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PRODUCTIVITY PREDICTION OF FRACTURED HORIZONTAL WELLS WITH LOW PERMEABILITY FLOW CHARACTERISTICS

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Abstract. Horizontal well and large-scale fracturing are revolutionary technologies in petroleum industry. The technologies bring obvious economic benefits to exploiting unconventional oil and gas reservoirs with low permeability, ultra-low permeability and shale gas. With the increasingly extensive application of these technologies, other correlated technologies have also gained great development. However, low-permeability reservoirs exhibit complicated features and horizontal well fractures have complex shape. The existing methods for the productivity prediction of fractured horizontal well in low-permeability reservoirs rarely consider the influencing factors in a comprehensive manner. In this paper, a horizontal well seepage model of casing fracturing completion was established according to the superposition principle of low-permeability reservoir and the relationship between potential and pressure, by which model the seepage characteristics of low-permeability reservoirs could be fully described. Based on the established new seepage model, a new targeted model with coupling seepage and wellbore flow was established for the productivity prediction of low-permeability fractured horizontal well. Finally, the new targeted model was verified through field experiment. The experimental results confirmed the reliability of productivity prediction by the proposed model. Sensitivity analysis was then performed on the parameters in the proposed model.

Keywords: horizontal well, seepage model, casing fracturing completion, low-permeability reservoir, productivity prediction, coupling seepage and wellbore flow.

Introduction

With continuous oil demand in domestic and foreign markets, fewer and fewer easy-to-develop blocks or oilfields, development of horizontal wells, large-scale fracturing and availability of other development technologies, people are gradually turning their attention to the development of unconventional reservoirs such as low permeability, ultra-low permeability as well as shale gas. Productivity prediction of horizontal well, which is an important activity during development of an oilfield, provides significant evidence for optimization design of oil well, drawing up reasonable development blueprint and dynamic analysis as well as adjustment of development. Nevertheless, there are still few productivity prediction studies that fully consider the mechanism of low permeability seepage (there are many studies on the gas productivity prediction of fractured horizontal wells, and there are few studies on the oil productivity prediction of fractured horizontal wells). For example, some studies

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(Belyadi, Aminian, Ameri, & Boston, 2010; Shen, 2011; Wang et al., 2014; Yang, Zhang, Liu, Tian, & Xu, 2014) were based on the production of dynamic numerical model research without considering starting pressure gradient, multiphase flow, and other factors in low-permeability reservoirs. Although considerable studies (Zhang, Su, Wang, & Sheng, 2015; Jiang, Xu, Sun, Guo, & Zhao, 2014; Zhao, L. H. Zhang, Luo, & B. N. Zhang, 2014) have established fracturing productivity models for certain categories of unconventional oil and gas, such as shale oil and gas, shale oil and gas is an area with ultra-low permeability and non-flowing in experiments, and their formation production mechanisms which are mainly adsorption and exchange transport are not entirely the same as that in this paper, so they can't be applied to low permeability oil and gas reservoirs which their flow mechanisms are seepage flow.

There are few models which consider the characteristic features, such as starting pressure gradient and multiphase seepage, of low permeability reservoirs. Based on the generalized Darcy's law, by considering the effects of the starting pressure gradient and the pressure-sensitive, with the elliptic seepage theory and the law of the conservation of the average mass, the calculation formula of the production of the fractured horizontal well was deduced through the equivalent well diameter principle by Hao et al. (Hao, Wang, & Hu, 2011; Hao, Hu, Liu, Wei, & Zhuang, 2013) but without accounting for the wellbore pressure drop. Li, Lan, Ying, and Dong (2006) applied the reset potential theory and the potential superposition principle, and deduced the seepage flow model in the reservoir with both fractures and horizontal sections. Based on the hydrodynamics theory, the mass conservation and momentum theorem, the calculation model of the pressure drop in the wellbore considering the fluid inflow and fracture fluid inflow along the horizontal wellbore is derived. On this basis, the productivity model of coupling of seepage flow in reservoir and tube flow in wellbore is established, and its solution method is given. However, the seepage characteristics of low permeability reservoirs are not considered. Z. Wang, Zhu, Gao, Zhang, and C. Wang (2012), Wang et al. (2012) established a non-Darcy production capacity prediction model for hydraulic fracturing open hole horizontal wells with multiple horizontal cracks interfering with each other, but ignored the consideration of wellbore pressure drop. Zhu, Yang, Wang, and Liao (2013) established a model which considered the nonlinear seepage in the low permeability and tight reservoir by coupled numerical solution of the four divided flow fields, namely, nonlinear elliptic flow in the medium, linear flow and radial flow in the fracture, hydraulic pressure drop flow in the horizontal wellbore based on the theory of elliptic flow, superposition and equivalent radius principle. But they only considered the low seepage reservoir nonlinear seepage characteristics, without calculating the pressure sensitivity and other factors. At present, there is no research on the productivity prediction considering the starting pressure gradient, pressure-sensitive effect and multiphase seepage in the low permeability formation. However, there are still considerable studies (Luo, Long, Xie, & Liu, 2007; Dong, Feng, & Zhao, 2007; Zhang, Peng, & Gu, 2012; Qiu et al., 2016) on the size of the starting pressure gradient of low permeability reservoir and its sensitivity influencing factors. These studies have laid a good foundation for the productivity prediction of low permeability fracturing horizontal well.

Accurate productivity prediction of horizontal well involves with two aspects: reservoir multiphase seepage mechanism and horizontal wellbore variable mass flow law. According to the research of variable mass flow of horizontal wellbore in recent 30 years, the horizontal wellbore pressure drop cannot be neglected, especially for high formation permeability, high production and long horizontal well length. Therefore, the horizontal well cannot be regarded as an integral equipotential body in the formation, which makes the oil layer seepage law of horizontal wells extremely complex, especially for the low permeability fracturing horizontal wells. In particular, the formation seepage law of low permeability fractured horizontal wells not only needs to consider the influence of fracture parameters, but also needs to consider the effects of low permeability characteristics such as start pressure gradient, pressure-sensitive effect, multiphase seepage, and finally coupled with the flow in the horizontal wellbore, so as to determine the oil well production more accurately.

In view of the fact that the existing methods are more or less not considering one factor or some factors, this paper fully considers the various seepage characteristics of low permeability reservoirs. To begin with, models to calculate the potential of the wellbore micro-element and the potential of the fracture were developed. Based on the superposition principle of potential and the corresponding relationship between potential and pressure, the seepage model of casing fractured completion horizontal well is developed.

Using the seepage model, a new productivity prediction model of coupling formation seepage and wellbore flow in low permeability fractured horizontal well is also developed. Finally, the new model is utilised to investigate productivity prediction of fractured horizontal well considering low permeability start pressure gradient, pressure-sensitive effect, multiphase seepage characteristics.

1. Analysis of seepage characteristics of low permeability reservoirs

There are numerous differences between low permeability reservoirs and conventional oil reservoirs in their seepage characteristics. This paper takes the Luo 1 well area of Changqing Oilfield as an example to illustrate some seepage flow characteristics usually found in low permeability reservoirs. The Chang 8 oil reservoir in the Luo 1 well area is an ultra-low permeability reservoir, which does not only have the characteristics of low permeability, but also possesses the following complex properties, measured by

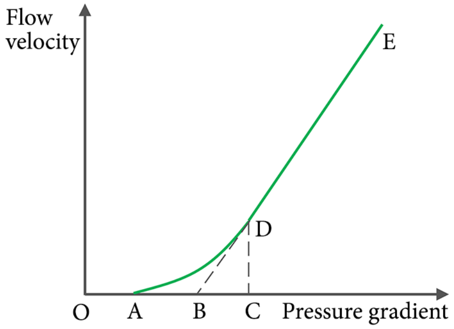


Figure 1. Schematic diagram of nonlinear seepage characteristics of low permeability reservoir

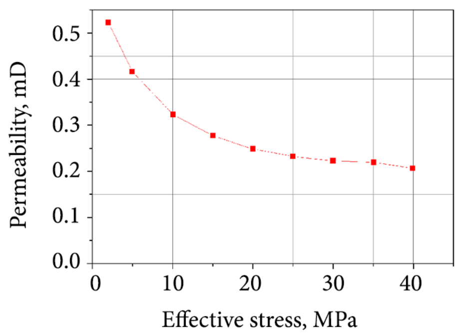


Figure 2. Reservoir rock stress sensitivity

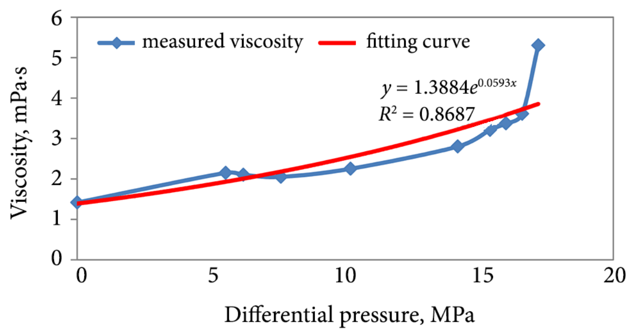


Figure 3. The relationship and the fitting equation between the fluid viscosity of the reservoir and the pressure difference

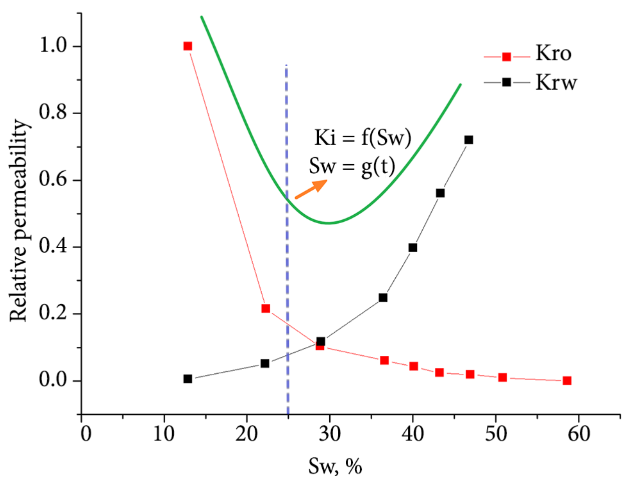


Figure 4. Oil-water two-phase seepage curve in oil layer

laboratory experiments: a. Non-linear seepage, where the fluid seepage in the core does not conform to the Darcy flow law. The seepage curve is not a straight line passing through the origin. The fluid flow needs to overcome the starting pressure gradient, as shown in Figure 1. Reservoir rock stress sensitivity, during the development process, the oil layer permeability is not a constant, but decreases as the reservoir pressure drops, as shown in Figure 2. Formation degassing, the crude oil viscosity will rise with the gas out, as shown in Figure 3, and gas-liquid two-phase seepage affects the total relative permeability (integrated relative permeability), as shown in Figure 4.

Consider the relationship between the affected and original properties as follows.

(1) Reservoir nonlinear seepage follows the below formula (Figure 1).

$$v=\begin{cases} 0 & \dfrac{dp}{dr}<\lambda \\ -\dfrac{K}{\mu}\left(\dfrac{dp}{dr}-\lambda\right) & \dfrac{dp}{dr}\geq\lambda \end{cases}, \tag{1}$$

where v is seepage velocity, m/s; $\dfrac{dp}{dr}$ is pressure gradient, MPa/m; λ is start pressure gradient, MPa/m; K is formation permeability, $K_{surface}$; μ is fluid viscosity, mPa·s.

(2) Stress sensitivity of the reservoir rock, the permeability obeys the following function (Figure 2).

$$K=K_{surface}e^{-\alpha_k p_{effect}}; \tag{2}$$

$$K=K_i e^{\alpha_k \Delta p}=0.5204e^{0.0286\Delta p}, \tag{3}$$

where $K_{surface}$ is the permeability under the ground standard condition, $K_{surface}$; α_k is variation coefficient of permeability, 1/MPa; p_{effect} is effective pressure $p_{effect}=p_i-\Delta p$, MPa; p_i is the reservoir initial pressure, MPa; Δp is pressure difference, MPa; K_i is initial permeability at the initial formation pressure, 10^{-3} μm^2.

(3) The relationship between fluid viscosity and pore pressure (Figure 3). In the low-speed seepage conditions, the fluid viscosity also alters with the pore pressure changes. Especially, when the pressure is below the saturation pressure, the gas is precipitated out of the oil and the crude oil viscosity will become larger. The fitting analysis of the experimental data shows that the two have an exponential relationship with the pore pressure. That is, the fluid viscosity is following the function.

$$\mu=\mu_i e^{\alpha_\mu(p_i-p_{effect})}=\mu_i e^{\alpha_u \Delta p}, \tag{4}$$

where α_μ is the deformation coefficient of the viscosity, 1/MPa; μ_i is the initial fluid viscosity under the reservoir initial formation pressure, mPa·s.

The relationship between viscosity and pressure is obtained from the data in Figure 3:

$$\mu=1.3884e^{0.0593\Delta p}. \tag{5}$$

Considering the permeability and fluid viscosity at the same time, that is, the flow changes with the pressure, the expression of flow degree can be obtained as presented in Equation (6) by combining Equations (4) and (5):

$$\frac{K}{\mu} = \frac{K_i}{\mu_i} e^{a_c \Delta p}, \tag{6}$$

where $\alpha_c = \alpha_k - \alpha_\mu$, K is the permeability under actual pressure condition, $K_{surface}$.

(4) Multi-phase seepages such as oil-water, oil-gas, oil-gas-water. For example, oil-water two-phase seepage is shown in Figure 4. It can be seen from the figure that as the water saturation in the formation increases, the relative permeability of the crude oil gradually decreases, and the relative permeability of water increases gradually. But as the water saturation increases, the total comprehensive relative permeability (the sum of the relative permeability of water and oil) first decreases and then increases, and the increased comprehensive relative permeability does not reach the initial comprehensive relative permeability. The reservoir comprehensive relative permeability is consistent with the below functions.

$$K_i = f(S_w); \tag{7}$$

$$S_w = g(t), \tag{8}$$

where $f(S_w)$ is the function of water saturation; S_w is water saturation, %; $g(t)$ is the function of time.

From the relationship among the four characteristics, at a certain stage in the production process, the water saturation can be taken as a fixed value. Then, the integrated reservoir permeability can be obtained from the relative permeability curve. Next, we can calculate the fluidity under current conditions based on the pressure-sensitive effect of permeability and fluid viscosity. Finally, with the starting pressure gradient of the nonlinear relationship, we can obtain the comprehensive relationship considering the low permeability of the four seepage characteristics.

2. Productivity prediction model development for low permeability casing completion fracturing horizontal well

For low permeability seepage reservoirs with the low permeability characteristics, the productivity prediction needs to consider these factors. Based on the basic principle of reservoir seepage and the hydropower similarity, the calculation model of the potential of fractures was first developed without considering the low permeability characteristics (Ning et al., 2002). Then, the horizontal well seepage model of casing fracturing completion was developed according to the superposition principle of low permeability reservoir and the relationship between the potential and the pressure which fully accounting accounts for the various seepage characteristics of low-permeability reservoirs in this paper. Finally, the coupling productivity prediction model of seepage and wellbore flow in low permeability fractured horizontal well is established. Model assumptions are: (1) The reservoir is an upper and lower enclosed formation. A horizontal well is in the center of the formation with the length of the wellbore L_h, adopting casing completion without shooting holes in horizontal sections; (2) The fluid in the reservoir is a single-phase incompressible fluid with stable seepage flow, while the reservoir temperature is constant, regardless of the effect of gravity; (3) In horizontal wellbore fracturing operation, "n" transverse fractures are fractured along the horizontal wellbore, and the fractures pass through the entire reservoir thickness, and are symmetrical with respect to the horizontal wellbore. Fractures are distributed unequally along the horizontal wellbore, and the length, width, permeability and production of each fracture are different; (4) Part of the fluid first flows from the formation into the fractures, and then flows along the fractures into the wellbore; (5) When casing completion is considered, it can be assumed that both friction pressure drop and acceleration pressure drop exist during the flow of fluid in the horizontal wellbore, therefore the conventional pipe flow model can be employed.

2.1. Fracture formation seepage flow model development

(1) A single fracture formation permeability model

Assuming that there is a 2L-long fracture in the formation, which breaks the thickness of the whole formation, and the fracture production is Q. The process of crude oil flowing to the fracture in the Z-plane coordinate system is shown in Figure 5.

Take the transform function:

$$z = Lchw. \tag{9}$$

Substitute $z = x + iy$, $w = u + iv$ into Equation (9)

$$x + iy = Lch(u + iv) = L(chu\cos v + ishu\sin v), \tag{10}$$

where z is the Z-plane coordinate; L is half length of fracture, m; w is the W-plane coordinate; x is the Z-plane abscissa; y is the Z-plane ordinate; u is the W-plane abscissa; v is the W-plane ordinate;

According to the corresponding relationship:

$$\begin{cases} x = Lchu\cos v \\ y = Lshu\sin v \end{cases}. \tag{11}$$

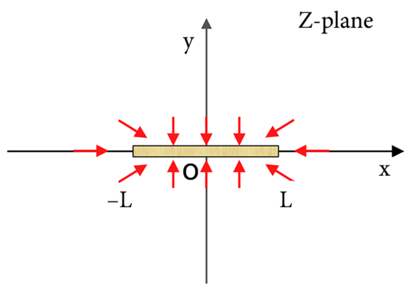


Figure 5. A single fracture flow diagram in the Z-plane coordinate system

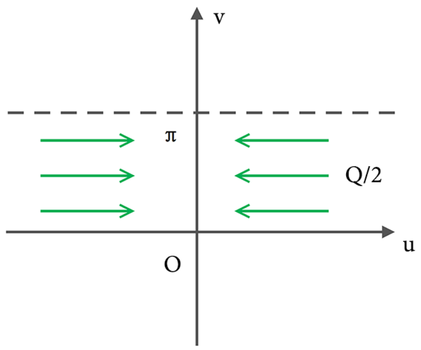


Figure 6. Parallel flow diagram of W-plane

Through the transformation function $z = Lchw$, the Z-plane upper half-plane formation is converted into a semi-infinite formation with W-plane whose bandwidth is π, and the $2L$ length of the fracture becomes the drainage channel with the width π. Similarly, the Z-plane lower half-plane formation can be altered into a semi-infinite formation with W-plane whose bandwidth is π, as shown in Figure 6. Through the conformal transformation, the Z-plane fracture flow (ignore the width, fractures as a straight line) converts into a one-way parallel flow of the W-plane.

From the Equation (11), we can obtain:

$$\frac{x^2}{L^2 ch^2 u} + \frac{y^2}{L^2 sh^2 u} = \cos^2 v + \sin^2 v = 1\,. \tag{12}$$

Since $ch^2 u - sh^2 u = 1$, so:

$$u = arcch\frac{1}{\sqrt{2}}[1 + \frac{x^2+y^2}{L^2} + \sqrt{(1+\frac{x^2+y^2}{L^2})^2 - 4\frac{x^2}{L^2}}]^{\frac{1}{2}}\,. \tag{13}$$

Since the conformal transformation does not change the formation properties, the low permeability formation still should have low permeability after conformal transformation. The existence of starting pressure gradient and stress sensitivity in low permeability reservoirs has a great influence on the production capacity, and it is necessary to consider the factors such as starting pressure gradient in the study of fluid flow in low permeability reservoirs. The flow of single phase flow of W-plane is:

$$\frac{Q}{2} = \pi h \frac{K}{\mu}(\frac{dp}{du} - G)\,, \tag{14}$$

where Q is the production rate of parallel flow of drainage channel, m^3/s; h is formation thickness, m; K is the average formation permeability, Because the formation may be heterogeneous, some authors have explored and summarized a number of treatment methods to calculate the average permeability (Guo & Du, 2004; Wang, Xue, Gao, & Tong, 2012), there are geometric average, arithmetic average, and harmonic average. In consideration of anisotropy, this study selected the permeability of geometric average as $K = \sqrt{K_h K_v}$, m^2; μ is formation crude oil viscosity, Pa·s; $\frac{dp}{du}$ is pressure drop per unit length, Pa/m; G is start pressure gradient, Pa/m.

As $\frac{K}{\mu}\left(\frac{dp}{du} - G\right) = \frac{d\varphi}{du}$, the Equation (14) can be changed into:

$$\frac{Q}{2\pi h} = \frac{d\varphi}{du}\,. \tag{15}$$

Separate variables and integrate:

$$\varphi = \frac{Q}{2\pi h}u + C\,, \tag{16}$$

where φ is the potential of parallel flow of drainage channel; C is integral constant.

Substitute Equation (13) into Equation (16):

$$\varphi = \frac{Q}{2\pi h} arcch\frac{1}{\sqrt{2}}[1 + \frac{x^2+y^2}{L^2} + \sqrt{(1+\frac{x^2+y^2}{L^2})^2 - 4\frac{x^2}{L^2}}]^{\frac{1}{2}} + C. \tag{17}$$

Equation (17) is the potential distribution function of one vertical fracture at any point in Z plane in a low permeability reservoir. The productivity prediction formula of horizontal wells with multiple fractures can be derived by the principle of potential superposition.

(2) Potential distribution function of multiple fractures in casing completion well

The heel is considered the origin of horizontal well, and the parallel direction of the fracture is the X-axis. While the horizontal wellbore is the Y-axis, and the rectangular coordinate system is established, as shown in Figure 7. From the origin, the fractures are denoted as F_1, F_2, ..., F_n, respectively. The lengths of the half fractures are L_1, L_2, ..., L_n respectively. The widths of the fractures are similarly defined as w_1, w_2, ..., w_n, respectively; the values of permeability are respectively denoted as K_1, K_2, ..., K_n, respectively; the productivities are Q_1, Q_2, ..., Q_n, respectively; the middle points of the fractures (ie, the junction of the fracture and the wellbore) are denoted as y_1, y_2, ..., y_n, respectively.

Since the fracture is small relative to the formation, the fracture can be considered to be equipotential. For the sake of calculation, the potential at the midpoint of the fracture is taken as the potential at the end of the fracture. According to the potential distribution function of one single fracture, the potential distribution function of the j-th fracture ($j = 1, 2, \ldots, n$) at any point (x, y) on the Z plane can be obtained by:

$$\varphi = \frac{Q_j}{2\pi h} arcch\frac{1}{\sqrt{2}}[1 + \frac{x^2+(y_j-y)^2}{L_j^2} + \sqrt{(1+\frac{x^2+(y_j-y)^2}{L_j^2})^2 - 4\frac{x^2}{L_j^2}}]^{\frac{1}{2}} + C. \tag{18}$$

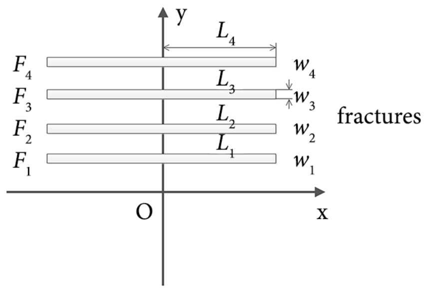


Figure 7. Schematic diagram of fracture distribution

For a casing well, the potential of all fractures at the midpoint of the j-th fracture is obtained by:

$$\varphi_j(0,y_j)=\sum_{k=1}^{n}\frac{Q_k}{2\pi h}arcch(1+\frac{(y_j-y_k)^2}{L_k^2})^{\frac{1}{2}}+C, \quad (19)$$

where Q_j is the production of the *j* fracture, m^3/s; Q_k is the production of the k fracture, m^3/s; y_j is the vertical ordinate of the midpoint of the j fracture, m; L_j is the half length of the j fracture, m; y_k is the vertical ordinate of the midpoint of the k fracture, m; L_k is the half length of the k fracture, m.

Set the point (0, r_e), which is farther away from the origin in the Y-axis, so the potential of the supply boundary is:

$$\varphi_e(0,r_e)=\sum_{k=1}^{n}\frac{Q_k}{2\pi h}arcch(1+\frac{(r_e-y_k)^2}{L_k^2})^{\frac{1}{2}}+C, \quad (20)$$

where r_e is supply radius, m.

According to Equation (19) and Equation (20):

$$\varphi_e(0,r_e)-\varphi_j(0,y_j)=\sum_{k=1}^{n}\frac{Q_k}{2\pi h}[arcch(1+\frac{(r_e-y_k)^2}{L_k^2})^{\frac{1}{2}}-arcch(1+\frac{(y_j-y_k)^2}{L_k^2})^{\frac{1}{2}}]. \quad (21)$$

Based on $\frac{K}{\mu}\left(\frac{dp}{du}-G\right)=\frac{d\varphi}{du}$, Equation (21) can change into:

$$p_e-p_j-G(r_e-\sqrt{y_j^2+L_j^2})=\sum_{k=1}^{n}\frac{\mu Q_k}{2\pi hK}[arcch(1+\frac{(r_e-y_k)^2}{L_k^2})^{\frac{1}{2}}-arcch(1+\frac{(y_j-y_k)^2}{L_k^2})^{\frac{1}{2}}], \quad (22)$$

where p_e is the pressure at the supply boundary, Pa; p_j is the pressure at the end of the j fracture, Pa.

The fractures usually have a high flow conductivity compared with the low permeability stratigraphy, so it is not necessary to consider the starting pressure gradient, stress sensitivity for the penetration of the fluid in the fractures. Since the half-length of the fracture is usually larger than the thickness of the formation, it is also much larger than the horizontal wellbore radius. When ignoring the influence of gravity, the process of crude oil flowing from the edge of the fracture into the horizontal wellbore can be regarded as the point sink of the upper and lower closed boundary with the flow radius L_j, the formation thickness w_j and the bottom flow pressure is pwf_j. The flow from the fracture to the wellbore can be shown in Figure 8.

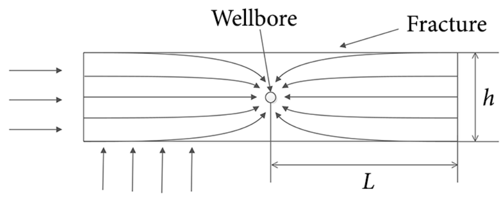


Figure 8. Schematic diagram of fluid flow from formation to fractures and from fractures to horizontal wells

Without considering the pressure drop caused by the skin factor of the fracture, the pressure drop of the fracture end to the wellbore, from the production formula of a well in a straight-line infinite well array, can be expressed as follows:

$$p_j-p_{wfj}=\frac{\mu Q_j}{2\pi K_j w_j}\left(\frac{\pi L_j}{h}+\ln\frac{h}{\pi r_w}\right),$$

$$(j=1, 2, \ldots, n), \quad (23)$$

where p_{wfj} is the center pressure of the j fracture, Pa; Q_j is the production of the j fracture, m^3/s; K_j is the permeability of the j fracture, m^2; w_j is the width of the j fracture, m; r_w is wellbore radius, m.

Based on the actual situation in which the fluid flows to the fractures in the formation, the fluid cannot flow completely from the end into the midpoint of the fracture, and the above calculation model is derived from the hypothetical flow not consistent with the fact. So the following calculation method is proposed:

$$p_j-p_{wfj}=\frac{\mu Q_j}{2\pi K_j w_j}\left(\frac{\pi L_{const}}{h}+\ln\frac{h}{\pi r_w}\right);$$

$$\begin{cases} L_{const}=3.5h & L_{const}>3.5h \\ L_{const}=L_j & L_{const}\le 3.5h, \end{cases} \quad (24)$$

where L_{const} is the effective length of the flow resistance calculation along the fracture, m.

Additional pressure drop caused by fractured skin factors can be calculated by the formula proposed by Hemanta Mukherjee and Michael J. Economides (Mukherjee & Economides, 1991):

$$\Delta p_s=\frac{\mu Q_j}{2\pi K_j w_j}\left(\ln\frac{h}{2r_w}-\frac{\pi}{2}\right). \quad (25)$$

Thus, the pressure drop from the supply boundary to the midpoint of the *j* fracture is:

$$p_e-p_{wfj}-G(r_e-\sqrt{y_j^2+L_j^2})=\sum_{k=1}^{n}\frac{\mu Q_k}{2\pi hK}[arcch(1+\frac{(r_e-y_k)^2}{L_k^2})^{\frac{1}{2}}-arcch(1+\frac{(y_j-y_k)^2}{L_k^2})^{\frac{1}{2}}]+\frac{\mu Q_j}{2\pi K_j w_j}(\frac{\pi L_{const}}{h}+\ln\frac{h}{\pi r_w}+\ln\frac{h}{2r_w}-\frac{\pi}{2}). \quad (26)$$

That is:

$$p_e - p_{wfj} - G(r_e - \sqrt{y_j^2 + L_j^2}) - \frac{\mu Q_j}{2\pi K_j w_j}(\frac{\pi L_{const}}{h} + \ln\frac{h}{\pi r_w} + \ln\frac{h}{2r_w} - \frac{\pi}{2}) = \sum_{k=1}^{n}\frac{\mu Q_k}{2\pi h K}[arcch(1+\frac{(r_e - y_k)^2}{L_k^2})^{\frac{1}{2}} - arcch(1+\frac{(y_j - y_k)^2}{L_k^2})^{\frac{1}{2}}],$$

$$(j=1,\ 2,\ \dots,\ n). \tag{27}$$

2.2. Coupling model development and solution

For the fracturing productivity prediction of the casing completion horizontal well, the flow in the wellbore is the conventional pipe flow, whose model can be used to calculate the pressure drop. From the flow condition in the wellbore and the flow in the formation, the coupling equation is established. Next, the coordinated production is obtained, which follows two flow rules: (a) three-dimensional steady-state seepages flow of the fluid must exist in the reservoir, (b) there must exist fluid flow in the wellbore. The two flows in the respective conduits must interact with each other.

The pressure is p_{wfj} of the j fracture in the horizontal well; the pressure at the center in the horizontal well can be calculated according to the calculation method of section 2.1:

$$p_e - p_{wfj} - G(r_e - \sqrt{y_j^2 + L_j^2}) - \frac{\mu Q_j}{2\pi K_j w_j}(\frac{\pi L_{const}}{h} + \ln\frac{h}{\pi r_w} + \ln\frac{h}{2r_w} - \frac{\pi}{2}) = \frac{\mu}{K}A,$$

$$(j=1,\ 2,\ \dots,\ n), \tag{28}$$

where

$$A = \sum_{k=1}^{n}\frac{Q_k}{2\pi h}[arcch(1+\frac{(r_e - y_k)^2}{L_k^2})^{\frac{1}{2}} - arcch(1+\frac{(y_j - y_k)^2}{L_k^2})^{\frac{1}{2}}], \tag{29}$$

The fractures divide the casing into n+1 sections, each section is divided into m sections, and the casing completion wellbore section is not inflow. With the conventional pipe flow model, the pressure drop can be calculated in the wellbore and the pressure at the midpoint of the j fracture is:

$$p_{wfj} = p_{2,k}\ (k = m\cdot j,\ j = 1,2,\cdots,n), \tag{30}$$

where $p_{2,m\cdot(n+1)} = p_{wf}$, p_{wf} is the flow pressure on the heel of wellbore.

$$p_{1,j+1} = p_{2,j} = p_{1,j} - dp_{w,j}\ (j = 1,2,\dots,m(n+1)). \tag{31}$$

Total well production

$$Q_o = \frac{(Q_1 + \cdots + Q_n)}{B_o}, \tag{32}$$

where B_o is the crude oil volume coefficient.

In the coupled model, unknown values of Q_j and p_{wfj} can be solved by iterative method. First we can assume a set of values for p_{wfj}, then figure out Q_j using Equation (28), which are taken into conventional pipe flow pressure drop model to update p_{wfj} from the heel to the toe with Equation (31) and Equation (30). Next, the new Q_j can be calculated from Equation (28), and this process is repeated until the calculated results of Q_j and p_{wfj} all reached relatively small changes. Finally, the total well production is obtained from Equation (32).

3. Comparison and analysis of examples

In this paper, the derived production equation is verified using four examples of wells in the literatures (Yuan, 2011; Liang, 2015).

Well Maoping 1 is fractured into four symmetrical distributed transverse fractures. The fractures have the same half-length and the same spacing distribution. The well parameters are shown in Tables 1–3.

Well Saiping 1 is fractured into four symmetrical distributed transverse fractures. The fractures have different half-length and different spacing distribution. The well parameters are shown in Tables 1–4.

Well Baibao 1 is fractured into four symmetrical distributed transverse fractures. The fractures have the same half-length and the same spacing distribution. The well parameters are shown in Tables 1–3.

Well Baibao 2 is fractured into four symmetrical distributed transverse fractures. The fractures have the same half-length and the same spacing distribution. The well parameters are shown in Tables 1–3.

Table 1. The first part of wells parameters

Well Name	Formation permeability (mD)	Formation thickness (m)	Original formation pressure (MPa)	Bottom flow pressure (MPa)	Supply radius (m)	Horizontal well length (m)	Fracture number	Fracture permeability (μm^2)
Maoping 1	7.5	12	11.83	3	350	556	4	30
Saiping 1	3.6	12	9.6	6.5	180	236.17	4	30
Baibao 1	1.3	24	17	13	250	400	4	30
Baibao 2	1.51	21	17	13	250	420	4	30

Table 2. Other wells parameters

Well Name	Starting pressure gradient (MPa/m)	Crude oil volume factor	Stratigraphic crude oil viscosity (mPa·s)	Crude oil density (g/cm^3)	Wellbore radius (m)	Pipe wall absolute roughness (m)	Fracture width (mm)	Fracture half-length (m)
Maoping 1	0.005	1.084	4.8	0.87	0.12	2×10^{-5}	5.84	75
Saiping 1	0.0078	1.1	2.3	0.85	0.062	2×10^{-5}	5	
Baibao 1	0.0097	1.2	1.1	0.822	0.1	2×10^{-5}	4.8	110
Baibao 2	0.0083	1.2	1.1	0.822	0.15	2×10^{-5}	4.5	90

3.1. Method verification

The basic parameters of the four oil wells are shown in Tables 1–4. The results of the four well production calculations are shown in Table 5 (regardless of the pressure-sensitive effect). It can be seen from Table 6 that compared with the previous calculation method, the error of the productivity prediction method in this paper is the smallest and the prediction accuracy is the highest, with an average of 18.7%.

3.2. Analysis of sensitive parameters of low permeability fracturing horizontal wells

Taking the Maoping 1 well as an example, and selecting a reasonable value range of different parameters (usually the value), analyses of the sensitivity of the factors affecting the production rate according to the established model (without considering the pressure-sensitive effect) are shown in Figure 9.

In Figure 9(a), when the other parameters remain unchanged, the starting pressure gradient is 0.0 MPa/m, 0.005 MPa/m, 0.01 MPa/m, 0.02 MPa/m, 0.03 MPa/m, 0.035 MPa/m, respectively, and the production of fractured horizontal wells is predicted. The results are plotted on the graph. As the starting pressure gradient increases, the production first drops significantly and then slowly decreases. The results indicate that the existence of fluid flowing in the formation is the key of the well having production rate. According to research on certain categories of unconventional oil and gas (e.g. shale gas), it is difficult for oil and gas to flow in unconventional oil and gas formations, thus oil well production rate should be zero. In fact, the production rate still exists in these categories of unconventional oil and gas well because the flow mechanisms of these categories of unconventional oil and gas in the formations which are mainly adsorption and exchange transport are different from those of low-permeability formations.

In Figure 9(b), other parameters remain unchanged, the position of the first fracture remains unchanged, other fracture positions are adjusted, and the production of fractured horizontal wells is predicted when the positions between fractures are 40 m, 60 m, 80 m, 100 m, 120 m and 140 m, respectively. The results are plotted in the graph, and the production increases as the fracture spacing increases. The findings indicate that the larger the

Table 3. Distribution of fractures

Wells	1 before wellbore	between 1 and 2	between 2 and 3	between 3 and 4
Maoping 1	100	100	100	100
Saiping 1	31	46.4	91.26	37
Baibao 1	80	80	80	80
Baibao 2	84	84	84	84

Table 4. Saiping 1 fracture half-lengths

Fracture half-lengths	Values (m)
1th	130
2th	109
3th	83
4th	109

Table 5. Statistics of productivity prediction

Well name	Ning's method (Ning et al., 2002) (m^3/d)	Liang's method (Liang, 2015) (m^3/d)	Our method (m^3/d)	Actual output (m^3/d)
Maoping 1	50.2	28.9	19.3	20.4
Saiping 1	36.7	16.2	10.5	21.8
Baibao 1	51.5	18.5	14.7	14.9
Baibao 2	51.4	21.9	14.6	17.5

Table 6. Statistics of calculation errors

Well name	Ning's method (Ning et al., 2002) (%)	Liang's method (Liang, 2015) (%)	Our method (%)
Maoping 1	145.9	41.6	5.4
Saiping 1	68.8	25.7	51.8
Baibao 1	246.7	24.2	1.3
Baibao 2	193.6	25.3	16.4
Average	163.7	29.2	18.7

control formation area of the oil well circulation channel, the higher the oil well production rate.

In Figure 9(c), the fracture conductivity is directly proportional to the fracture width and permeability. Under the unchanged conditions of other parameters, the fracture conductivity takes different values to predict the production of fractured horizontal wells. The results are plotted on the graph. As the fracture conductivity increases, the production first increases rapidly and then tends to be flat. The results demonstrate that the fluid flow is easier in the circulation channel and the oil well production rate is higher.

In Figure 9(d), under the unchanged conditions of other parameters, total well production is given by 50 m, 100 m, 150 m and 200 m half-length fractures, and the results are plotted. It is shown that the production of horizontal wells increases with the half-length increase of fractures. The results indicate that the larger the contact area between the oil well circulation channel and the formation, the higher the oil well production rate.

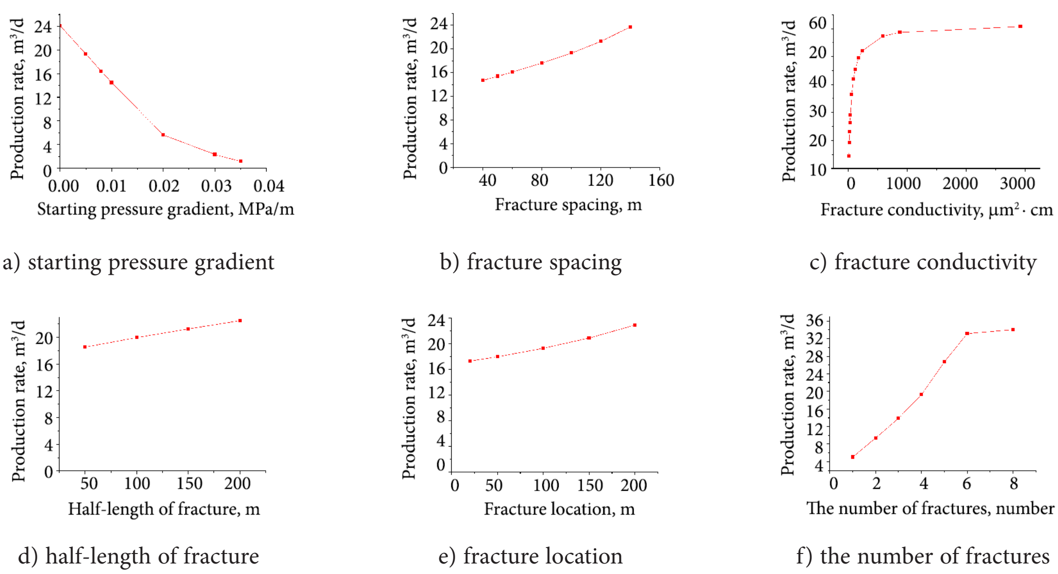
In Figure 9(e), under the unchanged conditions of other parameters, the spacing between fractures is constant, and all fractures move along the wellbore from heel to toe. When the distance between the first fracture and heel is 20 m, 50 m, 100 m, 150 m and 200 m respectively, the production of fractured horizontal wells is predicted and plotted in the graph. It can be seen that as the distance between the fracture and the heel is larger, the horizontal well production is higher (the moderate increase). This is the same as the fracture spacing, and it also can indicate that the larger the control formation area of the oil well circulation channel, the higher the oil well production rate.

In Figure 9(f), under the unchanged conditions of other parameters, the fractures are evenly distributed on the horizontal wells. When the number of fractures is 1, 2, 3, 4, 5, 6, and 8, respectively, the production of fractured horizontal wells is predicted. The results are plotted on the graph. It can be seen that as the number of fractures increases, the horizontal well production increases initially. When the fracture increases to a certain value, the horizontal well production has a limited increase with the number of fractures. This is the same as the half-length of fractures, which also can show that the larger the contact area between the oil well circulation channel and the formation, the higher the oil well production rate.

Taking the oil-water two-phase as an example to illustrate the effect of multi-phase seepage on the production of oil wells, as shown in Figure 10. Under the unchanged conditions of other parameters, the production of fractured horizontal wells under different water-saturated conditions in the formation is considered. The results are shown in the figure. With the increase of water saturation in the formation, the relative permeability of crude oil gradually decreases rapidly, the liquid production decreases first and then increases, and the oil production decreases rapidly, indicating that formation water saturation is also an important factor affecting production. This is the same as the starting pressure gradient, which also reflects the difficulty of crude oil flowing in the formation.

The permeability-sensitivity coefficient (0.0286) and the viscosity-sensitive effect coefficient (0.0593) described above are taken as examples. In Figure 11, the production of fractured horizontal wells is predicted when only permeability pressure sensitivity, only crude oil viscosity pressure sensitivity, both of them and none of them are taken into account respectively. The results show that the permeability pressure sensitivity of rock and the viscosity pressure sensitivity of crude oil have certain influence on the production, but the influence is not great. This is the same as the starting pressure gradient, which also reflects the difficulty of crude oil flowing in the formation.

From the trend (slope size) in the Figure 9 and the comparison in Figure 10 and Figure 11, we can see that the factors affecting the production rate of low permeability



a) starting pressure gradient

b) fracture spacing

c) fracture conductivity

d) half-length of fracture

e) fracture location

f) the number of fractures

Figure 9. Relationship between oil well production and sensitive parameters

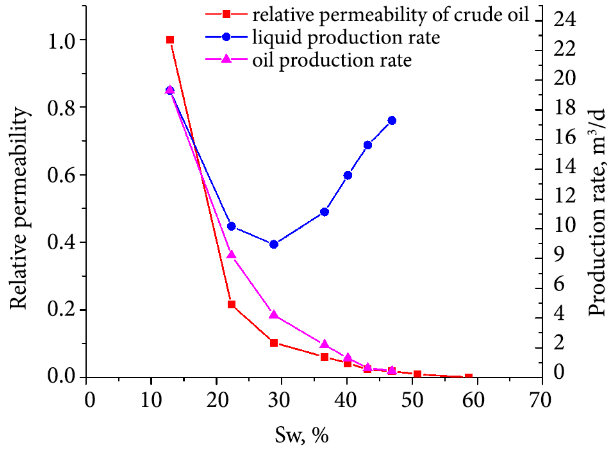


Figure 10. Comparison of the calculated production for different water saturation conditions

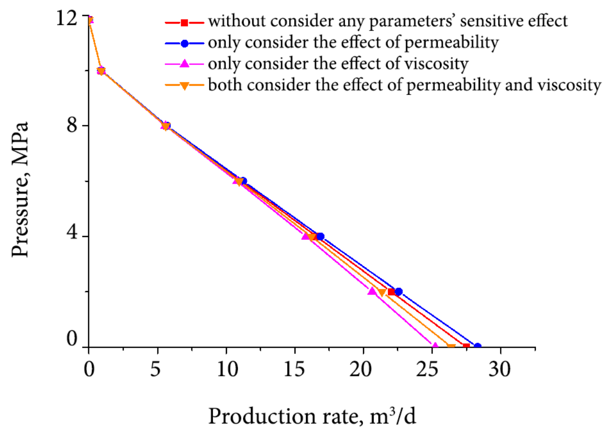


Figure 11. Comparison of the calculated production for different conditions of pressure-sensitive effect

fractured horizontal wells in reducing order of magnitude are: fracture conductivity, number of fractures, multiphase seepage, starting pressure gradient, fracture spacing, fracture location, fracture half-length, viscosity pressure sensitivity (without considering multiphase seepage) and permeability pressure sensitivity.

Conclusions

By fully considering characteristics of seepage flow, which include pressure gradient, pressure-sensitive effect, heterogeneous seepage, etc., of low permeability reservoirs, the seepage potential calculation model of horizontal well formation of multi-fracture casing is established from the basic principle of reservoir seepage and the similar principle of hydropower model. Next, the fractured horizontal well productivity prediction model is established by coupling with the wellbore flow. The model is validated and important sensitivity parameters are analyzed. Finally, the main conclusions are obtained in the following.

(1) A productivity prediction model for horizontal well with casing fracturing completion is derived, which considers seepage characteristics of low permeability reservoirs, such as multiphase seepage, starting pressure gradient, permeability pressure sensitivity, viscosity pressure sensitivity, etc.

(2) The model is validated using the measured data obtained from several wells. The model established in this paper is in good agreement with the measured data, and the calculation error is small. It is proved that the model is reliable.

(3) According to the sensitivity analysis, the factors significantly affecting the production of low permeability fractured horizontal wells in reducing order of magnitude are: fracture conductivity, number of fractures, multiphase seepage, starting pressure gradient, fracture spacing, fracture location, fracture half-length, viscosity pressure sensitivity (without considering multiphase seepage) and permeability pressure sensitivity.

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References

Belyadi, A. M., Aminian, K., Ameri, S., & Boston, A. L.-F. (2010). Performance of the hydraulically fractured horizontal wells in low permeability formation. In *SPE Eastern Regional Meeting*, 13–15 October, Morgantown, West Virginia, USA. Society of Petroleum Engineers. https://doi.org/10.2118/139082-MS

Dong, D. P., Feng, W. G., & Zhao, J. F. (2007). A method for relative permeability calculation considering start-up pressure gradient. *Natural Gas Industry*, *27*(10), 95-96.

Guo, X., & Du, Z. (2004). Impacts of permeability heterogeneity on horizontal well productivity. *Petroleum Exploration and Development*, *31*(1), 91-93.

Hao, M., Hu, Y., Liu, X., Wei, C., & Zhuang, Y. (2013). Predicting and optimising the productivity of multiple transverse fractured horizontal wells in ultra-low permeability reservoirs. In *International Petroleum Technology Conference*, 26–28 March (Vol. 124, pp. 11230–11234). Beijing, China. https://doi.org/10.2523/IPTC-16891-Abstract

Hao, M. Q., Wang, X. D., & Hu, Y. L. (2011). Productivity calculation of multi-fractured horizontal well in ultra-low permeability pressure-sensitive reservoirs. *Journal of China University of Petroleum*, *35*(6), 99-104.

Jiang, R. Z., Xu, J. C., Sun, Z. B., Guo, C. H., & Zhao, Y. L. (2014). Rate transient analysis for multistage fractured horizontal well in tight oil reservoirs considering stimulated reservoir volume. *Mathematical Problems in Engineering*, *2014*, Article ID 489015, 11. https://doi.org/10.1155/2014/489015

Liang, Q. (2015). *Study on productivity of fractured horizontal wells in low permeability oil reservoirs* (Master's dissertation). Yangtze University.

Li, L. T., Lan, L. C., Ying, W. U., & Dong, X. Y. (2006). A new way to calculate fractured horizontal well's productivity in low permeability oil reservoirs. *Journal of China University of Petroleum*, *30*(2), 48-52.

Luo, Y., Long, Z., Xie, J., & Liu, G. (2007). An improvement of darcy's law with threshold pressure gradient. *Well Testing*, *16*(5), 10-11.

Mukherjee, H., & Economides, M. J. (1991). A parametric comparison of horizontal and vertical well performance. *SPE Formation Evaluation*, *6*(2), 209-216. https://doi.org/10.2118/18303-PA

Ning, Z. F., et al. (2002). Productivity calculation method of fractured horizontal wells in low permeability oil or gas field. *Acta Petrolei Sinica*, *23*(2), 68-71.

Qiu, Z.-p., Yang, S.-l., Ren, S.-s., Liu, J., Chen, R.-b, Xu, B., Li, Y., Ma, Q.-z., & Han, W. (2016). Analysis and calculation of threshold pressure gradient based on capillary bundle model. *Science Technology* & *Engineering*, *16*(17), 127-132.

Shen, R. (2011). *Research on flow characteristic and reservoir engineering of horizontal wells in low permeability reservoirs* (Doctoral dissertation). Graduate school of Chinese academy of sciences.

Wang, H., Liao, X. W., Zhao, X. L., Xiao feng, L. I., Wang, M., & Song, J. X. (2014). Flow regimes of segmented multi-cluster fractured horizontal well in ultra-low permeability reservoir. *Journal of Shaanxi University of Science* & *Technology*, *21*(6), 107-110.

Wang, Z., Zhu, W., Gao, Y., Zhang, Y., & Wang, C. (2012). Parameters optimization of fractured horizontal well of overall fracturing well patterns in ultra-low permeability oilfield. In *International Conference on Computer Distributed Control and Intelligent Environmental Monitoring* (pp. 483-488). IEEE. https://doi.org/10.1109/CDCIEM.2012.120

Wang, Z. P., Zhu, W. Y., Yue, M., Gao, Y., Zhao, G. J., & Wang, H. Q. (2012). A method to predict the production of fractured horizontal wells in low/ultra-low permeability reservoirs. *Journal of University of Science* & *Technology Beijing*, *34*(7), 750-754.

Wang, H., Xue, S., Gao, C., & Tong, X. (2012). Horizontal well inflow performance in heterogeneous anisotropic reservoirs. *Journal of Northeast Petroleum University*, *36*(3), 79-85.

Yang, Z., Zhang, Z., Liu, X., Tian, W., & Xu, Q. (2014). Physical and numerical simulation of porous flow pattern in multi-stage fractured horizontal wells in low permeability/tight oil reservoirs. *Acta Petrolei Sinica*, *1*, 85-92.

Yuan, Z. B. (2011). *Research on productivity evaluation method for fractured horizontal well in low permeability reservoir* (Doctoral dissertation). Southwest Petroleum University.

Zhang, D., Peng, J., & Gu, Y. (2012). Experimental study on threshold pressure gradient of heavy oil reservoir. *Xinjiang Petroleum Geology*, *33*(2), 201-204.

Zhang, Q., Su, Y., Wang, W., & Sheng, G. (2015). A new semi-analytical model for simulating the effectively stimulated volume of fractured wells in tight reservoirs. *Journal of Natural Gas Science* & *Engineering*, *27*, 1834-1845. https://doi.org/10.1016/j.jngse.2015.11.014

Zhao, Y. L., Zhang, L. H., Luo, J. X., & Zhang, B. N. (2014). Performance of fractured horizontal well with stimulated reservoir volume in unconventional gas reservoir. *Journal of Hydrology*, *512*(10), 447-456. https://doi.org/10.1016/j.jhydrol.2014.03.026

Zhu, D., Yang, Z., Wang, X., & Liao, Z. (2013). New productivity evaluation model for segregated fracturing horizontal well in low permeability and tight reservoir. *Electronic Journal of Geotechnical Engineering*, *18*, 5981-5992.