LEARNING AND PROFESSIONAL DEVELOPMENT OF THE MATHEMATICS TEACHER IN RESEARCH COMMUNITIES

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ABSTRACT

The aim of this article is to identify, describe and understand the learning and professional development of teachers who participate in research communities. To analyse and interpret the process, we have chosen the case of a mathematics teacher who, throughout her career, participated in two professional and one academic research communities. The analysis is supported by the social theory of learning in communities, although there have been adaptations for professional and research communities and for teachers. We include descriptions of the learning contexts and a narrative analysis based on the teacher's history of participation and reification in those communities. The results show that as a result of her collaboration with critical partners in academic or professional research communities, the teacher developed professionally and gained a research-oriented attitude, constantly exploring new knowledge and opportunities regarding what is taught and learnt at school. She also, changed the way she worked and interacted with her pupils and dealt with mathematical and didactic knowledge, especially in classes of pupils with learning difficulties.

KEY WORDS

Professional learning; Mathematics teacher; Research communities; Professional development; Narrative analysis.

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INTRODUCTION

The aim of this article is to identify, describe and understand the learning and professional development of a particular mathematics teacher from her participation and reification in research communities.

This text begins with a brief description of the context – the research community – where the learning and professional development took place. We then present the theoretical basis of this study, highlighting the social theory of learning in communities of practice. We also present a bibliographical review on learning and professional development in professional and research communities.

Next, we broach the methodological side of this study, presenting the case of a teacher-researcher who participated in three research communities. We provide a narrative analysis of this teacher's learning process and professional development, based on her participation and reifications in the research communities.

Finally, we narrate the learning process and the ways in which the teacher developed as a result of her participation. From there in narrative style, we pinpoint and analyse certain episodes that took place in the communities.

THE LEARNING AND PROFESSIONAL DEVELOPMENT CONTEXTS OF THIS STUDY

Given the aims of this article, we shall first describe the three contexts of learning and professional development covered in this study. We have characterized these contexts as research communities, since they all carried out, in a collaborative environment, studies, analyses, research and the writing of articles on the process of teaching and learning mathematics in school.

The earliest of the three is the Research Group on Pedagogical Practices in Mathematics (PraPeM), which emerged, in 1995 as an academic research community associated with Unicamp's Post-Graduate Program in Education. Its aim was to offer theoretical-methodological support to Masters' and PhD students. It is a collaborative community with a university-school relationship, as it lends itself to the shared study of schoolteacher problems and demands. The group's research has centred on two main axes. One deals with teaching and learning mathematics in schools and includes ethnographical research on everyday schooling and/or the teachers' research into their own practice. The other centres on teacher training and professional development in a context of reflection, research and collaboration among educators and teachers.

The second context emerged in 1997 from a teacher in-service programme (Specialization Course) offered by PraPeM. At this time a group of five school teachers and two PraPeM educators was set up with the aim of collaboratively supporting teacher research into their own math teaching practices in school. The group continued after the course had ended until late 1999 when they published a book, entitled *Por Trás da Porta, que Matemática Acontece?* [What Kind of Mathematics Takes Place Behind Closed Doors?] (Fiorentini & Miorim, 2001a), containing the teachers' research in the form of narrative analyses.

In this research community, the collaborative process of researching one's own practice bears similarities with the Japanese *Lesson Studies* (Doig & Groves, 2011). As with the Japanese initiative, it included: an initial phase of group lesson planning for each teacher; a second phase of implementing the

lesson plan, accompanied by written records, audio recordings of classroom activities and documentation of the written student output; and a third phase of group analysis of the activities developed, where the records of the activities and the students' output were evaluated and collectively analysed, initially to plan new activities and later, to write the book (Fiorentini & Miorim, 2001b).

The third context features the Grupo de Sábado [Saturday Group] (GdS) which emerged in 1999, bringing together schoolteachers interested in studying, reflecting and researching mathematics teaching in schools; and academics (university teachers, masters' and PhD students) interested in researching the in-service teacher education process and the professional development of teachers in a collaborative context of reflection and research into teaching practice. The GdS is so named as it meets every fortnight on Saturday mornings.

Although the GdS was a subgroup of PraPeM, it was always run autonomously. Both have sought to discuss and carry out studies and provide theoretical-methodological contributions, with a socio-cultural perspective, that deal with (1) math pedagogy as a complex, multi-faceted practice involving multiple, constantly changing, dimensions; (2) the mathematics' teacher as a subject capable of producing and giving new meaning, through her practice, to her knowledge of professional activity and to her own professional development; (3) teacher training as an ongoing and always inconclusive process, which begins before a scholar obtains her degree and continues throughout her life, gaining strength, mainly through shared processes of reflection and research (Carvalho & Fiorentini, 2013). By 2013, GdS had published, along with articles in periodicals and annals, five books containing stories and research, which were, for the most part, *narrative analyses* of mathematics lessons.

These groups have been analysed in various studies. Among these, we highlight Jiménez (2002), Fiorentini *et al.* (2005) and Fiorentini (2009) who researched the learning of the GdS participants over the 12 years of its existence. A common characteristic of these communities is their heterogeneity, as they rely on the participation of school teachers, educators and university academics. This heterogeneity which never became hierarchical or unbalanced, featured participants with different knowledge and *overviews* (Bakhtin, 2003).

In relation to the future teachers, the schoolteachers, with their *overviews*, bring with them the classroom instructor's math teaching experience and their knowledge of the conditions and possibilities offered by certain tasks and teaching practices. The knowledge they mobilize and produce is based on

the complexity of their teaching practice. And their teaching experience is crucial to negotiating sense and meanings for the tasks they design, analysing episodes and situations of teaching-learning, appropriating and authenticating the knowledge gleaned from classroom practice and academic research.

The university educators, in turn, have *overviews* that feature theories and methodologies from which they produce analyses, interpretations and an understanding of actual classroom practice. Their aim is to question and break down these practices for analysis. Future teachers, who began participating in the GdS in 2003 display, more than any of the other participants, their skills at using information and communication technologies and a greater proximity to and understanding of the students' reference cultures.

Before going any further, we should clarify that our intention is not to question or defend learning and research communities. According to Hargreaves and Fink (2007), not every learning community brings about empowerment or greater professional autonomy for its participants; it depends on the reasons a community is set up and the activities it engages in. For example, communities, may be monitored, controlled or moved by external agents and/or by pragmatic motives that are contrary to the emancipation of students and teachers. On the other hand, communities of empowerment and of sustainable leadership tend to construct their own knowledge and motivations, moved by political-emancipatory principles or notions of inclusion and social justice, such as improving learning for all, that is, promoting inquisitive, wide-reaching, meaningful learning for all, not just a selected few young people.

These considerations shed light on the different kinds of teacher research communities, which can be academic, school-based or somewhere in between.

Academic research communities, which are monitored/governed institutionally by the university, may be endogenous, geared towards theoretical problems and unconnected to school practices. They may be colonizers of school practices, or collaborative, open to the problems and demands of school teachers and schools. They may be able to maintain a joint study agenda, as is the case of the PraPeM group and the collaborative group that emerged from the specialization course.

School-based communities, being governed from the schools themselves, may also be endogenous, open to collaboration and partnership with the university, or wish to benefit from university participation.

The *borderline* communities are on the border between school and university and normally have more freedom of action and ability to define their

own work and study agenda, since they are not institutionally monitored by the school or university. The border is a free place where interested parties from different communities can meet, venture forward, construct and question knowledge, and also carry out research. The borderline, however, is also a place of danger, a locale to transgress, a place to defy that which has been established in schools and academia. Since its participants come from various origins, the meetings tend to be interspersed with narratives of events that have occurred in the original communities. Still, what is produced and learnt in the borderline communities ends up having an appreciable impact on the personal and professional lives of each participant.

The GdS can be considered a borderline community. Although the teachers meet at the University, the meetings take place on Saturdays, a day when there are no formal academic activities or control over who attends and what is discussed. There is, however, a mutual commitment to build a pleasant study and research environment, and the freedom to suggest agendas that reflect common interests.

LEARNING IN PROFESSIONAL COMMUNITIES

From the perspective of *social learning theory* (Lave & Wenger, 1991; Wenger, 2001), all learning is situated in social practice that occurs through active participation in social community practices and the construction of identities within those communities. Knowledge in a practicing community is produced and evidenced through the shared forms of doing and understanding within the community, which results from the dynamics of negotiation involving full *participation* or *legitimate peripheral* and *reification* in (or from) the community.

In our interpretation of the theory, participation is a process whereby the members of a community share, discuss and negotiate the meaning of what they are doing, saying, thinking and producing. To participate, however, means engaging in the activity of the community; appropriating its practice, knowledge and values; and also contributing to the development of the community, especially to its members and to its repertoire of knowledge (Fiorentini, 2009).

Reification, according to Wenger (2001), means *turning into a thing*, and does not only refer to material or concrete objects (texts, tasks, manipulative materials). It also includes concepts, ideas, routines, written records and theories

that give meaning to the community's practices. *Participation* and *reification* are, therefore, interdependent and essential to the learning and constitution of identities in (or of) a community.

The theory of learning situated in a community of practice, according to Lave (1996, p. 8), can be supported by four knowledge and learning premises:

(1) Knowledge always undergoes construction and transformation in use.
(2) Learning is an integral aspect of activity in and with the world at all times. That learning occurs is not problematic.
(3) What is learned is always complexly problematic.
(4) Acquisition of knowledge is not a simple matter of taking in knowledge; rather, things assumed to be natural categories, such as «bodies of knowledge,» «learners,» and «cultural transmission,» require reconceptualization as cultural, social products.

Given this theory, we posed the question: What would a teacher's learning be like in a community of mathematics teachers working in a school? What practices would be formative in that community? Within this context, in-service programmes, which focus primarily on analysing and problematizing the teaching and learning practices of the teachers involved, seem to make sense. In these types of programmes, the educators and teachers, together and collaboratively, can design teaching tasks or analyse classroom episodes, which may be videotaped, orally narrated or written down by the teachers who are taking part. Such programmes are warranted, because everyday practices (with their procedures, discussions and knowledge) are fraught with values, finalities and know-how which may be relevant to personal development, but because of their routine nature – as Foucault (1977) highlights – often become valid in and of themselves and hide deviations, ideologies and power relationships.

In the process of problematizing and denaturalizing the everyday practices of classroom teaching and learning, heterogeneous professional learning communities may be useful, especially if people with different knowledge and social practices are involved. According to Ponte *et al.* (2009), although heterogeneous communities may find it harder to construct a common language and coordinate their ideas and work methods, the different points of view and the diversity of experiences and knowledge present may empower the community even further, by promoting understanding, identifying and analysing nuances, potentials and limits in the practices the group is examin-

ing. That community diversity may, therefore, provide opportunities for more intense and meaningful learning experiences.

Ponte *et al.* (2009, p. 202), analysing three studies[2] on learning in communities of mathematics teachers, confirmed that an important and significant «variety of a learning community occurs when that community establishes itself as a community of inquiry, that is, when inquiring on some issue becomes part of the purpose of the whole group». It is, therefore, a powerful way for a community to construct knowledge and learning, as we shall see later.

LEARNING IN RESEARCH COMMUNITIES

Every research community is also a community of learning and practice. But not every community of learning, even if it is reflective, is a research community. Reflective practice differs from research practice. The latter requires a systematic procedure for treating a phenomenon or educational problem. That is, the teacher's research practice, according to Beillerot (2001) and Cochran-Smith and Lytle (2009), presupposes a methodical process of collecting and treating information concerning the phenomenon. The teacher-researcher needs, from a certain perspective (snippet, focus or research question), to make written records, organize her ideas and revise and analyse her practices. By doing so, she is seeking and producing a better understanding of her teaching. At the end of the process, she should «publicly present a final written report on the study developed» (Fiorentini & Lorenzato, 2006, p. 75).

For Jaworski (2008), the teacher who participates in an *inquiry community* does classroom research and, as part of her work, questions, explores and analyses her own teaching practice. Jaworski is a mathematics teacher who, in her studies and research, often uses the term «inquiry community». However, she believes that the term *inquiry* in the field of mathematics education holds two meanings. In one, inquiry is a teaching and learning tool, as is the case of mathematical research, or of research-teaching. In the other sense, inquiry is a way of being, in that that the identity of the individual or of the group within a research community is rooted in a form of inquiry:

2   Presented at the 15th ICMI Study: Fiorentini *et al.* (2005); Ponte & Serrazina (2005); Van den Heuvel-Panhuizen & De Goeij (2005).

> Developing inquiry as a way of being involves *becoming, or taking the role of,* an inquirer; becoming a person who questions, explores, investigates and researches within every day, normal practice. The vision has much in common with what Cochran-Smith and Lytle (1999) speak of as «inquiry as stance» – the stance of teachers who engage in an inquiry way of being (Jaworski, 2008, p. 312).

The identity constructed by teachers in a community, who reflect together and research their own practice, approximates what we have called, teaching professionalism based on a research attitude, as was espoused by Cochran-Smith and Lytle (2009, p. 57),

> The work of practitioner inquiry assumes that practitioners generate local knowledge of practice by taking an inquiry stance on both the knowledge generated by those outside the local context and the knowledge constructed through the joint efforts of practitioners working together in research communities.

This research-teaching professionalism, in the present study, becomes one of the signs of the teacher's development in a research community. This professionalism, however, cannot be defined or characterized merely as the underlying knowledge of a profession or by the professional's ability to identify and solve problems in a situation of uncertainty. It must also be seen from the perspective of the ethical-political principles and values cultivated by the professionals in a community (Fiorentini, 2009).

This raises questions about the power relationships that exist between the school community and the academic community and, especially, public policy.

> In an inquiry community, we are not satisfied with the normal (desirable) state, but we approach our practice with a questioning attitude, not to change everything overnight, but to start to explore what else is possible; to wonder, to ask questions, and to seek to understand by collaborating with others in the attempt to provide answers to them (Wells, 1999). In this activity, if our questioning is systematic and we set out purposefully to inquire into our practices, we become researchers (Jaworski, 2008, pp. 313-314).

Neither research professionalism nor research communities are born ready-made. They are built up mainly through questioning, problematizing and denaturaliz-

ing what is taught and learnt at school which apparently seems to be normal, and, later by systematically searching for an answer or a better understanding of the questioning. We understand that to change radically a school practice, it is necessary to unravel its continuity. This in not achieved by its overlapping with something new, but by problematizing or contrasting it with the traditional and current cultures of the classroom. It is during the process of problematizing current practices that the educators' input gains importance and relevance, especially at the beginning when a community is hoping to assume a research dimension. In time, all teachers who develop a research stance begin to question practices.

The student who begins a post-graduate program becomes part of academia, which is generally composed of small research communities or research groups. Here the students produce and negotiate the meaning of what they are learning and researching. They share their reflections and knowledge; they learn to produce scientific work; they are committed to carrying out research; and, to attain their aims, they use the resources and observe the requirements of the academic community. Although debates and oral communication are widely used in these situations, written language plays a prominent role as an instrument of learning and communication. In this sense, one can view the text of the master's or PhD thesis as the main *reification* of the teacher-researcher in the academic community.

PROFESSIONAL DEVELOPMENT AND LEARNING

In this study we view teacher development as a continuous process which continues throughout the person's professional life, and begins before the graduate obtains her degree. This development «happens in the multiple areas and moments of each of our lives, involving personal, family, institutional and socio-cultural aspects» (Rocha & Fiorentini, 2006, p. 146). It is, therefore, a complex process which involves the teacher as a total human imbued with feelings, desires, utopias, knowledge, values and social and political conditioning (Fiorentini & Castro, 2003).

Within this concept of professional development, the teacher is seen, according to Ponte (1998), as the principal protagonist of her own education and professional culture. She acts from the «inside out» in search of knowledge and improvement in her teaching practice.

Day (1999), referring to certain signs of teacher development, highlights that it is a process through which

> the teachers review, renew and extend their commitment as change agents to the moral purposes of teaching. It is also the means by which they acquire and develop critically the knowledge, skills and emotional intelligence essential to good professional thinking, planning and practice with children, young people and colleagues through each phase of their teaching lives (pp. 20-21).

Day (1999) understands professional development as a process involving multiple «spontaneous learning experiences,» that are indicators or markers in the teacher's development. However, the way in which teachers learn in communities has so far been the subject of little research into professional development. The indicators and markers have been noted thanks to the perceptions of the teachers themselves in interviews or oral and written narratives. They have also been taken from other studies, without being researched in detail in practice situations or in shared community analyses.

It is our belief that the circumstances and context in which a teacher learns play an important role in understanding the process of becoming a teacher and in the construction of the teacher's professionalism. The learning context may be a workshop, the classroom itself, and/or a homogeneous or heterogeneous collaborative group that discusses and analyses teaching and learning practices.

According to Cochran-Smith and Lytle (2009), teachers learn and develop professionally, when they reproduce their local, practical knowledge by participating in research communities, theorizing and linking their work to a wider social, cultural and political context. In this sense, professional development requires that the researcher come closer and draw away from that which is circumstantial or isolated in the learning process, as the process of becoming a teacher can only be perceived and understood by the researcher in a diachronic movement, that is, over the years. Oral history and written reifications by the teacher herself may help the researcher gain access to the feelings and meanings that each teacher attributes to her professional development.

Therefore, the researcher who is interested in understanding how teachers learn and develop needs to focus on isolated moments in the teacher's learning process as well as on the diachronic movement of the development process over the years. She must consider the context, practice and interac-

tions that may have contributed to the process of becoming a teacher. Next, we will explain our research procedures in more detail.

METHODOLOGICAL ASPECTS OF THIS STUDY

The analysis of case studies involving teachers who participate in research communities is useful in identifying, describing and understanding the learning opportunities that can arise from the participation of the teacher-researcher in these communities. The case study also helps us track the milestones of the teacher's personal development.

According to the social theory of learning (Lave & Wenger, 1991), community-based learning can best be described and analysed by examining the participation and reification of the participants in that community.

Based on the above-mentioned aim and presupposition, we have chosen the following research question as the beacon for the present study: *What learning is evidenced by the teacher who researches his own practice and participates in research communities, and how does she develop professionally through participation and reification in these contexts?*

One way of researching learning in research communities, according to Lave and Wenger (2002, p. 168), is to analyse the «historical production, transformation and change in people» who participate in them and how they evolve through time and constitute their identities.

One way of understanding and describing this process is through *narrative analysis*, which, according to Bolívar, Domingo and Fernández (2001), consists of narrating an event or a person's development process by means of attributing sense and meaning, highlighting the common and unusual elements which make up the history of each subject over time. The researcher's task in this type of analysis «is to configure the data elements in a history which unite and give significance to the data, with the objective of expressing, in an authentic way, the individual's life, without manipulating the voice of the participants» (p. 110).

With the aim of developing a narrative analysis with a certain depth, we opted to examine just one case. Our subject was a Brazilian math teacher (Eliane Matesco Cristovão)[3] who had a track record of participation and reification in three communities with research characteristics.

3   The teacher herself has kindly and gladly given us permission to use her real name.

Other teachers in our data bank had also taken part in research communities, but Eliane was the only one who had participated in three. Therefore, the research we are presenting here is a case study, as «it presents individual characteristics which makes it deserving of special research» (Fiorentini & Lorenzato, 2006, p. 110). We have also adopted a qualitative approach, since the work was submitted to a process of narrative analysis, which requires an appreciable degree of interpretation.

In the narrative analysis and interpretation of Eliane's learning process and professional development, we have made use of her participation and reification in three research communities, as we mentioned at the beginning of this article. The reifications include the elaboration and discussion of teaching tasks; recordings or classroom episodes narrated or documented by the teacher; narrative analyses of lessons; published texts such as chapters from books, journal articles, and conference proceedings; master's theses; and the minutes from or recordings of group meetings, etc. We shall also use statements made by the teacher herself, and her reflections and perceptions of her community learning experiences.

In recounting Eliane's professional development, some analytical and interpretive parameters were established: her historical-cultural background and the motives that led her to participate in each research community; her problematizing and negotiation of meanings within each community; the knowledge she mobilized in teaching and learning mathematics; the questions that informed her research; the shared analysis of the teaching and learning tasks and activities; identification of the main things she learned from her participation in the community, with an emphasis on her conceptual, didactic-pedagogic and curricular knowledge; and the identification and description of the perceived changes and the professional development she underwent.

Next, in narrative form, we shall analyse and interpret Eliane's path as a learner and a professional. This will be gleaned from the multiple oral and written reifications she produced in the three study and research communities after having obtained her degree in mathematics. Although the written narrative accompanies the teacher's professional life chronologically, the reifications selected are not in chronological order. For example, for convenience sake, we used a recent reification of the teacher's to narratively analyse her early career. The last year we used in describing Eliane's career path is 2013 when she was 42 years of age and had taught for 21 years, 20 of which were spent as a middle school teacher.

NARRATIVE ANALYSIS OF ELIANE'S
LEARNING AND PROFESSIONAL DEVELOPMENT

While participating in the PraPeM Research Group, which resulted in her master's thesis (Cristovão, 2007), Eliane reflected on her schooling process and career options. She stated that she had always studied at state-run schools and, from grade 9, attended night school to be able to work during the day. Influenced by her mathematics teacher, she began a night program in mathematics at Unicamp. She remembers having had great difficulty with calculus, which made her aware of her «lack of cultural and scientific baggage». She says she did not drop out of the course because she wanted to show her father that she could overcome her difficulties, as he did not place any value on studying (Cristovão, 2007, p. 7).

She started teaching in the third year of her degree program. She remembers that at the beginning she tried to emulate the best teachers she had in school and do the opposite of what the worst teachers did. This early start in teaching enabled her to do a specialization course at FE/Unicamp after she graduated. The course had a profound effect on her, and was influential in her professional development. It allowed her to carry out, as a final assignment, research into her own teaching, which was developed with the support of a collaborative group made up of four other course colleagues and two teacher educators who were the supervisors of the five teacher-students.

This group was her first research community. As she herself says, it was two years of meetings where «each of us could count on the collaboration of everyone to prepare and analyse teaching practice, to give a (new) meaning to the history of their own professional formation» (Cristovão, 2007, p. 9). The results of Eliane's first research experience (Cristovão, 2001) were published in a book, organized by the educators (Fiorentini & Miorim, 2001a).

With regard to the research process, Fiorentini and Miorim (2001b) observed that the group had adopted, an exploratory, problematizing approach with negotiation of meanings as a teaching methodology. As this was an innovative approach for the teachers, in that the students had a voice and were asked to record their mathematical ideas, thoughts, and rationales, the teachers were frequently in a panic and called the didactic-pedagogic approach into question. Hence, the group was important in promoting analysis and furthering awareness of the students' accomplishments and difficulties.

This collaborative process is well illustrated in a small episode taken from Eliane's grade 6 teaching-research project entitled «Along the Paths of a New Experience in Geometry Teaching», which covered a period of 36 class hours. In one of her first lessons, Eliane explored notions of geometry using the tangram, having asked her pupils to compare the middle triangle with the smaller square of the tangram pieces and to find a way to prove that one of them was bigger or equal to the other. Although one of her classes produced satisfactory results by folding, cutting or overlapping the figures, to prove that the area of the two figures was the same, Eliane was disappointed with the other class where quite a few students wrote nonsensical answers such as «The square because it has 4 equal sides and 4 straight lines and the triangle only has three points and three straight lines»; «The triangle is bigger than the square, its sides are longer…»; «The triangle is bigger because the angles are bigger», etc. (Cristovão, 2001, p. 63).

When the collaborative group questioned her about how she managed the activities in the two groups, Eliane remembered that in the class where the results were satisfactory, one of the pupils, right at the beginning of the activity, had asked «bigger, how's that?» She then negotiated the meaning of bigger with the students with regard to the quantity of paper or to the area of the geometric figures. This had not occurred in the other class.

The collective analysis of this episode helped Eliane learn that as well as setting constructive tasks or *mathematical challenges*, the teacher needs to observe the Potari and Jaworski (2002) teaching triad *being sensitive to what the pupils say* in their answers and meanings, challenging the students and *managing the learning* by negotiating the meanings that are required for the development of the classroom activities.

In this and other episodes Eliane interacted with other interested parties to gain a better understanding of teaching and learning mathematics. She interacted principally with the educators and university academics who, according to Bakhtin (2003), have a broad overview, of schoolteachers, various methods of teaching and learning mathematics, and the ability to link teaching with research. This, as the teacher herself has acknowledged, was crucial to her professional development.

> It was principally in these moments of discussion with the group… that I understood the importance of having someone to share the conflicts we went through when trying to be innovative. Alone it is difficult to innovate and, even more difficult to analyse the practice (Cristovão, 2001, p. 58).

The research perspective of the project required that each of the group's teachers produce a narrative analysis of her educational experience, describing in detail: the production and negotiation of meanings; the mathematical sense-making and learning of the pupils; the personal, class or school dilemmas and tensions present in the innovative process; and the new professional knowledge each teacher produced in this process (Fiorentini & Miorim, 2001b).

Participating in this research community, Eliane started to develop *research with an attitude* of questioning and analysis with regard to her teaching. This attitude is echoed in the studies of Cochran-Smith and Lytle (1999) and Jaworski (2008). This research practice/stance can be inferred from Eliane's analyses of the relationship between her teaching practice and her research practice.

> It is difficult being a teacher and a researcher at the same time. In this educational experiment I tried to reconcile the two things. My greatest concern as a teacher: to teach and learn geometry with understanding and pleasure. As for being a researcher: to analyse and understand the process of teaching and learning when we prioritize a practice of production and negotiation of meanings (Cristovão, 2001, p. 45).

Years later, Eliane theorized about the experience in which she shared the process with colleagues and wrote about herself as a way to reflect on and research her teaching practice. When she looked back, she realized that it had contributed to her developing a critical and questioning eye with regard to her lessons. It was yet another landmark in her professional development:

> That experience began the process that formed my very research attitude. It was when I began to understand the importance of sharing our classroom experiences by writing them down, and how the process of writing and rewriting allows us to reflect on our pedagogical practice. After the book was written, my awareness in the classroom, especially of the students, became more critical and questioning (Cristovão, 2009, pp. 18-19).

Another significant experience for Eliane was her participation in the Grupo de Sábado starting in 2003. Her principal motivation in joining the group arose from her previous participation in the community that wrote *Por Trás da Porta, que Matemática Acontece?»*, which she recognized as being a period of great learning.

She immediately identified with the practices of the GdS, recognizing that in the new community, she would be able to resume the processes of reflection and research into her practice in an environment of collaboration, using written, narrative analyses of mathematics lessons. Her recognition of the benefits of being in the GdS can be seen in the episode we relate below.

The first time she participated in the GdS, the group was questioning the concept of perimeter. A colleague named Rogério gave the group a task to complete (Figure 1).
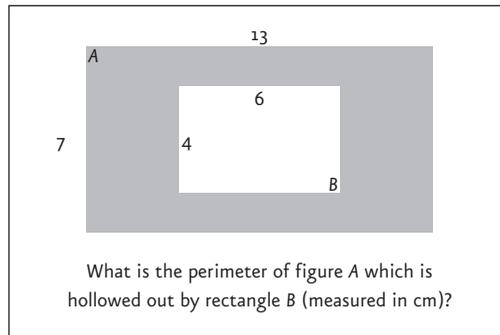

FIGURE 1 – TASK SET BY ROGÉRIO (EZEQUIEL, 2003, P. 32).

Eliane, some of her GdS colleagues and all of Rogério's grade 8 pupils found a measurement of 40cm for the perimeter of Figure A. Others, however, obtained 60cm, including the inside perimeter.

From these results, the group began to negotiate the meanings of perimeter. Some came up with the hypothesis that the tasks and definitions used in school textbooks created an incorrect understanding of perimeter, giving rise to what Brousseau (1986) called an «obstacle of didactic origin». With his 8th grade students, Rogério had also done a little research on how textbooks present tasks and definitions regarding perimeter. He confirmed that definitions such as «the perimeter is the sum total of the sides of a geometric figure» or that it is «the measurement of the contour of a geometric figure», as well as certain tasks involving figures that are not hollowed out, led to an imperfect understanding of the perimeter concept.

Meanwhile, Eliane and the academics in the group questioned how Rogério had set the task with the hollowed out figure. They argued that the way the task had been set was configured like a «trick», in which figure B could be considered overlapping figure A. This did not promote problematizing and recognition of the inside perimeter. Eliane felt motivated to set a task that

could explore or problematize the meaning of perimeter, putting a dent in the meaning that had been accepted by both teachers and students.

With this aim, she brought presented the group with a task with involving various geometric figures, some of them which were non-conventional (Figure 2). Her hypothesis was that, by negotiating meanings in the group, the pupils would manage, by means of negotiating meanings in the group, to reach a better solution and meaning definition for perimeter, remaking, by doing so, the concept they had of perimeter, thus reformulating the whole concept. The figures, in the first elaboration version of the task, contained the measurements of the sides. On being questioned by the academics about the need or relevance of this information, Elaine opted not to give this information specifically. This lesson is discussed in a narrative analysis of that experiment, in which she writes about how she justified making that option. She states that it, which is «part of the training process of the pupil, who learns to obtain the data of a problem-situation, therefore thus breaking with the *facilitation pedagogy*, that is, the pedagogy that gives hands everything over to the students already chewed up and digested over to the pupils» (Cristovão, 2003, p. 36).

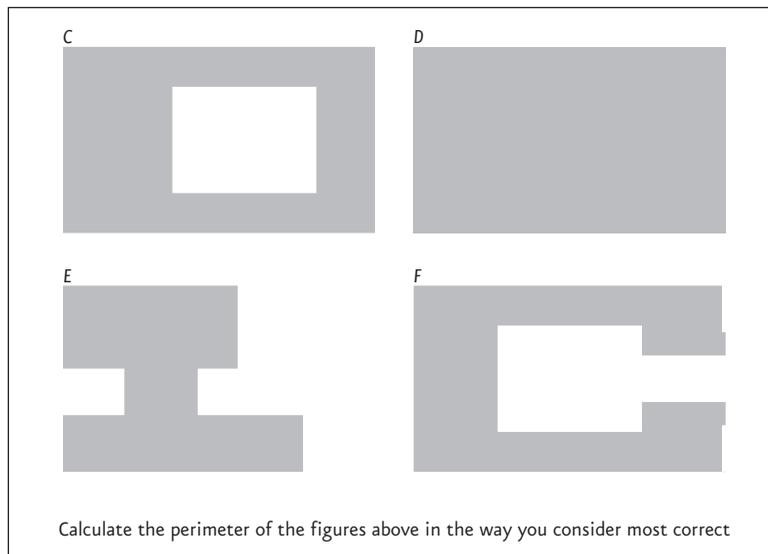Calculate the perimeter of the figures above in the way you consider most correct

FIGURE 2 – TASK SET BY ELIANE (CRISTÓVÃO, 2003, P. 35).

We should point out that the expression *facilitation pedagogy* was introduced and orally reified in the group in 2000 and was incorporated into the GdS's discursive repertoire. It was frequently used as a way of questioning and

denaturalizing the practice of mechanically using procedures or tips and was aimed at facilitating student performance.

Although the expression was already being used in the group, Jiménez Espinosa (2002), just two years after its emergence, was the first group participant to reify it in writing in his doctoral thesis, where he analysed one of the group's discussions on the meaning of facilitation pedagogy. In short, Jiménez pointed out that schoolteachers saw it as a «pedagogical culture that can create in pupils a mechanical attitude with little reflection in the light of knowledge», while the academics viewed it as obstacles of didactic origin which, according to Brousseau (1986), are simplifications used by teachers to help their students memorize a fact or computation procedure or solve a problem (Jiménez, 2002, p. 144).

Meanwhile, in the episode being analysed, Eliane had access to the expression from Rogério's narrative, as she had been a participant of the GdS since the year 2000. Eliane shows that not only did she appropriate this group's reification, but she also went on to use it in her own narrative analysis, having, however, produced her own reification: «facilitation pedagogy (…) is that which hands everything over to the students already chewed up and digested».

Looking back at Lave and Wenger (1991), we can say that, on authenticating her reification and her option not to give the measurements of the sides of the geometric figures (Figure 2), the GdS, from the very first meeting, recognized Eliane as a legitimate member of the research community, someone who identified with the practices of the group, and a person who stood to contribute to the community's development.

Applying the task that had been authenticated by the group (Figure 2) to her students, Eliane opted to cut out the figures in cardboard, so that her students would not construe the hollowed out part of figure C as one figure overlapping the other.

The results were positive and multi-faceted, and here we highlight geometric figure C. Some groups added the external and internal perimeters, giving a correct, single result for the perimeter. Others gave two separate results: the inside and the outside perimeter. Some groups only added up the outer sides, and one group presented the difference between the external and the internal perimeter as a final result.

To systematize the didactic experiment, and after discussing and negotiating the results and meanings with the whole class, Eliane used a questionnaire in which the students were to answer what a perimeter was and how

it is calculated when the figure is hollowed out or cut out. She presented the results to the GdS in the form of a written narrative analysis. After collecting reflections and analyses produced with the help of the group, a second GdS book (Cristovão, 2003) was published.

In the GdS, Eliane co-authored four books for the group. She published five narrative analyses of math lessons and three essays on systematization dealing with writing in mathematics, research classes. She also took part in the development of a research stance for math teachers. Her output demonstrates how rich and fraught with reifications her participation in GdS was.

Her activities in the two research communities described here and the path of her student and professional life helped Eliane to become constructively critical in relation to school practices, and sensitive and committed to children with learning difficulties, who are in danger of dropping out or failure. She believed that these children were capable of learning and that, with better public support policy and greater appreciation for the teachers' work, both the teacher and the school would be able to design alternative pedagogies to help these pupils become protagonists and subjects of learning. It was this conviction that motivated her to work toward her master's degree.

When she started the master's in 2005, she took this issue up with the PraPeM academic group, where she studied and dealt with the issue on a theoretical-pedagogical level. Based on the works of João Pedro da Ponte and Ole Skovsmose and on studies produced by the GdS and PraPeM, she became convinced that it was necessary to break with the exercise paradigm and promote open tasks and activities of an exploratory-research nature.

Other authors were also important to her understanding of the problem. Charlot (2000), for example, helped her to understand that «school failure does not exist, what exists is failed pupils, situations of failure, school stories that ended badly» (Cristovão, 2007, p. 43). Luiz Carlos de Freitas helped her to understand «the causes for exclusion experienced not just by her pupils, but also by a good number of those attending state schools» (Cristovão, 2007, p. 43) without, however, pointing at pedagogical alternatives in order to remedy the problem.

Michel de Certeau (2007) enabled her to understand that there are always possibilities of intervening and bringing about change in the daily practices of a school. The awareness of these possibilities merged with the knowledge she had gained in the two practice communities which in turn led her to question certain suppositions and academic stances, as witnessed by the following statement: «I couldn't sit with my arms folded and wait for what Freitas called

the «historical transforming project of the organizational foundations of the school and of society» (Cristovão, 2007, pp. 40-41).

The study of the pedagogical possibilities that promote the inclusion of students with math learning difficulties became the focus of her master's research. As there were Course Recovery classes for Cycle II[4], she partnered with two other teachers in designing an intervention project for her classes.

With her partners and other interested teachers she built up a local collaborative group – Grupo Colaborativo de Estudos em Educação Matemática [Collaborative Study Group in Mathematical Education] (GCEEM). However, for her master's research, she interacted with three collaborative groups (PraPeM, GdS and GCEEM). With the GCEEM she planned a series of exploratory-research tasks and lessons for the two classes. She worked together with the teachers in the classroom and acted simultaneously as a teacher and researcher in this experiment.

As a researcher, with the support of PraPeM, she looked for answers to the following question: What possibilities and contributions can exploratory-research practice involving the collaboration of a group of teachers, bring to the teaching processes and the math education of students in RCII classes that shows evidence of their school inclusion?» (Cristovão, 2007, p. 24).

She called this practice of teaching and research in the two classes **research-action of the 1ˢᵗ order**, as she shared and analysed the educational experiment with her partners and with GCEEM, also including time for discussion and analysis with the GdS and PraPeM.

The *meta-analysis* of this research carried out later just by Eliane, with the support of PraPeM, was dubbed **research-action of the 2nd order**. In this meta-study, she obtained evidence of the social inclusion of pupils through exploratory-research activities. Due to the quantity of material collected, she only analysed the experiment with the class of Teacher RE. The material for analysis was obtained from audio and video recordings, the pupils' portfolios, questionnaires, narratives of the teacher partners and the researcher's field diary.

She chose five approaches with which she developed analyses and interpretations of the output and relationship the students established with mathematical knowledge, with themselves, with others and with the teaching and learning process. These were: the mathematical output of the pupils; mobili-

4   In Brazil, Recovery classes in Cycle II are made up of pupils at the end of their Fundamental Schooling (currently grade 9), whose performance was not satisfactory enough to move on to middle school.

zation and (re)significance of their knowledge; their changes of attitude and stance; the role and active participation of the pupils; and their resistance and *negatricities*.

She carried out various exploratory-research activities in a grade 9 class in partnership with teacher RE, who was held back to recover the cycle. The following are two episodes dealing with geometry. The task aimed to review and explore the different types of triangles and the possibility of constructing them from an isosceles triangle (non-equilateral) on which they could only draw one section of straight line. There were several answers, but Eliane highlighted the justification given by students Gi and Ta on the impossibility of constructing an equilateral triangle from a non-equilateral isosceles triangle[5] with only one line.
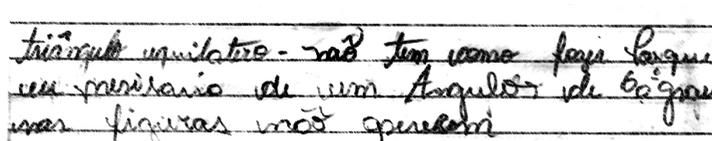
Another episode demonstrates the students' creativity and Eliane's ability to negotiate the authentication of the solution. It involved interpreting what two pupils (Da and Em) did with a task that Eliane set, inspired by an idea suggested by Ponte, Brocardo and Oliveira (2003) called «folds and cuts.» The pair understood that it was impossible to obtain a scalene triangle with two cuts on the folded sheet. So they initially made a cut in the lower right hand corner of the folded sheet and unfolded the cut part, obtaining an isosceles triangle. On this they made the second cut, obtaining a scalene triangle.

Seeing that Da and Em had managed to obtain a scalene triangle, Eliane and RE went to check how they had done it:

Eliane: How did you manage to make the shape?

Da: Two cuts, yeah? Now the one with the different sides, look…a cut, beautiful? [She folds the paper in half, makes a cut in the corner, as **Em** had done before, and begins to open the paper]

Eliane and RE: But it's all with it folded!

5   Transcript of the manual record: «Equilateral triangle – can't be done (built), because I'd need (at least) an angle of 60° (to then work with the other two, with a line, but) none appear in the figures» (Cristovão, 2007, p. 99) (my brackets).

> **Da**: No, no! Here you're not saying it's just folded, the… [Pointing to item 2 of the task]
>
> **Eliane**: [She reads the task sheet and agrees]…You're right…it should have said here: a sheet of paper

Eliane took these episodes to the GdS and PraPeM groups where other interpretations were added. In line with Bakhtin (2003), those with an overview were mobilized to understand the process of teaching and learning in children with significant learning difficulties. In PraPeM, for example, one of the educators found the concept of *negatricity* of the multi-referencing theory. He referred to the «incredible capacity» of the pupil to «shake things up, responding in an unpredictable and different way from the objectives outlined in our training action» (Borba, as cited in Cristovão, 2007, p. 10).

From this study the researcher concluded, among other things, that:

> Pupils of the RE are not mass consumers, who accept everything they are told. To work with them requires a change of attitude on the part of the teachers and the managers who need to see in them not just rebels who have no output, but consumers who are critical of the knowledge that is offered to them (Cristóvão, 2007, p. 15).

While Eliane was appropriating knowledge from the academic community to produce other meanings for teaching practice, she was also questioning the academic literature for not opening the possibility of other meanings and for not recognizing the complexity and richness of school practices and the students' own knowledge. Since she had experienced several episodes in which the students had surprised the teachers with their answers and creative, out-of-the-box solutions, Eliane concludes by posing a number of issues:

> Is it that students in a school failure situation do not produce knowledge or is it that they merely do not accept a rigid and closed school system where everything has to be done as prescribed and within a given time? Could it be that if we gave them more freedom to show their creativity, they would surprise us? Can school allow this type of work? In the RCII, wouldn't this be a path toward repairing these pupils' self-esteem and making them believe they are capable of learning mathematics? On interpreting EM's and DA's attitudes as the ability to argue against a set rule, we can appreciate even more

the use of an approach that would allow them to be the subjects of learning (Cristovão, 2007, p. 104).

Eliane's sensitivity to her students' learning styles brings us back to Lave (1996) when he affirms that learning is not a problem for pupils who engage and participate in educational activities. They always learn something in the process. And that something is complex and very often hidden, since it might not coincide with the aim of the instruction.

Years later, taking stock of her professional learning in collaborative groups and research communities, Eliane commented that in these communities: we research and share classroom experiences; we count on various views to better understand those experiences, their richness and their limitations; we find support to face our problems and challenges, searching for reading material and theoretical bases which meet our needs; we analyse and write about our practice; by writing it down, we reflect and provoke collective reflections, impacting other teachers; we become critical by not merely reproducing external suggestions and recommendations (academia and public policies); we become capable of constructing our own paths, of being authors of our own practice and our own ideas; we look for the personal and professional development that we want and believe in, bearing in mind our commitment to the quality of teaching that we consider to be the most suitable for our students (Cristovão, 2009).

The research carried out by Eliane, first in professional communities and later in an academic community, made her, in Day's (1999) words, an «agent of change», engaged in reviewing, renewing and broadening her commitment to the emancipatory suggestions of children and young people. It also gave her the authority and skill to question the knowledge of others outside the local context, an aspect Cochran-Smith and Lytle (2009) broached in their work. Eliane's professional development became even more evident when she and another colleague from the GdS led a movement of protest and resistance to the Education Secretary's curricular policies in 2008 and 2009. The new policy forwarded a proposal for the entire São Paulo public network, whereby bonuses would be given to teachers whose students did the best on standardized tests. This policy was implemented without consulting the teachers and without factoring in different realities and local necessities.

Led by Eliane and her colleague, the GdS community, mindful of the results of the studies carried out by the group, opposed this *homogenizing* policy

(that featured hand-outs containing ready-made lessons for the teachers to apply), and called for conditions for teachers to organize into groups and communities, so that they could design and implement projects to improve teaching, based on an assessment of local needs. The GdS claimed that, in addition to supporting these groups, the State, should make it viable for universities to take part by mobilizing educators and future teachers to act in partnership with practicing teachers.

Starting in 2008, Eliane also began working in higher education, motivated by the desire to share what she knew with future teachers. In 2011, when she began her PhD program, she stopped teaching school because her study grant was more than her salary as a school teacher. In the first semester of 2013, seeking work stability and a schedule that would be compatible with an academic career where she could also do research, she passed the test and became a math education teacher at the Universidade Federal de Itajubá.

CONCLUSIONS AND FINAL CONSIDERATIONS

The narrative analysis of learning and professional development shown in this study illustrates that the process of becoming a teacher-researcher in research communities is unique, unusual and complex. So much depends on the practices promoted by these communities, the conditions, and the inclination of each teacher to participate and throw herself into the educational experience of working, studying and researching.

Eliane's participation and reification is evidence that her initial participation in two research communities with a more professional orientation motivated her to enter a community that was more academically oriented. Here she could see the opportunity to further her understanding of issues relating to her school teaching.

After finding an academic community that was open to this type of problematizing, Eliane was able to discern other opportunities for pupils who were considered weak in math. One angle was to engage them in exploratory-research activities, in which the students, in small groups, took mathematical hypotheses and conjectures which they authenticated, tested and later reified in short written reports that were presented to and authenticated by the whole class.

Supported by authors such as Bernard Charlot, Michel de Certeau, João Pedro da Ponte and Cochran-Smith and Lytle, and by collaborating with

school-based research partners and crucial partners in the PraPeM and GdS communities, Eliane carried out a positive, incisive study on the progress of pupils who were considered to have failed or were seen as having difficulties in mathematics. This analytical, interpretative study also featured important concepts such as: exploratory-research activities; *negatricity*; school failure; school inclusion/exclusion; negotiation of meanings, which, given the evolution and intellectual development of pupils with learning difficulties, became a way for the school to promote the inclusion of young people with different mind-sets.

As far as the research process was concerned, the *narrative analysis* proved to be an important methodological tool in describing learning situations and providing signs of the teacher's professional development. It melded interpretations and meanings for the researcher and the teacher being researched concerning events that impacted her classroom teaching and professional education over the years. In fact, the narrative analysis on Eliane's trajectory throughout the research communities proved that her participation and reification in those communities had exercised a fundamental role in the understanding and transformation of her teaching practice and professional development. This was especially true when it came to the construction of her way of being and working in the profession, all of which highlights her professionalism, which was born of reflection and research.

Eliane's attitude toward research, an expression of professionalism that she developed within the research communities, highlights and clarifies what Jaworski (2008) and Cochran-Smith and Lytle (1999, 2009) said about the concept. In short, research as an ingrained attitude toward one's own teaching professionalism can be witnessed in the way one permanently questions, problematizes, documents, analyses and gives fresh meaning to one's own pedagogical practice and that of others in a professional or academic research community, thus valuing the overview of critical partners even when one is not intentionally doing research.

It was not just Elaine who developed professionally. The community itself developed and continues to develop, as it produces and presents its research, and interacts with other communities, thus forming a broader learning network. This allows local communities to acquire power and recognition from the wider educational community and subsequently gain the clout to negotiate the course of education with society and the State. This is precisely what occurred with Eliane and the GdS community when they led the protest move-

ment against the curricular policies that had been imposed by the Secretary of State for São Paulo.

The study demonstrates advances and evidence that it is important for the teacher to research her practice and participate in research communities. It provides a rich context for learning and professional development and a way to improve teaching practices, academic achievement, school culture and public policy in Brazil.

Yet the powers that be have still not come to value this type of professional. That was one of the reasons Eliane opted to work exclusively in higher education. With the exception of a few federal schools – where the teachers have a salary and work, study and research conditions equivalent to those of university teachers – the great majority of Brazilian schools continue not valuing the teacher who wants and likes to examine both the finer points and brass tacks of her own teaching. Unfortunately the normal school setting in Brazil has become a no man's land for teachers wishing to enhance their professionalism through research.

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