COMPARISON OF APPROACHES TO 10 BAR TRUSS STRUCTURAL OPTIMIZATION WITH INCLUDED BUCKLING CONSTRAINTS

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Abstract:
The complex problem of truss structural optimization, based on the discrete design variables assumption, can be approached through optimizing aspects of sizing, shape, and topology or their combinations. This paper aims to show the differences in results depending on which aspect, or combination of aspects of a standard 10 bar truss problem is optimized. In addition to standard constraints for stress, cross section area, and displacement, this paper includes the dynamic constraint for buckling of compressed truss elements. The addition of buckling testing ensures that the optimal solutions are practically applicable. An original optimization approach using genetic algorithm is verified through comparison with literature, and used for all the optimization combinations in this research. The resulting optimized model masses for sizing, shape, and topology or their combinations are compared. A discussion is given to explain the results and to suggest which combination would be best in a generalized example.

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1. INTRODUCTION

Structural truss optimization is a complex process used in the fields of mechanical, civil, and structural engineering. Structural optimization determines the best design for a specified problem subjected to certain restrictions. This process is very beneficial, as it can lead to lighter and more inexpensive structures, while maintaining structural integrity, through optimizing different parameters. The basic truss optimization types are sizing, shape, and topology and their combinations. Most of the optimization studies on truss problems are based on the discrete design variables assumption, where each member of the truss structure is treated as having separate design variables (i.e. length thickness, cross-sectional area) [1]. Truss sizing optimization views cross section geometries as variables, shape optimization varies the set geometrical configuration's shape, and topology optimization creates new geometrical configurations.

Optimization of one, or a combination, of these types has been the subject of numerous studies over the years using a wide range of optimization methods, most notably heuristic optimization methods due to their favourable constraints.

Genetic algorithm (GA) with finite element analysis using an encoding technique proposed by Cazacu and Grama [2] showed good results for sizing optimization of benchmark problems. Authors [3-5] have used various methods to improve sizing optimization results of standard test examples using static stress constraints. Degertekin and Hayalioglu [6] analysed the use of teaching-learning based optimization (TLBO) for sizing of trusses with a fixed stress limit.

Combinations of sizing and topology as optimized in [1,7] give structures with elements missing in place of the thinnest elements of their counterparts which just use sizing.

A sizing and shape optimization combination as presented by authors in [8] gives the better results out of the two combinations with sizing due to the decrease of element lengths as well as their cross

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sections. Frans and Arfiadi [9], and previously [10], optimized sizing, shape, and topology of truss examples.

An important, yet rarely found constraint in literature is buckling [11,12]. Most published research on the topic of structural truss optimization has static constraints for member compression stress. This approach results in optimal solutions which do not meet buckling requirements. The addition of dynamic constraints for buckling increases complexity, requires longer calculation times, results in structures of greater mass, but ensures practical applicability of attained results.

Structural optimization is intended to optimize all three aspects (sizing, shape, and topology) of the truss. In engineering practice this is not always possible due to various restrictions. This paper aims to show and analyse what can be achieved through the choice of either one or any combination of these aspects for optimization when dynamic constraints for critical buckling forces are added.

2. PROBLEM FORMULATION

The problem of structural truss optimization, based on the discrete design variables assumption, implies the simultaneous optimization of sizing, topological, and shape aspects of the initial model. However, practice shows that the combination of all three of these aspects is not always possible. The goal of this research is to analyse and show the possibilities and results of optimizing any single, or any combination of these aspects on one of the most frequently used examples for truss optimization.

The objective functions of all optimization configurations aim to find the variable combination which would minimize the construction's weight. Many researchers put considerable effort to solve this problem investigating numerous optimization methods. For typical truss optimization found in literature the minimum weight design problem can be defined as:

$$\begin{cases} minW(A,\, n,\, l) = \sum_{i=1}^{i=n} \rho_i A_i l_i \text{ with } A = (A_1, \ldots, A_n) \\ \text{subjected to} \begin{cases} A_{min} \le A_i \le A_{max} & \text{for } i = 1, \ldots, n \\ \sigma_{min} \le \sigma_i \le \sigma_{max} & \text{for } i = 1, \ldots, n \\ u_{min} \le u_j \le u_{max} & \text{for } j = 1, \ldots, k \end{cases} \end{cases} \quad (1)$$

where n is the number of truss elements, k is the number of nodes, l_i is the length of the i^{th} element, A_i is the area of the i^{th} element cross section, σ_i is the stress of the i^{th} element, u_j is displacement of the j^{th} node. This objective function depending on which combination or single optimization is conducted, the function criteria changes accordingly, while the constraints remain the same for all problems.

2.1 The design problem

The 10 bar truss is one of the most commonly used examples for truss optimization. The initial model bar and node layout is given in Fig.1. This cantilever truss has 10 independent variables. The material of the truss elements is Aluminium 6063-T5 whose characteristics are: Young modulus 68947 MPa, and density of 2.7 g/cm^3. Point load is F=444.82 kN, as shown in Fig.1. The model is limited to a maximal displacement of ±0.0508 m of all nodes in all directions, axial stress of ±172.3689 MPa for all bars, and minimum area of all members is limited to 0.6426 cm^2.

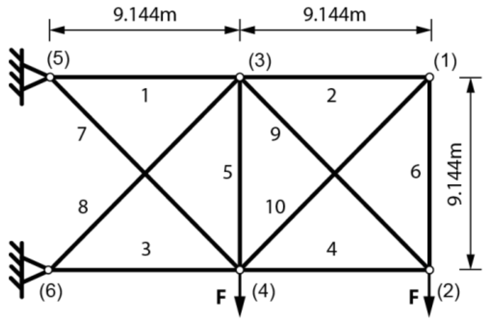

Fig.1. Configuration of 10 bar truss problem

The initial cross section area for all calculations is 452.3893 cm^2. This is also the cross section area for examples which do not consider sizing, as this is the minimal rounded up diameter (240 mm) of elements which meets buckling constraints. The initial model with these bars has a weight of 13019.482 kg. In order to allow for shape optimization coodrinates of nodes 1 and 3 are variables in examples which optimize this aspect of the truss. Node 5, as it is a support is not set as a variable, as found in [9]. Topology is limited to the removal of 6 elements at most.

2.2 Euler buckling

Optimal solutions of structural truss optimization problems have elements subjected to compression forces which need to be lower than critical buckling values. Since the Euler critical buckling load equation (3) considers axial compression force, cross sectional characteristics, and bar length, buckling needs to be checked for all bars for each iteration. The proposed Euler buckling constraint defined by Euler's critical load is given in the following expressions:

$$\left|F_{Ai}^{comp}\right| \le F_{Ki} \text{ for } i \quad 1,...,n \tag{2}$$

$$F_{Ki} = \frac{\pi^2 \cdot E_i \cdot}{l_i^2} \tag{3}$$

where F_{Ai}^{comp} is the axial compression force, F_{Ki} is Euler's critical load, E_i is the modulus of elasticity, and I_i is the minimum area moment of inertia of the cross section of the of the i^{th} element. The condition from equation (1) will be added to the existing constraints. Since the buckling constraint changes with each iteration, this constraint is considered a dynamic constraints, and its addition drastically increase the complexity of the optimization problem.

2.3 Optimization

The optimization method selected for the purposes of this research is genetic algorithm (GA). GA is a heuristic optimization method whose operation is based on imitating natural processes [13].

The algorithm consists of three basic operators: selection, crossover, and mutation (Fig.2). Selection is the process of transferring genetic information through generations. Crossover represents the process (operations) between two parents, where an exchange of genetic information is done, and new generations are created. A random change in the genetic structure of some individuals for overcoming early convergence is created by the mutation operator.

Algorithm operation is based on survival of the fittest individuals through evolution which exchange genetic material. Selection is used to rank individuals in the population using values from the fitness function, which defines the ability (quality) of the individual.

The parametric model and optimization in this research are all done in Rhinoceros 5.0 software using Grasshopper, Galapagos optimization, and Karamba plugins. An original files were created in this program which allows for the choice of optimization type, and/or combination of types, as well as the choice of constraints used. Galapagos optimization uses GA as its optimization method.

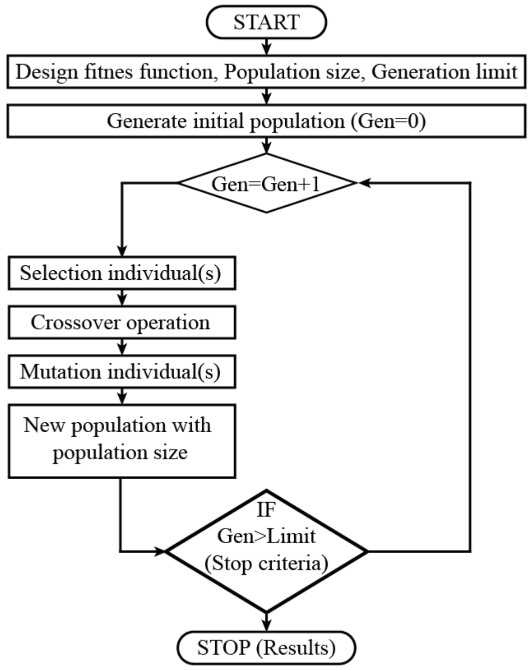

Fig.2. Genetic algorithm

3. RESULTS

Since there is no research found which gives buckling constrained results for 10 bar trusses, in order to verify the method and results existing examples found in literature were repeated using the original file used for this paper without including buckling constraints. In [3] sizing optimization gives an optimal weight of 2294.568 kg, and the file used for this research gives 2314.164 kg. The example of sizing and shape optimization conducted in [8] gave an optimal weight of 2322.080 kg, and 2300.590 kg in this paper. The sizing, topology and shape optimization result of 2122.6222 kg [9] compared to this papers 2257.239 kg validate the file setup, method and result.

All three individual aspects of the 10 bar truss were optimized in the proposed file with included dynamic constraints for buckling (Fig.3). The three possible combinations of two followed (Fig.4). Finally the sizing, topology and shape simultaneous optimization was done (Fig.5). Table 1 gives the new coordinates for nodes 1 and 3 for all combinations. In cases were shape was not optimized data is not given, as the coordinates are the same as in the initial model. Coordinates of nodes which do not connect to the model with bars, as a result of topology optimization, are not given. Optimal areas of bars and weights of all seven variants are given in Table 2.

Table 1. Optimal node coordinates by type

Optimization variant	Node 1 (x, y) [m]	Node 3 (x, y) [m]
Sizing	-	-
Topology	-	-
Shape	(10.584, 1.603)	(10.178, 2.853)
Sizing and topology	-	-
Sizing and shape	(12.079, 3.887)	(8.954, 5.879)
Topology and shape	(9.423, 2.956)	-
Sizing, topology and shape	-	(12.799, 3.966)

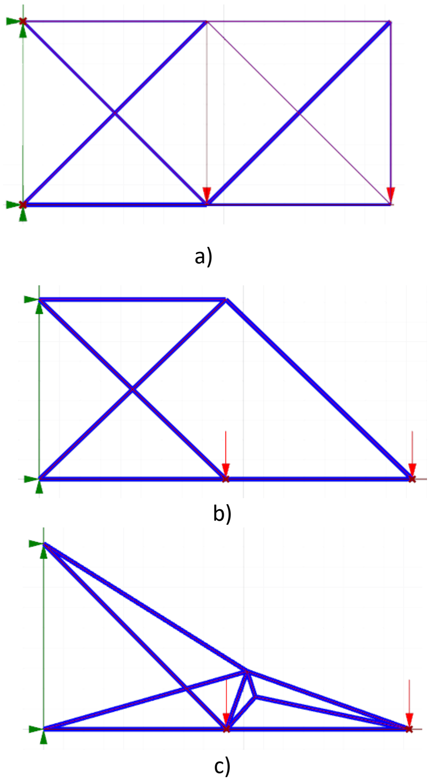

Fig.3. Optimal configurations for a) sizing, b) topology, c) shape

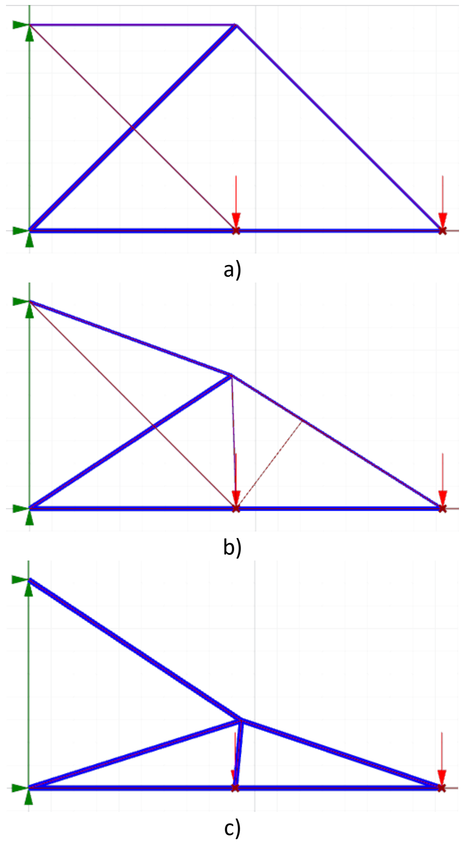

Fig. 4. Optimal configurations for a) sizing and topology, b) sizing and shape, c) topology and shape combinations

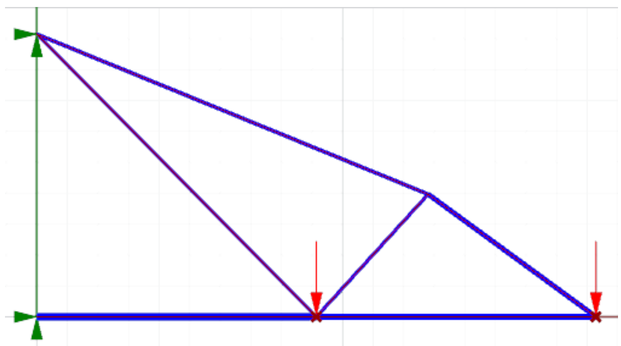

Fig. 5. Optimal configuration for the sizing, topology and shape simultaneous optimization

Table 2. Bar areas and optimized weight for all optimization combinations with buckling constraint

Area of bar [cm^2]	Sizing	Topology	Shape	Sizing and toplogy	Sizing and shape	Topology and shape	Sizing, topology and shape
1	74.58352	452.3893	452.3893	89.08818	179.1678	452.3893	125.6289
2	52.71413	-	452.3893	-	10.34957	-	-
3	425.1333	452.3893	452.3893	370.3749	368.7339	452.3893	454.3156
4	157.3157	452.3893	452.3893	262.2361	330.5622	452.3893	308.5585
5	0.741299	-	452.3893	-	28.70441	452.3893	102.5278
6	61.05257	-	452.3893	-	11.99239	-	-
7	169.4642	452.3893	452.3893	36.44082	33.7946	-	74.89887
8	267.758	452.3893	452.3893	441.3566	339.8973	452.3893	-
9	27.82731	452.3893	452.3893	111.2438	116.4499	452.3893	215.1459
10	352.5366	-	452.3893	-	0.690409	-	-
Weight [kg]	**4759.458**	**8089.269**	**9371.591**	**3838.440**	**3715.950**	**6321.673**	**3172.868**

4. CONCLUSION

Optimization results which use, the here proposed, buckling dynamic constraints give significantly larger weights of models compared to ones used to validate the method. However, the examples from literature which do not have this constraint all have at least one bar which does not meet buckling criteria, and thereby are rendered unsuitable for use in practice.

Of the single aspect optimizations sizing gives significantly better results than topology and shape alone. This is due to the large initial cross sections whose weight cannot be significantly decreased through shortening as a result of node relocation, or element removal which still need to meet buckling criteria. The shape and topology combination gives the worst results out of the combinations of two for those same reasons. The benefit of sizing when combined with shape, and with topology are evident. These combinations give lower model weights, and similar configurations to their individual optimizations without sizing respectively. The complete structural optimization of sizing, topology and shape as expected gives the optimal model with the smallest weight. Through combination of decreasing the number of elements, their lengths, and cross sections the least amount of material can be used in a configuration which does not exceed any of the set constraints.

Noticeably the optimal weights in all variants are lowest in combinations which include sizing optimization. As it is not always possible to optimize all three aspects of a truss it can be concluded that of the single aspect optimization sizing can give the best results for similar types of structures. In addition, a better solution if a complete structural optimization is not possible, sizing or topology optimizations in combination with simultaneous sizing optimization are expected to give better results than when combined with each other.

Further research in this field will include the application of dynamic constraints for Euler buckling in other planar, and space trusses to see if these results carry over into more complex structures.

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