ$, it may be written as

$$-\log B = a \log \Delta^\circ + b$$

Substituting the latter into (1) we obtain

$$M = \log A_{20} + a \log \Delta^\circ + b + C$$

Rearranging we get the form

$$Y = M - \log A_{20} = a \log \Delta^\circ + c' \tag{3}$$

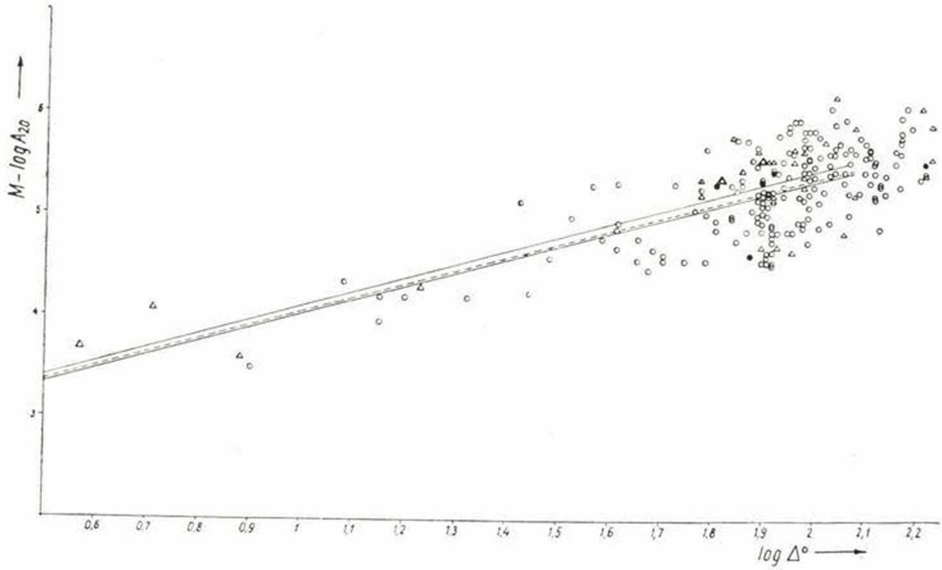

Fig. 1. The lines of the magnitude equations for Budapest. The circles represent magnitudes determined by Pasadena, the triangles those determined by different stations. The lower line refers to the circles, the upper one to the triangles, the dotted line to the mean value of the former ones

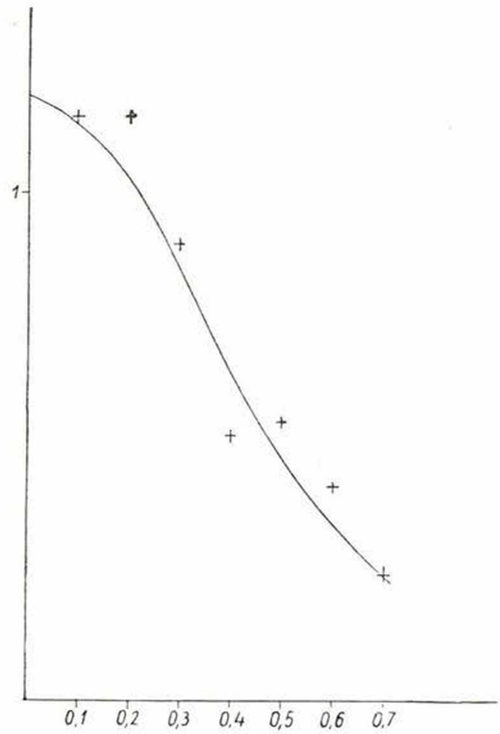

Fig. 2. Frequency of the deviations (Δ M) with respect to Equation (4)

and on the hand of known pairs of Y and $\Delta°$ we may proceed to compute a and C. In the knowledge of the latter we may determine the magnitude of any shock of an epicentral distance of $10° < \Delta° < 180°$.

By the method just described we have, using the data of 229 shocks, determined the magnitude equation of the Wiechert-pendulum of Budapest. a and c' were computed by the well-known method of least squares.

The magnitudes of 191 of the 229 shocks were determined by Pasadena. The magnitude equation for Budapest, as determined by (3) (Fig. 1) is

$$M = \log A_{20} + 1{,}37 \log \Delta° + 2{,}67 \quad (4)$$

Subdividing the two sides of the line into zones of the width of two tenths of a unit, we may determine the scatter of the data. The equation of the distribution function is

$$\gamma = \frac{1{,}86}{\sqrt{\pi}} e^{-1{,}78^2 x^2} \quad (5)$$

(see Fig. 2.). This is not much different from a normal Gaussian distribution, the mean error $\sqrt{\frac{[xx]}{n}}$ of the magnitude determination being $\pm$ 0,34 M. 81 per cent of the points is within a scatter range of 0,5 M units, 50 per cent within one of 0,26 M units.

When preparing on the basis of (4) the table of the values $\Delta M = M_{\text{Budapest}} - M_{\text{Pasadena}}$ and plotting the values on a map, we obtain Fig. 3.

Considering our map we see that proceeding from Greece eastwards, first of all the Grecian shocks are more intense, those of the area southeast of the Kaspian somewhat less strong than indicated by Pasadena. There is no systematic deviation for the shocks of Central Asia. Most of the Japanese shocks are felt to be more intense, while those of the East Indies as well as the Alaskan ones come in less strong. The American shocks are generally weaker than the Asiatic ones.

On writing the logarithms of the amplitudes beside the appropriate point in Fig. 1 we obtain a set of data being characterized by an inverse proportionality of A to M — $\log A_{20}$ belonging to one and the same value of $\log \Delta^\circ$. In other words, the Wiechert-pendulum shows the great shocks to be somewhat greater, the small ones to be somewhat smaller than they are actually. If it will be possible to find a relation between $\log \Delta_{20}$ and $x = \delta(\log A_{20})$, x being the distance of the point from the straight line, it will be possible to reduce the error in determining M. Of course, the upper and lower limits of x will depend on the epicentral distance of the shocks. For this reason we have chosen 10 distances characterized by the greatest abundance of points — and have plotted the values belonging thereto by the notation $y = \log A_{20}$, $x = \delta(\log A_{20})$ and approximating the set of points thus obtained by a straight line $y = a_0 x + a_1$ (Fig. 4). The slope of all the straight lines is negative and a_1 is dependent on Δ°.

Thus it may be written that

$$\log A_{20} = a\,\delta(\log A_{20}) + b(\Delta^\circ) \tag{6}$$

and

$$b(\Delta^\circ) = \alpha \log \Delta^\circ + \beta \tag{7}$$

or, substituting (7) into (6),

$$\log A_{20} = a\,\delta(\log A_{20}) + \alpha \log \Delta^\circ + \beta \tag{8}$$

Rearranging

$$\log A_{20} - \alpha \log \Delta^\circ = a\,\delta(\log A_{20}) + \beta \tag{9}$$

α being equal to 1,37 (from [4]), it is possible in the knowledge of A_{20} and Δ° to determine a and β by means of least squares. Thus (8) becomes

$$\log A_{20} = -1{,}2\,\delta(\log A_{20}) - 1{,}39 \log \Delta^\circ + 4{,}7 \tag{10}$$

Rearranging and dividing by 1,2 :

$$\delta(\log A_{20}) = -0{,}83 \log A_{20} - 1{,}16 \log \Delta^\circ + 3{,}81 \tag{11}$$

Thus (4) obtains the correction term (11),

$$M = \log A_{20} + 1{,}37 \log \Delta^\circ + 2{,}67 + \delta(\log A_{20}) \tag{12}$$

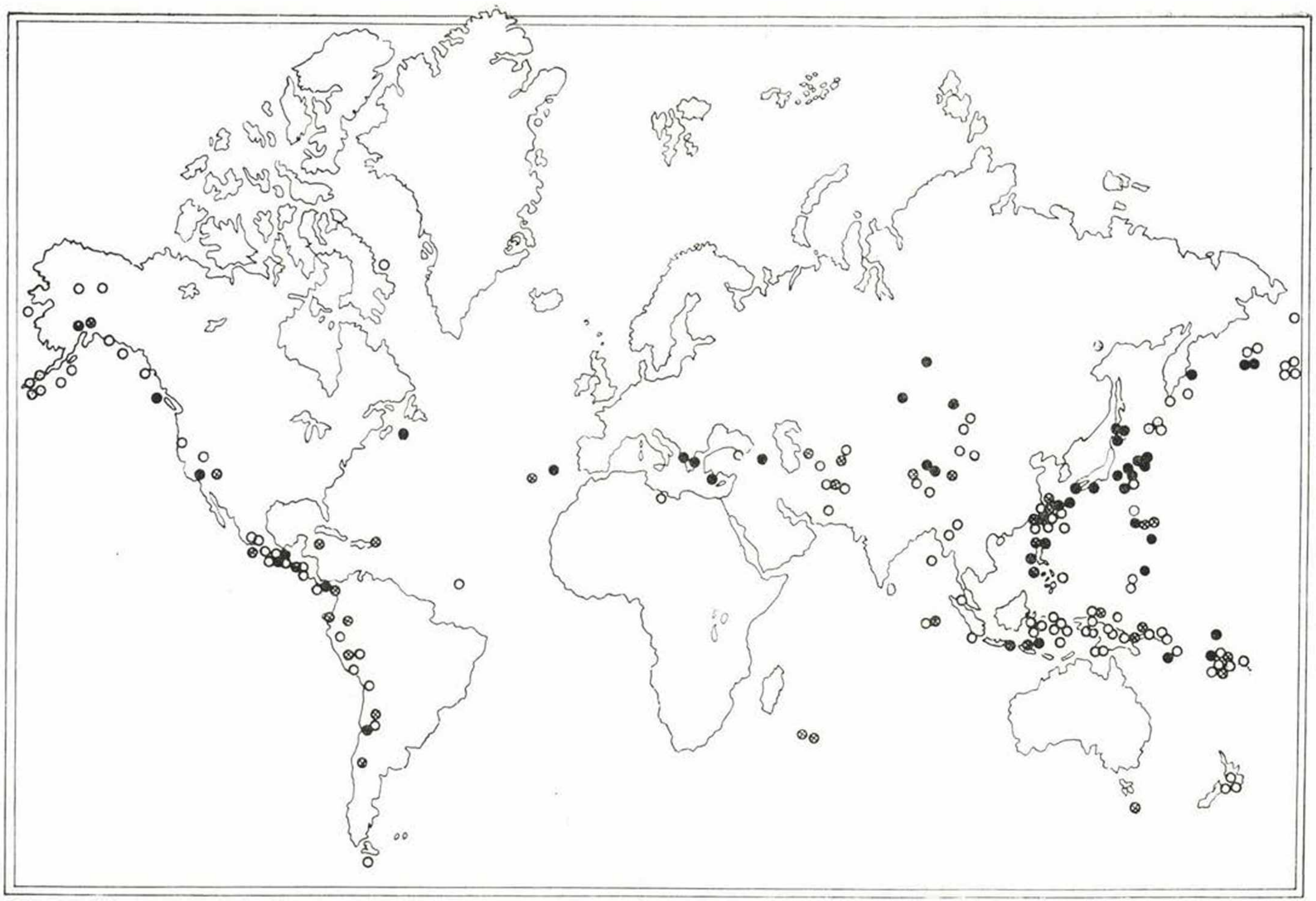

Fig. 3. The regional distribution of Δ M. Full circles represent $\Delta M > 0{,}26$ M, empty circles Δ M $< 0{,}26$ M, cross in circle $-0{,}26$ M $> \Delta$ M $> 0{,}26$ M

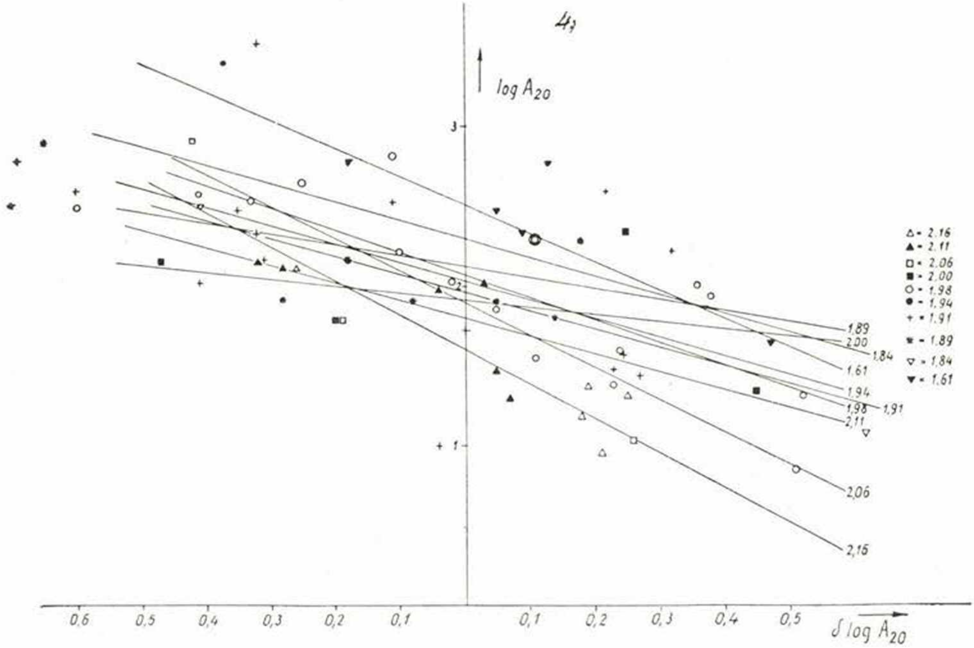

Fig. 4. log A_{20}) versus δ (log A_{20}). The numbers beside the straight lines denote the epicentral distance log Δ°

Substituting (11) into (12):

$$M = 0{,}17 \log A_{20} + 0{,}23 \log \Delta^\circ + 6{,}44 \tag{13}$$

The great value of the constant term is caused by the fact that the data used to determine (10) were all between $1{,}61 < \log \Delta^\circ < 2{,}21$. However, in this distance range, shocks weaker than 6,5 *M* pass unnoticed by the Budapest Wiechert-pendulum.

When applying formula (13) to the shocks of the years 1929-30-31, the points will of course be situated differently (Fig. 5). The mean error computed for these 28 shocks is 0,39 *M*, greater than the value of 0,34 *M* obtained for the entire lot. However, after applying the correction, the mean error will decrease to 0,22. The advantageous effect of the correction is visible at a glance in Fig. 5.

The magnitude determination of earthquakes was based by Richter [2] on the amplitudes of the long waves of shallow shocks. As already stated, the amplitude registered is distance-dependent and also influenced by the characteristics of the instrument.

By applying (4), good results are obtained even for epicentral distances as small as $\Delta^\circ = 10^\circ$. However, the magnitudes of nearer shocks will be overestimated by this method. A good illustration of this fact is the sequence of Grecian shocks, some 12 degrees of Prague, whose magnitudes were determined by Athens as well as by Prague. We have compared the magnitudes of the 22 shocks of the time span 1953-1955 (Fig. 6), with the result that the Athens data are throughout greater 0,4 M than those of Prague.

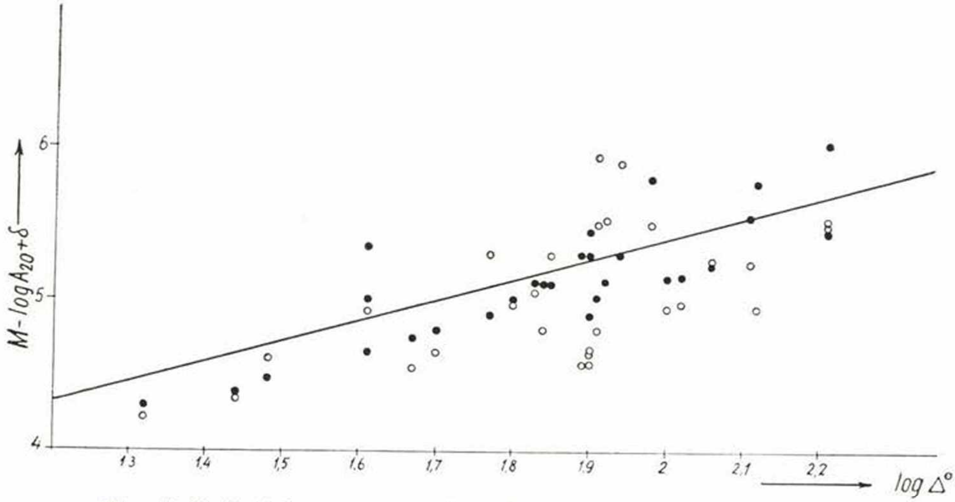

Fig. 5. Full circles represent the values corrected by Equation (11)

The study of a number of Hungarian shocks of small epicentral intensity has given a similar result. By applying the Budapest magnitude equation values greater by about one-half unit than the expected ones were obtained.

As the frequency of Hungarian shocks registered at distances above 5-6 degrees is about one for 20 to 30 years, it was impossible to determine a magnitude equation for the weak shocks in our country. On the other hand, even if the magnitudes of some near shocks would have been known, these could

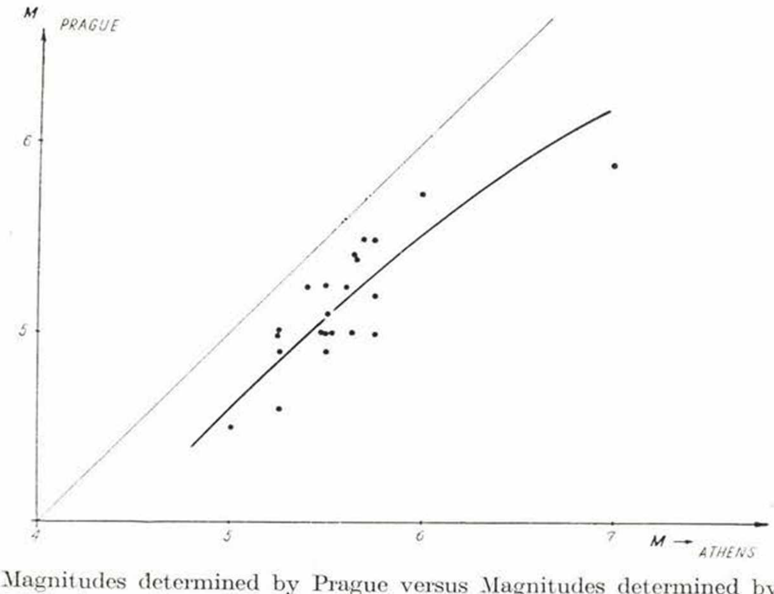

Fig. 6. Magnitudes determined by Prague versus Magnitudes determined by Athens. If the magnitude equations of the two stations would be equivalent, the points should lie along a straight line of 45 degrees slope

not have been used for the method outlined, as the seismogram of a near shock is almost illisible with paper velocities of 20 mm/min. Thus the determination of the period, of importance in the classical method, is impossible. Moreover, the signal is far from being sinusoidal so that the application of the formula

$$a = \frac{A.\,V}{\sqrt{4h^2\,u^2 + (u^2 - 1)^2}}$$

is incorrect. Because of all this it was necessary to try a new approach to magnitude determination.

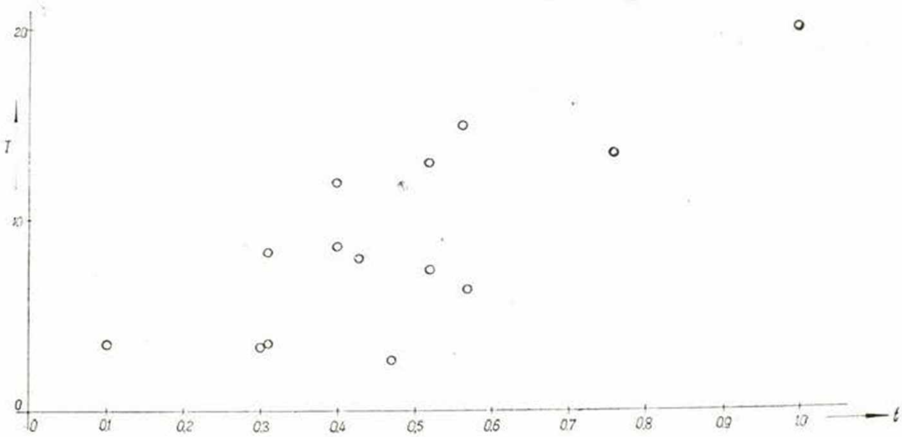

Fig. 7. Explosion registered at a distance of 27 kilometers. Mass of explosive (T) plotted against duration of surface wave (t)

A great help to this work was given by the Prague Seismological Report [3] [4] [5] of the years 1953-55, containing the data of 25 blasts of known amount of explosive and wave duration. Of these we have selected those of identical locality so as to have the amount of explosive as the only variable parameter. The study of these data has shown that the duration of the soil movement generated by the blast increase with the amount of dynamite sprung (Fig. 7).

If there is some kind of similarity between the artificial and natural shock, it is correctly assumed that with the latter a similar relation between the energy and the duration of soil movement must hold. The appropriateness of this assumption is proven by the following line of thought.

If the duration t of the decay of the tremor at a given epicentral distance depends on the amount of explosive, D by

$$D = C\, t^{\beta}$$

and if a proportionality of the elastic energy E generated to D is assumed, i. e.

$$E = \gamma\, D$$

then the relation

$$\log E = \log \gamma + \log D = \log \gamma + \log C + \beta \log t$$

will hold, wherein, applying the notation

$$\log \gamma + \log C = \alpha$$

we obtain

$$\log E = \beta \log t + \alpha$$

Comparing with Gutenberg's

$$\log E = a\, M + b$$

(M being the magnitude), we obtain

$$M = c \log t + d$$

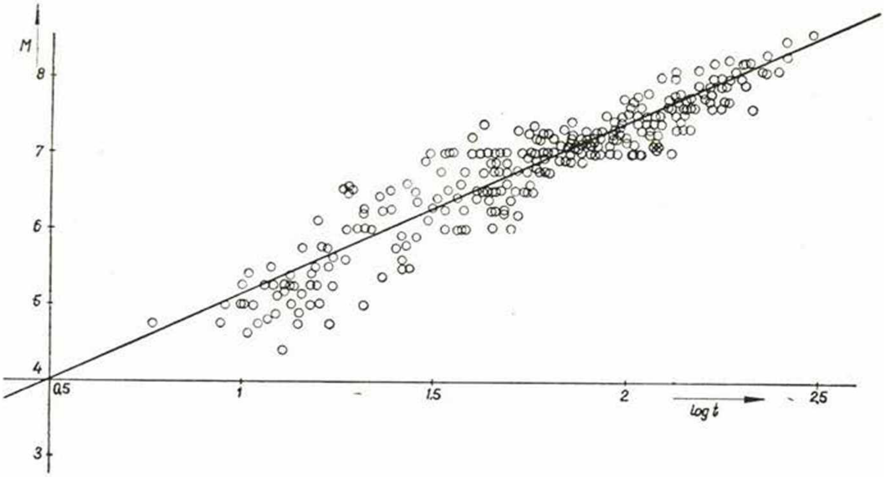

Fig. 8. t : duration of surface wave, M : magnitude of shock

According to the deduction the magnitude is a linear function of the logarithm of the decay time of the earth waves.

In case of a natural shock the picture will be somewhat more confused because of the dispersion of the individual phases and the changes in duration dependent thereon. Of course, the fact that the change of duration caused by dispersion is distance-dependent had to be taken into account also in this case.

The most reliable pendulum in Hungary is the Wiechert one in Budapest, (V $\sim$ 190, T $\sim$ 10, E $\sim$ 5). Therefore the studies to be described were based on the data of this instrument. In the period 1931 to 1955, 295 shocks whose magnitudes were given by Prague or Pasadena were registered by the Budapest pendulum. It was insufficient to apply Pasadena data only, as it was necessary to include the weak near shocks of the Balkans. Of the 295 shocks mentioned the magnitudes of 208 ones were determined by Pasadena, of 2 ones by Budapest and of the rest by Prague.

For these shocks we have computed the duration in minutes of the surface wave by the formula $F - eL$, F being the time of the ceasing of the tremor, as usual in seismology, and eL the time of arrival of the surface wave. The magnitudes of the shocks were plotted, with no regard to epicentral distance, against the logarithm of wave duration (Fig. 8).

It was seen at a first glance that scatter was no greater than with the usual methods of magnitude determination. This was somewhat striking as the epicentral distance which ranged from 4 to 160 degrees was completely left out of account.

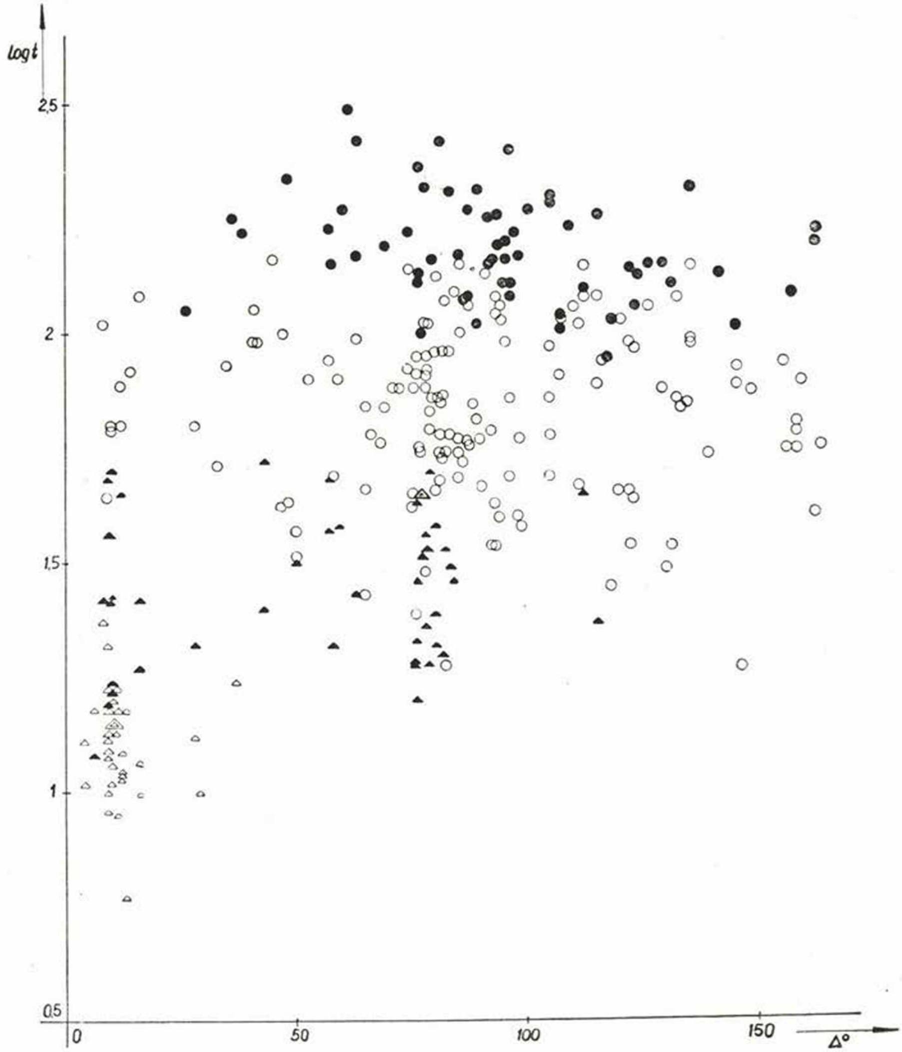

Fig. 9. t : duration of surface wave, Δ : epicentral distance in degrees. $M \geqq 7{,}5$: black dots. $6{,}5 \leqq M < 7{,}5$: circles. $5{,}5 \leqq M < 6{,}5$: black triangles. $M < 5{,}5$: empty triangles

Subsequently we have plotted the logarithmus of the surface wave duration against epicentral distance for four magnitude categories, namely $M < 5{,}5$; $5{,}5 \leqslant M < 6{,}5$; $6{,}5 \leqslant M < 7{,}5$; $7{,}5 \leqslant M$; with four different. signs (Fig. 9).

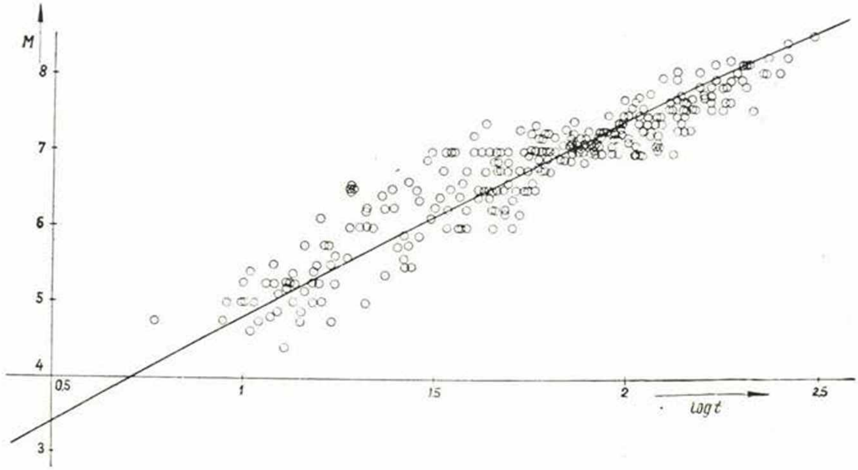

Fig. 10. t : duration of surface wave, M : magnitude of shock

It is seen that the distribution of points is almost insensitive to distance.

Then, as a first approximation, the points of Fig. 8 were approximated by a straight line. The equation thus obtained was

$$M = 2{,}25 \log t - 0{,}001\,\Delta^\circ + 2{,}92 \tag{14}$$

M being the magnitude of the shock, t the duration of the superficial wave as defined above, Δ° the epicentral distance in degrees. In (14) the coefficient of Δ° is very small. This was of a great advantage as the equation has permitted to extrapolate for near shocks without committing too great an error. The mean error of (14) is

$$\sqrt{\frac{[xx]}{n}} = 0{,}32\ M$$

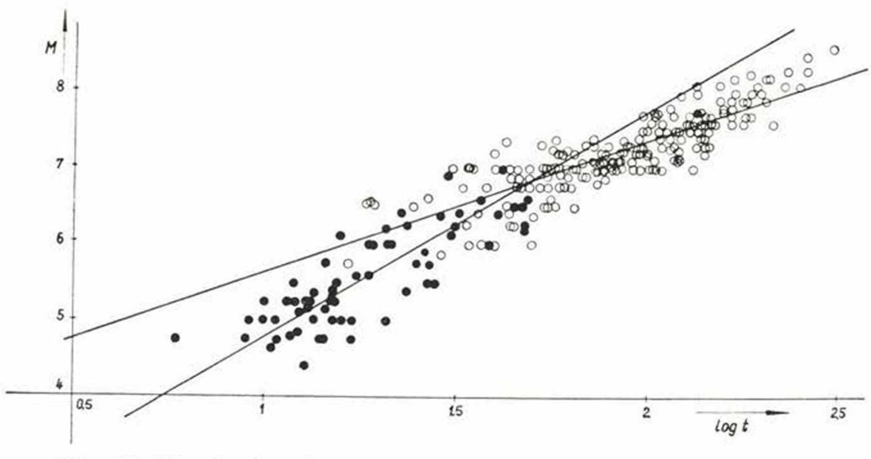

Fig. 11. Magnitudes given by Pasadena designated by circles, those given by Prague by dots

Fig. 8 has given the impression that an approximation by a parable would be more to the point. The quadratic approximation was performed and the equation

$$0{,}0063\, M^2 + 0{,}306\, M - 0{,}623 = \log t \tag{15}$$

obtained. Here, the coefficient of $\Delta°$ was neglected. However, this approximation is actually no better than the former one, the mean error being 0,33 M.

After a prolonged series of investigations it was found that even (14) does not yield magnitude values of sufficient accuracy for small magnitudes. As mentioned, this equation was based on Pasadena and Prague data. The magnitudes given, if plotted against log t, do not lie along a straight line (Fig. 11) but rather along a pair of intersecting straight lines. The usual procedure is to apply Pasadena values only. However, in our case these would have been insufficient as the Budapest pendulum is insensitive to shocks weaker than $M = 6$ registered by Pasadena ; on the other hand, because of the great distance between Pasadena and Budapest, the weak shocks around Budapest are not registered by Pasadena. The magnitude interval of mutually registered shocks is $6 \leq M \leq 8{,}5$. This interval is much too small and makes the determination of shocks of e. g. $M = 3$ too uncertain.

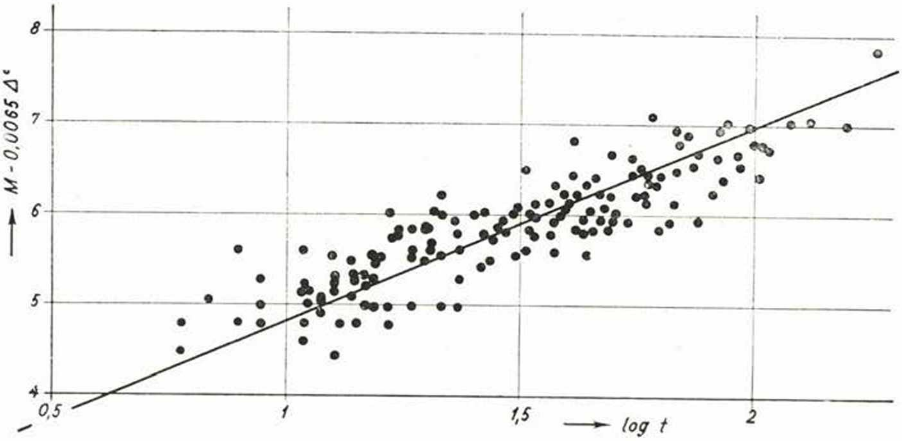

Fig. 12. Magnitude equation (16)

Therefore it was deemed best to use Prague data only, although the accuracy of these is less than that of the Pasadena ones.

To determine the magnitude equation, we have used the data of 170 shallow shocks given by Prague ([3], [4], [5]) for the interval 1953-55. The epicenters of these have been more than 10° off Prague. Therefore the relative error of their magnitudes is within $\frac{1}{4}$ M.

The coefficients of the equation

$$M = a \log t + b\, \Delta° + c$$

were determined by the method of least squares. The values $F - eL$ were derived from "Rapport microseismique de l'Institut National Seismologique de Hongrie 1953—1955". This procedure is subject to errors due to eventual misprinting, but it has the advantage that it is easily checked and precludes personal likes and dislikes of the operator.

The equation obtained is (Fig. 12)

$$M = 2{,}12 \log t + 0{,}0065\, \Delta^\circ + 2{,}66 \tag{16}$$

The mean error of magnitude determination is 0,32 M, thus the probable error is within the usual range of $\frac{1}{4}$ M.

(16) yields very good results even for weak shocks, as compared with the values obtained by the equation

$$M = 0{,}6\, J_0 + 1 \tag{17}$$

of Gutenberg, where I_0 is the epicentral intensity according to the Mercalli—Sieberg scale. The constant in (17) is 1, as the average depth of Hungarian earthquake foci is about 8 kms.

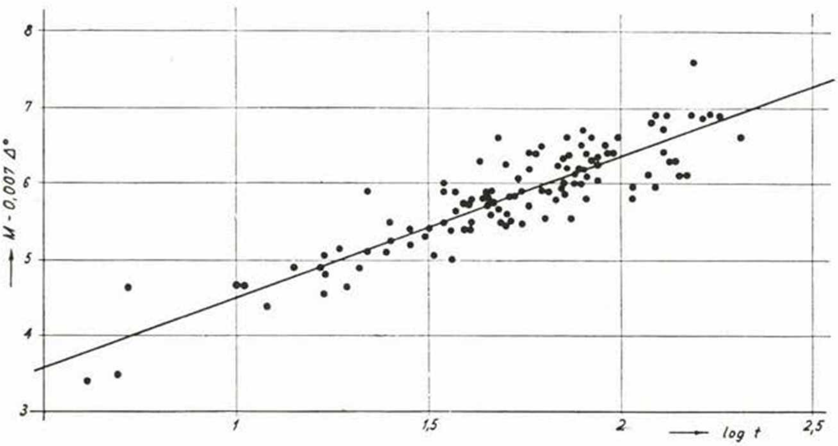

Fig. 13. Magnitude equation for Prague (18)

On comparing (16) with equation (3) in [6], relating to the Prague Wiechert-pendulum, and established on data taken from the report [3], [4], [5], the agreement is found to be very good. The mentioned equation (3) reads :

$$M = 1{,}85 \log t + 0{,}007\, \Delta^\circ + 2{,}66 \tag{18}$$

The coefficient of Δ differs by 5 ten thousandths, while the constant is identical up to two decimals. The discrepancy of the coefficient of log t is due to the greater sensitivity of the Prague pendulum. The coefficient of Δ° may be neglected for small distances for both instruments. Thus the magnitudes of Hungarian shocks may be readily calculated by equation

$$M = 2{,}12 \log t + 2{,}66$$

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