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Research Paper

Estimating the Mode Shapes of a Bridge Using Short Time Transmissibility Measurement from a Passing Vehicle

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Abstract. This paper reports on the analysis of the signals sent by accelerometers fixed on the axles of a vehicle which passes over a bridge. The length of the bridge is divided into some parts and the transmissibility measurement is applied to the signals recorded by two following instrumented axles. As the transmissibility procedure is performed on the divided signals, the method is called Short Time Transmissibility Measurement. Afterwards, a rescaling process is accomplished in order to estimate the bridge mode shapes. The numerical results indicate that the method can calculate the mode shapes of the bridge accurately. It is demonstrated that short time transmissibility method does not depend on the excitation characteristics contrary to the other related methods which assume that the excitation should be white noise. Generally, the bridge mode shapes may be invisible due to the excitation exerted by the road profile. This issue is also resolved by subtracting the signals from the successive axles. Finally, the signals are contaminated with noise and the robustness of the method is investigated.

Keywords: Transmissibility measurement, Vehicle bridge interaction, Passing vehicle, Bridge mode shapes, Road profile.

1. Introduction

Main parameters of a bridge, namely frequency, mode shape, and damping, which are also known as the modal parameters are to be considered for engineering applications. For instance, one may desire to measure the few frequencies of the bridge for model updating [1] or to utilize damping ratios [2, 3] and mode shapes [4] as indicators for non-destructive damage detection. Moreover, it has the potential to contribute to the future design of similar bridges. A clear understanding of the structural condition of a bridge can definitely help to rehabilitate it over to extend its service life.

There have been many direct methods which are conventionally employed to estimate the modal parameters of a bridge including on-site measurement methods such as the ambient vibration test, forced vibration test, and impact vibration test. Such methods are called direct bridge measurement methods as they depend directly on the bridge responses, which necessitates the installation of quite a large number of vibration sensors on the bridge. Abundant research has been conducted in these areas and the on-going traffic or controlled vehicular movement has been reported as the source of excitation in some of the early studies [5, 6]. To employ the direct approach, which is normally designed on a one-system-per-bridge basis, and if the mode shapes of the bridge are wanted, a large number of vibration sensors should be fixed on a single bridge. Additionally, it should be taken into account that on-site instrumentation on the bridges is expensive, risky and more significantly not maintenance free. Moreover, only a small number of the in-used bridges are equipped with some monitoring systems, whereas for the equipped bridges the

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large amount of information generated by the sensing devices has proved an unsurmountable problem. As a consequence, one can hardly guarantee that the monitoring system along with the sensors and central data logger mounted on a bridge can metaphorically speaking, live as long as the bridge itself have a lifespan longer than that the bridge.

Yang et al. tried the idea of extracting the bridge frequencies from a passing test vehicle to deal with such a problem in 2004 when they considered only the first frequency of the bridge [7]. It took, however, no more than one year for the feasibility of the idea to be verified by a field test [8]. Such a technique which makes use of the vehicle-collected data could be grossly called as the indirect method for bridge measurement as no sensors are needed to be used on the bridge and only a small amount of sensors is required on the vehicle. In comparison to the direct method, the technique enjoys such advantages as mobility, economy, and efficiency.

Oshima et al. [9], for example, employed independent component analysis (ICA) to estimate the bridge frequencies indirectly from the response of a vehicle. To preprocess vehicle measurements so as to increase the visibility of the second frequency of the bridge, Yang and Chang [10] used the empirical mode decomposition (EMD) technique. Optimization, one the other hand, has been recently used as a technique to extract the bridge frequency from a vehicle which passes over a bridge. A typical search method in optimization, the Generalized Pattern Search Algorithm (GPSA), is developed by Li et al. [11] as a new theoretical method. Malekjafarian and Obrien [12], in a theoretical study, utilized frequency domain decomposition (FDD) to make use of vehicle response. FDD is a generally known operational modal identification technique which employs singular value decomposition (SVD) of the power spectrum density (PSD) of the response. The FDD method, in the study, was employed for identification of frequency of the bridge together with the vehicle frequency from the signals of the two following axles.

Indirect estimation of the mode shapes plays an important role in the bridge engineering tasks. What adds to its significance is that the damage generates the discontinuities in the mode shapes which includes discontinuities of the slope at the points of localized damage. It has been reported that mode shape curvatures may come in handy in finding these discontinuities [13, 14]. Yang et al. [15] have introduced a new method which is built upon the instantaneous amplitudes taken from the Hilbert transform of the vehicle response which is band-pass filtered. They pointed out in the method that compared to the conventional method, the indirect approach can prepare desirable spatial resolution in mode shapes since the vehicle plays a role as a moving sensor. Moreover, Oshima et al. [16], theoretically examined a damage method that requires the extracted mode shapes from the vehicles which have been moving on a bridge. Zhang et al. [17] have carried out research which can be viewed as the pioneer efforts to identify bridge mode shapes indirectly. The method they used, however, is based on employing a machine which both excites and measures the exciting force that seems to be challenging while performing in a real case. Oshima et al. [18] have tested a similar apparatus in a field experiment and some interesting ideas [15, 16, 19] have also been proposed which are only relied on the response measured from a travelling vehicle. The method by Yang et al. [15] provides high-resolution mode shape with appropriate level of accuracy, particularly for the first mode shape. Malekjafarian and OBrien [19] have also proposed the utilization of short time frequency domain decomposition (STFDD) to extract bridge mode shapes indirectly by making use of the responses measured from two consecutive axles.

When it comes to the road profile, the problem of identification will be more challenging. McGetrick et al. [20] suggested the lower speed for the vehicle in order to identify the damping of the bridge more accurately when a road profile exists. At a higher speed, the vehicle will be excited more than ordinary and the peaks related to the vehicle frequencies will be increased in the spectrum. Chang et al. [21] declared the same statement as [20] and confirmed that the influence of the vehicle frequencies are raised higher when the road profile exists.

To increase the mode shape resolution and overcome noise difficulties, He et al. [22] conducted research on extracting the mode shapes of beam structures from a passing vehicle by Hilbert transform. However, the need for reference data has been resolved by utilizing an index which is based on RMSC over the damaged and undamaged state. Meanwhile, He et al. [23] extracted the mass-normalized bridge mode shapes by taking advantage of the load-bearing properties of the bridge. The approach was approved to be low sensitive to road roughness, unlike many other indirect methods. Although the mode shapes with high-resolution were obtained with only one sensor, the vehicle stiffness was assumed to be infinity and the vehicles were also parked on the bridge during data acquisition.

Malekjafarian and O'Brien [24] utilized an external excitation in order to estimate the mode shapes of a bridge from a moving vehicle. The frequency of the excitation was close to at least one of the bridge natural frequencies and the measurements were conducted at two axle points. As the amplitude of the signal may contain data about the operational deflected shape, the bridge mode shapes can be constructed using the amplitude of the signal derived from the Hilbert Huang Transform. However, a rescaling procedure may be conduct in order to estimate the global mode shapes. Recently, Yang et al. [25] conducted a review of indirect modal identification and damage detection of bridges which has been published since 2004. Useful recommendations and drawbacks along with the ideas for future research were pointed out.

As an alternative operational modal parameters identification of the structures, other methods have also been proposed recently which work based on the transmissibility concept [26-28]. What makes these direct approaches [26-28] more advantageous is the independence of the characteristics of the exerted forces on the structures, unlike previous methods in the frequency domain have made the white noise excitation as a preliminary assumption. Moreover, another indirect damage detection research [29] have been performed using the transmissibility of a Vehicle Bridge Interaction (VBI). It should be noted that in [29], recording data should be fulfilled when both vehicles are stopped at the end of each stage or at least one of them is supposed to be as a fixed reference.

In this paper, Short Time Transmissibility Measurement (STTM) is presented in order to indirectly achieve the bridge mode shapes. It is demonstrated that the transmissibility method does not depend on the excitation characteristics contrary to the other related methods such as FDD which assume that the excitation should be white noise. In the STTM method, the vehicle passes over the bridge without stopping or pausing and records the signals continuously. By defining some segments in the bridge, a multi-stage measurement is followed through the segments. The Singular Value Decomposition method is applied to transmissibility matrices of acceleration responses of the two following axles in each stage. Accordingly, the local mode shapes are estimated corresponding to each stage. Thereafter, the global mode shape vectors of the bridge are constructed by a certain procedure called rescaling. The method is numerically investigated by case studies. Road profile and noise are also included in the simulations and by making use of the models such as three following quarter cars and truck-trailers, the proposed method is confirmed to extract the mode shapes with acceptable accuracy.

2. Theory

2.1. Vehicle Bridge Interaction model

The quarter-car models (Fig. 1) are considered here in order to investigate the interaction between the vehicle and bridge.

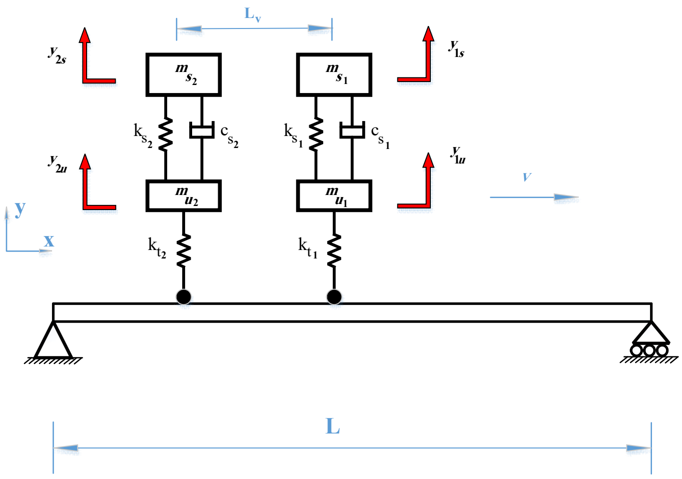

Fig. 1. Vehicle bridge interaction model.

The first and second axles of the vehicle are recognized by 1 and 2 indices, respectively, and the moment of inertia is neglected for simplification. m_s refers to the half of the vehicle body mass (sprung) and m_u, k_s, k_t, c_s stand for mass of the axle (unsprung), suspension stiffness, tire stiffness, and suspension damping, respectively. y_s is the time-dependent vertical displacements of sprung mass while y_u is the time dependent vertical displacements of un-sprung mass. v is the constant velocity of the vehicle and L_v denotes the distance between two axles. During the passage over the bridge, dynamic interaction may cause vibration in such a way that the responses of the masses are influenced by the response of the bridge.
The equations of motion in the interaction between the quarter-car models and bridge which are shown in Fig. 1 are as follows:

$$m_{s1}\ddot{y}_{s1} + c_{s1}\left(\dot{y}_{s1} - \dot{y}_{u1}\right) + k_{s1}\left(y_{s1} - y_{u1}\right) = 0, \tag{1}$$

$$m_{u1}\ddot{y}_{u1} - c_{s1}\left(\dot{y}_{s1} - \dot{y}_{u1}\right) - k_{s1}\left(y_{s1} - y_{u1}\right) + k_{t1}\left(y_{u1} - y_b|_{x=vt+L_v}\right) = 0, \tag{2}$$

$$m_{s2}\ddot{y}_{s2} + c_{s2}\left(\dot{y}_{s2} - \dot{y}_{u2}\right) + k_{s2}\left(y_{s2} - y_{u2}\right) = 0, \tag{3}$$

$$m_{u2}\ddot{y}_{u2} - c_{s2}\left(\dot{y}_{s2} - \dot{y}_{u2}\right) - k_{s2}\left(y_{s2} - y_{u2}\right) + k_{t2}\left(y_{u2} - y_b|_{x=vt}\right) = 0, \tag{4}$$

$$\begin{aligned} \bar{m}\ddot{y}_b + EIy_b'''' &= f_{c1}(t)\delta(x - vt - L_v) + f_{c2}(t)\delta(x - vt), \\ f_{c1}(t) &= k_{t1}\left(y_{u1} - y_b|_{x=vt+L_v}\right) - \left(m_{u1} + m_{s1}\right)g, \\ f_{c2}(t) &= k_{t2}\left(y_{u2} - y_b|_{x=vt}\right) - \left(m_{u2} + m_{s2}\right)g \end{aligned} \tag{5}$$

where y_b denotes the bridge displacement and $\bar{m}$, E, I, $f_{c1}(t)$ and $f_{c2}(t)$ are the mass per unit length, elastic modulus, moment of inertia of the bridge, and contact forces, respectively. As the vertical displacements are measured from static equilibrium, no gravity force appears in the dynamic equations of motion. Using modal superposition and considering the bridge as a simply supported beam, the displacement can be expressed as follows:

$$y_b(x,t) = \sum_n \left[\sin\frac{n\pi x}{L} q_{bn}(t)\right], \tag{6}$$

Making use of the orthogonality relations between the modal shapes, the nth modal equation will be as follows:

$$\ddot{q}_{bn} + \omega_{bn}^2 q_{bn} = \frac{f_{c1}(t)\sin\dfrac{n\pi(vt+L_v)}{L} + f_{c2}(t)\sin\dfrac{n\pi vt}{L}}{\bar{m}\dfrac{L}{2}}, \tag{7}$$

where L is the total length of bridge and ω_{bn} denotes the frequency of vibration of the nth mode shape as:

$$\omega_{bn} = \frac{n^2\pi^2}{L^2}\sqrt{\frac{EI}{\bar{m}}}, \tag{8}$$

Assuming the masses of the quarter-car models are much less than the mass of the bridge $\bar{m}L$; i.e., $(m_{s1}+m_{u1})/\bar{m}L = 1$ and $(m_{s2}+m_{u2})/\bar{m}L = 1$, Equation (5) can be rewritten as:

$$\ddot{q}_{bn} + \omega_{bn}^2 q_{bn} = -\frac{2(m_{s1}+m_{u1})g}{\bar{m}L}\sin\frac{n\pi(vt+L_v)}{L} - \frac{2(m_{s2}+m_{u2})g}{\bar{m}L}\sin\frac{n\pi vt}{L}, \tag{9}$$

Using zero initial conditions and substituting q_{bn} for the bridge coordinate into Eq. (2) and assuming a harmonic response, the body and axle responses of the first quarter-car is given by [30]:

$$\begin{Bmatrix} q_{u1} \\ q_{s1} \end{Bmatrix} = \left([K_1] - \omega^2[M_1] + i\,\omega[C_1]\right)^{-1}\begin{Bmatrix} f_1(t) \\ f_2(t) \end{Bmatrix}, \tag{10}$$

in which:

$$[M_1] = \begin{bmatrix} m_{u1} & 0 \\ 0 & m_{s1} \end{bmatrix},\ [K_1] = \begin{bmatrix} k_{t1}+k_{s1} & -k_{s1} \\ -k_{s1} & k_{s1} \end{bmatrix},\ i = \sqrt{-1}\ ,$$
$$[C_1] = \begin{bmatrix} c_{s1} & -c_{s1} \\ -c_{s1} & c_{s1} \end{bmatrix},\quad \begin{Bmatrix} f_1(t) \\ f_2(t) \end{Bmatrix} = \begin{Bmatrix} k_{t1}\sum_n q_{bn}\sin\dfrac{n\pi(vt+L_v)}{L} \\ 0 \end{Bmatrix} \tag{11}$$

In a similar way, the body and axle responses of the second quarter-car model can be obtained.

2.2. Theory of Short Time Transmissibility Measurement

In this part, indirect estimation of the mode shapes of the bridge is proposed. The transmissibility measurement of the responses of the quarter-car models and constructing the global mode shapes from estimated local mode shapes will be discussed in several steps. First, a short theoretical background of transmissibility measurement is presented.

2.2.1. Theoretical background to modal parameter identification using SVD of power spectrum density transmissibility matrices

In the case of random processes, the power spectrum density transmissibility (PSDT) is defined [31] as a ratio between the power spectrum density (PSD) of points, say, o and j with reference to point, say, k , as follows:

$$T_{x_o x_j}^{k}(i\omega) = \frac{S_{x_o x_k}(i\omega)}{S_{x_j x_k}(i\omega)}, \tag{12}$$

in which i stands for $\sqrt{-1}$, $S_{x_o x_k}$ is the PSD of outputs x_o and x_k , $S_{x_o x_k}$ is the PSD of outputs x_j and x_k . By using the relation between the PSDs of the outputs and the inputs (forces), the above equation will change into [31]:

$$T_{x_o x_j}^{k}(i\omega) = \frac{\sum_{r=1}^{N}\sum_{s=1}^{N} H_{x_o f_r}^{*}(i\omega) H_{x_k f_s}(i\omega) S_{f_r f_s}(i\omega)}{\sum_{r=1}^{N}\sum_{s=1}^{N} H_{x_j f_r}^{*}(i\omega) H_{x_k f_s}(i\omega) S_{f_r f_s}(i\omega)} \tag{13}$$

in which H^* and H are the transfer functions of the system and * represents the complex conjugate. By using the modal contribution representations and the assumptions such as the mode shapes to be well separated, modal damping ratios to be small, the dynamical response of the system at the resonant frequency to be dominated by the contribution of the corresponding mode shape and the other mode shapes to have negligible contributions, one may have [31]:

$$\lim_{i\omega\to i\omega_l} T_{x_o x_j}^{k}(i\omega) = \lim_{i\omega\to i\omega_l}\frac{S_{x_o x_k}(i\omega)}{S_{x_j x_k}(i\omega)} = \frac{\psi_{ol}^{*}\sum_{r=1}^{N}\sum_{s=1}^{N}\hat{H}_{z_l f_r}^{*}(i\omega_l) S_{f_r f_s}(i\omega_l)\hat{H}_{z_l f_s}(i\omega_l)\psi_{kl}}{\psi_{jl}^{*}\sum_{r=1}^{N}\sum_{s=1}^{N}\hat{H}_{z_l f_r}^{*}(i\omega_l) S_{f_r f_s}(i\omega_l)\hat{H}_{z_l f_s}(i\omega_l)\psi_{kl}} = \frac{\psi_{ol}^{*}}{\psi_{jl}^{*}}, \tag{14}$$

where ψ_{kl} and ψ^*_{ol} are the amplitudes of mode l; k and o stand for the degrees of freedom and ω_l is the natural frequency of mode l. In a similar way, by defining a different reference z, the transmissibility converges to the same ratio when $i\,\omega = i\,\omega_l$ [31]:

$$\lim_{i\omega\to i\omega_l} T^{z}_{x_o x_j}(i\,\omega) = \lim_{i\omega\to i\omega_l} \frac{S_{x_o x_z}(i\,\omega)}{S_{x_j x_z}(i\,\omega)} = \frac{\psi^*_{ol}}{\psi^*_{jl}}, \tag{15}$$

Similarly, by varying different points o_i, a transmissibility matrix can be constructed as following:

$$\left[T^{z_i}_{x_{o_i} x_j}(i\,\omega)\right] = \begin{bmatrix} T^{z_1}_{x_1 x_j}(i\,\omega) & T^{z_2}_{x_1 x_j}(i\,\omega) \\ T^{z_1}_{x_2 x_j}(i\,\omega) & T^{z_2}_{x_2 x_j}(i\,\omega) \end{bmatrix}, \tag{16}$$

By coming close to the system poles, the transmissibility matrices will converge to the matrix such that:

$$\lim_{i\omega\to i\omega_l}\left[T^{z_i}_{x_{o_i} x_j}(i\,\omega)\right] = \begin{bmatrix} \dfrac{\psi^*_{1l}}{\psi^*_{jl}} & \dfrac{\psi^*_{1l}}{\psi^*_{jl}} \\ \dfrac{\psi^*_{2l}}{\psi^*_{jl}} & \dfrac{\psi^*_{2l}}{\psi^*_{jl}} \end{bmatrix} \quad \text{where } \frac{\psi^*_{o_i l}}{\psi^*_{jl}} = 1 \text{ if } x_{o_i} = x_j, \tag{17}$$

The columns of the matrix in Eq. (17) are linearly dependent, therefore, the rank of the proposed transmissibility matrix (Eq. (17)) is equal to one when $i\,\omega = i\,\omega_l$. By performing the singular value decomposition, the second singular value will converge to zero. Then, the inverse of the second singular value can be used to obtain the natural frequencies of the system [31]. The SVD of the transmissibility matrices $T^{z_i}_{x_{o_i} x_j}(i\,\omega)$ is defined as:

$$\left[T^{z_i}_{x_{o_i} x_j}(i\,\omega)\right] = \left[U^j(i\,\omega)\right]_{2\times 2}\left[\Sigma^j(i\,\omega)\right]_{2\times 2}\left[V^j(i\,\omega)\right]^*_{2\times 2}, \tag{18}$$

The left and right singular vectors of $[T^{z_i}_{x_{o_i} x_j}(i\,\omega)]$ are defined, respectively, as $[U^j(i\,\omega)]$ and $[V^j(i\,\omega)]$ and * indicates the conjugate transpose. The singular values of $[T^{z_i}_{x_{o_i} x_j}(i\,\omega)]$ are arranged in a diagonal matrix, say $[\Sigma^j(i\,\omega)]$, such a way that $\sigma_1(i\,\omega) \geq \sigma_2(i\,\omega)$. For each transmissibility matrix $[T^{z_i}_{x_{o_i} x_j}(i\,\omega)]$, the singular values are shown as $\Sigma(i\,\omega)^j = [\sigma(i\,\omega)^j_1 \quad \sigma(i\,\omega)^j_2]$. The inverse of the second singular value is define as follows:

$$\pi(i\,\omega) = \frac{1}{\sigma(i\,\omega)_2}, \tag{19}$$

The function $\pi(i\,\omega)$ depicts the resonant peaks which are the natural frequencies of the system. Therefore, modal parameter identification using singular value decomposition of power spectrum density transmissibility matrices (PSDTM-SVD) utilizes a singular value to evaluate natural frequencies. Since the singular vectors $[U(i\,\omega)]$ are the eigenvectors of the matrix $[T^{z_i}_{x_{o_i} x_j}(i\,\omega)] \times [T^{z_i}_{x_{o_i} x_j}(i\,\omega)]^T$, the first singular vector $[U(i\,\omega)]_1$ will converge to the mode shape of the system if the matrix $[T^{z_i}_{x_{o_i} x_j}(i\,\omega)] \times [T^{z_i}_{x_{o_i} x_j}(i\,\omega)]^T$ tends toward the system poles [31]. Determining the resonant frequencies of the system, one could obtain the mode shapes using the first left singular vector $[U^j(i\,\omega)]_1$ in the natural frequency, $i\,\omega_l$.

2.2.2. Short Time Transmissibility Measurement (STTM)

The idea of rescaling the local mode shapes of the bridge which are extracted from short time measurement was first proposed by Malekjafarian and O'brien [19] based on the FDD method. The aim of the short time transmissibility measurement is the construction of the approximated global mode shapes from the local mode shapes of the bridge. In order to estimate the local mode shapes, the recorded signals from the two following quarter-car models in short time durations are imported into transmissibility relation. The distance between the two axles is used for dividing the bridge. The acceleration matrix is defined as:

$$\ddot{Y} = \begin{bmatrix} \ddot{y}_{i,2} \\ \ddot{y}_{i+1,1} \end{bmatrix} \quad i = 1 : N-1, \tag{20}$$

where $\ddot{y}_{i+1,1}$ and $\ddot{y}_{i,2}$ denotes the acceleration of the first and second axles when moving along the $(i+1)$th and ith segment, respectively. Also, N is the number of segments. The local mode shape vector related to $(i+1)$th and ith segment is calculated by performing the transmissibility procedure on the acceleration matrix, $\ddot{Y}$. The procedure of the method is shown in Fig. 2

schematically.

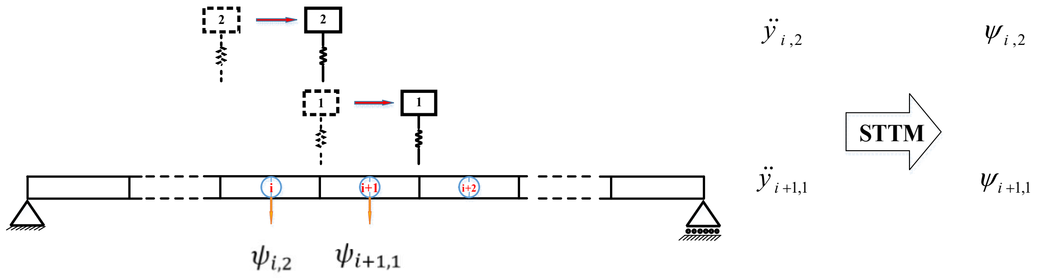

Fig. 2. Two following axles for performing the short time transmissibility measurement.

Fig. 2 shows the procedure of obtaining the ith acceleration matrix, $\ddot{Y}$, and the related local mode shape vector which is estimated from STTM as ψ:

$$\psi = \begin{Bmatrix} \psi_{i,2} \\ \psi_{i+1,1} \end{Bmatrix}, \tag{21}$$

After the vehicle passed over the bridge, all of the local mode shape vectors are estimated. As the local mode shape vectors are not extracted simultaneously, these vectors must be scaled in order to achieve the global mode shape vectors. Thus, a consecutive scaling scheme should be utilized. In this scheme, the first local mode shape vector is assumed to be global, as follows:

$$\phi_1 = \psi_{1,2} \tag{22}$$

$$\phi_2 = \psi_{2,1} \tag{23}$$

in which ϕ_1 and ϕ_2 stand for the global mode shapes of the first and the second segments of the bridge, respectively. As the acceleration corresponding to the second segment of the bridge is recorded by the front and the rear axles at different times, two different values of mode shape are extracted for the same segment. By considering $\psi_{2,2}$ as the local mode shape of the second segment estimated by the rear axle, the scaling ratio would be calculated as $\phi_2/\psi_{2,2}$. Using the scaling ratio and considering the local mode shape of the third segment, $\psi_{3,1}$, the global mode shape of the third segment can be estimated by $\psi_{3,1}\phi_2/\psi_{2,2}$. In the same way, the other global mode shape elements can be found using their own scaling ratios which leads to:

$$\phi = \begin{Bmatrix} \phi_1 & \phi_2 & \psi_{3,1}\dfrac{\phi_2}{\psi_{2,2}} & \cdots & \psi_{i+1,1}\dfrac{\phi_i}{\psi_{i,2}} \end{Bmatrix}^T \qquad i = 3:N-1\,, \tag{24}$$

where ϕ is the global mode shape element related to the mid-point of the ith segment. It seems that more mode shape elements might be extracted along the bridge when more segments are considered. However, if the number of segments is increased, the lengths of the acceleration signals will be decreased so that the accuracy of the transmissibility measurement is reduced.

3. Numerical Simulations

3.1. Two Following Quarter-Cars

In this section, indirect estimation of the mode shapes of a bridge using short time transmissibility measurement is tested numerically. The properties of the vehicles are listed in Table 1. The vehicle properties have been chosen from [19], but the body and axle masses together with the distance between two axles have been changed in order to excite the mode shapes sufficiently. The bridge is assumed to be a simply-supported beam which properties are listed in Table 2. The first three natural frequencies of the bridge are available in Table 3 for further consideration.

The equal division of the bridge is considered to be ten as displayed in Fig. 3. As mentioned in the last section, the global mode shapes are assumed to be equidistant between two points for each segment. Therefore, the short time transmissibility measurement, here, will result in ten-element mode shape vectors.

Nine stages are considered here for the ten segments to make use of the procedure in section 2.2.2. The vehicle is passing over the bridge at the velocity of $3m/s$ and the simulations are performed by making use of the vehicle bridge interaction system illustrated in section 2.1. The time histories of acceleration responses from the two following quarter-car models are then calculated with a time interval, $dt = 0.001s$. The short time transmissibility measurement is applied to the shortened pieces of acceleration signals. The SVD diagrams are depicted from the nine stages of the short time transmissibility measurement (Fig. 4). Not considering the road effect in this case, make the two peaks of the bridge frequencies to be obvious to be detected in Fig. 4.

Table 1. Properties of the quarter-car models [19].

Properties	Symbol	Value
Body mass (kg)	m_{s1}, m_{s2}	17300
Axle mass (kg)	m_{u1}, m_{u2}	700
Suspension stiffness (N/m)	k_{s1}, k_{s2}	4×10^5
Suspension damping $(N.s/m)$	c_{s1}, c_{s2}	10×10^3
Tyre stiffness (N/m)	k_{t1}, k_{t2}	1.75×10^6
Distance between vehicles (m)	L_v	2.5

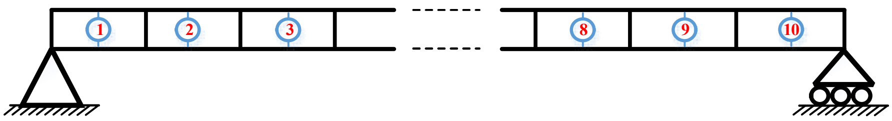

Fig. 3. Discretization of bridge into ten equal segments.

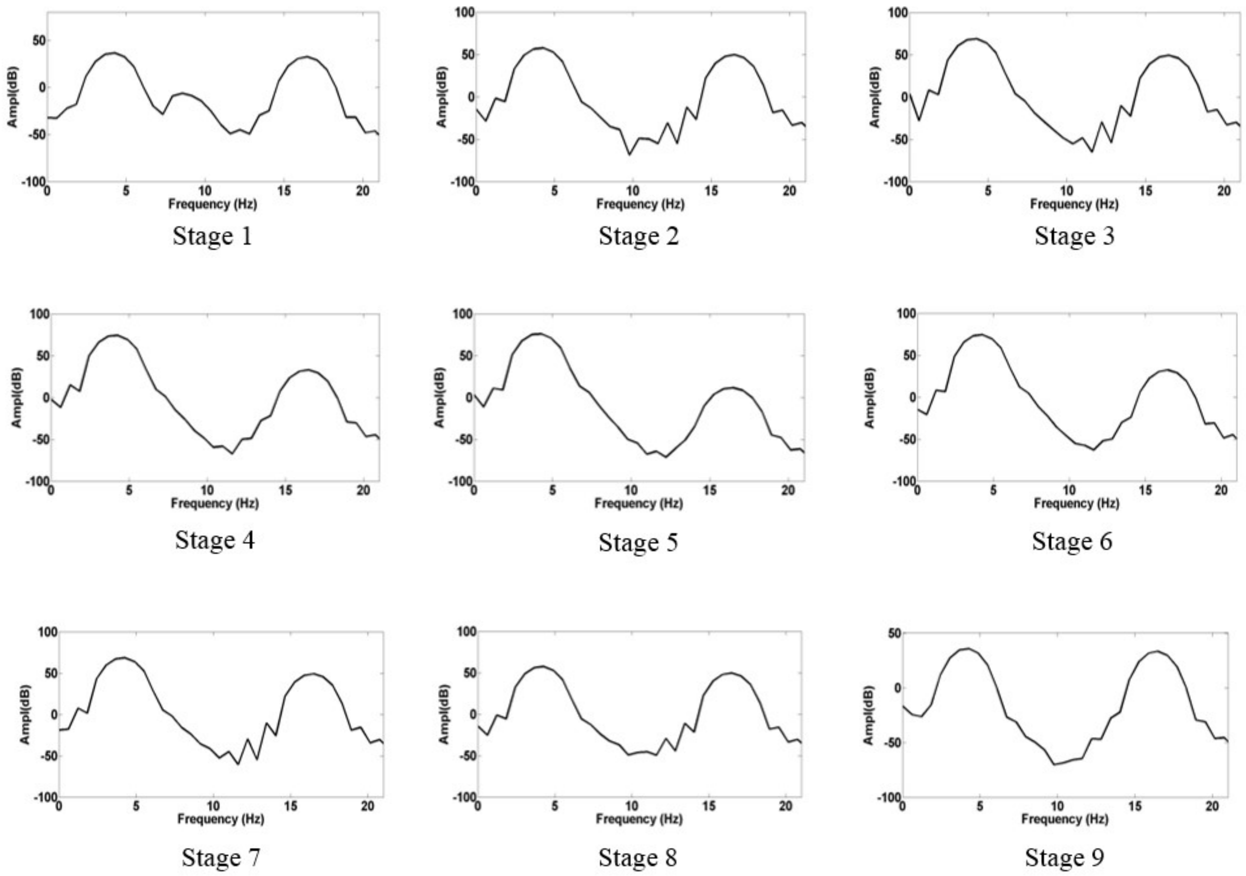

Fig. 4. SVD diagrams obtained from nine stages.

The local mode shape in each segment is defined by the singular vectors corresponding to their own singular values which are the peaks of the diagrams. In order to extract the first two global mode shapes of the bridge, the rescaling process of the local mode shapes would be carried out (Fig. 5).

Table 2. Properties of the bridge [20].

Properties	Symbol	Value
Mass per unit (kg/m)	$\bar{m}$	18358
Length (m)	L	25
Modulus of elasticity (MPa)	E	35000
Second moment of area (m^4)	I	1.3901

Table 3. First three natural frequencies of the bridge.

Mode no.	1	2	3
Natural frequency (Hz)	4.09	16.36	36.82

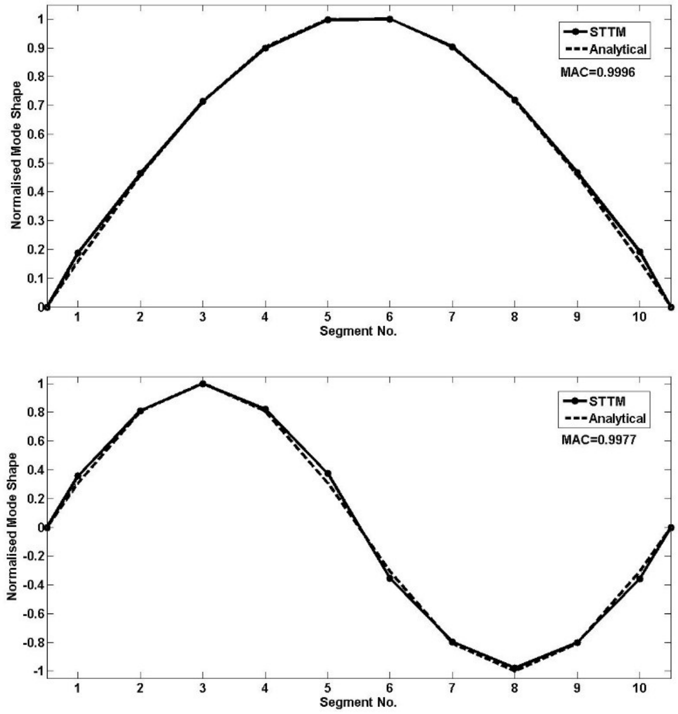

Fig. 5. The first and second mode shapes of the bridge.

By employing the Modal Assurance Criterion (MAC), the calculated mode shapes can be quantitatively compared with the exact analytical mode shapes [30]:

$$MAC = \frac{\left|\phi_{STTM}^{T}\,\phi_{A}\right|^{2}}{\left|\phi_{STTM}^{T}\,\phi_{STTM}\right|\left|\phi_{A}^{T}\,\phi_{A}\right|}, \tag{25}$$

where ϕ_{STTM} is the global mode shape extracted by the proposed method and ϕ_A is the exact analytical mode shape and "T" denotes transpose of the vector. Acceptable agreement can be deduced from the MAC results (Fig. 5).

3.2 Estimation of the Bridge Mode Shapes Considering Road Profile

3.2.1. Three following quarter-cars

Identifications of bridge parameters would be unpleasant when the road profile is considered in the simulations [7, 21, 32, 33]. Subtraction of the signals is suggested by Yang et al. [34] as an idea to overcome this obstacle. Two axles with the same properties were utilized in order to subtract the measured acceleration responses. It was also shown by Yang et al. [35] that heavy traffic may result in visibility of bridge frequency. However, this idea would be applicable in long-span bridges where the presence of a huge number of vehicles on the bridge is very likely. In the proposed STTM method, the responses of two axles are necessary to estimate the mode shape in each stage. So, three following axles should be available for subtraction idea at each stage (Fig. 6).

Nine difference matrices of acceleration are obtained from the ten stages as follows:

$$\ddot{X} = \begin{bmatrix} \ddot{y}_{i,2} - \ddot{y}_{i,3} \\ \ddot{y}_{i+1,1} - \ddot{y}_{i+1,2} \end{bmatrix} \quad i = 1:N-1, \tag{26}$$

The first and second row in the above matrix are related to the ith and $(i+1)$th segment of the bridge, respectively. The second index of $\ddot{y}$ is related to the axle number. Also, N is the number of segments. The STTM is then applied and the global mode shapes are obtained by rescaling procedure. The irregularities of the profile are randomly generated according to the ISO standard [36] for a road class 'A' (very good) profile. The VBI models are generally divided into two groups of coupled and uncoupled models. In this paper, the coupled system of equations [37] are considered and the solution is numerically calculated at the time steps.

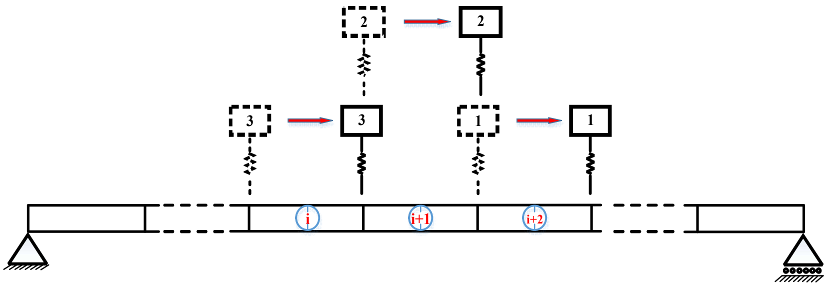

Fig. 6. Three following axles for subtraction of measured responses considering road profile.

A simply supported bridge by the length of $25\,m$ is discretized into 10 beam elements with 4 degrees of freedom. The system of the equation of motion for the response of the discretized beam model is given by:

$$M_b\ddot{y}_b + C_b\dot{y}_b + K_b y_b = f_{int}, \tag{27}$$

where $M_b, C_b, K_b, \ddot{y}_b, \dot{y}_b$ and y_b indicate global mass, damping and stiffness matrices, the vectors of nodal bridge accelerations, velocities, and displacements, respectively. Assuming that the damping ratio, ξ, is small and as a kind of Rayleigh damping which is given by:

$$C_b = \alpha M_b + \beta K_b, \tag{28}$$

in which, α and β are constants and the damping ratios of the modes are considered to be the same as each other. α and β can be estimated by the following relations [38]:

$$\alpha = \frac{2\xi\omega_1\omega_2}{\omega_1+\omega_2}, \ \beta = \frac{2\xi}{\omega_1+\omega_2} \tag{29}$$

where ω_1 and ω_2 are the first and second natural frequencies of the bridge, respectively. For illustrating the vehicle model, a system of two quarter-car model (Fig. 1) used here as:

$$M_v\ddot{y}_v + C_v\dot{y}_v + K_v y_v = f_{int} \tag{30}$$

The vehicle and the bridge are coupled at the tyre contact points via the interaction force vector. Combining Eqs. (27) and (30), the coupled equation of motion is formed as:

$$M_g\ddot{u} + C_g\dot{u} + K_g u = F \tag{31}$$

where M_g and C_g are the mass and damping matrices of the combined system, respectively, K_g stands for the coupled time-varying system stiffness matrix and F represents the system force vector. The vector, $u = \{y_v \ , y_b\}^T$, indicates the displacement vector of the system. Wilson-Theta integration scheme [39] is employed to solve the above equation for the coupled system and the optimal value of the parameter $\theta = 1.420815$ guarantees unconditional stability. The initial conditions such as displacement, velocity and acceleration are set to be zero in all simulations.

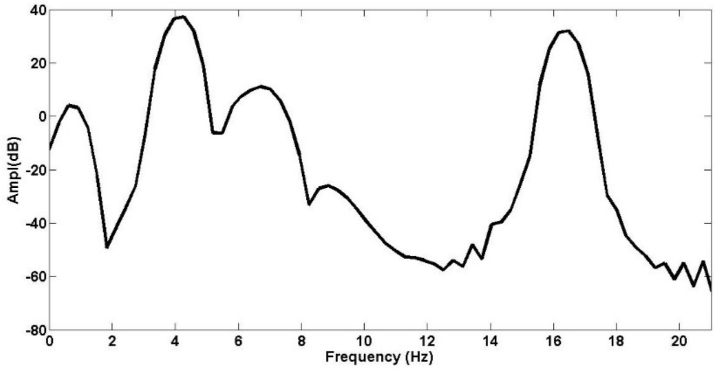

Fig. 7. The SVD diagram of the first Stage in the case of three quarter cars without considering noise when Class A profile is present.

As the signals in the engineering applications are always present either in excitations in the environment, interferences in a test or etc., the sensitivity of the algorithm to the noise is of great importance for engineering applications such as health monitoring the structures. Measuring the response and excitation signals at the same position on the bridge lead to robustness to noise in extracting the dynamic properties of the bridge from a passing vehicle [17]. In this study, the response signals of the vehicles are contaminated with noise to investigate the measurements accompanied by noise, as follows:

$$w = w_{calc} + E_p N_{noise} \sigma\left(w_{calc}\right) \quad (32)$$

where w denotes the polluted acceleration, E_P stands for the noise level, N_{noise} is a standard normal distribution vector with zero mean value and unit standard deviation, w_{calc} is the calculated displacement and $\sigma(w_{calc})$ points out the standard deviation of the calculated displacement. Then, the short time transmissibility measurement is applied with different levels of noise and the mode shape vectors are constructed for each level. The SVD diagram, say for stage 1, and the first two mode shapes are presented in Fig. 7 and Fig. 8, respectively.

Increasing the noise level up to 15% and 10% cannot prevent the method to obtain the first and second mode shapes, respectively, with reasonable accuracy (Fig. 9). The MAC values are reported in Fig. 9 for various noise levels of 5%, 10% and 15%.

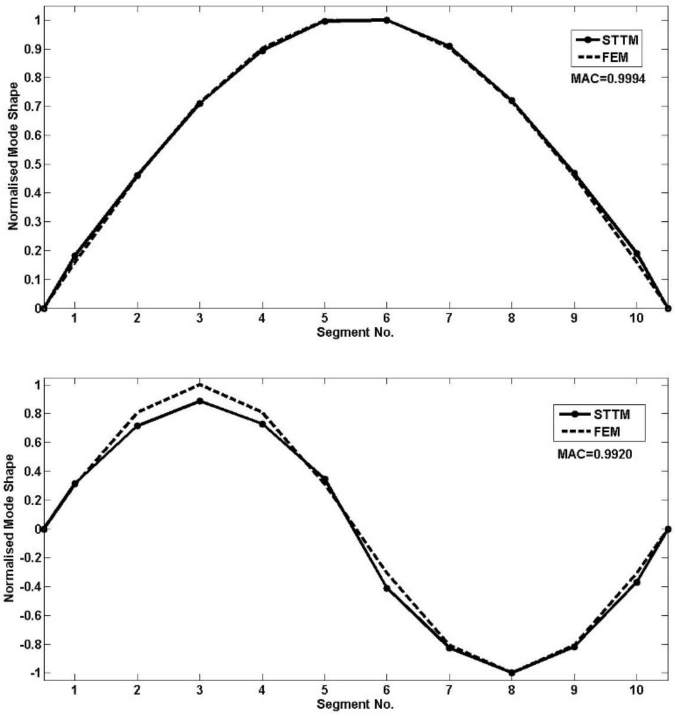

Fig. 8. The 1st and 2nd mode shapes of bridge in the case of three quarter cars by considering Class A profile without any other noise.

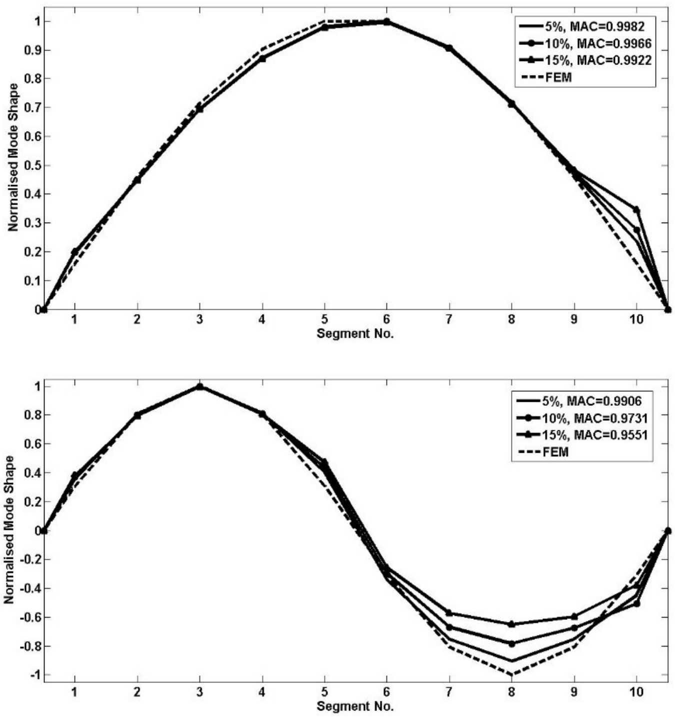

Fig. 9. The 1st and 2nd mode shapes of bridge in the case of three quarter cars by considering Class A profile accompanied by noise.

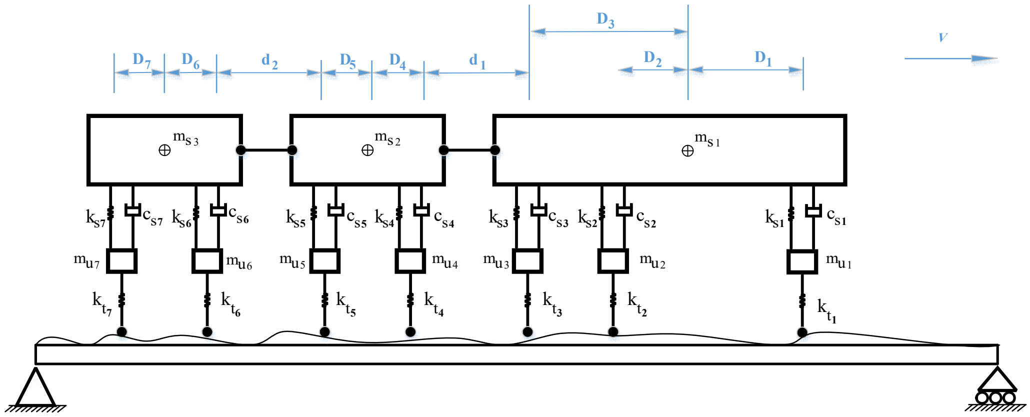

Fig. 10. The truck-trailers model.

Table 4. Properties of the truck.

Properties	Symbol	Value
Body mass (kg)	m_{s1}	54200
Axle mass (kg)	m_{u1}	700
Suspension stiffness (N/m)	k_{s1}	4×10^5
	$k_{s2}=k_{s3}$	1×10^6
Suspension damping $(N.s/m)$	c_{s1}	10×10^3
	$c_{s2}=c_{s3}$	20×10^3
Tyre stiffness (N/m)	k_{t1}	1.75×10^6
	$k_{t2}=k_{t3}$	3.5×10^6
Moment of inertia $(kg.m^2)$	I_{s1}	3×10^5
Distance of axle to center of gravity (m)	D_1	4.75
	D_2	1.43
	D_3	3.23
Body mass frequency (Hz)	$f_{body,1}$	0.93
Axle mass frequency (Hz)	$f_{axle,1}$	8.82
	$f_{axle,2}$	10.17
	$f_{axle,3}$	10.18

Table 5. Properties of the trailers.

Properties	Symbol	Value
Body mass (kg)	$m_{s2}=m_{s3}$	16600
Axle mass (kg)	$m_{u4}=m_{u5}=m_{u6}=m_{t7}$	50
Suspension stiffness (N/m)	$k_{s4}=k_{s5}=k_{s6}=k_{s7}$	4×10^5
Suspension damping $(N.s/m)$	$c_{s4}=c_{s5}=c_{s6}=c_{s7}$	10×10^3
Tyre stiffness (N/m)	$k_{t4}=k_{t5}=k_{t6}=k_{t7}$	1.75×10^6
Moment of inertia $(kg.m^2)$	I_{s2}	95765
Distance between axle and center of gravity (m)	$D_4=D_5=D_6=D_7$	1.25
Distance between two trailers and truck to trailers (m)	$d_1=d_2$	1
Body mass frequency (Hz)	$f_{body,2}$	0.9967
Axle mass frequency (Hz)	$f_{axle,4}=f_{axle,6}$	33.00
	$f_{axle,5}=f_{axle,7}$	33.00

3.2.2. Truck-trailers

In this section, modal identification will be proceeded using a truck-trailers over the aforementioned bridge with Class A road profile. Excitation of the bridge is assigned to heavy truck and the accelerometers are mounted on the two following trailers with identical vibrational characteristics. Fig. 10 illustrates the schematic model for the interaction between the bridge and truck-trailers which travels with a speed of $2m/s$. Table 4 and Table 5 give an insight into the dynamical properties of the truck and trailers.

The accelerometers mounted on the fourth to seventh axles are dedicated to record the responses. In order to attenuate the effect of road profile, the two signals are defined as follows:

$$\ddot{X} = \begin{bmatrix} \ddot{y}_{i,6} - \ddot{y}_{i,4} \\ \ddot{y}_{i+1,7} - \ddot{y}_{i+1,5} \end{bmatrix} \quad i = 1 : N - 1, \tag{33}$$

The first and second row in the above matrix are related to the ith and $(i+1)$th segment of the bridge, respectively. The second index of $\ddot{y}$ is related to the axle number. Also, N is the number of segments. The STTM is applied to these response signals and the SVD diagrams are obtained, as for instance stage 6 in Fig. 11. The first and second mode shapes of the bridge are extracted from the STTM procedure with good accuracy (Fig. 12).

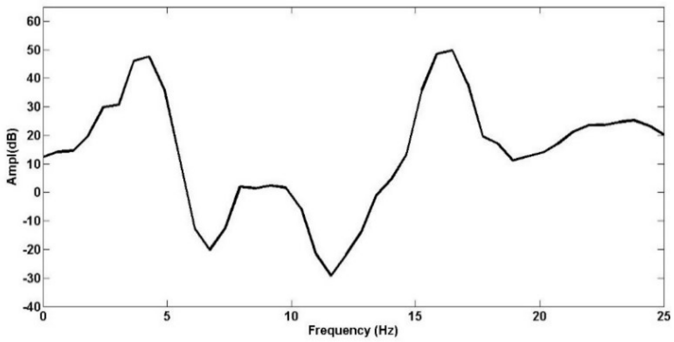

Fig. 11. The SVD diagram of the first Stage in the case of truck-trailers when considering Class A profile without any other noise.

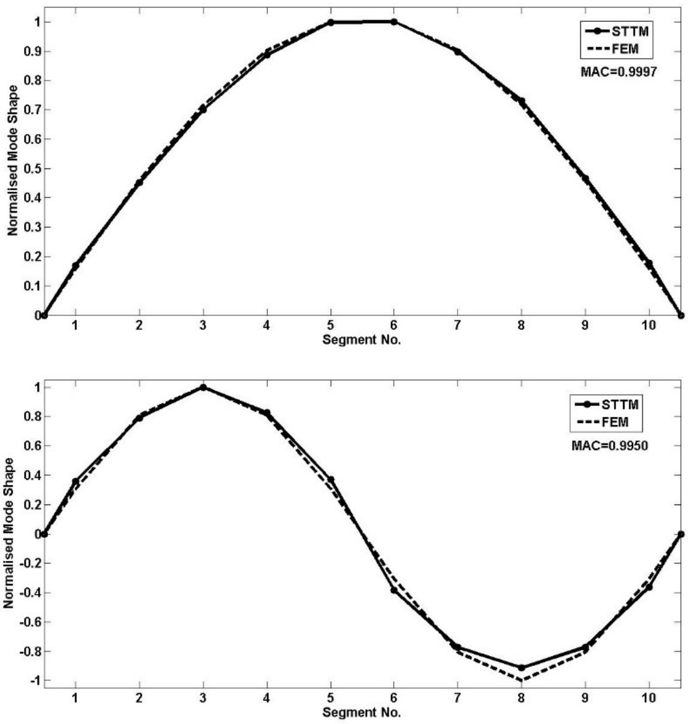

Fig. 12. The 1st and 2nd mode shapes of the bridge in the case of truck-trailers by considering Class A profile without any other noise.

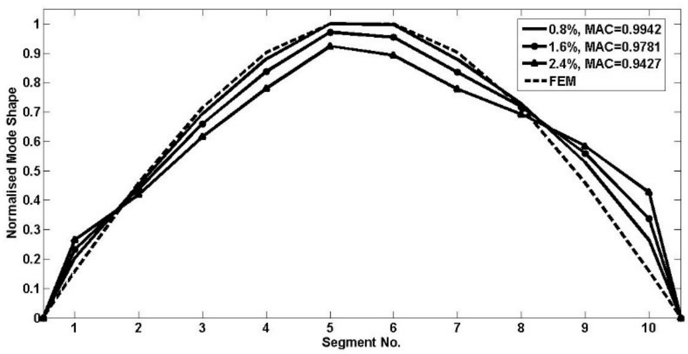

Fig. 13. The 1st and 2nd mode shapes of the bridge in the case of truck-trailers by considering Class A profile accompanied with noise.

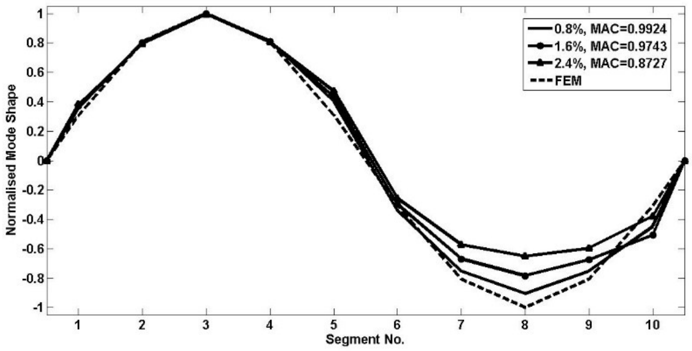

Fig. 14. Continued.

Fig. 14 shows the sensitive behavior of these mode shapes to noise pollution. Both mode shapes behave the same as each other by increasing the noise. Choosing the dynamic properties of the truck-trailer may result in this manner and high-accuracy accelerometers may be a choice to overcome this obstacle.

4. Conclusion

A new method for indirect estimation of the bridge mode shapes using transmissibility measurements is illustrated in this paper. The independence of the characteristics of the forces can be mentioned as an advantage of this method in comparison with the previous methods which assume white noise excitation. As a highlight difference between this method and the other transmissibility method it should be stated that there is no need to use reference vehicle, here, and also the vehicles are moving without any stop duration. In order to make the identification of the mode shapes applicable, two following axles are served as the recorder of the acceleration signals. Having divided the signals into some parts, the transmissibility measurement is put into service to make use of the short time signals obtained from each stage. A rescaling procedure is the last step after which the global mode shapes are extracted from local mode shapes. Moreover, subtraction of signals of the three identical axles is shown to be a good choice in order to diminish the effect of road profile. As another solution to resolve this drawback, a truck followed by two trailers is employed to serve both as exciter and receiver. The results of the numerical simulations show reasonable accuracy in obtaining the mode shapes of the bridge in the presence of low levels of noise.

Conflict of Interest

The author(s) declared no potential conflicts of interest with respect to the research, authorship and publication of this article.

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