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## Numerical Simulation of MHD Fluid Flow inside Constricted Channels using Lattice Boltzmann Method

M. Jamali Ghahderijani<sup>1,2</sup>, M. Esmaeili<sup>3</sup>, M. Afrand<sup>1,2†</sup> and A. Karimipour<sup>1,2</sup>

<sup>1</sup> Department of Mechanical Engineering, Najafabad Branch, Islamic Azad University, Najafabad, Iran

<sup>2</sup> Modern Manufacturing Technologies Research Center, Najafabad Branch, Islamic Azad University, Najafabad, Iran

<sup>3</sup> Department of Mechanical Engineering, faculty of Engineering, Kharazmi University, Tehran, Iran.

†Corresponding Author Email: [masoud.afrand@pmc.iaun.ac.ir](mailto:masoud.afrand@pmc.iaun.ac.ir)

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### ABSTRACT

In this study, the electrically conducting fluid flow inside a channel with local symmetric constrictions, in the presence of a uniform transverse magnetic field is investigated using Lattice Boltzmann Method (LBM). To simulate Magnetohydrodynamics (MHD) flow, the extended model of D2Q9 for MHD has been used. In this model, the magnetic induction equation is solved in a similar manner to hydrodynamic flow field which is easy for programming. This extended model has a capability of simultaneously solving both magnetic and hydrodynamic fields; so that, it is possible to simulate MHD flow for various magnetic Reynolds number ( $Re_m$ ). Moreover, the effects of  $Re_m$  on the flow characteristics are investigated. It is observed that, an increase in  $Re_m$ , while keeping the Hartman number ( $Ha$ ) constant, can control the separation zone; furthermore, comparing to increasing  $Ha$ , it doesn't result in a significant pressure drop along the channel.

**Keywords:** Lattice Boltzmann method; Magnetohydrodynamics; Constrictions; Magnetic Reynolds number; Hartmann number.

### NOMENCLATURE

$a$	analytical value	$m$	average
$B$	magnetic induction field	$P$	local pressure
$B_0$	constant transverse magnetic field	$Re$	Reynolds number
$b$	simulated Boltzmann value	$Re_m$	magnetic Reynolds number
BGK	Bhatnagar–Gross–Krook	$u$	flow velocity
CFD	Computational Fluid Dynamics	$w_i$	weighted factor indirection $i$
$C_s$	sound speed	$x_0$	location of minimum cross-section
$e$	lattice velocity in hydrodynamic field	$x$	cartesian coordinate of x-direction
$F$	uniform forcing in the along-channel direction	$y$	cartesian coordinate of y-direction
$f$	particle mass distribution function		
$f_i^{eq}$	equilibrium distribution function	$\delta$	constriction height
$g$	vector distribution function	$\eta$	magnetic resistivity
$H$	channel Height	$\rho$	local flow density
$Ha$	Hartmann number	$\tau$	relaxation time
$k$	constriction width	$\tau_m$	magnetic relaxation time
$L$	channel Length	$\nu$	kinematic viscosity
LBM	Lattice Boltzmann Method	$\Xi_i$	lattice velocity in magnetic field
MHD	magnetohydrodynamics	$\Omega_i$	collision operator in direction $i$

## 1. INTRODUCTION

Flow in constricted channels and tubes are observed in some fluidic devices (orifices, valves) and have wide applications in engineering as well as physiological applications. Because of such industrial and physiological applications, it is of no surprise that flow through local constrictions has been the subject of many studies in the past, in both experimental and numerical domains (Ahmed and Giddens, 1981; Deshpande *et al.*, 1976; Lee and Fung, 1970; Neren, 1992; Young, 1979; Giddens *et al.*, 1993; Lee, 1994; Chouly and Lagree, 2012; Bandyopadhyay and Layek, 2011; 2012). Such studies have already revealed that normal blood flow can be affected by factors such as wall shear stress, pressure fluctuations, and local velocities (Ahmed and Giddens, 1981; Deshpande *et al.*, 1976; Lee and Fung, 1970; Neren, 1992; Young, 1979). The degree of importance of the shape, breadth, and height of the constriction has also been revealed through such studies (Giddens *et al.*, 1993; Lee, 1994; Bandyopadhyay and Layek, 2011).

The main objective of previous studies has been to better understand flow irregularities caused by such local constrictions. The Present paper follows another objective which is to see if flow separation occurring downstream a partially-constricted passage can be controlled through the use of an external magnetic field. This idea has been investigated in several studies (Bandyopadhyay and Layek, 2011; Cramer and Pai, 1973; Esmaeili and Sadeghy, 2009; Gad-el-Hak and Bushnell, 1991; Midya *et al.*, 2004) especially in biology and medicine (Barnothy, 1964; Vardanyan, 1973; Ponalagusamy and Tamil, 2013; Sinha and Misra, 2012; Ikbal *et al.*, 2009; Mustapha *et al.*, 2009; Tashtoush and Magableh, 2007; Tzirtzilakis, 2005) but the novelty of current study is the application of a different numerical method for simulation of the mentioned idea.

There are several approaches to numerical simulation of MHD flow (Afrand, 2017; Afrand *et al.*, 2015; Afrand *et al.*, 2015; Afrand *et al.*, 2014; Afrand *et al.*, 2014; Afrand *et al.*, 2017;). Among them the lattice Boltzmann method (LBM) is a new and popular approach (He and Luo, 1997). In contrast to traditional computational fluid dynamics (CFD) methods which solve Navier–Stokes equations and compute macroscopic variables such as velocity and pressure in the LBM the particle mass distribution function  $f$  on the mesoscopic level is obtained by an approximation of the kinetic equation using the lattice Boltzmann equation (Mei *et al.*, 1999). Then macroscopic variables are computed by this mass distribution function.

The simplicity and suitability for parallel programming have made the lattice Boltzmann algorithms so popular. In addition, they can be easily incorporated for simulating fluid flows on complex geometries with complicated boundary conditions (Agarwal *et al.*, 2009). Among several LBM algorithms the Bhatnagar–Gross–Krook (BGK) model with a simple scalar relaxation parameter and mass distribution function is the

simplest one. In recent years the LBGK model for MHD flow has been developed. The earlier LBGK model in this area was presented by Chen *et al.* (1991). This model was an extension of lattice gas cellular automata MHD proposed by Chen and Matthaeus, 1987 and Chen *et al.*, 1988. In their model, a two-index particle distribution function was used corresponding to separate microscopic velocities for velocity and magnetic field. At streaming step each particle moves along with one of the two velocity vectors which are chosen randomly. Martinez *et al.* (1994) have proposed another model which reduces the number of necessary particle states and computation effort as a result. Bouchut (1999) has introduced a vector-valued distribution function instead of the scalar probability distribution function. Dellar (2002) has also characterized an approach based on Bouchut model. In this model a separate vector-valued magnetic distribution function based on a vector Boltzmann–BGK equation gives the magnetic field. This model can solve induction equation in a manner similar to fluid flow so it is appropriate for programming. The model proposed by Dellar (2002) has been used in the current study.

The paper has been organized as follows: we will start by presenting the computational domain which is laminar, incompressible fluid flow between two parallel plates containing a Gaussian symmetric, local constriction. It is also assumed that the fluid flow temperature is constant. We will then proceed with describing the lattice Boltzmann method as the method of solution. The magnetic induction equations which can be solved in a similar manner to the hydrodynamic equations (using a vector distribution function) are described next in details. Typical numerical results will be presented demonstrating magnetic field effects on flow characteristics in constricted channels. The paper is concluded with presenting a brief summary of the main findings of the work.

## 2. METHOD AND SIMULATION

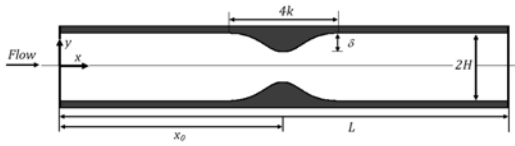
We consider laminar flow of an electrically conducting fluid flow between two parallel plates having an overall length of  $L$  which are separated by a distance  $2H$  from each other (see Fig. 1). As can be seen in Fig. 1, there is a local symmetric constriction somewhere between the two plates; it is seen to have amplitude of  $\delta$  (constriction height) and a span of  $4k$  (constriction width). The shape of constriction, assumes to be Gaussian type; that is

$$y = H - \delta \exp\left[-\left(\frac{x - x_0}{k}\right)^2\right] \quad (1)$$

In Eq. (1)  $x_0$  is the location of minimum cross-section (at the throat of channel).

In this paper we assumed that the minimum cross-section is located in the middle of channel. Also, the ratio of channel length ( $L$ ) to channel height ( $2H$ ) is 16. For simulation electrically conducting fluid flow through constricted channel (above mentioned geometry) the LBM is used which is

discussed in the following section.



**Fig. 1. Schematic showing the constricted channel shape used in the present study.**

## 2.1 Lattice Boltzmann Method

The LBM is a numerical scheme in mesoscopic scale which solves the LB equation for obtaining macroscopic flow field properties. Alternatively, this method can be used to simulate complex flow and transport phenomena especially in cases where direct solution of Navier-Stokes equations is not feasible (Martinez *et al.*, 1994; Bouchut, 1999; Dellar, 2002). The LBM has already been successfully used in various fluid flow problems, such as multiphase and multicomponent fluid flows (Bao and Schaefer, 2012; Yan *et al.*, 2011), thermal flow (Attar and Körner, 2011; Chen *et al.*, 2012; Lin *et al.*, 2012; Rong *et al.*, 2010), flows through porous media (Rong *et al.*, 2010; Hirabayashi *et al.*, 2012; Boek and Venturoli, 2010), solid particle suspensions (Hirabayashi *et al.*, 2012), non-Newtonian fluids (Chai *et al.*, 2011; Ohta *et al.*, 2011), reaction diffusion systems (Bresolin and Oliveira, 2012), and magneto-hydrodynamics, (Chen *et al.*, 1991; Chen and Matthaeus, 1987; Chen *et al.* 1988; Martinez *et al.*, 1994; Bouchut, 1999; Dellar, 2002;2011; Kefayati *et al.*, 2012).

To conserve mass and momentum and ensuring that the fluid is isotropic, in the LBM the particle distribution functions,  $f_i(x, t)$  at a point  $x$  and time  $t$ , move synchronously on a regular lattice (Boyd and Buick, 2007). Fig. 2 shows the D2Q9 lattice model used in this paper. Only the five speeds 0, 1, 2, 3, 4, shown with thick lines, are used for the magnetic field. The distribution functions are given by the Lattice Boltzmann equation (Chen and Doolen, 1998):

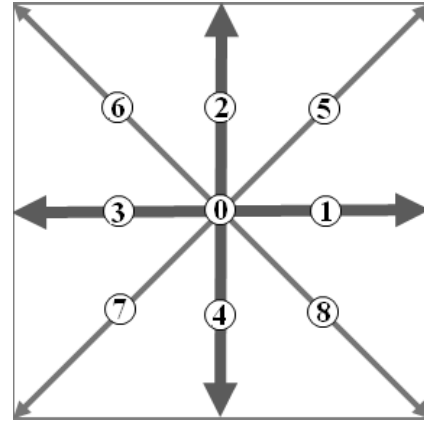
$$f_i(x + e_i \Delta t, t + \Delta t) = f_i(x, t) + \Omega_i(x, t) \quad (2)$$

Where in the case of D2Q9 lattice

$$e_0 = (0,0); \quad (i = 0) \quad (3)$$

$$e_i = \left[ \cos\left(\frac{\pi}{2}(i-1)\right), \sin\left(\frac{\pi}{2}(i-1)\right) \right]; \quad (i = 1,2,3,4) \quad (4)$$

$$e_i = \sqrt{2} \left[ \cos\left(\frac{\pi}{2}(i-1) + \frac{\pi}{4}\right), \sin\left(\frac{\pi}{2}(i-1) + \frac{\pi}{4}\right) \right]; \quad (i = 5,6,7,8) \quad (5)$$



**Fig. 2. The D2Q9 lattice model for MHD flow.**

The collision operator  $\Omega$  is given by the Bhatnagar–Gross–Krook approximation (Ohta *et al.*, 2011; Bhatnagar *et al.*, 1954):

$$\Omega_i = -\frac{1}{\tau} [f_i(x, t) - f_i^{eq}(x, t)] \quad (6)$$

In the above equation  $\tau$  is a relaxation time and  $f_i^{eq}$  is the equilibrium distribution function given by the local fluid density,  $\rho$ , and flow velocity,  $u$ , as follows:

$$f_i^{eq}(x) = w_i \rho(x) \left[ 1 + 3e_i u + \frac{9}{2}(e_i u)^2 - \frac{3}{2}u^2 \right] \quad (7)$$

Where the weight factors  $w_i$  are as below:

$$\begin{aligned} w_i &= 4/9 & i &= 0 \text{ (the rest particles)} \\ w_i &= 1/9 & i &= 1, 2, 3, 4 \\ w_i &= 1/36 & i &= 5, 6, 7, 8 \end{aligned}$$

In this model an ideal gas equation of state is used (i.e.  $p = \rho c_s^2$ ) where  $c_s^2 = 1/3$  is the speed of sound.

The fluid density  $\rho$  and velocity  $u$  can be easily obtained by  $\rho = \sum_i f_i$  and  $\rho u = \sum_i f_i e_i$ , respectively at each node. It is worth mentioning that the kinematic viscosity can be expressed by  $\nu = (2\tau - 1)/6$ .

## 2.2 Magneto-hydrodynamics Approach

Calculation of the magnetic field in MHD flow can be done using the magnetic induction equation (Moreau, 1990):

$$\frac{\partial B}{\partial t} + \nabla \cdot (uB - Bu) = \eta \nabla^2 B \quad (8)$$

Where  $B$  and  $\eta$  represent magnetic induction field and magnetic resistivity, respectively. The magnetic induction equation can be solved in a similar manner to the hydrodynamic equations, using a vector distribution function,  $g$ . The evolution of this function is given by (Dellar, 2002)

$$g_{i\beta}(x + \Xi_i \Delta t, t + \Delta t) = g_{i\beta}(x, t) - \frac{1}{\tau_m} [g_{i\beta}(x, t) - g_{i\beta}^{eq}(x, t)] \quad (9)$$

$$g_{i\beta}^{eq} = W_i [B_\beta + 3\Xi_{i\alpha} (u_\alpha B_\beta - B_\alpha u_\beta)] \quad (10)$$

$$B_\alpha = \sum_{i=0}^4 g_{i\alpha} \quad (11)$$

Where  $\Xi_i$  is lattice velocity in magnetic field and  $\tau_m$  is magnetic relaxation time. The five-point lattice weight factors  $W_i$  used for magnetic field are as below:

$$W_i = 1/3 \quad i=0 \text{ (the rest particles)}$$

$$W_i = 1/6 \quad i=1, 2, 3, 4$$

In two space dimensions the magnetic resistivity is related to the magnetic relaxation time by  $\eta = (2\tau_m - 1)/6$ . Assuming  $B = (B_x, B_y, 0)$ , a suitable equilibrium distribution is

$$f_i^{eq}(x) = w_i \rho(x) \left[ 1 + 3e_i u + \frac{9}{2} (e_i u)^2 - \frac{3}{2} u^2 \right] + \frac{9}{2} w_i \left[ \frac{1}{2} |B|^2 |e_i|^2 - (e_i B)^2 \right] \quad (12)$$

It is worth mentioning that the velocity field is coupled with magnetic field using above equation. To solve the governing equations, we need to impose suitable boundary conditions. At the inlet we impose fully developed velocity profile (Eq. (15) for MHD problem and a Poiseuille velocity profile for  $Ha=0$ ) according to [Zou and He \(1997\)](#) method. Also, for solid walls a mid-grid bounce back boundary condition ([Succi, 2001](#)) is used. For the magnetic boundary conditions, the bounce-back method is also used, but with the sign reversed to enforce  $B_x = 0$  at the wall boundaries ([Dellar, 2002](#)).

At the outlet, we assume that the downstream exit length is chosen long enough so that the flow streamwise variations in the flow will become sufficiently small. As a result the zero-derivative boundary conditions can be applied. This zero gradient condition is based on linear extrapolation ([Kefayati et al., 2012](#)), as follows:

$$f_i(x, t + e_i \Delta t) = 2f_i(x + e_i \Delta t, t + \Delta t) - f_i(x + 2e_i \Delta t, t + \Delta t) \quad (13)$$

### 2.3 Hartmann Flow

Hartmann flow comprises a steady unidirectional flow of viscous, electrically conducting fluid through a channel containing a constant transverse magnetic field. The fluid, assumed incompressible and all relevant quantities, except the pressure, are a function of only the transverse coordinate,  $u = (u(y), 0, 0)$ ,  $B = (B_x(y), B_0, 0)$  where  $B_0$  is the constant magnetic field transverse to the channel length. A uniform and time independent pressure gradient is maintained along the channel direction

to drive the fluid. The walls are located at  $y = -H$  and  $y = H$  ([Martinez et al., 1994](#)). For this case, the incompressible MHD equations can be simplified to the following linear system ([Dellar, 2002](#))

$$F + \rho_0 \nu \frac{d^2 u}{dy^2} + B_0 \frac{dB}{dy} = 0 \quad (14)$$

$$B \frac{du}{dy} + \eta \frac{d^2 B}{dy^2} = 0$$

In the above equation  $F$  is spatially uniform force acting along channel direction, such as a pressure gradient. There is an analytical solution for Eq.14 if non-slip boundary conditions is applied for the velocity field and accompanied by  $B_x(-H) = B_x(H) = 0$  for the magnetic field.

$$B(y) = \frac{FH}{B_0} \left( \frac{\sinh(Ha y/h)}{\sinh(Ha)} - \frac{y}{H} \right)$$

$$u(y) = \frac{FH}{\sqrt{\rho} B_0} \sqrt{\frac{\eta}{\nu}} \coth(Ha) \left( 1 - \frac{\cosh(Ha y/h)}{\cosh(Ha)} \right) \quad (15)$$

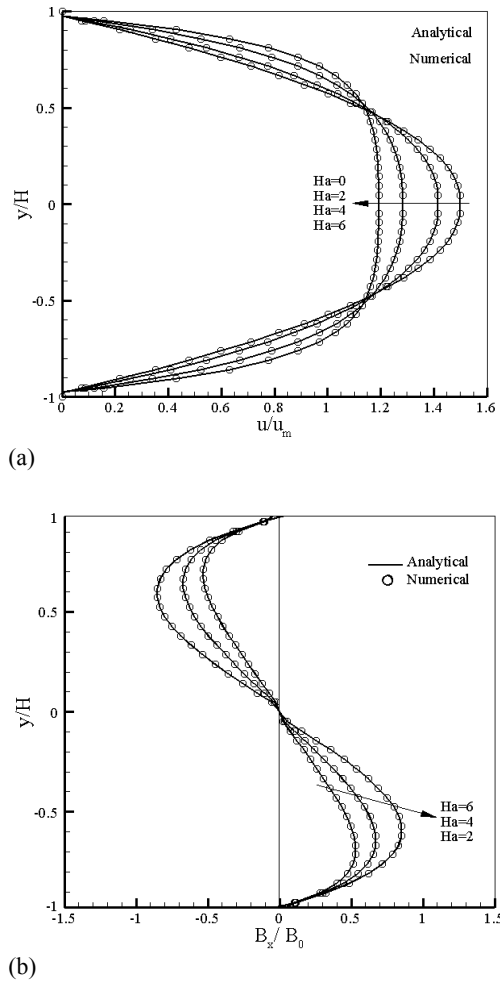
where  $Ha$  is a dimensionless Hartmann number which represents the ratio of Lorentz force to viscous force and can be defined by  $Ha = B_0 H / \sqrt{\rho_0 \eta \nu}$ .

When the Hartmann number is large, as is typical in liquid metal MHD applications, the magnetic field maintains a nearly uniform velocity  $u$  over the bulk of the channel ([Dellar, 2002](#)). Conversely, for zero Hartmann number which means no external magnetic field the solution will reduce to the parabolic velocity profile of the Poiseuille flow.

### 3. VALIDATION

To examine the validity of the code, the obtained velocity profile and axial component of induced magnetic field,  $B_x$ , in a straight channel are compared with analytical solution of Eq. (15) (see Fig. 3). The Reynolds number and magnetic Reynolds number defined by  $Re = U_m H / \nu$  and  $Re_m = U_m H / \eta$  where  $U_m$  denotes average velocity at inlet of the channel and  $\eta$  denotes magnetic resistivity. It is assumed that the Reynolds number and magnetic Reynolds number are constant ( $Re=50$ ,  $Re_m=5$ ) along different Hartmann numbers. To reduce compressibility error inlet Mach number ( $Ma_{in} = U_m / C_s$ ) set to be equal to 0.026.

For any given flow rate, a decrease in the velocity near the centerline must be accompanied by an increase in the velocity near the walls, as can be inferred from Fig. 3a. Moreover, in Fig. 3b the axial component of induced magnetic field grows from zero on the wall to its maximum adjacent to the surface and then experiencing a point of inflection once again falls to zero in the centerline. The mentioned maximum value decreases with increasing  $Ha$  number.



**Fig. 3. Comparison between analytical and numerical results of (a) Velocity profile and (b) induced magnetic field profiles.**

It is worth mentioning that a good agreement is observed between numerical results and analytical solutions. The second order accuracy of MHD Lattice Boltzmann method (Dellar, 2002) was also investigated. The simulations were finished when the following criterion was satisfied.

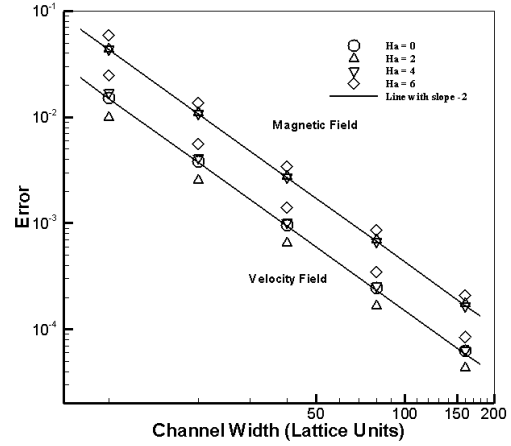
$$\sum_x \|u(x,t) - u(x,t-1)\| < \varepsilon \quad (16)$$

where  $\varepsilon$  is a small number assumed to be  $\varepsilon=1 \times 10^{-10}$ . The global error can be calculated by Eq. (17).

$$\zeta = \frac{\sum_x \|u_b(y) - u_a(y)\|}{\sum_x \|u_a(y)\|} + \frac{\sum_x \|B_b(y) - B_a(y)\|}{\sum_x \|B_a(y)\|} \quad (17)$$

In the Eq.17 the subscript  $a$ , denotes results of the exact analytic solution while the subscript  $b$  refers to the obtained results from LBM simulation. Results for the global error have been shown in Fig.

4. The black lines in Fig. 4 represent lines of slope  $-2$ , indicating second-order behavior. Although, the amount of error for axial magnetic field is higher than velocity field, it can be seen that for various Hartmann number the presented data closely match the slope of those lines. These results prove that the code can well represent MHD flow in channels.



**Fig. 4. Global error for MHD Lattice Boltzmann Method.**

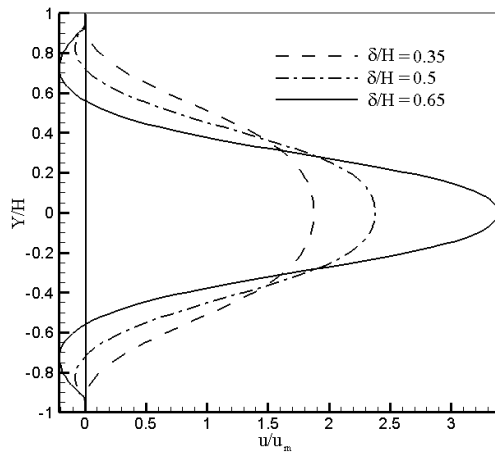
#### 4. RESULTS AND DISCUSSION

Having found the fully-developed velocity profiles in the previous section, one can proceed with calculating the velocity profiles in the constriction region. To achieve this goal, we used a uniform grid consisting 125 nodes along the channel height and 2000 nodes along the channel length. To produce various Reynolds number, we changed viscosity,  $\nu$ , so that the relaxation time would be changed too. For  $Re=50$  the relaxation time,  $\tau$ , and the inlet Mach number,  $Ma_{in}$  is set to be 0.5562 and 0.026 respectively. To perform various magnetic Reynolds number,  $Re_m$ , the magnetic resistivity,  $\eta$ , was changed. For  $Re_m=5$ , magnetic relaxation time is set to be 1.0625.

Figure 5 illustrates the effect of the constriction height,  $\delta$ , on the streamwise velocity profile at  $x/H=17$  (i.e., downstream the throat), a constriction with  $k/H=0.4$  for  $Re=50$  and  $Ha=0$ . The negative values of the velocity imply the recirculating flow region. As the height of the constriction increases the greater separation zone in the upper and lower walls of the channel can be observed.

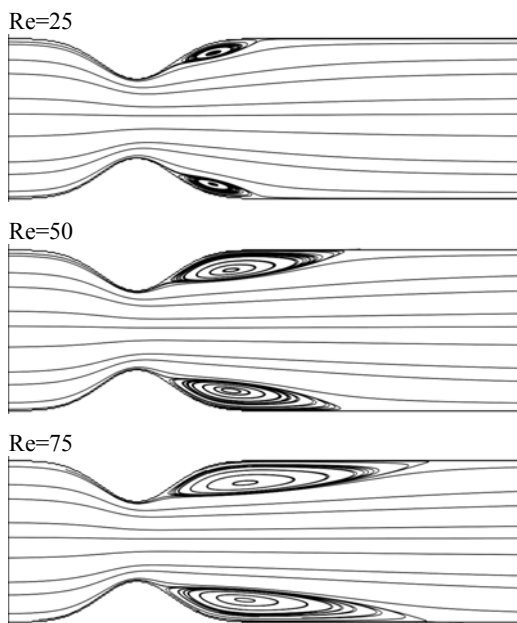
Figure 6 shows a history of the development of the streamlines inside a constricted channel (with  $\delta/H=0.5$  and  $k/H=0.4$ ) for  $Ha=0$  at  $Re=25, 50$  and  $75$ . A flow reversal (i.e., separation) downstream of the throat is obvious in this figure. Evidently, once the flow enters the diverging section of the channel, it is vulnerable to separation because of the influence of the adverse pressure gradient in this part of the channel. As expected, separated zone grows further with the increase of Reynolds number (the inertia forces become more important than the viscous forces), and at  $Re=75$  (Fig. 6c) it occupies

main portion of the channel.



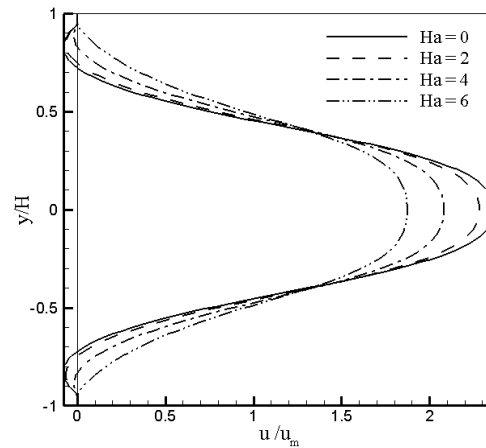
**Fig. 5. Effect of constriction height,  $\delta$ , on the velocity profiles at  $x/H = 17$  for  $Re=50$  and  $Ha=0$ .**

Here, the effect of Hartman number on the velocity profile at  $x/H = 17$ ,  $Re=50$  and  $Re_m=5$  for a constriction with  $\delta/H=0.5$  and  $k/H=0.4$  is investigated.



**Fig. 6. Streamline pattern at various Reynolds number,  $Re$ , at  $Ha=0$ .**

As can be observed in Fig. 7, the Lorentz force, as induced by the magnetic field, is predicted to have a retarding effect on fluid elements near the axis of the constriction. Close to the wall, however, fluid elements accelerate to keep the mass flow rate constant. This increase in the fluid velocity near the walls with increasing  $Ha$  number enables the fluid to overcome the influence of any adverse pressure gradient (downstream of the constriction) and thereby enhances the ability of the fluid to withstand flow separation near the wall.



**Fig. 7. Velocity profiles at various Hartman number for  $x/H = 17$ ,  $Re=50$  and  $Re_m=5$ .**

The effect of Hartman number on the streamline pattern is shown in Fig. 8. As can be seen in this figure, with an increase in the magnetic field, the flow separation zone becomes smaller in size and for greater Hartman numbers; the separation zone is vanished totally. These results are in line with the velocity profiles presented in Fig. 7. Evidently, the accelerating effect of the magnetic field (through the action of the Lorentz force) can compensate for the decelerating effect of the adverse pressure gradient near the wall and downstream the throat more.

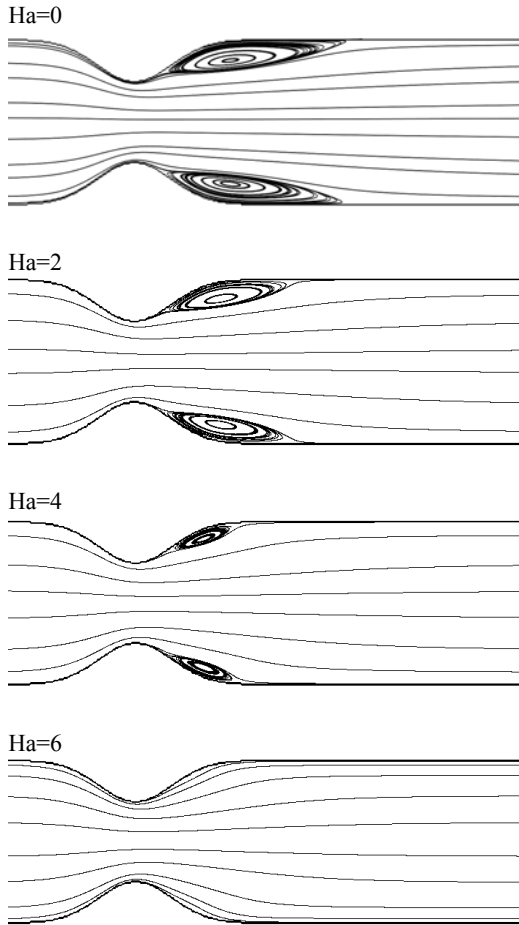
**Table1 Effect of Hartmann number on the overall wall friction**

Ha	$C_f \cdot Re$
0	6.195621
2	6.859182
4	9.889191
6	13.14766
straight channel	6

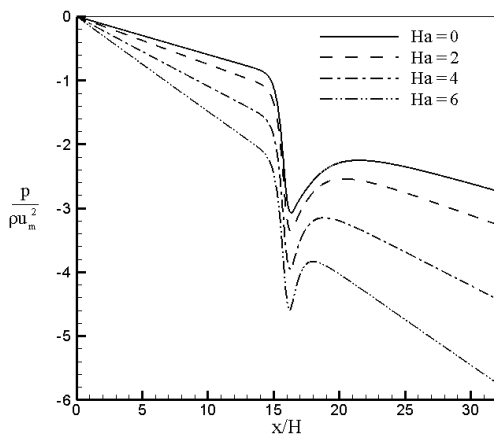
The effect of Hartmann number on overall wall friction (it can be obtained with integrating of  $C_f \cdot Re$  on the wall surface) is summarized in Table 1. It is clear from this table that the overall wall friction increases with an increase in the strength of magnetic field. An increase of the wall friction with an increase of the Hartman number can be attributed to the boundary layer becoming thinner due to the higher Hartman number resulting in an increase in the velocity gradient near the wall (see Fig. 3a).

Figure 9 represents the effect of Hartman number on the pressure variation for  $Re=50$  and  $Re_m=5$ . It is concluded from this figure that by increasing the Hartmann number from 0 to 6, the pressure drop increases dramatically. This result is attributed to increase of wall shear stress due to the increase of velocity gradient near the wall. In this connection it is important to mention that since in the real situations the externally applied pressure gradient will need to be adjusted appropriately, we cannot

strictly conclude that more increase in  $Ha$  number is a completely effective way to cause a delay in separation.



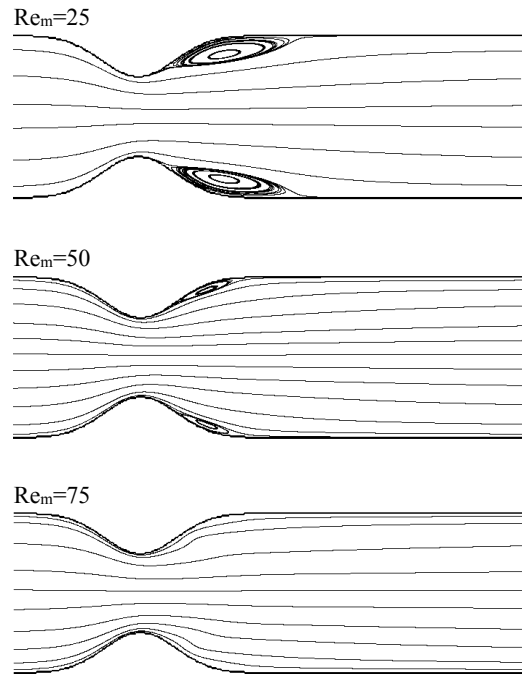
**Fig. 8. Streamline pattern at various Hartmann number,  $Ha$ , for  $Re=50$  and  $Re_m=5$ .**



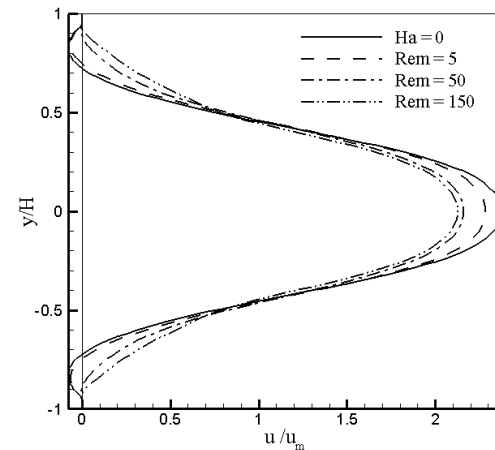
**Fig.9. Effect of the Hartmann number,  $Ha$ , on the dimensionless pressure at the axis of channel  $Re=50$  and  $Re_m=5$ .**

Figure 10 shows the effect of magnetic Reynolds number on streamline patterns at constant Hartmann number ( $Ha=2$ ). It can be seen that, by increasing the magnetic Reynolds number from 5 to 150, the

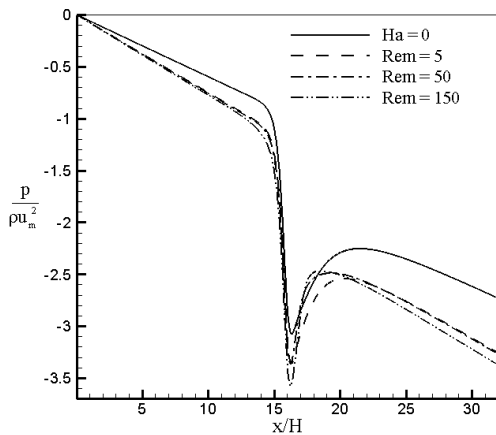
diffusion term in Eq. (8) decreases (because of the magnetic resistivity,  $\eta$ , reduction) and convection term becomes more important. As a result, the induced magnetic field increases in comparison with the uniform transverse magnetic field and the separation zone becomes smaller in size. In Fig. 11 the variation of magnetic Reynolds number with stream wise velocity is depicted at  $x/H= 17$ . This confirms that, similar to Hartmann number,  $Re_m$  has an increasing effect on velocity of fluid elements close to the wall. However, as can be seen in Fig. 12 the pressure drop due to increasing magnetic Reynolds number is not significant. In fact, increasing  $Re_m$  just results in increasing induced magnetic field in the constrictions region which accelerates fluid flow near wall. So, flow doesn't have to tolerate large velocity gradient on the wall along the whole channel length which causes considerably less pressure drop.



**Fig. 10. Streamline pattern at various magnetic Reynolds number,  $Re_m$ , for  $Re=50$  and  $Ha=2$ .**



**Fig. 11. Velocity profiles at various magnetic Reynolds number for  $x/H = 17$  and  $Re=50$ .**



**Fig. 12. Effect of the Magnetic Reynolds number,  $Re_m$ , on the dimensionless pressure at the axis of channel  $Re=50$  and  $Ha=2$ .**

## 5. CONCLUSION

In this paper, numerical simulation of electrically conducting fluid flow inside a locally constricted channel using lattice Boltzmann method is investigated. Simultaneously solving magnetic and hydrodynamic fields, makes it possible to study the effect of various magnetic Reynolds number in the simulation. It is observed that flow separates at the lee of constriction and this separated zone will be larger when the Reynolds number increases. To control flow separation, two scenarios are suggested which include increasing  $Ha$  and  $Re_m$ . It is viewed that increasing  $Ha$  can reduce the separated zone in downstream of constrictions whereas considerable pressure drop along channel length occurs. However, while  $Ha$  is kept constant, an increase in  $Re_m$  not only could delay the separation phenomena but also significant pressure drop does not occur.

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