

## Blind Channel Equalization of Star QAM using Dual Dispersion MCMA Algorithm

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**Abstract.** An algorithm for blind channel equalization is presented for 16 and 32 Star QAM, namely, Dual Dispersion MCMA algorithm. The algorithm taking the concept from MCMA, uses the Dual Dispersion minimization approach for blind channel equalization. As Star QAM constellation contains two rings, so instead of one, dual dispersion minimization approach is used for its both rings. With modification in MCMA cost function, the new algorithm results improved performance in convergence rate of Residual ISI and MSE against MCMA algorithm. By incorporating decision directed approach, the performance increases drastically. Simulation results show effectiveness of proposed algorithm in removing the ISI and correcting the errors in symbols of received signal.

### 1 Introduction

Recent advancement in wireless communication has refocused the attention of researchers to problems such as Equalization which is one of the signal processing issue in digital communication over distorted channel with Inter Symbol Interference (ISI). The selection of modulation scheme is the big challenge for engineers today because there is no such ideal scheme as different schemes are well suited for different type of communication. The limitation in availability of power, strict bandwidth allocation by regulatory authorities and varying nature of wireless channel demands an attention for selection of better modulation schemes. The selection process depends mainly on three parameters, implementation complexity, error rate and spectral efficiency.

Quadrature Amplitude Modulation (QAM) is a modulation scheme which is spectral efficient due to which higher transmission rate can be achieved. It is widely used in broadband multimedia transmission and in digital video broadcasting (DVB) standards but it has some disadvantages. The Square MQAM will have a high chance of false phase locking in channels where both amplitude and phase of the transmitted signal varies [1]. The traditional QAM suffers from high peak to average power ratio (PAPR) which can be overcome by Star QAM [2].

Star QAM is a constellation scheme in which symbol points are arranged as Phase Shift Keying (PSK) distributed over rings of different amplitude. The M ary Star QAM is better suited in vehicular environments [1], gives a better performance in heavy fading channels like the Rayleigh channel, also simplifies the receiver structure as

automatic gain control (AGC) and carrier recovery is not required [3].

In wireless communication, every signal that is transmitted undergoes some changes when received at the receiver. During the transmission in channel, distortion and noise are introduced. These distortions are due to two main factors, multi-path propagation and band limited channel. These effects cause the signals to distort and to introduce the ISI at the receiver. This process creates complexity for the receiver to distinguish between different information symbols in constellation which are originally transmitted by transmitter. In order to mitigate this effect of ISI, an equalizing filter is used at receiver end which models the inverse of channel so that the impairments introduced by channel can be cancelled out for the detection of actual information symbols which are transmitted. The equalizer is a transversal filter that tries to estimate inverse of channel, so that to mitigate the effects of channel. The equalizer uses the adaptive filtering algorithm which is based on minimization of certain cost function with the help of stochastic gradient algorithm.

There are two types of equalizer, training based equalizer and blind equalizer. Traditionally, training based equalizer relies on transmitter assisted training sessions to recover the reference signal. It was first developed by Lucky for telephone channels [4, 5]. A Least Mean Square (LMS) algorithm [6] was also proposed which starts adaptation with the aid of training sequence known to both transmitter and receiver. The disadvantage of these algorithms is that they add additional payload of training sessions on signal bandwidth hence limiting the total bandwidth of the system. Now recently there has been more research on blind equalizers where no training sessions are needed. In Blind Equalization, the equalizer exploits the signal statistics to model the inverse of channel response.

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A blind adaptive algorithm runs at the receiver end and changes the tap weights of equalizer so as to inverse model the channel. The research on these algorithms started with the publication by Sato [7]. The Godard and Thirion proposed Reduced Constellation Algorithm (RCA) [8] which minimizes the cost function comprising of fewer points. Then Godard invented Constant Modulus Algorithm (CMA) [9] which minimizes the certain cost function around a circle contour. After that Modified Constant Modulus Algorithm (MCMA) is proposed by [10, 11]. Instead of CMA, which tries to move the equalizer output around a circle, the MCMA tries to move the equalizer output to lie on four points in real and imaginary plane. After that Multi modulus algorithm (MMA) [12] and Square Contour Algorithm (SCA) [13] for square QAM constellation are proposed. An improved versions of SCA is also proposed [14–16]. For odd bit QAM constellations, namely, Cross QAM, constant cross algorithm (CXA) and sliced constant cross algorithm (SCXA) are proposed by [17]. For odd bit Rectangular QAM (RQAM), two algorithms namely, rectangular contour algorithm (RCA) and improved rectangular contour algorithm (IRCA) are proposed [18].

The performance of MCMA is not good for equalization of Star QAM and it has slow convergence rate to achieve the required ISI and MSE floor. By taking an inspiration from MCMA approach, we have modified the cost function of MCMA, instead of minimizing with only four points, the modified cost function tries to minimize with eight points, four for each rings of Star QAM which we called it Dual Dispersion minimization. Simulation result shows the improved performance of Dual Dispersion MCMA and by incorporating the decision directed approach with it, the performance enhances as compared to MCMA. The paper has been divided in to five sections. Second section deals with Star QAM and its merits. Third section summarizes the MCMA algorithm. Fourth section gives the information about the proposed algorithm. Fifth section gives the simulation results and comparison with existing algorithms.

## 2 Star QAM

Star QAM is a constellation scheme in which symbol points are arranged as Phase Shift Keying (PSK) distributed over rings of different amplitude. It can also be called as Amplitude and Phase Shift Keying (APSK).

Star QAM has some merits when compared to QAM constellation and for this reason we have chosen this constellation for its blind channel equalization. The merits of Star QAM are:

1. M-ary Star QAM scheme gives a better performance in heavy fading channels like the Rayleigh channel and also simplifies the receiver structure as automatic gain control (AGC) and carrier recovery are no longer required [3].

2. The authors in [1] have contended that the M-ary Star QAM modulation scheme will be better suited for vehicular environments.
3. The M-ary Star QAM overcomes the problem of high peak-to-average power ratio (PAPR) present in square and rectangular QAM schemes [2].

The Star QAM with first and second ring having amplitude of 1 and 2 is shown below:

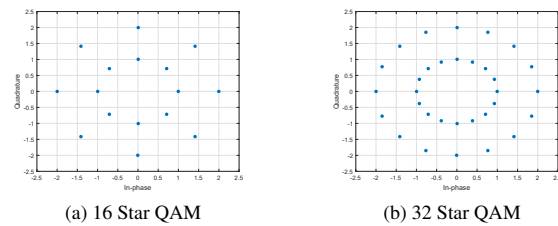


Figure 1. Star QAM Constellation

## 3 Communication System Model and Modified Constant Modulus Algorithm (MCMA)

A typical baseband communication system can be represented as shown:

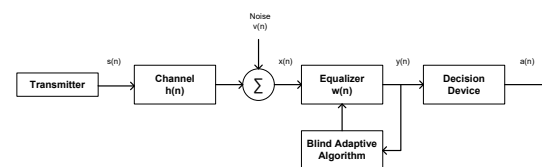


Figure 2. Simplified Baseband Communication System Model

In this figure

- $s(n)$  is the transmitted signal data
- $h(n)$  is the channel impulse response
- $v(n)$  is the noise
- $x(n)$  is the received signal
- $w(n)$  are the equalizer tap weights
- $y(n)$  is the equalizer output
- $a(n)$  is the actual transmitted signal symbols

The channel  $h(n)$  is time varying and the goal is to achieve the estimate of  $s(n)$  using  $y(n)$  without using training signal at the receiver. To achieve this, modified constant modulus algorithm (MCMA) is proposed by Kil Nam Oh and Yong Ohk Chin [10]. In this algorithm, the blind equalization and carrier phase recovery both are achieved. Since the CMA converges independently

of carrier recovery, the MCMA solves the problems in CMA by using modified cost function of CMA. The cost function of CMA is modified to form two cost functions of real and imaginary parts of equalizer output  $y(n) = y_R(n) + j y_I(n)$ .

From the cost function of CMA as given below:

$$J_{CMA} = E \left[ (|y(n)|^2 - R_{CMA})^2 \right] \quad (1)$$

The modified cost function equation of MCMA for  $p = 2$  is given as:

$$J(n) = J_R(n) + J_I(n) \quad (2)$$

$$J_R(n) = E \left[ (|y_R(n)|^2 - R_{2,R})^2 \right] \quad (2a)$$

$$J_I(n) = E \left[ (|y_I(n)|^2 - R_{2,I})^2 \right] \quad (2b)$$

The  $R_{2,R}$  and  $R_{2,I}$  are the constants determined for real and imaginary parts of input transmitted signal constellation data  $s_n = s_R + j s_I$  can be calculated as:

$$R_{2,R} = \frac{E \left[ |s_R|^4 \right]}{E \left[ |s_R|^2 \right]} \quad (3)$$

$$R_{2,I} = \frac{E \left[ |s_I|^4 \right]}{E \left[ |s_I|^2 \right]} \quad (4)$$

The taps update equation is:

$$w(n+1) = w(n) - \mu \cdot \nabla J(n) \quad (5)$$

$$w(n+1) = w(n) - \mu \cdot e(n) \cdot X^*(n) \quad (6)$$

where error  $e(n) = e_R(n) + j e_I(n)$  is given as:

$$e_R(n) = y_R(n) \cdot (y_R(n)^2 - R_{2,R}) \quad (7a)$$

$$e_I(n) = y_I(n) \cdot (y_I(n)^2 - R_{2,I}) \quad (7b)$$

### 3.1 MCMA with Decision Directed (DD)

In this operation, the output error is reduced further by using DD algorithm after the initial convergence by MCMA algorithm that's why MCMA algorithm is used with the decision directed (DD) mode [11]. The algorithm with the decision directed approach shows improved performance against MCMA as expected. The MCMA is effective in all aspect but a switch to DD mode result in increased convergence speed and less output error. The MCMA operation will remain dominant in blind mode with DD mode. It automatically switches between these two modes according to the equalizer output error level. The DD mode working is based on the confidence zones where equalizer output have to lie in them for deciding the switch between DD

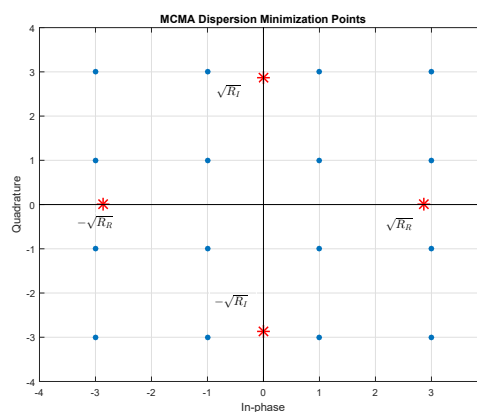
mode and MCMA. The error signals used in this approach is given:

$$e_R(n) = \begin{cases} y_R(n) - dec(y_R(n)) & \text{if } y_R(n) \in C_R \\ y_R(n) \cdot (y_R(n)^2 - R_{2,R}) & \text{otherwise} \end{cases} \quad (8)$$

$$e_I(n) = \begin{cases} y_I(n) - dec(y_I(n)) & \text{if } y_I(n) \in C_I \\ y_I(n) \cdot (y_I(n)^2 - R_{2,I}) & \text{otherwise} \end{cases} \quad (9)$$

The first criteria error equations in both  $e_R(n)$  and  $e_I(n)$  are of DD mode and the other two error equations are of MCMA algorithm. The  $C_R$  and  $C_I$  are confidence zones for real and imaginary part of the equalizer output and  $Z_R$  and  $Z_I$  are their parameters such that  $0 < Z_R, Z_I < 1$ .

The MCMA Dispersion Minimization Points are shown below:



**Figure 3.** MCMA Dispersion Minimization Points

From figure 3, the MCMA tries to move the equalizer output to lie on four points in real and imaginary plane, instead of CMA which penalize the deviation of equalizer output around a circle.

## 4 Dual Dispersion Modified Constant Modulus Algorithm (DD-MCMA)

By taking the motivation from MCMA algorithm, which tries to move the equalizer output to lie on four points in real and imaginary plane, we have introduced the eight points instead of four points for minimization of cost function which we called it Dual Dispersion Minimization approach for blind channel equalization of Star QAM. The cost function of MCMA algorithm is modified such that it tries to move the equalizer output to lie on eight points, as the Star QAM constellation consist of two rings, so four points for one ring and four points for second ring. The cost function of DD-MCMA with the criteria to switch between these points is given as:

Cost Function of Dual Dispersion MCMA is:

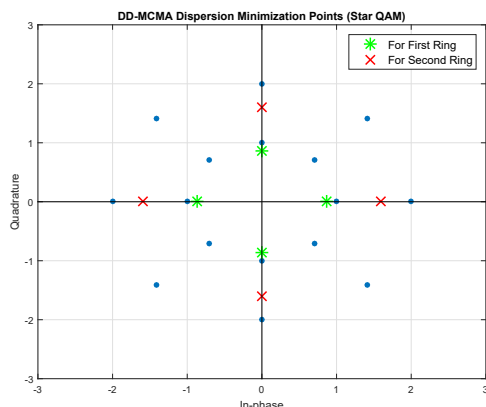
$$J_{DD-MCMA} = E \left[ (|y_R(n)|^2 - R_{VR})^2 \right] + E \left[ (|y_I(n)|^2 - R_{VI})^2 \right] \quad (10)$$

The dispersion constants  $R_{VR}$  and  $R_{VI}$  is chosen based on following criteria automatically during the execution of algorithm:

$$R_{VR} = \begin{cases} RR1 & \text{if } y_R(n)^2 \leq RR1 \\ RR2 & \text{if } y_R(n)^2 \leq RR2 \\ RR2 & \text{otherwise} \end{cases} \quad (11)$$

$$R_{VI} = \begin{cases} RI1 & \text{if } y_I(n)^2 \leq RI1 \\ RI2 & \text{if } y_I(n)^2 \leq RI2 \\ RI2 & \text{otherwise} \end{cases} \quad (12)$$

The DD-MCMA Dispersion Minimization Points are shown below:



**Figure 4.** DD-MCMA Dispersion Minimization Points

The green colored points in asterisk (\*)  $\sqrt{R_{R1}}$ ,  $\sqrt{R_{I1}}$  represents first ring and red colored points in cross (x)  $\sqrt{R_{R2}}$ ,  $\sqrt{R_{I2}}$  represents second ring. The  $RR1, RR2$  and  $RI1, RI2$  are dispersion constants for real and imaginary parts of input transmitted signal constellation data  $s_n = s_R + j s_I$  can be calculated as:

$$R_{R1} = \frac{E \left[ |s_{R1}|^4 \right]}{E \left[ |s_{R1}|^2 \right]} \quad (13)$$

$$R_{R2} = \frac{E \left[ |s_{R2}|^4 \right]}{E \left[ |s_{R2}|^2 \right]} \quad (14)$$

$$R_{I1} = \frac{E \left[ |s_{I1}|^4 \right]}{E \left[ |s_{I1}|^2 \right]} \quad (15)$$

$$R_{I2} = \frac{E \left[ |s_{I2}|^4 \right]}{E \left[ |s_{I2}|^2 \right]} \quad (16)$$

The  $s_{R1}, s_{I1}$  are the symbols of input transmitted signal constellation on first ring only, and the  $s_{R2}, s_{I2}$  are the symbols of input transmitted signal constellation on both rings.

In order to obtain the stochastic gradient adaptive algorithm tap update equation, we differentiate the cost function in equation (10) with respect to equalizer tap weight vector  $w$  and approximating the expectation with the instantaneous values gives

$$\frac{\nabla J_{DD-MCMA}}{\nabla w(n)} = \{ [2(y_R(n)^2 - R_{VR}) \cdot 2 \cdot y_R(n)] + j \cdot [2(y_I(n)^2 - R_{VI}) \cdot 2 \cdot y_I(n)] \cdot x^*(n) \} \quad (17)$$

The taps update equation is given as:

$$w(n+1) = w(n) - \mu \cdot \nabla J_{DD-MCMA}(n) \quad (18)$$

$$w(n+1) = w(n) - \mu \cdot e(n) \cdot X^*(n) \quad (19)$$

where error  $e(n) = e_R(n) + j e_I(n)$  is given as:

$$e_R(n) = y_R(n) \cdot (y_R(n)^2 - R_{VR}) \quad (20a)$$

$$e_I(n) = y_I(n) \cdot (y_I(n)^2 - R_{VI}) \quad (20b)$$

#### 4.1 DD-MCMA with Decision Directed

We have also implemented a DD-MCMA with decision directed (DD) in which DD-MCMA is used with decision directed algorithm. This helps reducing output error more after the convergence by DD-MCMA algorithm. This DD algorithm is also used with MCMA algorithm described in section 3.1. The error equations used in this algorithm are given:

$$e_R(n) = \begin{cases} y_R(n) - dec(y_R(n)) & \text{if } y_R(n) \in C_R \\ y_R(n) \cdot (y_R(n)^2 - R_{VR}) & \text{otherwise} \end{cases} \quad (21)$$

$$e_I(n) = \begin{cases} y_I(n) - dec(y_I(n)) & \text{if } y_I(n) \in C_I \\ y_I(n) \cdot (y_I(n)^2 - R_{VI}) & \text{otherwise} \end{cases} \quad (22)$$

The first criteria error equations in both  $e_R(n)$  and  $e_I(n)$  are the DD mode of operation and the other two error equations are of DD-MCMA algorithm. From [19], an open eye condition can be expressed as:

$$|y(n) - s(n)| < \frac{D}{2} \quad \text{for all } n, \quad (23)$$

We have calculated the value of  $D$  for above equation to use it in DD mode. The value of  $D$  is chosen based on distance between symbols on consecutive rings of Star QAM. The value of  $Z_R$  and  $Z_I$  used for simulation is 0.25 for best results.

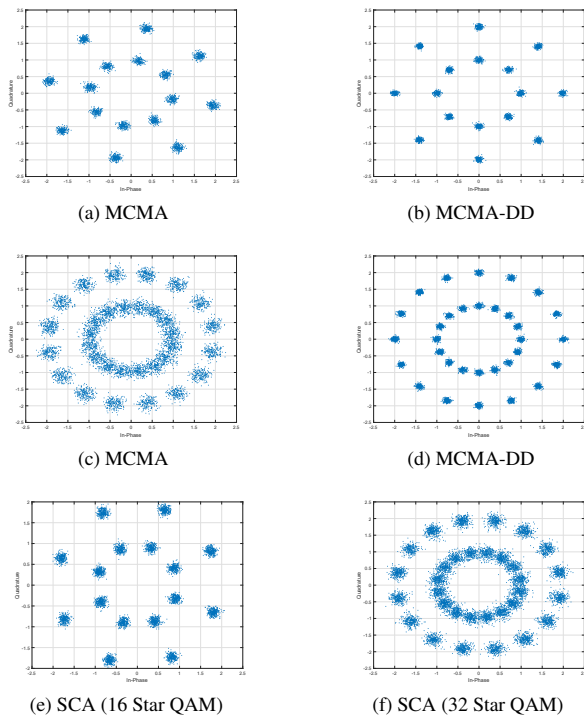
## 5 Simulation Results

The comparison of algorithms is done using computer simulations on MATLAB software. The channel real and imaginary tap weights are shown below in a table:

**Table 1.** Simulated 7 tap channel impulse response

Tap No.	Real Part	Imaginary Part
1	-0.005	-0.004
2	0.009	0.030
3	-0.024	-0.104
4	0.854	0.520
5	-0.218	0.273
6	0.049	-0.074
7	-0.016	0.020

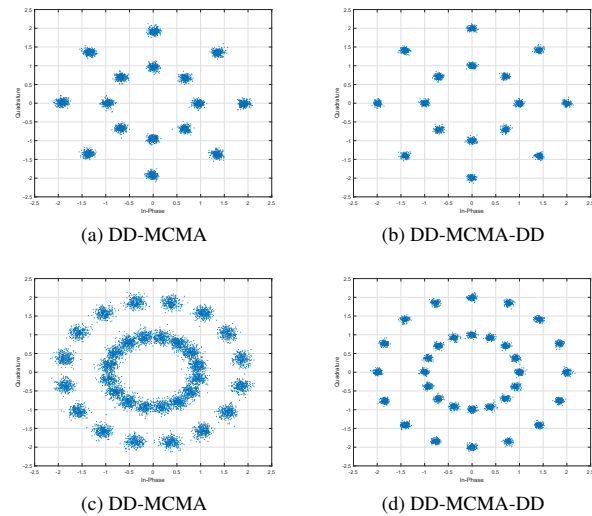
A typical voice band communication channel consisting of 7-taps is used [20] with the zero forcing condition i-e center tap is set to 1 while other taps are zero. The output symbols equalized results of MCMA, MCMA-DD and SCA algorithms are shown below:



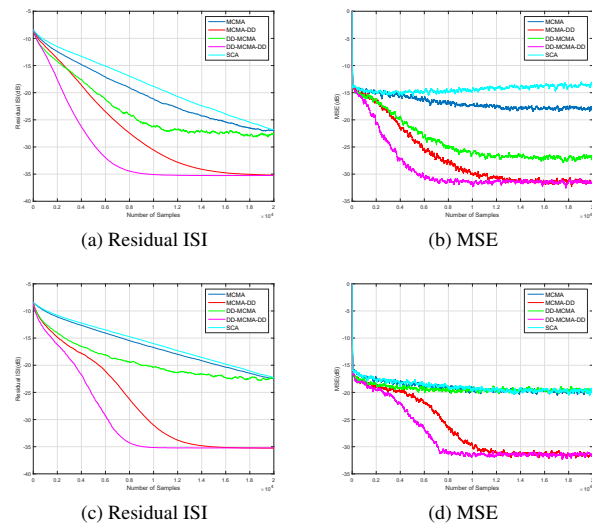
**Figure 5.** Equalized Star QAM Results of 16 Star QAM (a,b and e) and 32 Star QAM (c,d and f)

For fair comparison, algorithms having phase correction ability are used. Three algorithms, namely, MCMA algorithm, MCMA with decision directed and SCA are compared with the DD-MCMA algorithm and DD-MCMA with decision directed. Signal to Noise ratio (SNR) is maintained at 30 dB.

Next the equalized symbols, Residual ISI and MSE results of DD-MCMA and DD-MCMA with decision directed are also shown next:



**Figure 6.** Equalized Star QAM Results of 16 Star QAM (a and b) and 32 Star QAM (c and d)



**Figure 7.** Residual ISI and MSE results of 16 Star QAM (a and b) and 32 Star QAM (c and d)

In the figures above of Residual ISI and MSE, it is clear that DD-MCMA and DD-MCMA with decision directed is performing good in convergence rate against the MCMA, MCMA with decision directed and SCA. By using the Dual Dispersion minimization technique, and addition of decision directed approach to DD-MCMA, we are able to achieve the fast convergence rate, the convergence rate increases further and reaches the Residual ISI and MSE floor faster as compared to MCMA and MCMA with decision directed.

## 6 Conclusion

In this paper, we presented the new algorithm by modifying the MCMA algorithm for blind channel equalization

of Star QAM. The new algorithm uses Dual Dispersion minimization approach, which results in improved performance in convergence rate of Residual ISI and MSE against MCMA and SCA algorithm. The convergence rate of the proposed algorithm is further improved when used jointly with decision directed approach.

The dispersion constants and step size values used in these algorithms during simulations are given in a tables:

**Table 2.** Values of Dispersion Constant for 16 Star QAM

Algorithms	M=16
SCA	$R_{SCA} = 3.3113$
MCMA	$R_R = 2.55$ and $R_I = 2.55$
DD-MCMA	$R_{R1} = 0.75$ and $R_{R2} = 2.55$ $R_{I1} = 0.75$ and $R_{I2} = 2.55$

**Table 3.** Values of Dispersion Constant for 32 Star QAM

Algorithms	M=32
SCA	$R_{SCA} = 3.3750$
MCMA	$R_R = 2.55$ and $R_I = 2.55$
DD-MCMA	$R_{R1} = 0.75$ and $R_{R2} = 2.55$ $R_{I1} = 0.75$ and $R_{I2} = 2.55$

**Table 4.** Values of Step Size ( $\mu$ )

Algorithm	Step Size ( $\mu$ )	
	M=16	M=32
SCA	$\mu = 40.3e - 7$	$\mu = 30.7e - 7$
MCMA	$\mu = 1.00e - 4$	$\mu = 0.55e - 4$
MCMA(DD)	$\mu = 2.55e - 4$	$\mu = 3.85e - 4$
DD-MCMA	$\mu = 2.50e - 4$	$\mu = 2.55e - 4$
DD-MCMA(DD)	$\mu = 4.90e - 4$	$\mu = 7.40e - 4$

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