INFLUENCE OF THE INTERNAL LAYER CRACKS ON THE CRACKING OF FLEXURAL THREE-LAYER CONCRETE MEMBERS

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Abstract. Three-layer structures made of concrete-type materials usually have an internal layer of a less strength, which serves as thermal insulation and makes the structure more light. Therefore cracks may appear in the internal layer of a bended structure earlier than in the external tensile layer. This paper deals with an influence of cracks in the internal layer and its deformational properties on the calculation of layered structures made of concrete-type material. A model is presented for calculating flexural three-layer members for crack resistance taking into account the internal layer cracking. Analytical procedures are described for calculating the depth of the tensile zone and cracking of the internal layer. The paper also presents an analysis of the influence of deformational properties of the material (concrete) on the stress-strain relationship in the layers.

Keywords: layered structures, reinforced concrete, elasticity, strength, crack, stress, strain.

1. Introduction

Three-layer slabs made of concrete-type materials are used for the roof, ceiling and other parts of buildings. The internal layer of such structures usually serves as thermal insulation. Also this layer increases the stiffness of the structure. Usually the materials used for internal layers are of a less strength compared with materials used for external layers. Therefore the cracks may appear in the internal layer [1]. The external layer cracks in most cases are inadmissible because they may influence the thermal insulation properties of the internal layer (due to the soak of moisture) [2, 3]. Cracks in the internal layer change the stress-strain state and in this way they change the behaviour of the external tensile layers under loading [2, 4, 5]. However, the analysis of literature sources shows that the calculation of structures with cracked internal layer has not been investigated enough. For the first time this question was discussed in [2], although only conditions of possible cracking of the internal layer were given. The calculation for crack resistance of layered structures made of concrete-type materials in general has not been investigated in detail.

It is important to determine the deformational properties of tensile concrete as accurately as possible when analyzing the stress-strain relationship under ultimate load conditions. Plastic deformations of concrete members can be estimated via Poisson's ratio $v_c = \varepsilon_{c,el} / \varepsilon_c$, where $\varepsilon_{c,el}$ is the elastic strain of the concrete and ε_c is the full deformation of the concrete [2, 4, 6-8]. For the ideal

elastic material the Poisson's ratio $v = 1$ and for the ideal plastic material $v = 0$. The deformational properties of concrete depend upon many factors [9-11]. Although the actual influence of deformational properties on the results of calculating layered structures neither has been investigated sufficiently. That's why the main purpose of this article is to determine the influence of internal layer cracking and its deformational properties on crack resistance of the external tensile layer.

2. Defining a suitable model for numerical calculations

For more than 60 years the layered structures of stiff layers have been calculated using the so-called 'method of transformed cross-sections' [3, 4, 10]. However, this method cannot estimate all the deformational properties of reinforced concrete. Therefore scientists in different countries, including Lithuania, have proposed various methods for estimating the actual strain distribution within the section depth [3, 10, 12]. The actual stress-strain relationship $\sigma - \varepsilon$ of the concrete is not linear (Fig 1, b). However the precise description of $\sigma - \varepsilon$ relationship does not exist or the calculations are very complicated. Consequently, a simpler method for evaluating deformational properties of concrete in layered structures is required.

In our research on flexural layered structures we, as other researchers [3, 5, 12, 13], follow several widely known assumptions: 1) validity of the hypothesis of plane

sections, ie strain distribution within the section depth is linear; 2) compressive concrete is working in the elastic stage; 3) stress distribution curves in the layers may take different shapes; 4) plastic deformations appear in the tensile concrete; 5) influence of shrinkage is ignored.
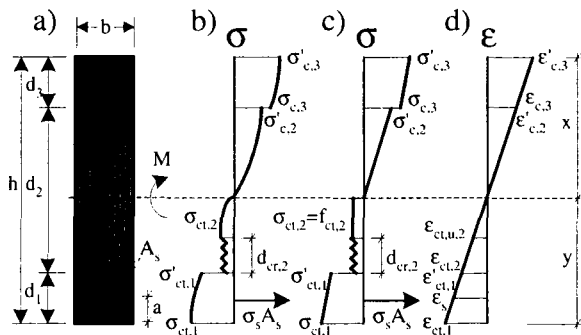

Fig 1. The model for numerical calculations of three-layer beam when the material of the internal layer is of a less strength than that of the external layer: a) basic geometry of the cross-section; b) real shape of stress distribution within the section depth; c) simplified stress distribution within the section depth; d) strain distribution within the section depth

Following these classical assumptions a simple model can be created for calculating layered structures when their internal layer is cracked and their external layer is not cracked (Fig 1, b, c).

It is not difficult to prove that a simplified model of stress distribution is very close to the real shape and corresponds to the theoretical shape of stress-strain distribution within the depth of the section of layered structures made of concrete-type materials with internal layer of less strength.

In order to simplify the calculation model it' is assumed that the stress in the most tensile fibre of the external layer reaches the ultimate tensile strength, ie $\sigma_{ct,1} = f_{ct,1}$ and $\varepsilon_{ct,1} = \varepsilon_{ct,u,1}$. Meanwhile, the stress distribution within the depth of section in the tensile zone of the internal layer could take one of the three simplified shapes: triangular (when the external tensile layer reaches its ultimate tensile strength earlier than the internal one), rectangular (when the external tensile layer and the internal layer reach their ultimate tensile strength at the same time), and cracked rectangular (when the cracks in the internal layer appear before the external tensile layer reaches its ultimate tensile strength). The shape of stress distribution within the section depth mostly depends on the constitutive deformational properties of the internal layer material (the ratio between the deformation modulus $E_{c,2}$ and the tension strength $f_{ct,2}$ of the internal layer) [2, 10, 14].

In order to verify the correspondence of the selected model to the constitutive deformational properties and the geometrical characteristics of the flexural composite beam it is necessary to check these conditions (Fig 1):

$$\varepsilon_{ct,2} < \varepsilon_{ct,u,2}, \tag{1}$$

$$\varepsilon_{ct,2} = \varepsilon_{ct,u,2}, \tag{2}$$

$$\varepsilon_{ct,2} > \varepsilon_{ct,u,2}, \tag{3}$$

$$\varepsilon_{ct,ul,i} = \frac{f_{ct,i}}{E_{ct,i}} = \frac{f_{ct,i}}{\upsilon_{ct,i} E_{c,i}}, \tag{4}$$

$$\varepsilon_{ct,2} = \varepsilon_{ct,u,1} \frac{y - d_1}{y}, \tag{5}$$

where $\varepsilon_{ct,i}$ is the concrete tensile strain at the i point in the section; $\varepsilon_{ct,u,i}$ is the ultimate tensile strain of the concrete.

If the depth of the tensile zone of the cross-section y could be known, it is possible to determine what shape of stress distribution within the depth of section the internal layer (its tensile zone) will take:

a) if the condition in Eq 1 is valid, the external tensile layer reaches its ultimate tensile strength earlier than the internal one;

b) if the condition in Eq 2 is valid, the external tensile layer and the internal one reach their ultimate tensile strength at the same time;

c) if the condition in Eq 3 is valid, the cracks in the internal layer appear before the external tensile layer reaches its ultimate tensile strength, ie the model shown in Fig 1 is valid. According to our research the validity of the condition in Eq 3 could be possible when the ratio $E_{c,2} / f_{ct,2} \geq 1 \times 10^5$ and the ratio $f_{ct,1} / f_{ct,2} \geq 3,5$. As long as the depth of the tensile zone of cross-section is unknown, it is not possible to verify the selected model completely in this stage of calculations. Consequently, two interdependent quantities - the depth of the tensile zone of the cross-section y and the depth of the crack in the internal layer $d_{cr,2}$ - should be determined.

3. Calculation of the tensile zone depth and the crack depth in the internal layer

During the calculation of the depth of cross-section tensile zone and the depth of the crack in the internal layer the taken assumptions and the model shown in Fig 1 are followed. According to the assumptions that the beam compressive zone is working in the elastic stage and the plastic deformations appear in the tensile concrete, the equilibriums for the stresses have the following expressions:

$$\sigma_{c,i} = E_{c,i} \varepsilon_{c,i}, \tag{6}$$

$$\sigma_{ct,i} = \upsilon_{ct,i} E_{c,i} \varepsilon_{ct,i}. \tag{7}$$

Since the stress distribution within the section depth is linear (Fig 1, c), it is enough to determine the stresses acting in every side of each layer and the reinforcing steel:

$$\left.\begin{array}{r}\sigma'_{c,3} = E_{c,3}\varepsilon'_{c,3} \\ \sigma_{c,3} = E_{c,3}\varepsilon_{c,3} \\ \sigma'_{c,2} = E_{c,2}\varepsilon'_{c,2} \\ \sigma_{ct,2} = \upsilon_{ct,2}E_{c,2}\varepsilon_{ct,2} = f_{ct,2} \\ \sigma'_{ct,1} = \upsilon_{ct,1}E_{c,1}\varepsilon'_{ct,1} \\ \sigma_{ct,1} = \upsilon_{ct,1}E_{c,1}\varepsilon_{ct,1} = f_{ct,1} \\ \sigma_s = E_s\varepsilon_s \end{array}\right\} \qquad (8)$$

The stress in the tensile zone of the internal layer $\sigma_{ct,2}$ acting above the crack equals $f_{ct,2}$. The stress in the cracked zone equals zero.

It is noted that the calculation of stress distribution according to the calculated strains is used in Eurocode [15] and in most research proceedings of various authors [1, 2, 3, 9, 10, 11, 13].

By the hypothesis of plane sections and using the known $\varepsilon_{ct,u,1}$ quantity (Eq 4), it is possible to calculate the relative linear strains of the concrete in every section point (Fig 1, d):

$$\varepsilon_{ct,i} = \varepsilon_{ct,u,1}\frac{y_i}{y}, \qquad (9)$$

where y_i differs in tensile and compressive zones: in the tensile zone y_i means the distance between $\varepsilon_{ct,u,1}$ and $\varepsilon_{ct,i}$, and in the compressive zone it means the distance between $\varepsilon_{c,i}$ and neutral axis (Fig 1, d).

When $\varepsilon_{ct,u,1}$ is a known quantity (Eq 4), linear strains on the every side of each layer and in the reinforcing steel will be as follows:

$$\left.\begin{array}{r}\varepsilon'_{c,3} = \varepsilon_{ct,u,1}\dfrac{d_1+d_2+d_3-y}{y} \\[2mm] \varepsilon_{c,3} = \varepsilon'_{c,2} = \varepsilon_{ct,u,1}\dfrac{d_1+d_2-y}{y} \\[2mm] \varepsilon_{ct,2} = \varepsilon'_{ct,1} = \varepsilon_{ct,u,1}\dfrac{y-d_1}{y} \\[2mm] \varepsilon_s = \varepsilon_{ct,u,1}\dfrac{y-a}{y} \end{array}\right\} . \qquad (10)$$

In order to calculate the tensile zone depth and the crack depth in the internal layer, the equilibrium equation of the forces acting around the neutral axis

$\int_A \sigma_i dA = 0$ is constructed:

$$\left.\begin{array}{r}0.5(\sigma_{c,3}+\sigma'_{c,3})d_3b_3 \\ +0.5\sigma'_{c,2}(d_1+d_2-y)b_2 \\ -0.5\sigma_{ct,2}(y-d_1-d_{cr,2})b_2 \\ -0.5(\sigma_{ct,1}+\sigma'_{ct,1})d_1b_1 - A_s\sigma_s = 0 \end{array}\right\} . \qquad (11)$$

The strain expressions in Eq 10 put into the stress equations in Eq 8, and the obtained expressions put into the equilibrium Eq 11, the following is obtained after some mathematical rearrangements:

$$[E_{c,2}d_2(1-2\upsilon_{ct,2})]y^2$$
$$+\left[\begin{array}{c}\left(\begin{array}{c}-E_{c,3}d_3b_3 \\ -E_{c,2}b_2\left(\begin{array}{c}d_1+d_2-2\upsilon_{ct,2}d_1- \\ 2\upsilon_{ct,2}d_{cr,2}\end{array}\right) \\ -E_{c,1}d_1b_1\upsilon_{ct,1}-A_sE_s \end{array}\right)\end{array}\right]y$$
$$+\left[\begin{array}{c}E_{c,3}d_3b_3(2d_1+2d_2+d_3) \\ +E_{c,2}b_2\left(\begin{array}{c}d_1^2+d_2^2+2d_1d_2 \\ -2\upsilon_{ct,2}d_1^2- \\ -4\upsilon_{ct,2}d_1d_{cr,2} \\ -2\upsilon_{ct,2}d_{cr,2}^2 \end{array}\right) \\ +E_{c,1}d_1^2b_1\upsilon_{ct,1}+2A_sE_sa \end{array}\right]=0. \qquad (12)$$

There are two unknown quantities in Eq 12: y and $d_{cr,2}$.

The equation for calculating the crack depth in the internal layer $d_{cr,2}$ is obtained following the hypothesis of plane sections and Fig 1, d:

$$d_{cr,2} = y-d_1-\frac{\varepsilon_{ct,u,2}(y-d_1)}{\varepsilon_{ct,2}}. \qquad (13)$$

Consequently, two equations (Eqs 12, 13) with two unknown interdependent quantities (y and $d_{cr,2}$) are obtained. We suggest using the iteration method for calculating these unknown quantities. For the first iteration the zero value may be used for the crack depth in the internal layer $d_{cr,2}$.

When the tensile zone initial depth y is being calculated (with $d_{cr,2} = 0$), the validity of the model shown in Fig 1 must be verified by given conditions (Eqs 1, 2 and 3). If the selected model is valid, ie Eq 3 is satisfied, the depth of the crack in the internal layer $d_{cr,2}$ with obtained initial y quantity could be calculated by Eq 13.
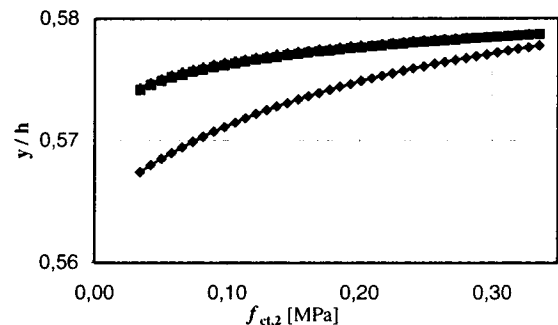

Fig 2. The relationship between $f_{ct,2}$ quantities and the ratio y/h, following the three iterations of calculation (—◆— first iteration - crack depth equals zero; —■— second iteration - with the crack depth obtained in first iteration; —▲— third iteration - with the crack depth obtained in second iteration)

When the second iteration of the calculations is followed, ie y value is calculated using Eq 12 with obtained $d_{cr,2}$ value. Then the model validity in Fig 1 must be verified again and the next iteration could be followed if required.
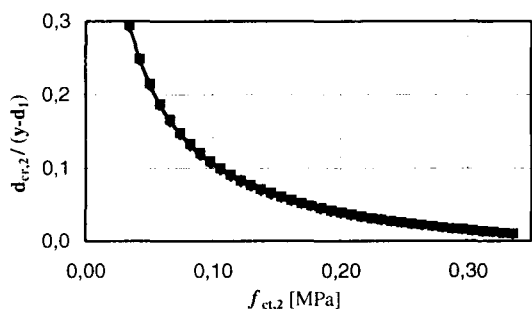

Fig 3. The relationship between $f_{ct,2}$ quantities and the ratio $d_{cr,2}/(y-d_1)$, following three iterations of calculation (—◆— first iteration - y calculated without crack; —■— second iteration - y calculated with the crack depth obtained in first iteration; —▲— third iteration - y calculated with the crack depth obtained in second iteration)

The analysis concerning the influence of the number of followed iterations on the accuracy of calculations in general is made. The calculations are done with various tensile strength characteristics ($f_{ct,2}$) of the material (concrete) of the internal layer. By the results obtained (Figs 2, 3) it is determined that there is no sense of following more than two iterations because both $d_{cr,2}$ and y values become stable after the second iteration.

When the correct $d_{cr,2}$ and y values are obtained, the final model for further calculating the cracking moment of the flexural composite structure could be compiled.

4. Determination of the cracking moment of the tensile external layer and its dependence upon the internal layer properties

The cracking moment $M_{cr,1}$ of the flexural three-layer structures made of concrete-type material consists of bending moments carried by the tensile zone layers, including the reinforcing steel:

$$M_{cr,1} \leq \sum M_{ct,i} + M_s .\tag{14}$$

These moments may be calculated according to the resultant axial forces, acting in concrete layers and reinforcing steel, and their positions (Fig 4):

$$\left.\begin{array}{l} M_{ct,i} = N_{ct,i}(z + z_{t,i}) \\ M_s = N_s(z + y - a) \end{array}\right\}\tag{15}$$

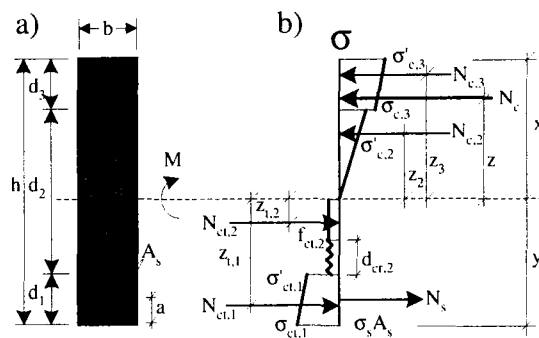Resultant axial forces and their positions may be calculated using the calculation model shown in Fig 4, b:

Fig 4. A model for calculation of the cracking moment: a) basic geometry of the cross-section; b) resultant axial forces and their positions

$$N_{ct,2} = f_{ct,2}\left.\begin{array}{l} N_{c,3} = \dfrac{\sigma_{c,3} + \sigma'_{c,3}}{2} b_3 d_3 \\[2mm] N_{c,2} = \dfrac{\sigma'_{c,2}}{2}(x - d_3)b_2 \\[2mm] (h - x - d_1 - d_{cr,2})b_2 \\[2mm] N_{ct,1} = \dfrac{\sigma_{ct,1} + \sigma'_{ct,1}}{2} b_1 d_1 \\[2mm] N_s = c\dfrac{E_s}{E_{c,1}} f_{ct,1} A_s \end{array}\right\},\tag{16}$$

$$\left.\begin{array}{l} z_3 = x - \dfrac{d_3(2\sigma_{c,3} + \sigma'_{c,3})}{3(\sigma_{c,3} + \sigma'_{c,3})} \\[3mm] z_2 = \dfrac{2(x - d_3)}{3} \\[3mm] z_{t,2} = \dfrac{h - x - d_1 - d_{cr,2}}{2} \\[3mm] z_{t,1} = h - x - \dfrac{d_1(2\sigma'_{ct,1} + \sigma_{ct,1})}{3(\sigma'_{ct,1} + \sigma_{ct,1})} \\[3mm] z = \dfrac{\sum(N_{c,i} z_i)}{\sum N_{c,i}} \end{array}\right\}.\tag{17}$$

Here factor c depends upon the structure and deformational properties of concrete [10]. In this article factor $c = 1/v_{ct,1} = 1/0,5 = 2$.

When flexural layered members are calculated, the relationship between deformational properties of concrete is very similar compared with the calculations of solid (one-layer) structures [1, 3, 11, 13]. The main difference is in the possibility for different layers to reach their tensile strength at a different time [2]. Therefore it is necessary to determine the stresses acting in different layers as accurately as possible. Then the precise Poisson's ratio for each layer must be calculated. Deformational properties of concrete are widely researched by many authors [2, 3, 8, 11, 12]. Simple semi-empirical formulae for calculating the actual Poisson's ratio $v_{ct,i}$ have been suggested in [2]:

$$\upsilon_{ct,i} = 1 - k\,\frac{\sigma_{ct,i}}{f_{ct,i}}, \qquad (18)$$

$$k = \left(1 - \frac{E_{c,i}}{A + E_{c,i}}\right), \qquad (19)$$

where A is the factor dependent upon the type and structure of the material (concrete): $A = 48 \times 10^3$ for the heavy concrete, $A = 18 \times 10^3$ for the light concrete and $A = 16 \times 10^3$ for the hollow concrete.
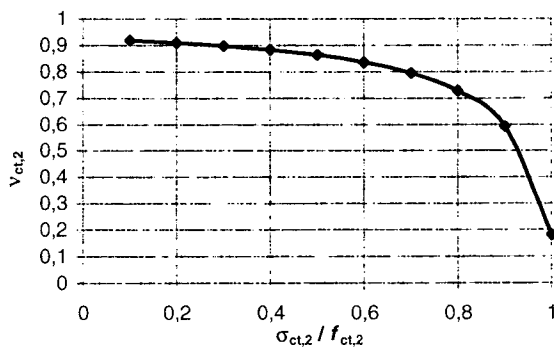

Fig 5. The influence of the intensity of tensile stress (ratio $\sigma_{ct,2} / f_{ct,2}$) on the Poisson's ratio of the internal layer $\upsilon_{ct,2}$

The analysis of the relationship between Poisson's ratio $\upsilon_{ct,i}$ and the intensity of tensile stress $\sigma_{ct,i} / f_{ct,i}$ is made (Fig 5). It is determined that the Poisson's ratio in the internal tensile layer could be $\upsilon_{ct,2} = 0,2$, because according to the model (Fig 4) the ratio $\sigma_{ct,i} / f_{ct,i} = 1$ in the tensile zone of the internal layer. The external layer is non-cracked, thus its Poisson's ratio may be taken according to the recommendations of design codes, ie $\upsilon_{ct,1} = 0,5$.
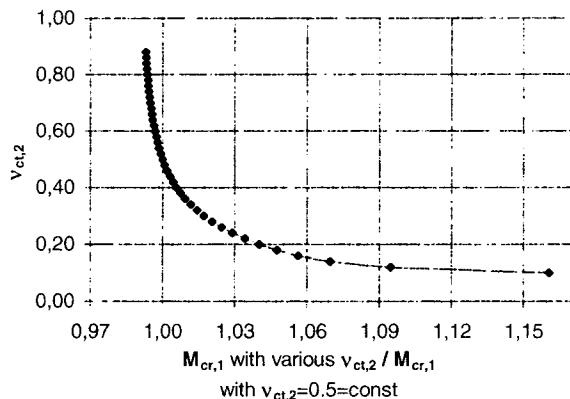

Fig 6. The influence of Poisson's ratio of the internal layer $\upsilon_{ct,2}$ on the ratio between the cracking moment of the external tensile layer $M_{cr,1}$ calculated with constant $\upsilon_{ct,2}$ and the same cracking moment $M_{cr,1}$ calculated with different $\upsilon_{ct,2}$

When the cracking moment of flexural layered structures is calculated, the Poisson's ratio of the internal layer $\upsilon_{ct,2}$ may have a significant influence on the results obtained. It is determined that when the Poisson's ratio $\upsilon_{ct,2}$ vary from 0,1 to 0,9, the cracking moment $M_{cr,1}$ may vary up to 20% compared to the same cracking moment which is calculated with constant value of the Poisson's ratio $\upsilon_{ct,2} = 0,5$ (Fig 6). Accordingly before calculating the cracking moment it is suggested determining the correct value of the Poisson's ratio for each layer according to the $\sigma_{ct,i} / f_{ct,i}$ ratio and the constitutive deformational characteristics of the material by using Eqs 18 and 19. The Poisson's ratio should be calculated by a corresponding calculation model.

5. Concluding remarks

1. While producing and using the layered structures with the internal layer of less strength, eg thermal insulation, cracks in the internal layer may appear earlier than in the external tensile layers.

2. The suggested method evaluates the stress-strain state and determines the acting stresses in each layer of such structures.

3. It is proved that when the influence of the tensile part of the internal layer is evaluated, the Poisson's ratio $\upsilon_{ct,2}$ of its material (concrete) may be taken equal to 0,2.

4. The method for calculating the crack resistance of three-layer flexural structures, determining the tensile zone depth and the crack resistance of the internal layer is suggested.

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