

*Journal of Applied Fluid Mechanics*, Vol. 11, No. 6, pp. 1679-1689, 2018.  
Available online at [www.jafmonline.net](http://www.jafmonline.net), ISSN 1735-3572, EISSN 1735-3645.  
DOI: 10.29252/jafm.11.06.29048

# The Influence of Pulsating Throughflow on the Onset of Electro-Thermo-Convection in a Horizontal Porous Medium Saturated by a Dielectric Nanofluid

D. Yadav<sup>†</sup>

*Department of Mathematical & Physical Sciences, University of Nizwa, Nizwa, P.O. Box 33, Postal Code 616, Oman*

<sup>†</sup>Corresponding Author Email: [dhananjayadav@gmail.com](mailto:dhananjayadav@gmail.com); [dhananjay@unizwa.edu.om](mailto:dhananjay@unizwa.edu.om)

(Received March 26, 2018; accepted May 25, 2018)

## ABSTRACT

The joint effect of pulsating throughflow and external electric field on the outset of convective instability in a horizontal porous medium layer saturated by a dielectric nanofluid is investigated. Pulsating throughflow alters the basic profiles for temperature and the volumetric fraction of nanoparticle from linear to nonlinear with layer height, which marks the stability expressively. To treat this problem, the Buongiorno's two-phase mathematical model is used taking the flux of volumetric fraction of nanoparticle is vanish on the horizontal boundaries. Using the framework of linear stability theory and frozen profile approach, the stability equations are derived and solved analytically applying the Galerkin weighted residuals method with thermal Rayleigh-Darcy number  $R_D$  as the eigenvalue. The effect of increasing the external AC electric Rayleigh-Darcy number  $R_e$ , the modified diffusivity ratio  $N_A$  and the nanoparticle Rayleigh number  $R_N$  is to favorable for the convective motion, while the Lewis number  $L_e$  and porosity parameter  $\phi$  have dual influence on the stability scheme in the existence of pulsating throughflow. The frozen profile method shows that the result of pulsating throughflow in both directions is stabilizing. An enlarged amplitude of throughflow fluctuations offers to increased stability by an amount that vary on frequency. It is also found that the oscillatory mode of convection is not favorable for nanofluids if the vertical nanoparticle flux is vanish on the boundaries.

**Keywords:** Nanofluids; Pulsating throughflow; Linear stability theory; Electro-convection; Galarkin method.

## 1. INTRODUCTION

In the last few years, substantial research has been carried out on nanofluids convection in a porous medium due to their numerous uses in diverse field such as geophysics, bioengineering, thermal buildings, food processing, oil reservoir representing, petroleum production, material processing and cooling in processors and electronic packing, to name but a few (Wong and Leon 2010; Saidur *et al.* 2011; Bég *et al.* 2013; Sheikholeslami and Ganji 2014; Leong *et al.* 2010; Wang 2007; Murshed *et al.* 2008; Sharifpur *et al.* 2016; Özerinç *et al.* 2010; Yadav 2014; Nield and Bejan 2013). The computational and scientific models depicted here contain two fundamental methodologies: homogeneous flow models and dispersal models. In 2006, Buongiorno (2006) observed that the homogeneous flow models does not fit to the test results and determined the lower nanofluid heat exchange coefficient, while the scattering impact is absolutely ignore due to the nanoparticle measure. He determined another model to clarify the irregular

convective heat exchange in nanofluids with the impact of Brownian movement and thermophoresis and also he detached the deficiencies of the homogeneous and dispersal models. Using this model, many researchers calculated the flow and stability of nanofluids in a porous medium. Nield and Kuznetsov (2009, 2013, 2014) studied the onset of nanofluid convection in a horizontal porous layer with the Darcy model. Later, Yadav *et al.* (2016a, 2017), Chand and Rana (2012), Rana *et al.* (2014), Umavathi *et al.* (2015) and Shivakumara *et al.* (2015) extended this problem in various situations.

In the last few years, the electro-thermo-convection in dielectric fluids or nanofluids has an increasing importance in practice; it may be used in the electrical equipments, such as shunt reactors, power transformers and potential technology (Asadzadeh *et al.* 2012; Bryan and Seyed-Yagoobi 1997; Lin and Jang 2005). Several investigations have been made to study the convective instability in a dielectric fluid or nanofluid with the direct usage of a vertical AC electric field and an upright

temperature gradient. Roberts (1969) studied the electrohydrodynamic convection problem taking the dielectric parameter as a linear function of temperature. Stiles (1991) considered the direct consequence of an upright temperature gradient and a vertical AC electric field on convective instability in a flat dielectric fluid. He observed that the induced instability is due to mainly the polarization body force in the case of a rapidly varying AC electric field. Yadav *et al.* (2016b) and Chand *et al.* (2016) studied the electro-thermo convection problem in a porous medium saturated by a dielectric nanofluid. The extension with internal heating was analyzed by Yadav (2017) by the Galerkin weighted residuals method.

Throughflow effect on the convective instability in a dielectric nanofluid layer is an important concept because of its uses in engineering, geophysics and electrohydrodynamic. In-situ processing of electronic components, chemical equipment, energy assets, petroleum, geo-thermal energy and numerous real-world problems frequently connected with throughflow in a porous medium (Vafai 2005; Nield and Bejan 2013). The consequence of convection problem in such situation may become vital when specific production is desired. In addition, the throughflow result in such circumstances may be of attention due to the opportunity of regulating the convective instability by regulating the throughflow with respect to gravity force. Throughflow changes the basic shapes for heat and volumetric fraction of nanoparticle from linear to nonlinear with height, which lettering the stability of the system considerably. Without the effect of electric field, the result of throughflow on the outset of nanofluid convection in a porous medium was calculated by Nield and Kuznetsov (2011) and Yadav *et al.* (2016c). Lately, Nield and Kuznetsov (2016) studied the result of pulsating throughflow on the onset of convection in a horizontal porous layer for regular fluid using the frozen profile approach. Such problem is relevant to the small frequency situation that is appropriate to electrical and hydrological conditions.

From the literature no examination has been noted which study about the result of pulsating throughflow on the outset of electro-thermo-convection in a horizontal porous medium saturated by a dielectric nanofluid. Therefore, it would be importance here to examine the joint impact of pulsating throughflow and electric field on the convective instability in a horizontal porous medium soaked by a dielectric nanofluid. By linear stability theory and frozen profile approach, the critical states for the outset of convection are derived analytically and discussed graphically. Many experimental results show that the linear stability analysis is sufficient to obtain the stability conditions (Chandrasekhar 1961; Fein 1973; Nield and Bejan 2013). For instance, Chossat and Iooss (1994) and Lewis and Nagata (2004) observed that the linear stability analysis successfully predicts experimentally observed critical conditions for the onset of convection.

## 2. PROBLEM FORMULATION

Examine an infinitely extended flat porous medium soaked by incompressible dielectric nanofluid, bounded by perfectly conducting plates at  $z^* = 0$  and  $z^* = L$ . The nanofluid layer is heated from below and the temperatures at the bottom and top plates are supposed to be  $\theta_1^*$  and  $\theta_2^*$ , respectively. It is supposed that the flux of the volumetric fraction of nanoparticle  $J_z^*$  is to be vanish on the plates. Under the gravitational field  $\vec{g}$ , the nanofluid layer is applied to an external vertical AC electric field, such that the bottom plane is grounded and the top plane is reserved at an intermittent potential whose root mean square is  $\psi_1^*$  (Fig. 1).

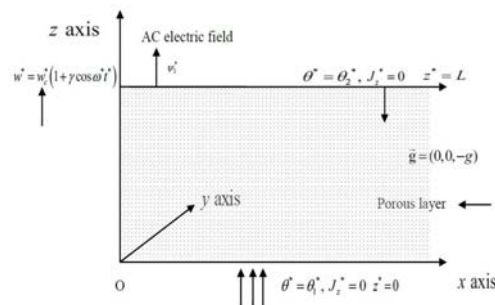


Fig. 1. Physical configuration of the problem.

It is accepted that the size ( $<50\text{nm}$ ) and the volumetric fraction of nanoparticles ( $<1\%$ ) are very small and nanoparticles are dispersed in the nanofluid utilizing either surfactant or surface charge innovation. This preserves the particles from accumulating on the porous lattice. The viscosity, thermal conductivity and specific heat of nanofluid are taken as uniform with respect to the reference temperature  $\theta_c^*$  of the cold wall apart from the dielectric parameter and the density in the Maxwell and the momentum equations, respectively. This approximation is valid for small volumetric fraction of nanoparticles (Tzou, 2008). In the present work, we choose  $\theta_c^* = \theta_2^*$ . The asterisks are taken to make dissimilar the dimensional variables from the dimensionless variables (with no asterisks). The extension of Navier-Stokes equations for nanoscales under the Oberbeck-Boussinesq approximation are (Buongiorno 2006; Yadav *et al.* 2016d,e; Chand *et al.* 2016):

$$\nabla^* \cdot \vec{\nabla}_D^* = 0 \tag{1}$$

$$0 = \left[ \phi^* \rho_p + \rho_0 (1 - \phi^*) \left\{ 1 - \beta (\theta^* - \theta_c^*) \right\} \right] \vec{g} + \vec{f}_e^* - \nabla^* P^* - \frac{\mu}{k} \vec{\nabla}_D^* \tag{2}$$

$$\left[ (\rho c)_m \frac{\partial}{\partial t^*} + (\rho c) (\vec{\nabla}_D^* \cdot \nabla^*) \right] \theta^* = K_m \nabla^{*2} \theta^* + \phi (\rho c)_p \left[ D_B \nabla^* \phi^* \cdot \nabla^* \theta^* + \left( \frac{D_T}{\theta_c^*} \right) \nabla^* \theta^* \cdot \nabla^* \theta^* \right] \tag{3}$$

$$\left[ \frac{\partial}{\partial t^*} + \frac{1}{\phi} (\bar{\mathbf{V}}_D^* \cdot \nabla^*) \right] \phi^* = D_B \nabla^2 \phi^* + \left( \frac{D_T}{\theta_c^*} \right) \nabla^2 \theta^* \quad (4)$$

where,  $K_m$  is the effective thermal conductivity of the porous medium,  $\nabla^*$  is the vector differential operator,  $\rho_0$  is the nanofluid density at reference temperature  $\theta_c^*$ ,  $\bar{\mathbf{V}}_D^*$  is the Darcy velocity of nanofluid,  $t^*$  is the time,  $P^*$  is the pressure,  $\mu$  is the viscosity of nanofluid,  $\rho$  is the density of nanofluid,  $\rho_p$  is the density of nanoparticles,  $\beta$  is the thermal expansion coefficient,  $\phi^*$  is the volumetric fraction of nanoparticles,  $(\rho c)_p$ ,  $(\rho c)$  and  $(\rho c)_m$  are the heat capacities of nanoparticles, nanofluid and overall porous medium, respectively,  $D_B$  is the Brownian diffusion coefficient,  $D_T$  is the thermophoresis diffusion coefficient and  $\bar{\mathbf{f}}_e^*$  is the force due to electrical field and given as (Landau and Lifshitz, 1960):

$$\bar{\mathbf{f}}_e^* = \rho_e \bar{\mathbf{E}}^* - \frac{1}{2} (\bar{\mathbf{E}}^* \cdot \bar{\mathbf{E}}^*) \nabla^* \varepsilon^* + \frac{1}{2} \nabla^* \left[ \rho \frac{\partial \varepsilon^*}{\partial \rho} (\bar{\mathbf{E}}^* \cdot \bar{\mathbf{E}}^*) \right] \quad (5)$$

where,  $\rho_e \bar{\mathbf{E}}^*$  is the Coulomb force and  $\left\{ \frac{1}{2} (\bar{\mathbf{E}}^* \cdot \bar{\mathbf{E}}^*) \nabla^* \varepsilon^* \right\}$  is the dielectrophoretic force.

Here,  $\rho_e$  is the charge density,  $\bar{\mathbf{E}}^*$  is the root mean square value of the electric field and  $\varepsilon^*$  is the dielectric parameter. Here the dielectric parameter  $\varepsilon^*$  is function of temperature. If AC electric field is applied then the second term in Eq. (5) cannot neglect. Since the second term depends on the gradient of  $\varepsilon^*$  so this force dominates when an AC electric field is imposed on a dielectric fluid. If we ignore the effect of the Coulomb forces compared with the other forces (Turnbull and Melcher 1969), then we will retain only the dielectrophoretic force term in the Eq. (2). This situation can be happened when an AC electric field will be associated at a frequency greatly higher than the reciprocal of the electrical relaxation time.

Considering the free charge density is zero, the related Maxwell equations are:

$$\nabla^* \times \bar{\mathbf{E}}^* = 0 \quad (6)$$

$$\nabla^* \cdot (\varepsilon^* \bar{\mathbf{E}}^*) = 0 \quad (7)$$

In sight of the Eq. (6),  $\bar{\mathbf{E}}^*$  can be written as:

$$\bar{\mathbf{E}}^* = -\nabla^* \psi^* \quad (8)$$

where  $\psi^*$  is the root mean square value of the electric potential.

The dielectric parameter  $\varepsilon^*$  is given as:

$$\varepsilon^* = \varepsilon_0 \left[ 1 - e (\theta^* - \theta_c^*) \right] \quad (9)$$

Here,  $\varepsilon_0$  is the dielectric parameter at reference temperature  $\theta_c^*$  and  $e$  is the thermal expansion coefficient of dielectric parameter (Roberts 1969). If the temperature is uniform and the flux of volumetric fraction of nanoparticle is vanish on the plates, then the boundary conditions for both plates are:

$$w^* = w_c^* (1 + \gamma \cos \omega t^*), \theta^* = \theta_1^*, J_z^* = 0 \text{ at } z^* = 0 \quad (10a)$$

$$w^* = w_c^* (1 + \gamma \cos \omega t^*), \theta^* = \theta_2^*, J_z^* = 0 \text{ at } z^* = L \quad (10b)$$

here,  $J_z^* = -\rho_p \left[ D_B \frac{\partial \phi^*}{\partial z^*} + (D_T / \theta_c^*) \frac{\partial \theta^*}{\partial z^*} \right]$  is the flux of volumetric fraction of nanoparticle,  $w^*$  is the vertical velocity of nanofluids,  $\gamma$  and  $\omega^*$  are the amplitude and angular frequency of pulsation, respectively. Since the considered plates are perfectly conducting so, Dirichlet boundary conditions for temperature (Eq. (10)) are realistic.

We initiate the succeeding dimensionless variables:

$$\begin{aligned} \bar{x} &= \frac{\bar{x}^*}{L}, t = \frac{\alpha_m}{\sigma L^2} t^*, \bar{V} = \frac{L}{\alpha_m} \bar{V}_D^*, \\ P &= \frac{k}{\mu \alpha_m} P^*, \theta = \frac{\theta^* - \theta_2^*}{\theta_1^* - \theta_2^*}, \phi = \frac{\phi^* - \phi_0^*}{\phi_0^*}, \\ \bar{E} &= \frac{1}{e \Delta \theta^* E_0^*} \bar{E}^*, \psi = \frac{1}{e \Delta \theta^* L E_0^*} \psi^*, \\ \varepsilon &= \frac{1}{\varepsilon_0} \varepsilon^*, J_z = \frac{L}{\rho_p D_B \phi_0^*} J_z^* \end{aligned} \quad (11)$$

where  $\alpha_m = \frac{K_m}{(\rho c)}$ ,  $\sigma = \frac{(\rho c)_m}{(\rho c)}$ ,  $\Delta \theta^* = \theta_1^* - \theta_2^*$ ,  $E_0^*$  is the root mean square value of the electric field at  $z^* = 0$  and  $\phi_0^*$  is a reference value for the nanoparticle volume fraction.

Considering the Eq. (11), the governing equations then take the form:

$$\nabla \cdot \bar{\mathbf{V}} = 0 \quad (12)$$

$$0 = -\nabla \left[ P + R_M z - \frac{1}{2} R_e \rho \frac{\partial \varepsilon}{\partial \rho} (\bar{\mathbf{E}} \cdot \bar{\mathbf{E}}) \right] - \bar{\nabla} - \frac{1}{2} R_e (\bar{\mathbf{E}} \cdot \bar{\mathbf{E}}) \nabla \varepsilon + (R_D \theta - R_N \phi) \hat{\mathbf{e}}_z, \quad (13)$$

$$\left[ \frac{\partial}{\partial t} + (\bar{\mathbf{V}} \cdot \nabla) \right] \theta = \nabla^2 \theta + \frac{N_B}{L_e} \left( \frac{\nabla \phi \nabla \theta}{+ N_A \nabla \theta \nabla \theta} \right) \quad (14)$$

$$\left[ \frac{1}{\sigma} \frac{\partial}{\partial t} + \frac{1}{\phi} (\bar{\mathbf{V}} \cdot \nabla) \right] \phi = \frac{1}{L_e} \nabla^2 \phi + \frac{N_A}{L_e} \nabla^2 \theta \quad (15)$$

$$\nabla \times \bar{\mathbf{E}} = 0 \quad (16)$$

$$\nabla \cdot (\varepsilon \bar{\mathbf{E}}) = 0 \quad (17)$$

$$\bar{E} = -\nabla \psi \tag{18}$$

$$\varepsilon = (1 - e \Delta \theta^* \theta) \tag{19}$$

$$J_z = -\left(\frac{\partial \phi}{\partial z} + N_A \frac{\partial \theta}{\partial z}\right) \tag{20}$$

In the non-dimensional form, the boundary conditions become:

$$\begin{aligned} w = Q(1 + \gamma \cos \omega t), \theta = 1, J_z = 0, \text{ at } z = 0, \\ w = Q(1 + \gamma \cos \omega t), \theta = 0, J_z = 0, \text{ at } z = 1, \end{aligned} \tag{21a,b}$$

The non-dimensional parameters that come out in Eqs. (12)-(21) are defined as:

$$L_e = \frac{\alpha_m}{D_B} \text{ is the Lewis number, } R_D = \frac{\rho_0 g \beta \Delta \theta^* k L}{\mu \alpha_m}$$

is the thermal Rayleigh-Darcy number,

$$R_M = \frac{[\rho_0(1 - \phi_0^*) + \rho_p \phi_0^*] g k L}{\mu \alpha_m} \text{ is the density}$$

$$\text{Rayleigh-Darcy number, } R_N = \frac{(\rho_p - \rho_0) \phi_0^* g k L}{\mu \alpha_m} \text{ is}$$

the nanoparticle Rayleigh-Darcy number,

$$R_c = \frac{\varepsilon_0 (e \Delta \theta^* E_0^*)^2 k}{\mu \alpha_m} \text{ is the AC electric Rayleigh-}$$

$$\text{Darcy number, } N_A = \left(\frac{D_T}{\phi_0^* D_B}\right) \left(\frac{\theta_1^* - \theta_c^*}{\theta_c^*}\right) \text{ is the}$$

$$\text{modified diffusivity ratio, } N_B = \varphi \frac{(\rho c)_p \phi_0^*}{(\rho c)}$$

$$\text{is the modified particle-density increment, } Q = \frac{L w_c^*}{\alpha_m}$$

$$\text{is the Péclet number and } \omega = \frac{\sigma L^2}{\alpha_m} \omega^* \text{ is the dimensionless angular frequency.}$$

### 2.1 Basic State

Assuming the basic state to be calm and given as:

$$\begin{aligned} \bar{V}_b = Q_1 \hat{e}_z, \theta_b = \theta_b(z), P_b = P_b(z), \phi_b = \phi_b(z), \\ \varepsilon_b = \varepsilon_b(z), \bar{E}_b = E_b(z) \hat{e}_z, \psi_b = \psi_b(z) \end{aligned} \tag{22}$$

$$\text{Here } Q_1 = Q(1 + \gamma \cos \omega t).$$

The solutions of the basic state are:

$$\theta_b = \frac{e^{Q_1} - e^{Q_1 z}}{e^{Q_1} - 1} \tag{23}$$

$$\phi_b = \frac{N_A [(e^{Q_2} - 1)(e^{Q_2 z} - 1)Q_1 - (e^{Q_1} - 1)(e^{Q_2 z} - 1)Q_2]}{(e^{Q_1} - 1)(e^{Q_2} - 1)(Q_1 - Q_2)} + \phi_0 \tag{24}$$

$$E_b = \frac{1}{e \Delta \theta^* (1 + e \Delta \theta^* z)} \tag{25}$$

$$\psi_b = -\frac{\log(1 + e \Delta \theta^* z)}{(e \Delta \theta^*)^2} \tag{26}$$

$$\varepsilon_b = 1 + e \Delta \theta^* z \tag{27}$$

$$J_{zb} = 0 \tag{28}$$

$$\text{where, } Q_2 = \frac{Q_1 L_e}{\varphi}, E_0^* = -\frac{e \Delta \theta^* \psi_1^*}{L \log(1 + e \Delta \theta^*)} \text{ and}$$

$$\phi_0 = \frac{\phi_b^*(0) - \phi_0^*}{\phi_0^*} \text{ is the relative volumetric fraction}$$

of nanoparticle at  $z = 0$ . If we consider that the mean value of the basic volumetric fraction of nanoparticles  $\phi_b^*$  in each section  $x$  is equal to its reference value  $\phi_0^*$ , then

$$\int_0^1 \phi_b dz = 0 \tag{29}$$

Equation (29) gives

$$\phi_0 = \frac{N_A}{Q_1 - Q_2} \left[ \frac{Q_1}{e^{Q_1} - 1} - \frac{Q_2}{e^{Q_2} - 1} \right] \tag{30}$$

Using Eq. (30) into Eq. (24), the basic solution for volumetric fraction of nanoparticle become

$$\phi_b = \frac{N_A}{Q_1 - Q_2} \left[ \frac{Q_1 e^{Q_1 z}}{e^{Q_1} - 1} - \frac{Q_2 e^{Q_2 z}}{e^{Q_2} - 1} \right] \tag{31}$$

The basic solutions for temperature and volumetric fraction of nanoparticle were obtained by neglecting

the terms containing  $\frac{N_A N_B}{L_e}$  and  $\frac{N_B}{L_e}$ . This

approximation is valid for nanofluids. Because for the majority of nanofluids, the Lewis number  $L_e$  is

large and of arrange  $10^2$  to  $10^3$ ,  $N_A$  is of arrange

$10^0$  to  $10^1$  and  $N_B$  is of arrange  $10^{-2}$  to  $10^{-5}$ , hence

the terms containing  $\frac{N_A N_B}{L_e}$  and  $\frac{N_B}{L_e}$  are very small

and can be neglected. In the nonappearance of throughflow i.e.  $Q \rightarrow 0$ , Eq. (31) match with that of

Wakif *et al.* (2018).

### 2.2 Perturbation Equation and Frozen Profile

We now take the frozen profile hypothesis. In the

expression for the basic solution (Eqs. (23) and

(31)), we write  $t_0$  for  $t$  and

$$\tilde{Q}_1 = Q(1 + \gamma \cos \omega t_0) \text{ and } \tilde{Q}_2 = \frac{\tilde{Q}_1 L_e}{\varphi} \tag{32}$$

We now suppose small perturbations on this basic solution in the form:

$$\begin{aligned} \vec{V} &= \vec{V}_b + \vec{V}', \quad J_z = J_{zb} + J_z', \quad P = P_b + P', \\ \theta &= \theta_b + \theta', \quad \varepsilon = \varepsilon_b + \varepsilon', \quad \vec{E} = \vec{E}_b + \vec{E}', \quad (33) \\ \psi &= \psi_b + \psi', \quad \phi = \phi_b + \phi', \end{aligned}$$

where  $\vec{V}'$ ,  $P'$ ,  $\theta'$ ,  $\phi'$ ,  $J_z'$ ,  $\varepsilon'$ ,  $\vec{E}'$  and  $\psi'$  are the perturbed quantities over their equilibrium counterparts and function of  $\mathbf{x}$  and  $t$ .

Using Eq. (33) into Eqs. (12)-(21), ignoring the multiple of prime measures, abolishing the pressure term from the momentum Eq. (13) and holding the vertical component, we get the following linear stability equations :

$$\nabla \cdot \vec{V}' = 0 \quad (34)$$

$$\nabla^2 W' = R_D \nabla_H^2 \theta' - R_N \nabla_H^2 \phi' + R_e \nabla_H^2 \left( \theta' - \frac{\partial \psi'}{\partial z} \right) \quad (35)$$

$$\frac{\partial \theta'}{\partial t} + \vec{V}' \cdot \nabla \theta_b + \vec{V}_b \cdot \nabla \theta' = \nabla^2 \theta' \quad (36)$$

$$\frac{1}{\sigma} \frac{\partial \phi'}{\partial t} + \frac{1}{\varphi} \vec{V}' \cdot \nabla \phi_b + \frac{1}{\varphi} \vec{V}_b \cdot \nabla \phi' = \frac{1}{L_e} (N_A \nabla^2 \theta' + \nabla^2 \phi') \quad (37)$$

$$\nabla^2 \psi' - \frac{\partial \theta'}{\partial z} = 0 \quad (38)$$

$$\varepsilon' = -e \Delta \theta' \theta' \quad (39)$$

$$J_z' = - \left( \frac{\partial \phi'}{\partial z} + N_A \frac{\partial \theta'}{\partial z} \right) \quad (40)$$

where  $\nabla_H^2 = (\partial^2 / \partial x^2 + \partial^2 / \partial z^2)$  is the horizontal Laplacian operator. Here, Eq. (36) is obtained by neglecting the terms containing  $\frac{N_A N_B}{L_e}$  and  $\frac{N_B}{L_e}$ .

This approximation is valid for nanofluid as explained above.

In dimensionless form, the boundary conditions become:

$$w' = \theta' = J_z' = \frac{\partial \psi'}{\partial z} = 0 \text{ at } z = 0, 1 \quad (41)$$

Allowing for the perturbation quantities to be of the form as:

$$\begin{aligned} (w', \theta', \phi', \psi') &= [W, \Theta, \Phi, \Psi](z) \\ &\times e^{[ik_x x + ik_y y + st]}, \end{aligned} \quad (42)$$

where  $k_x$  and  $k_y$  are the wave numbers along the  $x$  and  $y$  directions, respectively and  $s$  is the growth rate of disturbances. The growth rate  $s$  is in general a complex quantity such that  $s = s_r + is_i$ . The scheme with  $s_r < 0$  is balanced for all time, whereas it is unbalanced when  $s_r > 0$ . For neutral stability, the real part of  $s$  is zero. Hence, we consider  $s = is_i$ , where  $s_i$  is real and is a dimensionless frequency.

Using Eq. (42) into Eqs. (35)-(38), we have:

$$\begin{aligned} (D^2 - a^2)W + R_D a^2 \Theta - R_N a^2 \Phi \\ + R_e a^2 \left( \Theta - \frac{\partial \Psi}{\partial z} \right) = 0 \end{aligned} \quad (43)$$

$$-\frac{dI_b}{dz} W + [D^2 - s - a^2 - \tilde{Q}_1 D] \Theta = 0 \quad (44)$$

$$\begin{aligned} -\frac{1}{\varphi} \frac{d\phi_b}{dz} W + \frac{N_A}{L_e} (D^2 - a^2) \Theta \\ + \left[ \frac{1}{L_e} (D^2 - a^2) - \frac{s}{\sigma} - \frac{\tilde{Q}_1}{\varphi} D \right] \Phi = 0 \end{aligned} \quad (45)$$

$$(D^2 - a^2) \Psi - D \Theta = 0 \quad (46)$$

Here  $\frac{d}{dz} \equiv D$  and  $a = \sqrt{k_x^2 + k_y^2}$  is the resulting dimension-less wave number.

In the perturbed dimensionless form, the boundary conditions become:

$$\begin{aligned} W = 0, \quad \Theta = 0, \quad D\Psi = 0, \\ D\Phi + N_A D\Theta = 0 \text{ at } z = 0, 1, \end{aligned} \quad (47)$$

To obtain an analytical solution of Eqs. (43)-(47), the Galerkin weighted residuals method is applied. In view of that, the support functions  $W$ ,  $\Theta$ ,  $\Phi$  and  $\Psi$  are considered as:

$$\begin{aligned} W = \sum_{p=1}^N A_p W_p, \quad \Theta = \sum_{p=1}^N B_p \Theta_p, \\ \Phi = \sum_{p=1}^N C_p \Phi_p, \quad \Psi = \sum_{p=1}^N D_p \Psi_p, \end{aligned} \quad (48)$$

where  $W_p = \Theta_p = \sin p\pi z$ ,  $\Phi_p = -N_A \sin p\pi z$ ,  $\Psi_p = \cos p\pi z$ , (trial functions agreeable the corresponding boundary conditions),  $A_p$ ,  $B_p$ ,  $C_p$  and  $D_p$  are new coefficients, and  $p = 1, 2, 3, \dots, N$ . Using past expression for  $W$ ,  $\Theta$ ,  $\Phi$  and  $\Psi$  into Eqs. (43)-(46) and multiplying the resultant first equation by  $W_p$ , second equation by  $\Theta_p$ , third equations by  $\Phi_p$  and fourth equation by  $\Psi_p$  and integrating in the restrictions from 0 to 1, we obtained a system of  $4N$  linear algebraic equations in the  $4N$  unknowns  $A_p$ ,  $B_p$ ,  $C_p$  and  $D_p$ ,  $p = 1, 2, 3, \dots, N$ .

For the occurrence of non-trivial result, the determinant of coefficients matrix necessity to be zero, which provides the characteristic equation for the scheme, with the thermal Rayleigh- Darcy number  $R_D$  as the eigenvalue.

### 3. RESULTS AND DISCUSSION

To obtain analytical outcome, we choose  $N = 1$ , and eliminating the complex numbers from the

denominator we get an expression for the thermal Darcy-Rayleigh number  $R_D$  as

$$R_D = \Lambda_1 + is_i \Lambda_2, \tag{49}$$

where,

$$\Lambda_1 = \frac{J^2(4\pi^2 + \tilde{Q}_1^2)}{4a^2\pi^2} - \frac{a^2 R_e}{J} + \frac{N_A R_N \sigma [L_e^2 s_i^2 (L_e \tilde{Q}_1^2 - 4\phi\pi^2) - 4\phi J^2 (\phi + L_e) \pi^2 \sigma]}{(4\phi^2 \pi^2 + L_e^2 \tilde{Q}_1^2)(L_e^2 s_i^2 + J^2 \sigma^2)} \tag{50}$$

$$\Lambda_2 = \frac{J L_e N_A R_N \sigma [4\phi(\phi + L_e) \pi^2 - 4\phi\pi^2 \sigma + L_e \tilde{Q}_1^2 \sigma]}{(4\phi^2 \pi^2 + L_e^2 \tilde{Q}_1^2)(L_e^2 s_i^2 + J^2 \sigma^2)} + \frac{J(4\pi^2 + \tilde{Q}_1^2)}{4a^2 \pi^2} \tag{51}$$

Here,  $J = (a^2 + \pi^2)$  and  $\tilde{Q}_1 = Q(1 + \gamma \cos \omega t_0)$ .

Since  $R_D$  is a physical measure, it should be obligatorily genuine. Thus, it take after from Eq. (49) that either  $s_i = 0$  (stationary convection) or  $\Lambda_2 = 0$  ( $s_i \neq 0$  non-oscillatory convection).

### 3.1 Stationary Convection

Stationary convection happens when  $s_i = 0$ . For this situation, from Eq. (49), the stationary thermal Darcy-Rayleigh number  $R_D^S$  can be derived as:

$$R_D^S = \frac{(a^2 + \pi^2)^2}{a^2} \left(1 + \frac{\tilde{Q}_1^2}{4\pi^2}\right) - \frac{a^2 R_e}{a^2 + \pi^2} - \frac{4\phi(\phi + L_e) N_A R_N \pi^2}{4\phi^2 \pi^2 + L_e^2 \tilde{Q}_1^2}, \tag{52}$$

From Eq. (52), it is apparent that the thermal Darcy-Rayleigh number does not depends on the sign of pulsating throughflow parameter  $\tilde{Q}_1$ . The effect of pulsating throughflow parameter  $\tilde{Q}_1$  is stabilizing effect in both directions. This result was expected from the symmetry of the problem. The value of  $\tilde{Q}_1^2$  lies between a minimum  $Q^2(1 - \gamma)^2$  and a maximum  $Q^2(1 + \gamma)^2$  when  $t_0$  varies.

The expression for the stationary critical thermal Rayleigh-Darcy number  $R_{D,c}^S$  is derived corresponding to the critical value of the wave number  $a_c$  by setting  $\partial R_D^S / \partial \chi = 0$  which satisfies the following equation

$$(4\pi^2 + \tilde{Q}_1^2) \chi^4 + 2(4\pi^2 + \tilde{Q}_1^2) \chi^3 - 4R_e \chi^2 - 2(4\pi^2 + \tilde{Q}_1^2) \chi - (4\pi^2 + \tilde{Q}_1^2) = 0, \tag{53}$$

where  $a_c = \pi \sqrt{\chi_c}$ .

In the nonattendance of pulsating throughflow, i.e.  $\tilde{Q}_1 = 0$ , Eqs. (52) and (53) give:

$$R_D^S = \frac{(a^2 + \pi^2)^2}{a^2} - \frac{a^2 R_e}{a^2 + \pi^2} - \left(1 + \frac{L_e}{\phi}\right) N_A R_N \tag{54}$$

$$4\pi^2 \chi^4 + 8\pi^2 \chi^3 - 4R_e \chi^2 - 8\pi^2 \chi - 4\pi^2 = 0 \tag{55}$$

In the nonattendance of electric field, Eq. (54) is identical with the outcome of Yadav and Lee (2015) for a thermal equilibrium case with  $D_a = T_D = 0$  and Eq. (55) coincides with that of Chand et al. (2016).

For regular fluid and in the nonattendance of electric field, i.e.  $R_N = N_B = R_e = 0$ , Eqs. (52) and (53) give:

$$R_D^S = \frac{(a^2 + \pi^2)^2}{a^2} \left(1 + \frac{\tilde{Q}_1^2}{4\pi^2}\right) \tag{56}$$

$$a_c = \pi \tag{57}$$

From Eqs. (56) and (57), when pulsating throughflow is equal to one, i.e.  $\tilde{Q}_1 = 1$ , the value of critical thermal Rayleigh-Darcy number is 40.4784. Recently, Barletta et al. (2016) obtained analytical result for the consequence of through flow on the buoyancy-induced instability in a horizontal porous layer by considering pressure and temperature as the dependent variables and it is equal to 40.8751 when throughflow is equal to one. Hence the approximation formula used in this paper gives a value accurate to approximately 1%. This shows that the approximation formula used in this paper is satisfactory.

To examine the effect of the pulsating throughflow  $\tilde{Q}_1$ , the AC electric Rayleigh-Darcy number  $R_e$ , the nanoparticle Rayleigh-Darcy number  $R_N$ , the modified diffusivity ratio  $N_A$ , the Lewis number  $L_e$  and the porosity parameter  $\phi$  on the onset of stationary convection, we study analytically the behaviour of  $\partial R_D^S / \partial \tilde{Q}_1$ ,  $\partial R_D^S / \partial R_e$ ,  $\partial R_D^S / \partial R_N$ ,  $\partial R_D^S / \partial L_e$ ,  $\partial R_D^S / \partial N_A$  and  $\partial R_D^S / \partial \phi$ . Equation (52) gives

$$\frac{\partial R_D^S}{\partial \tilde{Q}_1} = \frac{(a^2 + \pi^2)^2 \tilde{Q}_1}{2a^2 \pi^2} + \frac{8\phi\pi^2 L_e^2 (\phi + L_e) N_A R_N \tilde{Q}_1}{(4\phi^2 \pi^2 + L_e^2 \tilde{Q}_1^2)^2} \tag{58}$$

$$\frac{\partial R_D^S}{\partial R_e} = -\frac{a^2}{(a^2 + \pi^2)} \tag{59}$$

$$\frac{\partial R_D^S}{\partial R_N} = -\frac{4\phi(\phi + L_e) N_A \pi^2}{(4\phi^2 \pi^2 + L_e^2 \tilde{Q}_1^2)} \tag{60}$$

$$\frac{\partial R_D^S}{\partial N_A} = -\frac{4\phi(\phi + L_e) R_N \pi^2}{(4\phi^2 \pi^2 + L_e^2 \tilde{Q}_1^2)} \tag{61}$$

$$\frac{\partial R_D^s}{\partial L_e} = \frac{4\phi N_A \pi^2 \left[ -4\phi^2 \pi^2 + L_e (2\phi + L_e) \tilde{Q}_1^2 \right] R_N}{\left( 4\phi^2 \pi^2 + L_e^2 \tilde{Q}_1^2 \right)^2} \quad (62)$$

$$\frac{\partial R_D^s}{\partial \phi} = -\frac{4L_e N_A \pi^2 \left[ -4\phi^2 \pi^2 + L_e (2\phi + L_e) \tilde{Q}_1^2 \right] R_N}{\left( 4\phi^2 \pi^2 + L_e^2 \tilde{Q}_1^2 \right)^2} \quad (63)$$

From Eq. (58), it is found that  $\partial R_D^s / \partial \tilde{Q}_1$  is always positive for a stationary convection. This established that the pulsating throughflow  $\tilde{Q}_1$  has always a stabilizing effect on a system. Similarly, from Eqs. (59-63), it is observed that the AC electric Rayleigh-Darcy number  $R_e$ , the nanoparticle Rayleigh-Darcy number  $R_N$  and the modified diffusivity ratio  $N_A$  have always destabilizing result whereas, the Lewis number  $L_e$  and the porosity parameter  $\phi$  have dual outcome on the stability of the system. If  $\frac{L_e (2\phi + L_e) \tilde{Q}_1^2}{4\phi^2 \pi^2} < 1$  then the Lewis number  $L_e$  and the porosity parameter  $\phi$  have destabilizing and stabilizing outcome on the stability scheme, respectively. However, in the nonattendance of the pulsating throughflow, i.e.  $\tilde{Q}_1 = 0$ , Eqs. (62) and (63), respectively show that the Lewis number  $L_e$  and the porosity parameter  $\phi$  have always destabilizing and stabilizing result on the stability scheme.

### 3.2 Oscillatory Convection

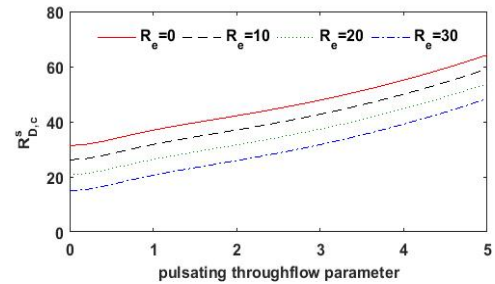
On behalf of oscillatory convection  $\Lambda_2 = 0$  and  $s_i \neq 0$ . By means of these in Eq. (49), the expressions for oscillatory Rayleigh-Darcy number  $R_D^{osc}$  and the frequency of oscillations  $s_i$  can be written as:

$$R_D^{osc} = \frac{N_A R_N \sigma \left[ L_e^2 s_i^2 (L_e \tilde{Q}_1^2 - 4\phi \pi^2) - 4\phi J^2 (\phi + L_e) \pi^2 \sigma \right]}{\left( 4\phi^2 \pi^2 + L_e^2 \tilde{Q}_1^2 \right) (L_e^2 s_i^2 + J^2 \sigma^2)} + \frac{J^2 (4\pi^2 + \tilde{Q}_1^2)}{4a^2 \pi^2} - \frac{a^2 R_e}{J} \quad (64)$$

$$s_i^2 = -\frac{4a^2 N_A \pi^2 R_N \sigma \left[ 4\phi (\phi + L_e) \pi^2 - 4\phi \pi^2 \sigma + L_e \tilde{Q}_1^2 \sigma \right]}{L_e (4\pi^2 + \tilde{Q}_1^2) (4\pi^2 \phi^2 + L_e^2 \tilde{Q}_1^2)} - \frac{J^2 \sigma^2}{L_e^2} \quad (65)$$

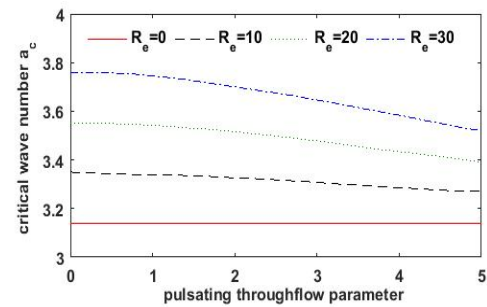
Here, it is important to be noted that for existing of oscillatory convection, the frequency of oscillation  $s_i$  must be positive. From Eq. (65) it is also found that the vertical AC electric field does not influence the occurrence of oscillatory convection. In the absence of nanoparticles, i.e.,  $R_N = 0$ , the value of the frequency of oscillation  $s_i$  is always negative. Hence oscillatory mode of convection is ruled out for regular fluid. However for nanofluids, according to *Yadav et al.*

(2016a,b), *Buongiorno (2006)* and *Siddheshwar et al. (2017)*, for the majority of nanofluids, the Lewis number  $L_e$  is on the arrange of  $10^1 \sim 10^3$ ,  $N_A$  is on the arrange of  $1 \sim 10$ , the nanoparticle Rayleigh-Darcy number  $R_N$  and  $\sigma$  are on the arrange of  $1 \sim 10$ , and hence from Eq. (65), the value of  $\omega_i^2$  will be always negative. Since  $\omega_i$  is real for oscillatory convection, therefore oscillatory convection cannot take place also for the case of nanofluid.



**Fig. 2. Effect of the pulsating throughflow parameter  $\tilde{Q}_1$  on the critical stationary thermal Rayleigh-Darcy number  $R_{D,c}^s$  for the different values of the AC electric Rayleigh-Darcy number  $R_e$  with  $R_N = 0.5$ ,  $L_e = 5$ ,  $N_A = 2$ ,  $\phi = 0.7$ .**

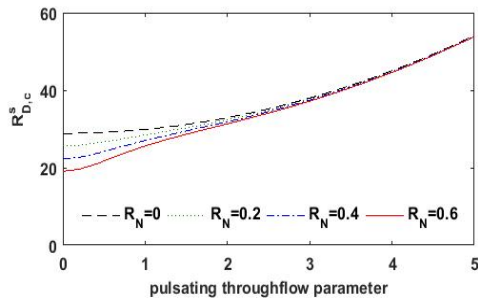
Analytically, the expression of the stationary thermal Rayleigh-Darcy number  $R_D^s$  (Eq. (52)) shows that the stability of the dielectric nanofluids varies on the values of six parameters which are  $\tilde{Q}_1$ ,  $R_e$ ,  $N_A$ ,  $L_e$ ,  $\phi$  and  $R_N$ .



**Fig. 3. Effect of the pulsating throughflow parameter  $\tilde{Q}_1$  on the critical wave number  $a_c$  for the different values of the AC electric Rayleigh-Darcy number  $R_e$  with  $R_N = 0.5$ ,  $L_e = 5$ ,  $N_A = 2$ ,  $\phi = 0.7$ .**

To study the effect of various values of the control parameters ( $\tilde{Q}_1$ ,  $R_e$ ,  $N_A$ ,  $L_e$ ,  $\phi$  and  $R_N$ ) on the onset of electroconvection, first we fix four parameters and then the variation of the critical thermal Rayleigh-Darcy number  $R_{D,c}^s$  and the critical wave number  $a_c$  are plotted for diverse values of the fifth parameter as a function of the pulsating throughflow parameter  $\tilde{Q}_1$  (see Figs. 2-11). According to *Buongiorno (2006)* and *Yadav et al.*

al. (2016d,e), we fix  $R_e = 20$ ,  $N_A = 2$ ,  $L_e = 5$ ,  $\varphi = 0.7$  and  $R_N = 0.5$ .



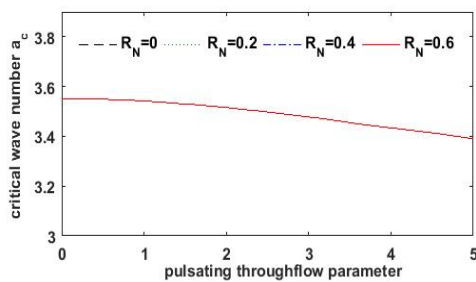
**Fig. 4.** Effect of the pulsating throughflow parameter  $\tilde{Q}_1$  on the critical stationary thermal Rayleigh-Darcy number  $R_{D,c}^S$  for the different values of the nanoparticle Rayleigh-Darcy number  $R_N$  with  $R_e = 20$ ,  $L_e = 5$ ,  $N_A = 2$ ,  $\varphi = 0.7$ .

The value of pulsating throughflow parameter  $\tilde{Q}_1$  lies between a minimum  $Q(1-\gamma)$  and a maximum  $Q(1+\gamma)$  when  $t_0$  varies. Therefore from the Figs 2-11 and Eq. (52), we can calculate two values of the critical Rayleigh-Darcy number as:

$$R_{D,c}(\min) = R_{D,c}^S \text{ at } \tilde{Q}_1 = Q(1-\gamma) \text{ and} \quad (66)$$

$$R_{D,c}(\max) = R_{D,c}^S \text{ at } \tilde{Q}_1 = Q(1+\gamma) \quad (67)$$

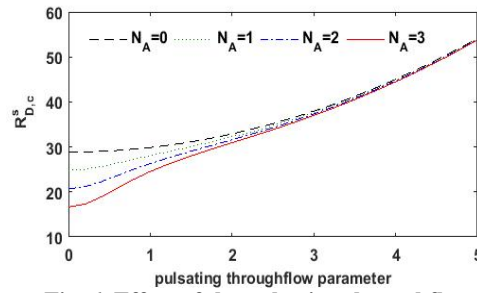
When the thermal Rayleigh-Darcy number  $R_D$  is less than  $R_{D,c}(\min)$ , no convection possible. When the thermal Rayleigh-Darcy number  $R_D$  is greater than  $R_{D,c}(\max)$ , continuous convection happens. For intermediate values of the thermal Rayleigh Darcy number  $R_D$ , convection occurs for just part of each cycle.



**Fig. 5.** Effect of the pulsating throughflow parameter  $\tilde{Q}_1$  on the critical wave number  $a_c$  for the different values of the nanoparticle Rayleigh-Darcy number  $R_N$  with  $R_e = 20$ ,  $L_e = 5$ ,  $N_A = 2$ ,  $\varphi = 0.7$ .

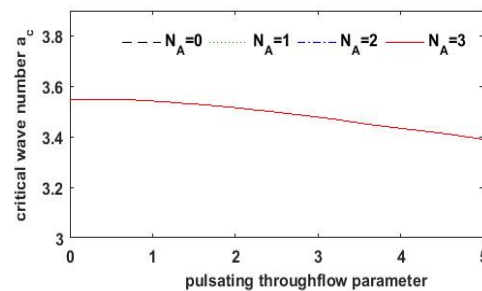
Figures 2 and 3 exhibit the effect of the AC electric Rayleigh-Darcy number  $R_e$  on the onset of electroconvection. From Fig. 2, it is found that the critical stationary thermal Rayleigh-Darcy number  $R_{D,c}^S$  decreases with the AC electric

Rayleigh-Darcy number  $R_e$ . Therefore, the inclusion of the external AC electric field decreases the stability of the scheme.



**Fig. 6.** Effect of the pulsating throughflow parameter  $\tilde{Q}_1$  on the critical stationary thermal Rayleigh-Darcy number  $R_{D,c}^S$  for the different values of the modified diffusivity ratio  $N_A$  with  $R_e = 20$ ,  $L_e = 5$ ,  $R_N = 0.5$ ,  $\varphi = 0.7$ .

From Fig. 2, it is also noted that on rising the value of the pulsating throughflow parameter  $\tilde{Q}_1$ , the critical value of the stationary thermal Rayleigh-Darcy number  $R_{D,c}^S$  raises; thus the result of  $\tilde{Q}_1$  is to delay the onset of convection.



**Fig. 7.** Effect of the pulsating throughflow parameter  $\tilde{Q}_1$  on the critical wave number  $a_c$  for the different values of the modified diffusivity ratio  $N_A$  with  $R_e = 20$ ,  $L_e = 5$ ,  $R_N = 0.5$ ,  $\varphi = 0.7$ .

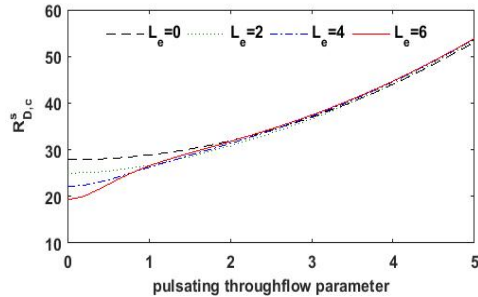
From Fig. 3, it is established that an increase in the values of the AC electric Rayleigh-Darcy number  $R_e$  and decrease in the pulsating throughflow parameter  $\tilde{Q}_1$  tend to boost  $a_c$  and therefore its influence is to decrease the size of convection cells.

Figures 4-7 display the effect of the nanoparticle Rayleigh-Darcy number  $R_N$  and the modified diffusivity ratio  $N_A$  on the stability scheme. From Figs. 4 and 6, it is observed that enlarge in the value of one of these parameters allows to reduce the critical values of the stationary thermal Rayleigh-Darcy number  $R_{D,c}^S$ .

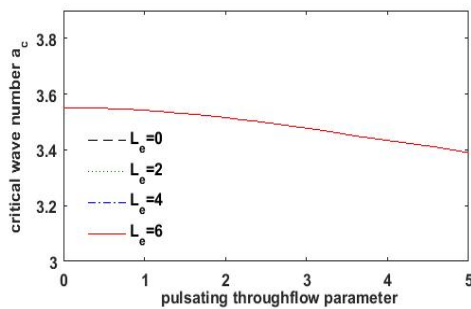
These show that those parameters allow to speeding up the onset of electroconvection in dielectric nanofluids. It happens because enhance in the nanoparticle Rayleigh-Darcy number  $R_N$  and the modified diffusivity ratio  $N_A$  enhanced the



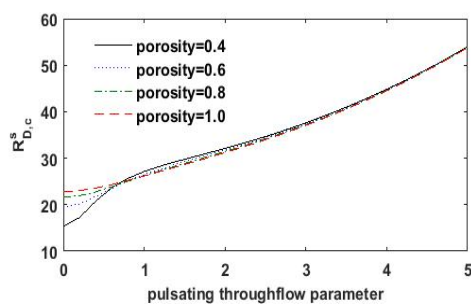
thermophoresis and also the Brownian motion of nanoparticles, and thus system is more unstable. From Figs. 5 and 7, we observed that the critical wave number  $a_c$  does not modify on the nanoparticle Rayleigh-Darcy number  $R_N$  and the modified diffusivity ratio  $N_A$ .



**Fig. 8. Effect of the pulsating throughflow parameter  $\tilde{Q}_1$  on the critical stationary thermal Rayleigh-Darcy number  $R_{D,c}^s$  for the different values of the Lewis number  $L_e$  with  $R_e = 20$ ,  $N_A = 2$ ,  $R_N = 0.5$ ,  $\varphi = 0.7$ .**



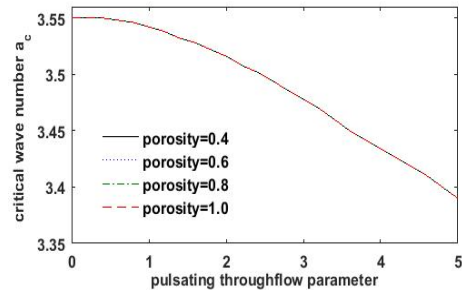
**Fig. 9. Effect of the pulsating throughflow parameter  $\tilde{Q}_1$  on the critical wave number  $a_c$  for the different values of the Lewis number  $L_e$  with  $R_e = 20$ ,  $N_A = 2$ ,  $R_N = 0.5$ ,  $\varphi = 0.7$ .**



**Fig. 10. Effect of the pulsating throughflow parameter  $\tilde{Q}_1$  on the critical stationary thermal Rayleigh-Darcy number  $R_{D,c}^s$  for the different values of the porosity parameter  $\varphi$  with  $R_e = 20$ ,  $N_A = 2$ ,  $R_N = 0.5$ ,  $L_e = 5$ .**

The impacts of the Lewis number  $L_e$  and porosity parameter  $\varphi$  on the onset of electroconvection are displayed in Figs. 8-11. From Figs. 8 and 10, it is established that the Lewis number  $L_e$  and porosity

parameter  $\varphi$  have dual effect on the onset of convection in the attendance of pulsating throughflow parameter  $\tilde{Q}_1$ . From Figs. 9 and 11, we found that the Lewis number  $L_e$  and porosity parameter  $\varphi$  have no considerable impact on the critical wave number  $a_c$ .



**Fig. 11. Effect of the pulsating throughflow parameter  $\tilde{Q}_1$  on the critical wave number  $a_c$  for the different values of the porosity parameter  $\varphi$  with  $R_e = 20$ ,  $N_A = 2$ ,  $R_N = 0.5$ ,  $L_e = 5$ .**

#### 4. CONCLUSION

The outcome of the pulsating throughflow and the external AC electric field on the convective instability in a porous medium layer soaked by a dielectric nanofluid is investigated utilizing the frozen profile approach. The study has been carried out for vanish flux nanoparticles condition at boundaries. The resultant eigenvalue problem is determined analytically and discussed graphically by means of the Galerkin weighted residuals method with thermal Rayleigh-Darcy number  $R_D$  as the eigenvalue. The frozen profile approach shows that an expanded amplitude of throughflow oscillations prompts stability in both directions by a sum that relies upon the frequency. The effect of increasing the modified diffusive ratio  $N_A$ , the external AC electric Rayleigh-Darcy number  $R_e$  and the nanoparticle Rayleigh number  $R_N$  is to speed up the onset of convection, whereas the Lewis number  $L_e$  and porosity parameter  $\varphi$  have dual effect on the stability scheme. The effect of enhance in the pulsating throughflow parameter  $\tilde{Q}_1$  and shrink in the external AC electric Rayleigh-Darcy number  $R_e$  is to diminish the critical wave number  $a_c$  and therefore to enlarge the size of convection cells. The critical wave number  $a_c$  does not vary on nanoparticle parameters and porosity parameter  $\varphi$ .

#### ACKNOWLEDGMENT

This work was supported by the University of Nizwa.

#### REFERENCES

Asadzadeh, F., M. N. Esfahany, and N. Etesami

- (2012). Natural convective heat transfer of  $\text{Fe}_3\text{O}_4$ /ethylene glycol nanofluid in electric field. *International Journal of Thermal Sciences* 62, 114–119.
- Barletta, A., M. Celli, A. V. Kuznetsov, and D. A. Nield (2016). Unstable forced convection in a plane porous channel with variable-viscosity dissipation. *ASME Journal of Heat Transfer* 138, 032601.
- Bég, O. A., T. A. Bég, M. M. Rashidi, and M. Asadi (2013). DTM-Padé semi numerical simulation of nanofluid transport in porous media. *International Journal of Applied Mathematics and Mechanics* 9, 80–107.
- Bryan, J. E., and J. Seyed-Yagoobi (1997). Heat transport enhancement of monogroove heat pipe with electrohydrodynamic pumping. *Journal of Thermophysics Heat Transfer* 11, 454–460.
- Buongiorno, J. (2006). Convective transport in nanofluids. *ASME Journal of Heat Transfer* 128, 240-250.
- Chand, R., and G. C. Rana (2012). On the onset of thermal convection in rotating nanofluid layer saturating a Darcy-Brinkman porous medium. *International Journal of Heat and Mass Transfer* 55, 5417–5424.
- Chand, R., G. C. Rana and D. Yadav (2016). Electrothermo Convection in a Porous Medium Saturated by Nanofluid. *Journal of Applied Fluid Mechanics* 9, 1081-1088.
- Chandrasekhar, S. (1961). *Hydrodynamic and Hydromagnetic Stability*, Oxford University Press, Oxford, UK.
- Chossat, P. and G. Iooss (1994). *The Couette-Taylor Problem*. *Applied Mathematical Sciences* 102, Springer-Verlag, New York.
- Fein, J. (1973). An experimental study of the effects of the upper boundary condition on the thermal convection in a rotating cylindrical annulus of water. *Geophys. Fluid Dynamics* 5: 213–248.
- Landau, L. D. and E. M. Lifshitz (1960). *Electrodynamics of Continuous Media*. Pergamon Press, London.
- Leong K. Y., R. Saidur, S. N. Kazi and A. H. Mamun (2010). Performance investigation of an automotive car radiator operated with nanofluid-based coolants (nanofluid as a coolant in a radiator). *Applied Thermal Engineering* 30, 2685–2692.
- Lewis, G. M., and W. Nagata (2004). Linear stability analysis for the differentially heated rotating annulus. *Geophysical & Astrophysical Fluid Dynamics* 98: 129-152.
- Lin, C. W. and J. Y. Jang (2005). 3D Numerical heat transfer and fluid flow analysis in plate-fin and tube heat exchangers with electrohydrodynamic enhancement. *Heat Mass Transfer* 41, 583–593.
- Murshed S. M. S., K. C. Leong and C. Yang (2008). Thermophysical and electrokinetic properties of nanofluids-a critical review. *Applied Thermal Engineering* 28, 2109–2125.
- Nield, D. A. and A. Bejan (2013). *Convection in Porous Media*. Springer.
- Nield, D. A. and A. V. Kuznetsov (2009). Thermal instability in a porous medium layer saturated by a nanofluid, *International Journal of Heat and Mass Transfer* 52 (25-26), 5796-5801.
- Nield, D. A. and A. V. Kuznetsov (2011). The effect of vertical through flow on thermal instability in a porous medium layer saturated by nanofluid. *Transp. Porous Med.* 87: 765–775
- Nield, D. A. and A. V. Kuznetsov (2013). Onset of convection with internal heating in a porous medium saturated by a nanofluid. *Transp. Porous Med.* 99, 73-83.
- Nield, D. A. and A.V. Kuznetsov (2014). Thermal instability in a porous medium layer saturated by a nanofluid: A revised model. *International Journal of Heat and Mass Transfer* 68, 211–214.
- Nield, D. A. and A. V. Kuznetsov (2016). The effect of pulsating throughflow on the onset of convection in a horizontal porous layer, *Transport in Porous Media* 111 (3), 731–740.
- Özerinç, S., S. Kakaç and A. Yazicioğlu (2010). Enhanced thermal conductivity of nanofluids: a state-of-the-art review. *Microfluidics and Nanofluidics* 8, 145–170.
- Rana G. C., R. C. Thakur and S. K. Kango (2014). On the onset of double-diffusive convection in a layer of nanofluid under rotation saturating a porous medium. *J Porous Med.* 17, 657-667.
- Roberts, P. H. (1969). Electrohydrodynamic convection. *Q. J. Mech. Appl. Math.* 22 211–220.
- Saidur, R., K. Y. Leong and H. A. Mohammad (2011). A review on applications and challenges of nanofluids. *Renewable and Sustainable Energy Reviews* 15, 1646-1668.
- Sharifpur, M., S. Yousefi and J. P. Meyer (2016). A new model for density of nanofluids including nanolayer. *International Communications in Heat and Mass Transfer* 78, 168-174.
- Sheikholeslami, M. and D. D. Ganji (2014). Heated permeable stretching surface in a porous medium using Nanofluids. *Journal of Applied Fluid Mechanics* 7(3), 535-542.
- Shivakumara I. S. and M. Dhananjaya (2015). Penetrative Brinkman convection in an anisotropic porous layer saturated by a nanofluid. *Ain Shams Engineering Journal* 6 (2), 703-713.
- Siddheshwar, P. G., C. Kanchana, Y. Kakimoto and A. Nakayama (2017). Steady finite-amplitude Rayleigh–Bénard convection in nanoliquids

- using a two-phase model: Theoretical answer to the phenomenon of enhanced heat transfer. *Journal of Heat Transfer* 139 (1), 012402.
- Stiles, P. J. (1991). Electro-thermal convection in dielectric liquids. *Chem. Phys. Lett.* 179 311–315.
- Turnbull, R. J. and J. R. Melcher (1969). Electrohydrodynamic Rayleigh-Taylor Bulk Instability. *Phys. Fluids*. 12, 1160–1166.
- Tzou, D. Y. (2008). Thermal instability of nanofluids in natural convection. *International Journal of Heat and Mass Transfer* 51, 2967–2979.
- Umavathi J. C., D. Yadav and M. B. Mohite (2015). Linear and nonlinear stability analyses of double-diffusive convection in a porous medium layer saturated in a Maxwell nanofluid with variable viscosity and conductivity. *Elixir Mechanical Engineering*. 79, 30407-30426.
- Wakif, A., Z. Boulahia and R. Sehaqui (2018). A semi-analytical analysis of electro-thermohydrodynamic stability in dielectric nanofluids using Buongiorno's mathematical model together with more realistic boundary conditions, *Results in Physics*.
- Wang, X. Q. (2007). Heat transfer characteristics of nanofluids: a review. *Int. J. Therm. Sci.* 46, 1–19.
- Wong, K. V. and O. D. Leon (2010) Applications of nanofluids: Current and Future. *Advances in Mechanical Engineering* 2010, Article ID 519659.
- Yadav, D. (2014). *Hydrodynamic and Hydromagnetic Instability in Nanofluids*. Lambert Academic Publishing Germany.
- Yadav, D. (2017). Electrohydrodynamic Instability in a Heat Generating Porous Layer Saturated by a Dielectric Nanofluid. *Journal of Applied Fluid Mechanics* 10 (3), 763-776.
- Yadav, D. and J. Lee (2015). The effect of local thermal non-equilibrium on the onset of Brinkman convection in a nanofluid saturated rotating porous layer. *Journal of Nanofluids* 4 (3), 335-342.
- Yadav, D., D. Lee, H. H. Cho and J. Lee (2016a). The onset of double-diffusive nanofluid convection in a rotating porous medium layer with thermal conductivity and viscosity variation: a revised model. *Journal of Porous Media* 19 (1), 31-46.
- Yadav, D., J. Lee and H. H. Cho (2016b). Electrothermal instability in a porous medium layer saturated by a dielectric nanofluid, *Journal of Applied Fluid Mechanics* 9 (5), 2123-2132.
- Yadav, D., J. Lee and H. H. Cho (2016c). Throughflow and quadratic drag effects on the onset of convection in a Forchheimer-extended Darcy porous medium layer saturated by a nanofluid. *Journal of the Brazilian Society of Mechanical Sciences and Engineering* 38, 2299–2309.
- Yadav, D., R.A. Mohamed, H. H. Cho and J. Lee (2016d). The effect of Hall current on the onset of MHD convection in a porous medium layer saturated by a nanofluid. *Journal of Applied Fluid Mechanics* 9 (5), 2379-2389.
- Yadav, D., R. A. Mohamed, J. Lee and H. H. Cho (2016e). Thermal convection in a Kuvshiniski viscoelastic nanofluid saturated porous layer. *Ain Shams Engineering Journal*.
- Yadav, D., J. Wang and J. Lee (2017). Onset of Darcy-Brinkman convection in a rotating porous layer induced by purely internal heating. *Journal of Porous Media* 20 (8), 691-706.
- Vafai, K. (2005). *Handbook of Porous Media*, Boca Raton: CRC Press.