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NONLINEAR VIBRATIONS AND LONG-TERM STRENGTH OF TURBINE BLADES

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ABSTRACT

The method of a durability estimation of rotating turbomachinery blades at forced flexural-flexural-torsional vibrations is offered. The method is based on the methods of Continuous Damage Mechanics and the accurate strain analysis of the pre-twisted blades at the nonlinear vibrations with moderate displacements. The method to solve the strain analysis problem and turbomachinery blades high-cycle fatigue damage estimation as a result of nonlinear vibrations is presented.

INTRODUCTION

Modernization of the existing steam turbines park is an actual problem. The decisions offered in this paper are connected with increasing the working pressure and changing the blades geometrical characteristics. In these operating conditions, the deflections of blades comparable with a thickness are possible. In this case, the new methods of an estimation of turbine' blades durability are necessary. The damage arising due to the vibrations at their geometrically nonlinear deformation were taking into account. Approaches offered earlier in [1-6] allow to estimate the deformed condition of blades at their geometrically nonlinear vibrations, and, on this basis to make the stress analysis. Following the damage theories, stated in works [7-9], it is possible to estimate the long term strength of blades.

The results of the nonlinear vibrations and the blade stress-strain analysis are considered in this paper. The estimation of blades durability by the number of cycles before failure due to high-cycle fatigue is presented. The fatigue damage of blades of steam turbine of type K-300-240 (Ukraine) was studied.

1. THE ANALYSIS OF NONLINEAR VIBRATIONS AND STRESS-STRAIN STATE OF BLADES

Following [1-6], the analysis of geometrically nonlinear vibrations of blades is made on the basis of pre-twisted beams with variable cross-section theory. As the beam has asymmetrical cross section, the gravity centre and the shear centre are in the different points. As the amplitudes of vibrations are commensurable with the blades thickness, the nonlinear geometric law for beam displacements and strains has to be used. In this case, strains are small and the strain-stress connection is linear. The equations of geometrically nonlinear vibrations of turbine blades were obtained in works [2-4].

The beam vibrations are considered with respect to the global coordinate system $(\hat{x},\hat{y},\hat{z})$ (fig.1). It is assumed, that the beam cross sections remain planar. The coordinate system $(\hat{\xi},\hat{\eta},\hat{\varsigma})$ is attached to a beam cross section to predict its motions. The origin of this coordinate system is placed in the gravity centre of the cross section. Then the dynamics of the cross section is reduced to analysis of motions of the coordinate system $(\hat{\xi},\hat{\eta},\hat{\varsigma})$ with respect to the global system $(\hat{x},\hat{y},\hat{z})$.

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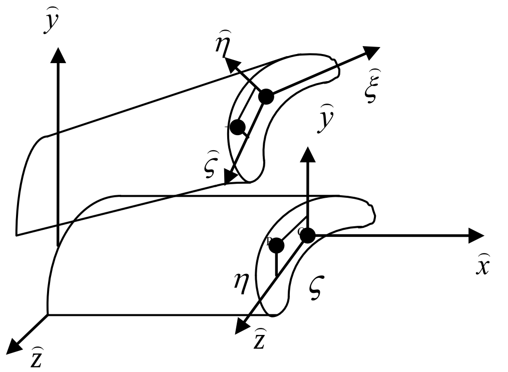

Fig. 1 The beam and the coordinates systems

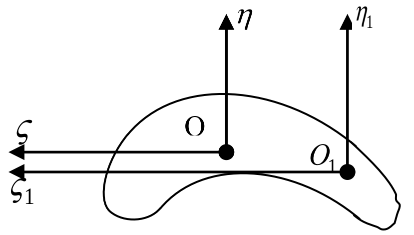

Fig. 2 The cross section of a beam

Cross section motions are described by displacements u, v, w in the directions $\hat{x}, \hat{y}, \hat{z}$ and by three successive rotations. The angles θ_z and θ_y describe the beam bending vibrations in two perpendicular planes. The third rotation θ_x takes place about the shear centre O_1 and describes the torsion of a beam. The new coordinate system $(\eta_1 O_1 \varsigma_1)$ with the origin O_1 in the shear centre is introduced to study this rotation (fig.2). Therefore, the cross section gravity centre O has the coordinates $O(\eta^{(1)}, \varsigma^{(1)})$ in the coordinate system $(\eta_1 O_1 \varsigma_1)$.

The Hamilton's principle is used to derive the equations of rotating beams vibrations. Then the partial differential equations of beam nonlinear vibrations with respect to the warping have the form:

$$\begin{gathered}
E\left(w'' J_{\varsigma\eta}\right)'' + E\left(v'' J_{\varsigma}\right)'' - 2E\left(\theta_x v'' J_{\varsigma\eta}\right)'' + E\left[\theta_x w''\left(J_{\varsigma} - J_{\eta}\right)\right]'' + m\ddot{v} - m\ddot{\theta}_x \varsigma^{(1)} = 0; \\
E\left(w'' J_{\eta}\right)'' + E\left(v'' J_{\varsigma\eta}\right)'' + E\left[\theta_x v''\left(J_{\varsigma} - J_{\eta}\right)\right]'' + 2E\left(\theta_x w'' J_{\varsigma\eta}\right)'' + m\ddot{w} + m\ddot{\theta}_x \eta^{(1)} + P(x)\cos(\Omega t) = 0; \qquad (1) \\
E\left(w'' J_{\varsigma\eta} + v'' J_{\varsigma}\right)w'' - E\left(w'' J_{\eta} + v'' J_{\varsigma\eta}\right)v'' - \left(D_{\xi}^{(1)}(x)\theta_x'\right)' - m\ddot{v}\varsigma^{(1)} + m\ddot{w}\eta^{(1)} + \ddot{\theta}_x\left[m\left(\varsigma^{(1)2} + \eta^{(1)2}\right) + I_{\xi}\right] = 0; \\
\dot{u} = -\int_0^x \frac{\partial}{\partial t}\left(\theta_x v'' \varsigma^{(1)} - \theta_x w'' \eta^{(1)}\right)dx - \int_0^x \left(v' \dot{v}' + w' \dot{w}'\right)dx.
\end{gathered}$$

where m is the weight of a beam length unit; E is the Young modulus; $P(x)\cos(\Omega t)$ is the loading operating from gas forces; $J_{\zeta\eta}, J_{\varsigma}, J_{\eta}, D_{\xi}^{(1)}$ are the geometrical characteristics of cross-section of a blade.

The Galerkin method is used for discretization of the nonlinear partial differential equations (1).The beam vibrations are considered in the form:

$$W(x,t) = \sum_{\nu=1}^{N_1} q_{\nu}(t) W_{\nu}(x); \; \theta_x(x,t) = \sum_{\nu=1}^{N_2} q_{\nu+N_1}(t)\theta_{\nu}(x); \; V(x,t) = \sum_{\nu=1}^{N_3} q_{N_1+N_2+\nu}(t) V_{\nu}(x),$$

where $q_1, \ldots, q_{N_1+N_2+N_3}$ are the generalized coordinates of a system; $W_{\nu}(x); \theta_{\nu}(x); V_{\nu}(x)$ are the forms of linear vibrations [3]. As the nonlinear vibrations of rotating beams close to the equilibrium position ($q_1^{(0)}, q_8^{(0)}, q_{15}^{(0)}$), then the change of the variables is used:

$$q_i = \theta_i + q_i^{(0)}; \; i = \overline{1,16}; \; q_1^{(0)} \neq 0; \; q_8^{(0)} \neq 0; \; q_{15}^{(0)} \neq 0; \; q_{\nu}^{(0)} = 0; (\nu = \overline{2,7}; \overline{9,14}; 16)$$

The following dynamic system is derived:

$$(M)\ddot{\theta} + (K)\theta + \Omega(G)\dot{\theta} + \Omega F(\theta, \dot{\theta}) + \Phi(\theta) = N\cos(\Omega t), \qquad (2)$$

$$\theta = \begin{bmatrix} \theta_1 \\ ... \\ \theta_{16} \end{bmatrix}; K'_{\nu\mu} = K_{\nu\mu} + \Omega^2 R_{\mu}^{(\nu)} + \tilde{K}_{\nu\mu}; (\nu = \overline{1,16}; \mu = \overline{1,16}) \ (K) = \begin{bmatrix} K'_{11} & ..... & K'_{1,16} \\ .... & ..... & .... \\ K'_{16,1} & ..... & K'_{16,16} \end{bmatrix};$$

$$f_\nu = \sum_{\mu=1}^{2} \sum_{j=1}^{14} D_{14+\mu,j}^{(\nu)} \theta_j \dot{\theta}_{14+\mu}, \ (\nu = \overline{1,7}); \ f_l = \sum_{\nu=1}^{16} \sum_{\mu=1}^{16} D_{\nu\mu}^{(l)} \dot{\theta}_\nu \theta_\mu, \ (l = \overline{8,16});$$

$$\varphi_\nu = \sum_{\mu=1}^{7} \sum_{j=1}^{2} A_{\mu+7,14+j}^{(\nu)} \theta_{\mu+7} \theta_{14+j} + \sum_{\mu=1}^{7} \sum_{j=1}^{7} A_{\mu+7,j}^{(\nu)} \theta_{\mu+7} \theta_j; \ (\nu = \overline{1,7};15,16);$$

$$\varphi_l = \sum_{r_1=1}^{7} \sum_{r_2=1}^{7} A_{r_1 r_2}^{(l)} \theta_{r_1} \theta_{r_2} + \sum_{r_1=1}^{2} \sum_{r_2=1}^{2} A_{14+r_1,14+r_2}^{(l)} \theta_{14+r_1} \theta_{14+r_2} + \sum_{r_1=1}^{2} \sum_{r_2=1}^{7} A_{14+r_1,r_2}^{(l)} \theta_{14+r_1} \theta_{r_2}, \ l = \overline{8,14};$$

$$N = [f_1 \gamma, 0, ..., 0]^T; \ f_1 = -\int_0^L W_1 W_\nu dx; \nu = 1, ..., 7,$$

where L is a length of a blade.

To investigate the force vibrations in (2) the method of nonlinear normal modes in the combination with Rausher procedure was applied in works [5, 6]. On this basis, the frequency-response curves of the system and values of blades estimation parameters are derived. The further specification of the given stress peak values in a blade is carried out for 3D models by use the program complexes.

2. LONG-TERM FATIGUE STRENGTH ESTIMATION

Following modern approaches of the Continuous Damage Mechanics (CDM) theories, stated in works [7, 8], we will define the number of cycles before failure due to the high-cycle fatigue, using the stress analysis data at nonlinear vibrations. The fatigue damage phenomenon represents the irreversible process of accumulation of micro cracks in a material. The damaged condition is represented as initiation of a macroscopically crack (for materials of 0.1-1 mm). The fatigue damage, caused by repeating action of stresses, is defined as a function of stress cycle's number. The description of processes of the materials hidden damage in the modern mechanics is described by the concept of damage parameter [7, 8].

Consider a damage scalar parameter of $D = D(N)$ in a point of a deformable body under the cyclic loading using hypothesis of isotropic damage. Here N is the number of loading cycles, and $0 \le D \le 1$, $D = 0$ for the undamaged material, $D(N_*) = 1$ answers the material failure in a body point. Then the kinetic equation for the damage parameter is written in the following form [9]:

$$dD = f(\sigma_a/(1-D))dN, \tag{3}$$

where $f(\sigma_a/(1-D))$ is the function defined by use of stress-rupture curves under the cyclically changing stresses with peak values σ_a.

In the case of high cycle fatigue, the most probable limiting number of loading cycles before the failure is $N_* > 5 \cdot 10^4 \div 5 \cdot 10^5$. For the description of the fatigue damage accumulation the classical laws used an amplitude of variable stresses σ_a [7].

Then the equation (3) can be concretized, using the auto- model law [7]:

$$\frac{dD}{dN} = \frac{F\sigma_a^k}{(1-D)^k}, \tag{4}$$

where F and k are the material's constants [11].

Processes of turbine blades loadings are characterized by static and variable loadings. For a case of joint action of static σ_0 and cyclically changing stresses the following dependence is used

$$\sigma_a = \sigma_0 + b\sigma_m ; b = \frac{\sigma_{-1}}{\sigma_a} , \tag{5}$$

where σ_m is an average stress of a cycle; $\sigma_в$ is an ultimate strength; σ_{-1} is a fatigue limit [10]. The equation (4) is being generalized to estimate the fatigue strength. Thus, various criteria of fatigue strength are used for equivalent stress $\sigma_в$:

$$\frac{dD}{dN} = \frac{F\sigma_e^k}{(1-D)^k} . \tag{6}$$

Sines criterion for equivalent stress is used [7]:

$$\sigma_e = \tau_{ocm}^a = 3b\overline{\sigma}_н , \tag{7}$$

where $\tau_{ocm}^a = \frac{\sqrt{2}}{3}\left(\frac{3}{2}\sigma_a\sigma_a\right)^{\frac{1}{2}}$ is an octahedronical stress amplitude; $\overline{\sigma}_н = \frac{1}{3}\left(\sigma_{11}^m + \sigma_{22}^m + \sigma_{33}^m\right)$ is a hydrostatic pressure component of the cycle stress.

It is possible to define the value of limiting number of cycles till the moment of the macro crack initiation by the integration of the kinetic damage equation (6) at constant stresses

$$N_* = \frac{1}{F\sigma_e^k (k+1)} . \tag{8}$$

Thus, the durability of turbine blades can be estimated by using data of the number of cycles before failure due to high cycle fatigue (8).

3. DURABILITY ESTIMATIONS OF STEAM TURBINE BLADES OF TYPE K-300-240

The following data are used for calculations of steam turbine blade: $E = 2.12\cdot10^5 MPa$; $G = 78\cdot10^5 MPa$; $\rho = 7.859\cdot10^3 \, kg/m^3$; $L = 0.342 \, m$; $J_\eta = 6.45\cdot10^{-8} m^4$; $J_\varsigma = 10.91\cdot10^{-8} m^4$; $A = 483.391\cdot10^{-6} m^2$; $\varsigma^{(1)} = -7.7\cdot10^{-3} m$. $F = 3.94\cdot10^{-38}\left(\kappa G / mm^2 \; {}^{-1}\right)^k$; k = 16.1.The blade's cross-section is shown in fig. 2.

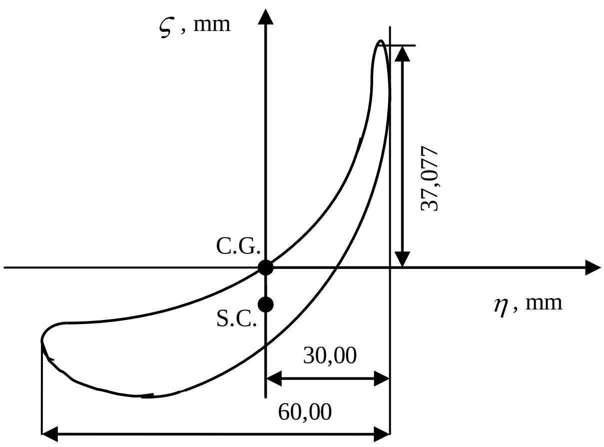

Fig. 2 The blade cross-section

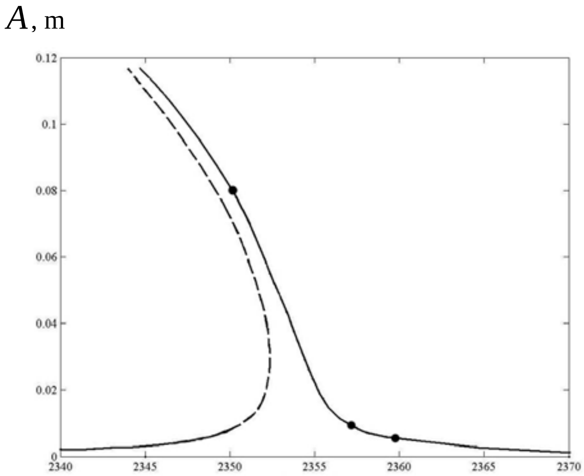

Fig. 3 The blade frequency-response curve

Applying the method of nonlinear normal modes in a combination with a Rausher procedure, the frequency-response curve of a system is presented in fig. 3. The frequency-response curve of a considered blade is soft. So the contribution of the blade rotatory inertia in the nonlinearity is bigger than the curvature of the axis displacements.

The specified analysis of the blade's 3D stress-strain state for three cases of forced vibrations amplitudes is made: 1) 0.08 m, 2) 0.006 m, 3) 0.0078 m. Calculations for the stress state in the first case have shown, that conditions of short-term durability are not satisfied (σ_{max} = 6420 MPa, h_{max} = $8{\cdot}10^{-2}$). For a second case the von Mises equivalent peak stress distribution on a blade's surface is presented in fig.5. The distribution of the stresses in a blade, which is under the action of a static steam pressure, is presented in fig. 6.

The results of calculations are shown in tab. 2.

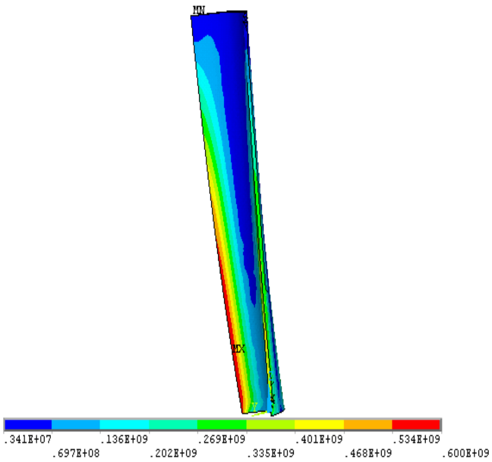

Fig. 5 The intensity values distribution of peak stresses in a blade, the maximum displacement on a free side is $0.78{*}10^{-2}$ m

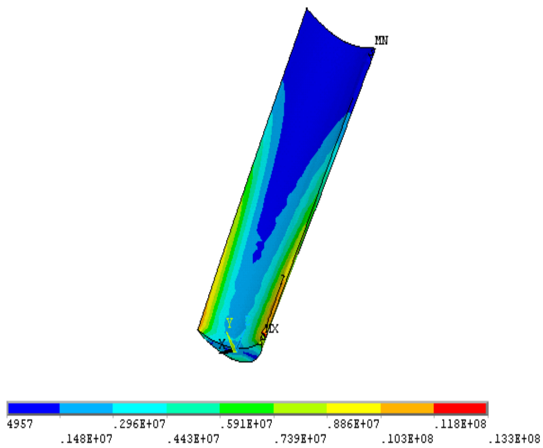

Fig. 6 Equivalent stresses in a blade under the pressure of a steam stream P=7 кPa

Table 2 Characteristics of the 3-rd stage blade of the powerful steam turbine of type К-300-240

Material (steel)	**T, K**	**Von Mises equivalent stress, MPa**	**Number of cycles to failure**
12X13 (1X13), 403 US	423	392.7	$7.62{\cdot}10^{8}$
		516.2	$2.95{\cdot}10^{6}$
12X13 (1X13), 403 US	723	392.7	$2.42{\cdot}10^{5}$
		516.2	-
322 US (aged and hardened)	423	392.7	$3.32{\cdot}10^{10}$
		516.2	$4.08{\cdot}10^{8}$
450 US	423	392.7	$2.65{\cdot}10^{9}$
		516.2	$1.03{\cdot}10^{7}$
ЕI437Б	973	392.7	$1.07{\cdot}10^{7}$
		516.2	$4.24{\cdot}10^{5}$

The analysis of results shows, that only the blades made of in a special way thermo processed steel 322 (USA), and working in a mode 3 (the forced vibrations amplitude is 8 mm), are agreed with the requirements of designers. In this case, the number of cycles before failure is equal to $4.08{\cdot}10^8$, that is smaller than $2{\cdot}10^7$ cycles. If requirements are softened to $N = 1{\cdot}10^7$, then the blades from a steel 450 (USA) also can maintain similar amplitudes. For the second mode (with amplitude of 6 mm) inadmissible are values of the operational temperatures exceeding 423 K for steel 12X13 (1X13), and for steel ЭИ437Б at T=973 K a life time value should be limited by the number of cycles $N = 1{\cdot}10^7$.

CONCLUSIONS

In this paper the methods of solving the nonlinear vibrations analysis problem and stress analysis problem were offered. These methods are applied to get estimations of turbine blades' low - cycle fatigue with respect to damages due to geometrically nonlinear vibrations. Using experimental curves of fatigue strength, it is possible to define the number of cycles to failure. The results are used to make conclusions on a choice of blades' materials that satisfy to the design requirements. The powerful steam turbine blades of type K-300-240 were used for investigations.

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