Improved online sequential extreme learning machine for simulation of daily reference evapotranspiration

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Abstract

Yubin, Z., Zhengying, W., Lei, Z., Qinyin, L., & Jun, D. (March-April, 2017). Improved online sequential extreme learning machine for simulation of daily reference evapotranspiration. *Water Technology and Sciences* (in Spanish), *8*(2), 127-140.

The traditional extreme learning machine has significant disadvantages, including slow training, difficulty in selecting parameters, and difficulty in setting the singularity and the data sample. A prediction model of an improved Online Sequential Extreme Learning Machine (IOS-ELM) of daily reference crop evapotranspiration is therefore examined in this paper. The different manipulation of the inverse of the matrix is made according to the optimal solution and using a regularization factor at the same time in the model. The flexibility of the IOS-ELM in ET0 modeling was assessed using the original meteorological data (*T*max, *Tm*, *T*min, *n*, *Uh*, *RHm*, ϕ, *Z*) of the years 1971–2014 in Yulin, Ankang, Hanzhong, and Xi'an of Shaanxi, China. Those eight parameters were used as the input, while the reference evapotranspiration values were the output. In addition, the ELM, LSSVM, Hargreaves, Priestley-Taylor, Mc Cloud and IOS-ELM models were tested against the FAO-56 PM model by the performance criteria. The experimental results demonstrate that the performance of IOS-ELM was better than the ELM and LSSVM and significantly better than the other empirical models. Furthermore, when the total ET0 estimation of the models was compared by the relative error, the results of the intelligent algorithms were better than empirical models at rates lower than 5%, but the gross ET0 empirical models mainly had 12% to 64.60% relative error. This research could provide a reference to accurate ET0 estimation by meteorological data and give accurate predictions of crop water requirements, resulting in intelligent irrigation decisions in Shaanxi.

Keywords: Daily reference evapotranspiration, extreme learning machine, online learning, matrix singularity.

Resumen

Yubin, Z., Zhengying, W., Lei, Z., Qinyin, L.,& Jun, D. (marzo-abril, 2017). Máquina de aprendizaje extremo secuencial en línea mejorada para la simulación de la evapotranspiración de referencia diaria. Tecnología y Ciencias del Agua, *8(2), 127-140.*

La máquina de aprendizaje extremo tradicional tiene desventajas significativas, tales como entrenamiento lento, dificultad en la selección de parámetros y dificultad en establecer la singularidad y la muestra de datos. Por lo tanto, en el presente artículo se examina un modelo de predicción de una máquina de aprendizaje extremo secuencial en línea mejorada (IOS-ELM) de la evapotranspiración de referencia diaria de cultivos. La diferente manipulación de la inversa de la matriz se hace de acuerdo con la solución óptima y utilizando un factor de regularización al mismo tiempo en el modelo. La flexibilidad de la IOS-ELM en la modelación de la ET0 se evaluó empleando los datos meteorológicos originales (Tmax, Tm, Tmin, n, Uh, RHm, ϕ, Z) de los años 1971–2014 en Yulin, Ankang, Hanzhong, y Xi'an en Shaanxi, China. Estos ocho parámetros se usaron como insumos o datos de entrada, mientras que los valores de la evapotranspiración de referencia fueron los datos de salida o el producto. Asimismo, se probaron los modelos ELM, LSSVM, Hargreaves, Priestley-Taylor, Mc Cloud y IOS-ELM contra el modelo FAO-56 PM mediante los criterios de desempeño. Los resultados experimentales demuestran que el desempeño de IOS-ELM fue mejor que le de ELM y LSSVM y significativamente mejor que los demás modelos empíricos. Más aún, al comparar la estimación total de ET0 de los modelos mediante el error relativo, los resultados de los algoritmos inteligentes fueron mejores que los modelos empíricos a índices inferiores a 5%, pero los modelos empíricos de ET0 bruta tuvieron un error relativo de 12 a 64.60%. Esta investigación podría proporcionar una referencia para la estimación precisa de ET0 mediante datos meteorológicos y proporcionar predicciones precisas de los requerimientos de agua de los cultivos, lo cual resultaría en decisiones de riego inteligentes en Shaanxi.

Palabras clave: *evapotranspiración de referencia diaria, máquina de aprendizaje extremo, aprendizaje en línea, singularidad de la matriz.*

Received: 30/04/2016
Approved: 26/102016

Introduction

The calculation of reference crop evapotranspiration is a key to intelligent irrigation systems. Therefore, accurate estimation of *ET*0 becomes important in irrigation schedules for planning and optimizing the agriculture area. Numerous methods have been put forward to estimate *ET*0. Of these, the Penman-Monteith 56 (PM) model has given the best results; it was officially recommended by the Food and Agriculture Organization of the United Nations (FAO) in 1998 (Allen *et al*., 1998). FAO selected the PM model as the standard equation for *ET*0 estimation because it can provide the most accurate results in the world. *ET*0 is also a key parameter in the design of intelligent irrigation for field crops. Engineers need to know the irrigation water consumption requirements for each crop so that they can calculate or estimate the remaining components of the water balance. Also, agriculturists need to obtain the specific water requirements of a crop so that they can generate a satisfactory yield. It is also necessary to know whether these specific requirements are being met with ordinary irrigation.

As described by Kisi (2008) and (Yubin *et al*., 2014), about 50 measures have been proposed for estimating evapotranspiration, which can be sorted into four types: radiation, temperature, synthetic and evaporating dish. FAO assumed the ET definition given by Smith, Allen and Pereira (1997), and adopted the FAO-56 PM as the sole equation for estimation of *ET*0.

However, the PM model requires a lot of meteorological data as input, and the calculation process involves complex and nonlinear regression among these factors. So, a simpler and more accurate simulation model needs to seek *ET*0 in the case of lack of meteorological data. Thus, artificial intelligence algorithms (*e.g*., neural networks) for reference evapotranspiration (*ET*0) modeling have been given more attention in recent decades. Feng and Cui (2015) found that an ELM model gave better results than empirical models in the area of central Sichuan. Kisi (2007) estimated daily ET0 using the ANN method and compared their calculation results with the other models. Ozgur Kisiet (2013) proposed a reference evapotranspiration model by LSSVM. Kisi (2011a) considered daily *ET*0 using wavelet regression model and compared this model to other empirical models. Kisi (2011b) modeled *ET*0 using evolutionary feed-forward neural networks. Marti, Gonzalez-Altozano and Gasque (2011) used ANN to estimate daily *ET*0 without local climatic data. Kumar, Raghuwanshi and Singh (2011) researched the application of ANN in estimating evapotranspiration modeling. Shiri *et al*. (2012) established an *ET*0 simulation model using GEP (gene expression programming) for Spanish Basque, and found that the GEP model performed better than the ANFIS, Hargreaves and Priestley-Taylor models. Wang, Traore and Kerh (2008), and Traore, Wang and Kerh (2010) estimated daily *ET*0 using BP-ANN. However, BP-ANN has major disadvantages, such as its slow speed of training and difficulty in selecting parameters. In recent years, new intelligent algorithms have appeared in the industrial field, such as extreme learning machines (ELM) and support vector machine, among others.

In the present study, ELM is proposed as an alternative to other models for predictive control. It can randomly choose hidden nodes and analytically determine the output weights of SLFNs. However, ELM cannot confirm the singularity of the output matrix of the hidden layer, and it also cannot make fine tuning according to the characteristics of the data set, which will affect its efficiency and stability.

The main purpose of this paper is to optimize the ELM approach in the modeling of daily *ET*0 using the original meteorological data. All previous studies have indicated that intelligent models can input the factors of FAO-56 PM as they estimate *ET*0. In fact, these factors will be another complex computing project by meteorological raw data, to avoid creating more severe error during multistage formula calculation. Also, the manipulation of the inverse of the

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• ISSN 0187-8336

matrix is adjusted with reference to the optimal solution and the regularization factor at the same time, which is motivated by the online learning method. In summary, an improved online sequential extreme learning machine (IOS-ELM) is designed, and the new algorithm can produce good generalization performance in a model of daily *ET*0 in an irrigation system.

Materials and case study

The study was conducted in Yulin (38.27° N, 109.78° E), Ankang (32.72° N, 109.03° E), Hanzhong (33.07°N, 107.7°E) and Xi'an (34.3° N, 108.9° E) in Shaanxi province in China, shown in Fig. 1. The area has a hot and dry climate for the greater part of the year.

Daily meteorological data used for this study was from the years 1971–2014. The following observed eight meteorological variables with daily temporal resolution were used: wind speed at 10m above the ground (Uh), mean temperature (T), mean relative humidity (RH), minimum temperature (Tmin), maximum temperature (Tmax), actual Sunshine duration (n), latitude (ϕ) and elevation (Z), which were downloaded from China meteorological data sharing service system (http://cdc.nmic.cn/home.do). Data from the first 29 years (1971-1999) was used to train the models. Data from the next ten years (2000-2009) was used for the test. The data from the remaining years was used for validation. It must be noted, however, that missing data was replaced by the average of the data from the day before and the day before. The regional climate characteristics are given in table 1.

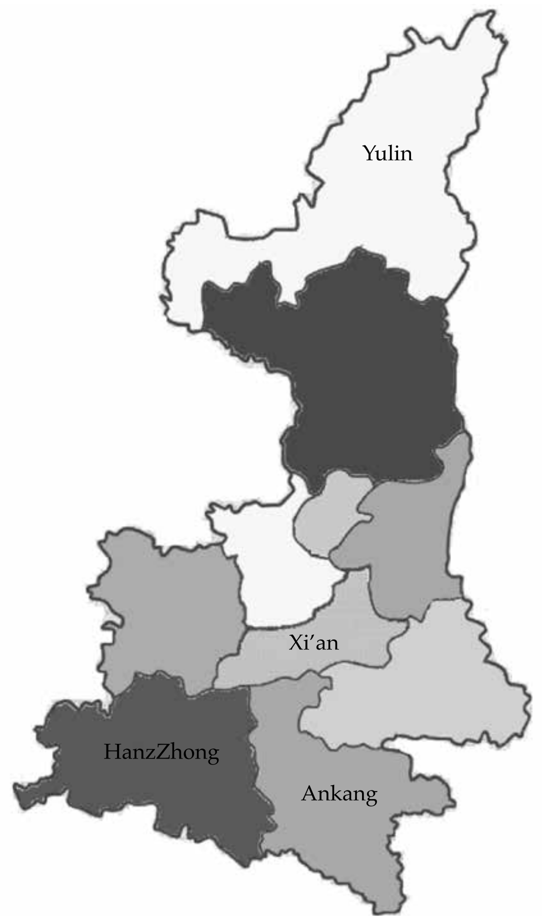

Figure 1. The location of the cities.

Methodology

Calculation models of reference crop evapotranspiration

The study focused on the comparison of the proposed IOS-ELM model with the ELM, LSSVM,

Table 1. Means of main variables.

	Uh	*T*	*RH*	*T* min	*T* max	*n*	ϕ	*Z*
Yulin	2.14	9.2	53.5	3.1	16.2	7.32	0.67	1157
An kang	1.35	15.9	74.1	12.1	21.4	4.58	0.57	290.8
Han zhong	1.15	15.1	78.5	11.5	19.8	4.03	0.58	509.5
Xi'an	1.60	14.6	64.3	9.7	19.6	4.54	0.60	397.5

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ISSN 0187-8336 •

Hargreaves, Mc-Cloud, and Priestley-Taylor models. First, the $ET0$ values of four cities were calculated using the FAO-56 PM. Then, the standard formula evapotranspiration calculation for all empirical models is shown.

(1) FAO-56 Penman-Monteith:

$$ET0 = \frac{0.418\Delta\left(R_s - G\right) + \gamma \frac{900}{T + \gamma 273} u_2 \left(e_s - e_a\right)}{\Delta + \gamma\left(1 + 0.34 u_2\right)} \quad (1)$$

The original meteorological data of T_{max}, T, T_{min}, n, Uh, RH_m, ϕ and Z were used in the model:

(2) Hargreaves:

$$ET0 = \frac{0.0023 \times R_a \left(T + 17.8\right)\left(T_{max} - T_{min}\right)^{\frac{1}{2}}}{\lambda} \quad (2)$$

T_{max}, T, T_{min}, n and ϕ were used in the model:

(3) Mc-Cloud:

$$ET0 = 0.254 \times 1.07^{(1.8T)} \quad (3)$$

Only T was referred to in the model:

(4) Priestley-Taylor

$$ET0 = 1.26 \frac{\Delta}{\Delta + \gamma} \cdot \frac{\left(R_n - G\right)}{\lambda} \quad (4)$$

T_{max}, T, T_{min}, n and ϕ were used.

However, these variables are obtained directly or indirectly from the meteorological raw data (Uh, T, RH, Tmin, Tmax, n, ϕ and Z). Furthermore, the calculation formula for them did not have a precise formula by estimation or experience.

Therefore, the inputs Uh, T, RH, Tmin, Tmax, n, ϕ and Z, the $ET0$ output were calculated by the FAO-56 PM method and used for the calibration of the IOS-ELM models. The mean absolute error (MAE), the root mean square error (RMSE), effectiveness index of the model (EF) and self-correlation coefficient (R^2) statistics were used for the assessment criteria of the models in this study. EF model efficiency mainly depends on the Nash coefficient EF values; as the values approach one, the efficiency of the model increases. The study adopted the calculation model of the validity index for EF by Nash and Sutcliffe.

Extreme learning machine (ELM)

For N random distinct samples (x_i, t_i) where $x_i = [x_{i1}, x_{i2}, \ldots, x_{in}]^T \in R^n$, $t_i = [t_{i1}, t_{i2}, \ldots, t_{in}]^T \in R^m$ and for the standard SLFNs ($\tilde{N}$ hidden nodes), the activation function $g(x)$ is expressed as:

$$\sum_{i=1}^{\tilde{N}} \beta_i g_i\left(x_i\right) = \sum_{i=1}^{\tilde{N}} \beta_i g\left(w_i \cdot x_j + b_i\right) = 0_j$$
$$j = 1, \ldots, N \quad (5)$$

where $w_i = [w_{i1}, w_{i2}, \ldots, w_{in}]^T$ is the weight vector connecting the ith hidden node and the input nodes, $\beta_i = [\beta_{i1}, \beta_{i2}, \ldots, \beta_{in}]^T$ is the weight vector connecting the ith hidden node and the output nodes and b_i is the threshold of the ith hidden node.

$$\sum_{i=1}^{\tilde{N}} \beta_i g\left(w_i \cdot x_j + b_i\right) = t_j \quad j = 1, \ldots, N \quad (6)$$

The above N equations can be written compactly as

$$H\beta = T \quad (7)$$

where

$$H\left(w, b, x\right) = \begin{bmatrix} g\left(w_1 \cdot x_1 + b_1\right) & \mathrm{L} & g\left(w_{\tilde{N}} \cdot x_1 + b_{\tilde{N}}\right) \\ \mathrm{M} & \mathrm{L} & \mathrm{M} \\ g\left(w_1 \cdot x_N + b_1\right) & \mathrm{L} & g\left(w_{\tilde{N}} \cdot x_N + b_{\tilde{N}}\right) \end{bmatrix}_{N \times \tilde{N}} \quad (8)$$

$$\beta = \begin{bmatrix} \beta_1^T \\ \mathrm{M} \\ \beta_{\tilde{N}}^T \end{bmatrix}_{\tilde{N} \times m}, \quad T = \begin{bmatrix} t_1^T \\ \mathrm{M} \\ t_N^T \end{bmatrix}_{N \times m} \quad (9)$$

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• ISSN 0187-8336

where H is called the hidden layer output matrix in the neural network, and the ith column of H is the ith hidden node output toward inputs $x_1, x_2,..., x_N$.

The aim is to solve the above issues and put forward an extreme learning machine for SLFNs.

A training set was provided as:

$\aleph = \left\{ \left(x_i, t_i \right) \middle| x_i \in R^n, t_i \in R^m \right\}_{i=1}^{N}$, the activation function $g(x)$, and hidden node number $\tilde{N}$.

Step 1: Randomly allocate input weight w_i and bias b_i, $i = 1, 2,..., \tilde{N}$.

Step 2: Calculate the hidden layer output matrix H.

Step 3: Calculate the output weight β.

$$\hat{\beta} = H^+ T \tag{10}$$

where $T = [t_1, ..., t_N]^T$, H^+ is a generalized inverse of MP.

Online sequential ELM (OS-ELM)

ELM is a relatively effective and simple algorithm that is also able to learn quickly and generalize well. However, meteorological data are difficult to collect and the data set is large, which may cause a decline in the performance of the *ET*0 model. Thus, the online sequential extreme learning machine (OS-ELM) by Liang (2006) was referenced in the previous research.

The output weight matrix $\hat{\beta} = H^+ T$ is a least-squares solution of (7). Meanwhile, the matter where $rank(H) = \tilde{N}$ the number of hidden nodes (Ao, Xiao, & Mao, 2009) is considered. So, $H^+$of (10) is given as:

$$H^+ = \left(H^T H \right)^{-1} H^T \tag{11}$$

If $H^T H$ tends to become fantastic, it can also be made nonsingular by increasing the number of data or choosing a smaller network size. Substituting (11) into (10) gives:

$$\hat{\beta} = \left(H^T H \right)^{-1} H^T T \tag{12}$$

Equation (12) is called the least-squares solution to $H\beta = T$. Sequential implementation of the least-squares solution of (12) gives the OS-ELM. However, the OS-ELM may have some deficiencies, especially the fact that solving the generalized inverse matrix MP of H may cost a huge amount of time in the training process. The general method of singular value decomposition is used to solve matrix H, but its computational complexity is $O(4N\tilde{N}^2 + 8\tilde{N}^3)$ (Brown, 2009).

Improved algorithm of OS-ELM (IOS-ELM)

This paper proposes an improved OS - ELM called IOS-ELM. This new model was developed by modifying and improving the singularity of the matrix. First, Equation $H\beta = T$ will be replaced by $H^T H\beta = H^T T$, which has at least one optimization solution. This reduces the computational complexity of solving the inverse, which results in a reduction of the training time. Second, the regular factor $1/\lambda$ is joined when calculating the output weights. Last, the subsequent online learning stage is added. In theory, this algorithm can provide good generalization performance at an extremely fast learning speed.

Step (1): Allocate random input weights w_i and bias b_i, initialize network and calculate the initially hidden layer output matrix H_0.

Step (2): Set $r = rank(H)$, if $r = N_0$, then calculate the initial weight matrix $\beta^0 = P_1 H_0^T T_0$.If $r = N$ then calculate the initial weight $\beta^0 = H_0^T P_2 T_0$.

Where $P_1 = \left(\frac{1}{\lambda} + H_0 H_0^T \right)^{-1}$, $P_2 = \left(\frac{1}{\lambda} + H_0^T H_0 \right)^{-1}$.

If $r \neq N_0$ and $r \neq \tilde{N}$, to solve the two optimization models:

$$\min_{B \in g^+} \left\| M - B \right\| \quad \text{and} \quad \min_{B \in R^{N0}} \left\| B^* \beta^0 - c \right\|$$

Then, the optimization solution B^* and β^0 can be obtained.

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ISSN 0187-8336 •

Where $c = H_0^T T_0$, $M = H_0^T H_0$, $g^+ = \{g \in R^{N_0 \times N_0}\}$, g is a positive definite symmetric matrix.

Step (3) Set $K = 0$; then, present the $(K + 1)$th chunk of new observations:

$$\aleph_{K+1} = \left\{\left(x_i, t_i\right)\right\}_{\substack{j=1 \\ i=\left(\sum_{j=1}^{K} N_j\right)}}^{K+1} \sum N_j$$

where N_{K+1} is the number of observations in the $(K + 1)$th chunk.

Step (4) Calculate the partially hidden layer output matrix H_{K+1} for the $(K + 1)$th chunk of data $\aleph_{K+1}$, as shown in (17):

$$H_{K+1} = \begin{bmatrix} g\left(w_1 \cdot x_1 + b_1\right) & \cdots & g\left(w_{\tilde{N}} \cdot x_1 + b_{\tilde{N}}\right) \\ \vdots & \cdots & \vdots \\ g\left(w_1 \cdot x_{N_{K+1}} + b_1\right) & \cdots & g\left(w_{\tilde{N}} \cdot x_N + b_{\tilde{N}}\right) \end{bmatrix}_{N_{K+1} \times \tilde{N}} \quad (13)$$

Step (5) According to step (1), calculate the output weight β^{K+1}.

Step (6) Set $k = k + 1$. Go to Step (3).

Application and results

IOS -ELM model under lack of data

The original eight meteorological parameters chose and combined with a different pattern in this section, which was taken as input values. Meanwhile, the calculation of FAO 56 Penman-Montieth was put as the output value. By this method, the IOS-ELM model is established. However, it needs to further consider the effectiveness of the combination pattern among the eight meteorological data. Therefore, the correlation between *ET*0 and the data was analyzed. In this way, ISO-ELM can choose reasonable meteorological parameters to complete the forecast even if there is a lack of meteorological data. This is shown in table 4.

It can be clearly seen from table 2 that the *ET*0 outperformed all eight meteorological parameters in terms of correlation. Although the data set is not similar for different cities, the correlation behaved in the same way. T_{max} is closely correlated with evapotranspiration for each city, followed by the average temperature, minimum temperature, the actual sunshine time and wind speed. The influence of the latitude and altitude were so small that they were negligible. Finally, the humidity is negative.

The simulation accuracy of the IOS-ELM model was discussed by referring to table 2 under lack of meteorological data. It should be noted that the latitude and altitude were eliminated because they had virtually no effect on the results. Taking Yulin city as an example, the first 10-year (1971-1999) span of data was used to train the models. Then, using different combinations, the error was analyzed, as well as the correlation coefficient and effectiveness of the prediction. This is shown in table 3.

The ISO-ELM model was applied by comparing the different parameters shown in table 3. It is immediately noticeable that the prediction results were the same for eight-parameter and six-parameter inputs. That is because the latitude and altitude almost have no effect on the prediction for the same station. Secondly, the temperature had the largest influence on the prediction, particularly the maximum temperature. As long as the temperature is one of

Table 2. Correlation of data and *ET*0.

	Uh	*T*	RH_m	T_{min}	T_{max}	*n*	ϕ	*Z*
Yulin	0.20	0.88	-0.36	0.79	0.92	0.50	-2e-15	NaN
Ankang	0.15	0.85	-0.34	0.71	0.93	0.70	-7e-16	
Hanzhong	0.21	0.85	-0.48	0.73	0.93	0.69	4e-15	
Xi'an	0.30	0.80	-0.43	0.72	0.85	0.66	1e-16	

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• ISSN 0187-8336

Table 3. Influence of different meteorological data combinations on *ET*0.

Model inputs	RMSE	R^2	EF%
All	**0.4132**	**0.9696**	**95.72**
T_{max}, T, T_{min}, n, Uh, RH	**0.4132**	**0.9696**	**95.72**
T_{max}, T, T_{min}, n, Uh	0.7865	0.9625	92.3
T_{max}, T, T_{min}, n, RH	**0.6737**	**0.9619**	**94.3**
T_{max}, T, T_{min}, Uh, RH	0.7868	0.9620	92.2
T_{max}, T, n, Uh, RH	0.9021	0.9496	89.8
T_{max}, T_{min}, n, Uh, RH	0.9135	0.9490	89.6
T, T_{min}, n, Uh, RH	0.9666	0.9423	88.3
T_{max}, T, T_{min}, n	0.8270	0.9591	91.4
T_{max}, T, T_{min}, Uh	0.7338	0.9673	93.2
T_{max}, T, T_{min}, RH	**0.6368**	**0.9651**	**94.9**
T_{min}, n, Uh, RH	1.2321	0.9060	81.1
T, n, Uh, RH	1.0048	0.9377	87.4
T_{max}, n, Uh, RH	0.8428	0.9560	91.1
T_{max}, T, T_{min}	0.8505	0.9566	90.9
n, Uh, RH	2.2612	0.6222	36.3
T_{max}, T, RH	0.6480	0.9753	94.7
T_{max}, T_{min}, n	0.8580	0.9548	90.8
T_{max}, T_{min}, Uh	0.7942	0.9616	92.1
T_{max}, Uh, RH	0.7436	0.9668	93.1
T_{max}, n, RH	**0.6298**	**0.9664**	**95.06**
T_{max}, n, Uh	0.8786	0.9513	90.3
T_{max}, T	0.9464	0.9432	88.8
T, T_{min}	0.9119	0.9502	89.64
T_{max}, RH	**0.6258**	**0.9670**	**95.12**
T_{max}, n	0.9012	0.9483	89.89
T_{max}, Uh	0.9999	93.59	87.55
n, RH	2.1127	0.6694	44.42
n, Uh	2.1611	0.6543	41.85
Uh, RH	2.6964	0.3395	9.47

the parameters, the model is accurate. Thirdly, when only two temperatures were used as the inputs, it still performed better than RMSE, EF and R^2 statistics. So, properly reducing some variables and adopting reasonable combinations of variables can improve the accuracy of prediction.

Comparison with other calculation formulas

The IOS-ELM model was compared with the ELM and LSSVM, as well as conventional models including Hargreaves, Mc-Cloud and Priestley-Taylor methods in respect of RMSE and MAE statistics in different cities in tables 4-5. There are six parameters as input variables in the model.

Tables 4-5 show that IOS-ELM outperformed all other models by all performance criteria. Compared with the intelligent and empirical models, the ISO-ELM performed the best value of RMSE<0.46 and MAE<0.41, and the ELM and LSSVM models performed better than the others. A few differences appeared among the Mc Cloud and Priestley-Taylor models. It was

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ISSN 0187-8336 •

Table 4. RMSE of the models in the test period.

Models	RMSE (mm/day)			
	Yulin	A kang	Hanzhong	Xi'an
IOS-ELM	041	0.45	0.45	0.41
ELM	0.86	0.72	0.88	0.78
LSSVM	0.96	1.38	1.30	1.05
Hargreaves	2.18	1.38	1.27	0.79
Mc Cloud	3.53	2.20	1.98	1.70
Priestley-Taylor	2.43	1.73	1.59	0.71

Table 5. MAE of the models in the test period.

Models	MAE (mm/day)			
	Yulin	Ankang	Hanzhong	Xi'an
IOS-ELM	0.40	0.35	0.33	0.31
ELM	0.52	0.68	0.62	0.55
LSSVM	0.77	1.20	1.13	0.93
Hargreaves	1.97	1.25	1.19	0.58
Mc Cloud	3.15	1.86	1.66	1.36
Priestley-Taylor	2.13	1.56	1.40	0.51

also discovered that the Hargreaves method provided better accuracy than other methods among the empirical models.

Although IOS-ELM, ELM and LSSVM models had better simulation effects, the running time is distinguishing, as shown in table 6.

It is clear from table 6 that the IOS-ELM model runs faster than ELM and LSSVM in the process of calculating by at least 24.8%.

In order to consider the portability and error causes of the IOS - ELM model, the estimates of each model for four cities are shown in figures 2-5 in the form of scatter plots in the validation period. It is generally clear from the scatter plots that the six input ISO-ELM estimates are closer to the corresponding FAO-56 PM *ET*0 values than other models. The fit line equations $y = ax + b$ and R^2 values indicate that the ISO-ELM model performed with better accuracy. Meanwhile, the a and b coefficients of the six-input ISO-ELM model were closer to 1 and 0, respectively, with a higher R^2 value than those of the other models.

For Yulin, ISO-ELM and ELM estimates were closer to the FAO-56 PM *ET*0 values than those of the other models ($R^2 > 0.96$). A slight difference exists between LSSVM, and Hargreaves was better than the surplus models. The Mc Cloud estimate had the least accuracy. It can be concluded that the ISO-ELM and ELM models are the best methods to use for daily *ET*0 estimation in Yulin.

Table 6. Running time of different models.

Model	Running time (*S*)			
	Yulin	Ankang	Hangzhong	Xi'an
IOS-ELM	16.5	17.9	13.8	17.1
ELM	28.8	23.8	21.2	25.8
LSSVM	22.4	30.5	19.9	36.5

Tecnología y Ciencias del Agua, vol. VIII, núm. 2, marzo-abril de 2017, pp. 127-140 • ISSN 0187-8336

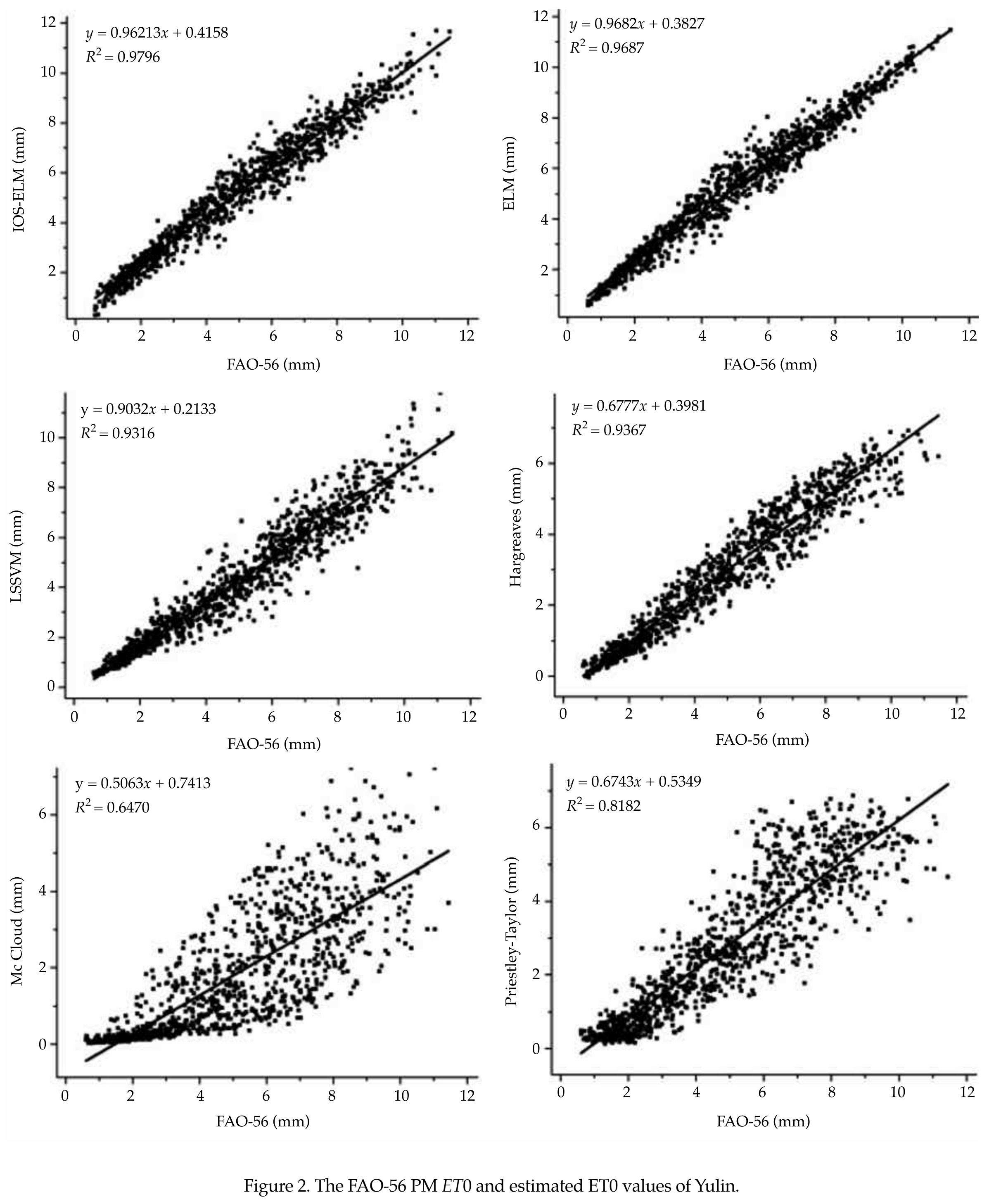

Figure 2. The FAO-56 PM *ET*0 and estimated ET0 values of Yulin.

For Ankang, ISO-ELM and Hargreaves were closer to the FAO-56 PM *ET*0 values, a slight difference exists between LSSVM, and ELM was better than the Mc Cloud model. In this city, the Mc Cloud estimate was also the least accurate ($R^2 = 0.6141$). This leads to the conclusion that in this city, the ISO-ELM and empirical Hargreaves models were the best.

ISSN 0187-8336 •

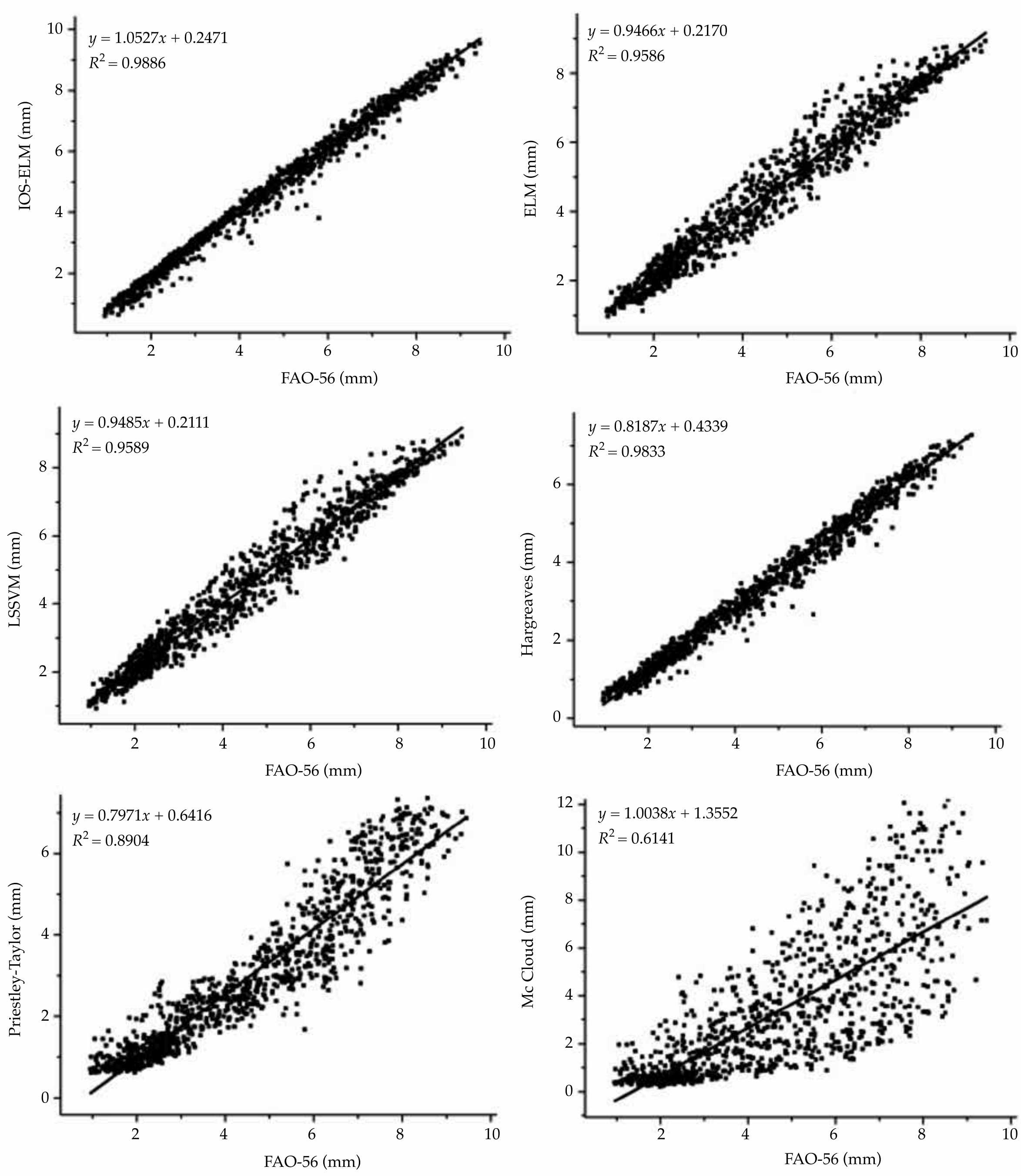

Figure 3. The FAO-56 PM *ET*0 and estimated ET0 values of Ankang.

For Hanzhong, ISO-ELM was closer to the FAO-56 PM *ET*0 of $R^2 = 0.9911$, followed by the Hargreaves, ELM, and LSSVM models. The Mc Cloud and Priestley-Taylor estimates were the least accurate.

For Hanzhong, ISO-ELM was closer to the FAO-56 PM *ET*0 with $R^2 = 0.9905$, followed by the ELM, LSSVM, Priestley-Taylor, Hargreaves and Mc Cloud models, which had R^2 values of 0.9547, 0.9488, 0.8804, 0.8335 and 0.5577, respectively.

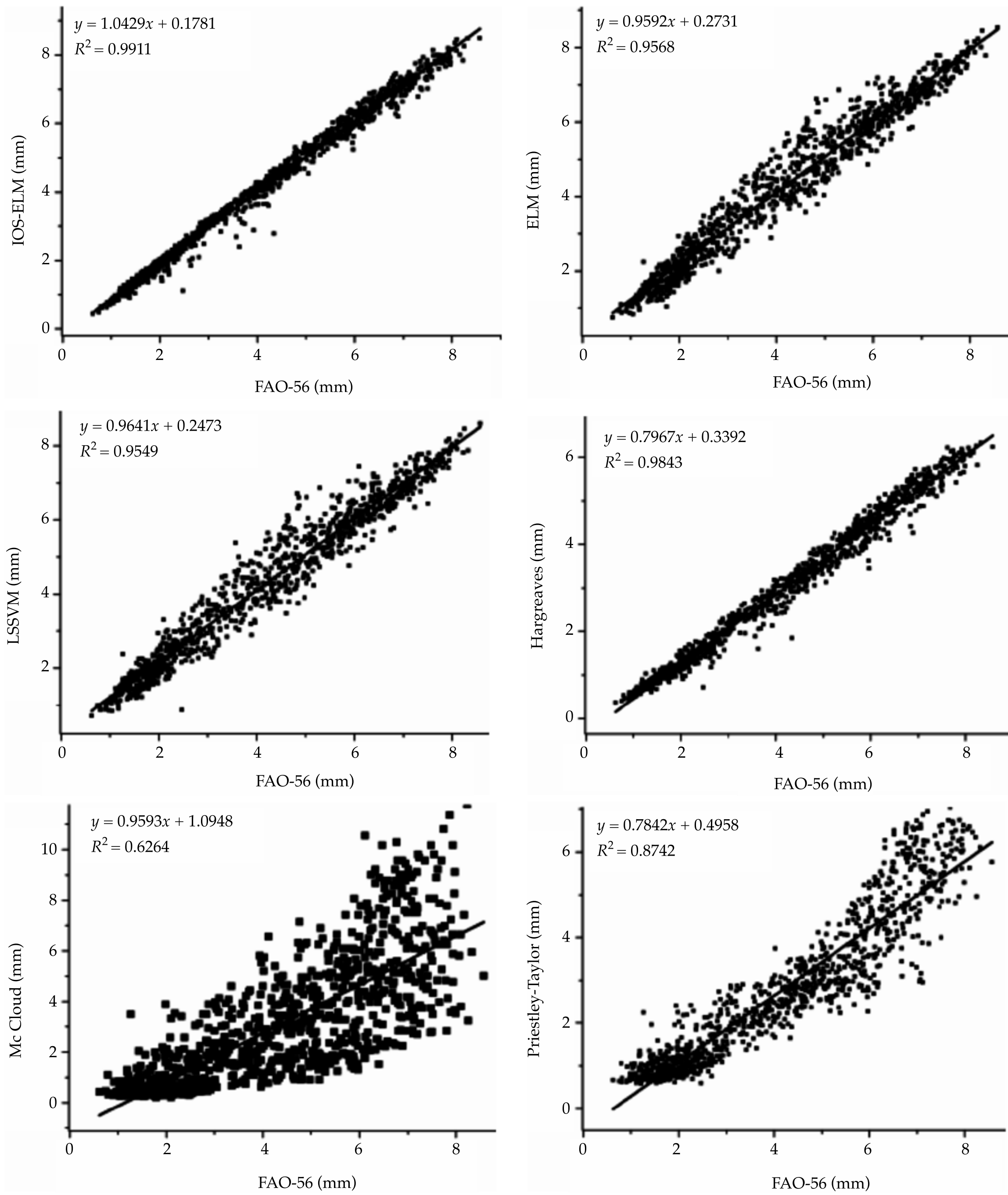

Figure 4. The FAO-56 PM ET0 and estimated *ET*0 values of Hanzhong.

The total *ET*0 estimation of every model is compared in table 7 because of its importance in irrigation management. The ISO-ELM clearly performed better than the other models from the relative error, which was 4.76, 0.23, 0.02 and 0.54%, respectively. In four of the cities, it gave the closest estimate to the total FAO-56 PM *ET*0 during the validation period.

For Yulin, the ELM and LSSVM had the same accuracy, which was the second best, and

ISSN 0187-8336 •

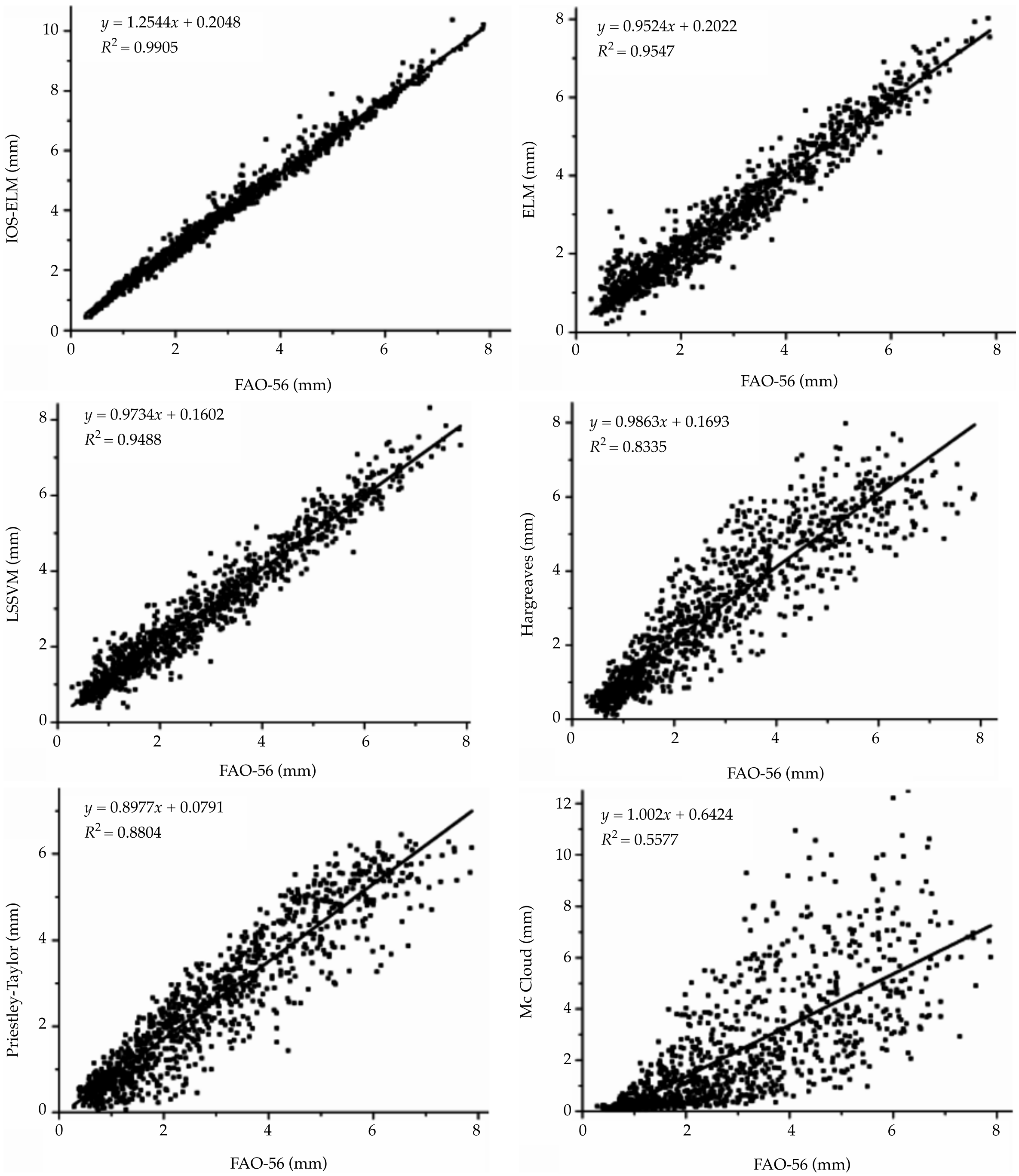

Figure 5. The FAO-56 PM ET0 and estimated *ET*0 values of Xi'an.

had 5274 and 5488 estimates lower than the 10% error, respectively. They were followed by Hargreaves, Priestley-Taylor and, lastly, McCloud (1955), which had the highest error at 64.60%. For Ankang, the LSSVM was ranked as the second best, followed by the ELM, Mc Cloud, Hargreaves and, finally, Priestley-Taylor. For Hangzhong, the LSSVM was ranked as the the second best, followed by ELM, Hargreaves, Mc Cloud, and Priestley-Taylor, which had 28.49,

Table 7. Performance statistics of the models in the validation period.

Models	Total ET0(mm)				Relative error (%)			
	Yulin	Ankang	Hanzhong	Xi'an	Yulin	Ankang	Hanzhong	Xi'an
Observed	5 039	4 661	4 307	3 134	-	-	-	-
ISO-ELM	5 279	4 650	4 306	3 117	4.76	0.23	0.02	0.54
ELM	5 274	4 636	4 413	3 207	4.66	0.54	2.46	2.33
LSSVM	5 488	4 639	4 408	3 226	8.91	0.47	2.35	2.94
Hargreaves	3 003	3 362	3 080	3 276	40.40	27.87	28.49	4.53
Mc Cloud	1 784	3 672	2 999	2 436	64.60	21.22	30.37	22.27
Priestley-Taylor	2 844	3 051	2 864	2 727	43.56	34.54	33.50	12.99

30.37 and 33.50% error, respectively. For Xi'an, the ELM, LSSVM, and Hargreaves were ranked the second best, followed Priestley-Taylor and, finally, Mc Cloud.

In short, for the different cities, the ISO-ELM performed better than the other models, and the other models had different degrees to adapt to the application.

Conclusion

The improved sequential extreme learning machine (IOS-ELM) is designed and applied for simulation of daily reference evapotranspiration through different manipulation of the inverse of the matrix and using the regularization factor and online learning method at the same time Experimental results demonstrated that the IOS-ELM can learn faster and achieve better performance than traditional ELM.

First, the IOS-ELM model effectively overcomes the defects of traditional ELM, such as slow training speed, difficult parameter decisions, difficulty in setting the singularity and effect of data samples.

Second, the potential of the ISO-ELM technique for the estimation of reference evapotranspiration was investigated for four areas in Shaanxi of China; particularly, eight meteorological data were used as inputs.

Third, it was demonstrated that intelligent algorithm models (IOS-ELM, ELM, and LSSVM) are widely applicable to different areas, but empirical models were limited to specific regions and required modification.

Fourth, in the different meteorological data combinations for $ET0$ estimation, as long as there was a temperature-related parameter calculation, the calculation accuracy of $ET0$ was over 94%, and Tmax was especially effective. These accurate calculations can be a valuable reference for the development of intelligent irrigation in water decision-making systems.

Acknowledgments

This work was supported by the National High Technology Research and Development Program of China (863 Program) under Grant Nos. 2011AA100509-01.

Notation

The following symbols are used in this paper:

$ET0$ = reference evapotranspiration (mm day^{-1}).
Δ = slope of the saturation vapor pressure function (kPaC^{-1}).
Rn = net radiation (MJ m^{-2} day^{-1}).
G = soil heat flux density (MJ m^{-2} day^{-1}).
c = psychometric constant (kPa C^{-1}).
T = mean air temperature (°C).
U2 = average24-h wind speed at 2 m height (ms^{-1}).
Rs = solar radiation (MJ m^{-2} day^{-1}).
es = the saturation vapor pressure (kPa).
ea = the actual vapor pressure (kPa).

ISSN 0187-8336 •

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Tecnología y Ciencias del Agua, vol. VIII, núm. 2, marzo-abril de 2017, pp. 127-140
• ISSN 0187-8336