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Numerical, Experimental and Analytical Studies on Fluid Flow through a Marsh Funnel

S. Sadrizadeh[1,2†], A. N. Ghafar[3], A. Halilovic[4] and U. Håkansson[5]

[1] *Lawrence Berkeley National Laboratory, 1 Cyclotron Road, Berkeley, California 947 20, USA*
[2] *Center for the Built Environment, University of California, Berkeley, California 947 20, USA*
[3] *KTH Royal Institute of Technology, Soil and Rock Mechanics, Stockholm 100 44, Sweden*
[4] *KTH Royal Institute of Technology, Fluid and Climate Technology, Stockholm 100 44, Sweden*
[5] *Skanska AB, 112 74 Stockholm, Sweden*

†*Corresponding Author Email: SSadrizadeh@lbl.gov*

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ABSTRACT

This paper presents the application of computational fluid dynamics technique in civil and underground industries to evaluate fluid behaviour in a Marsh funnel. The numerical approach, based on computational fluid dynamics, simulated an incompressible two-phase Newtonian flow by means of the Volume-of-Fluid method. A complementary analytical proposed which provided a quick, field-ready method to assess the fluid field in the Marsh funnel. A supplemental experimental effort evaluated the results obtained from both the analytical calculation and numerical simulation. Results showed that the application of computational fluid dynamics technique gives the desired results in studying fluid flows in civil and underground industries. Proposed analytical solution is also capable of accurately predicting the fluid flow and thus can complement the experimental and numerical approaches. Further, the proposed analytical approach can be an alternative method for faster evaluation of fluid, although it needs to be calibrated with either the numerical or the experimental studies.

Keywords: Multiphase flow; Marsh funnel; CFD; Analytical solution; Experimental approach; Fluid flow simulation; Discharge coefficient; Cement-based grout.

1. INTRODUCTION

Grouting is one of the most common engineering methods of improving soil and fractured hard rock sealing and strength properties. Cement-based grout is one of the most used materials for this purpose due to its several economic and environmental advantages (Stille, 2015; Stille, Gustafson, and Hassler, 2012). Success in a grouting operation highly depends on penetration length, whereas the grout rheological properties (i.e. the viscosity and yield stress) are among the governing parameters (Eriksson, Friedrich, and Vorschulze, 2004; Gustafson, Claesson, and Fransson, 2013; Gustafson and Stille, 2005; Hässler, Håkansson, and Stille, 1992b; Stille, 2015; Stille *et al.*, 2012). Due to the cement hydration, these properties are also time dependent (Hakansson, 1993; Rahman, Håkansson, and Wiklund, 2015; Schwarz, 1997). In addition, the cement-grout suspensions, in different water-to-solid ratios, can either exhibit Newtonian or non-Newtonian properties depending on their viscosity behavior as a function of shear rate, stress, and deformation history (Yang, Hou, and Guo, 2011a, 2011b). The chemical grouts such as colloidal and pure solutions are also categorized as evolutive or non-evolutive Newtonian fluids (Gafar, Soga, Bezuijen, Sanders, and Tol, 2009).

In one of the latest advancements in prediction of grout penetration, the so-called Real Time Grouting Control Method (RTGCM), the grout rheological properties are the inputs (Gustafson and Stille, 2005; Rafi, 2013; Rafi, Stille, and Bagheri, 2012; Stille, 2015; Stille *et al.*, 2012). Thus, accurate measurement of the grout rheological properties is of great importance. Therefore, several instruments and methods have been developed to measure grout properties, among which the Marsh funnel is the most common, applicable in both the lab and field in the construction and oil industries (Balhoff *et al.*, 2011; Benaicha, Jalbaud, Hafidi Alaoui, and

Burtschell, 2015; Guria, Kumar, and Mishra, 2013; Le Roy, 2004; Mohammed, Pusch, Knutsson, and Hellström, 2014; Nguyen, Rémond, Gallias, Bigas, and Muller, 2006; Pitt, 2000; Roussel and Le Roy, 2005; Schwarz, 1997). The Marsh funnel, developed in the late 1920s, is an orifice meter, that is, a simple draining cone under gravitational force without replenishment for measuring the viscosity of drilling mud, grout, and any sort of slurry or cement paste (Marsh, 1931).

Orifice meters, which rely on the pressure-velocity variation caused by area contraction, are the most practical fluid flow-measuring devices due to their ease of manufacture, assembly, and application (Muñoz-Díaz, Solorio-Ordaz, and Ascanio, 2012; Shah, Joshi, Kalsi, Prasad, and Shukla, 2012). After filling the Marsh funnel with usually 1.5 liters of fluid material, the time needed for one liter of material to flow out is recorded as a measure of apparent viscosity. This drainage time is often referred to as the Marsh funnel viscosity, which is interpreted as a ratio of shear rate (the fluid speed as it passes through the orifice) and shear stress (the fluid weight that is causing the flow). Although the Marsh funnel has been extensively used to make a rapid property measurement, its accuracy is a matter of considerable controversy. It is generally known that a time interval error of up to one second can be expected.

Increased interest in multiphase flow in different engineering applications, along with advancement in interface capturing techniques, have prompted a number of recent attempts to apply numerical simulation (Hässler, Håkansson, and Stille, 1992a; Mitsoulis, 2007), in particular computational fluid dynamics (CFD), to model two-phase Newtonian and non-Newtonian flows. The Volume-of-Fluid (VOF) model is usually employed to capture the interface between two fluids by solving a single set of Navier-Stokes equations indicating both air and liquid (Hirt and Nichols, 1981). This method was employed in previous studies to predict the two-phase flows in several industrial applications and produce numerical results within the engineering accuracy (Balcázar, Lehmkuhl, Jofre, Rigola, and Oliva, 2016; Gupta, Fletcher, and Haynes, 2009; Jiang, Long, Wang, Liu, and Chen, 2016; Shirani, Ghadiri, and Ahmadi, 2011).

Fluid flow through the Marsh funnel has been adequately studied, but seldom with numerical simulation (Nguyen *et al*., 2006) or analytical calculation (Hakansson, 1993; Le Roy, 2004; Roussel and Le Roy, 2005). To the authors' best knowledge, limited investigations have been made to propose an easy, fast, and reliable fluid viscosity analysis in terms of Marsh funnel discharge flow time. The purpose of the present study is therefore to propose an analytical approach and present a detailed numerical simulation for Newtonian fluids that complements the experimental analysis. The study will be extended to predict the behaviour of non-Newtonian fluid flow in the later stage of the project.

2. METHODS

2.1 Experimental Setup

Figure-1 shows a standard Marsh funnel in inches.

The test apparatus consisted of a standard Marsh funnel from Fann Instrument (*Fann Instrument*, n.d.), suspended from an S-shaped HBM RSCC C3/50kg load cell for registering the weight of the discharged fluid over time. The load cell was connected to an HBM Quantum X MX440A data acquisition system to convert the acquired analog signals into digital values. HBM's Catman software was also used to visualize the data (Fig. 2).

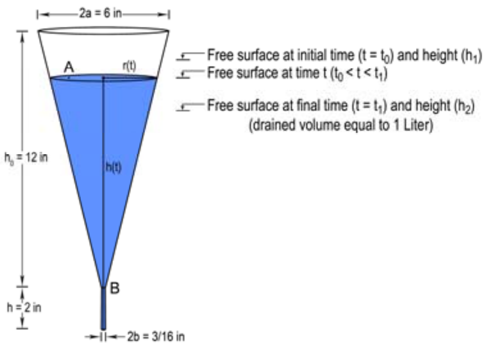

Fig. 1. a simple sketch of a Marsh funnel.

Fig. 2. Experimental setup.

The experiment used tap water at a temperature of 21 °C as a fluid. The test was reiterated 10 times to obtain a statistically reliable data set. In normal practice, the working volume of Marsh funnel is 1.5 liters.

3. ANALYTICAL APPROACH

Determining the fluid flow in a vessel with varying geometry, for example, a Marsh funnel is impossible to calculate analytically without making simplifying assumptions, even in the case of a Newtonian fluid.

Although the velocity during the transient stage changes over time, one can easily conclude that conservation of mass says that velocity has to be constant at any given instant along the length of the pipe. Calculating fluid flow in a varying geometry such as that of the Marsh funnel normally requires several assumptions (Hakansson, 1993; Lombardi, 1985). However, here, those assumptions have been minimized to propose an evaluation method that is more accurate.

In this study, the analytical solution was derived using Torricelli's theorem, which represents the relation between the speed of a gravitationally draining, un-replenished fluid out of an opening, and the height of the fluid above the opening. Torricelli's law can be written in a general form thusly:

$$v = c_d \sqrt{2gh} \tag{1}$$

where v is fluid velocity, g is the acceleration due to gravity, and h is the fluid's height in the cone. Here, c_d is a dimensionless discharge coefficient, ranged $0 \le c_d \le 1$, which represents the ratio of the actual to the theoretical discharge. If all friction losses as well as water contraction are ignored, then $c_d = 1$. Assuming the Marsh funnel (Fig. 1) contains fluid to an initial height ($V = 1.5\ lit$) at $t = t_0$ generates the following:

$$\frac{dV}{dt} = -A_B c_d \sqrt{2gh} \tag{2}$$

Here, $V(t)=dV/dt$ denotes the fluid volume at time t within the funnel and A_B is the bottom cross-section of cone (Point B). After some mathematical manipulation, Eq-2 will be:

$$A_h\, dh = -A_B c_d \sqrt{2gh}\, dt \tag{3}$$

In the above equation, A_h is the funnel cross-section at any arbitrary height, which can be calculated thusly:

$$A_h = \pi (r_h)^2 = \pi \left[h^2 \left(\frac{a-b}{h_0} \right)^2 + 2bh \left(\frac{a-b}{h_0} \right) + b^2 \right] \tag{4}$$

Here, h_0 denotes funnel height, b and a denote the bottom and top radius of the funnel, respectively. Implementing A_h from Eq-4 to Eq-3 produces the following:

$$\frac{h^2 \left(\frac{a-b}{h_0} \right)^2 + 2bh \left(\frac{a-b}{h_0} \right) + b^2}{\sqrt{h}} \pi\, dh = -A_B c_d \sqrt{2g}\, dt \tag{5}$$

and integration on both sides produces the following:

$$\Delta t = \frac{1}{c_d b^2 \sqrt{2g}} \left[\begin{array}{l} \frac{2}{5}\left(h_2^{5/2} - h_1^{5/2} \right) \left(\frac{a-b}{h_0} \right)^2 \\ + \frac{4}{3} b \left(h_2^{3/2} - h_1^{3/2} \right) \left(\frac{a-b}{h_0} \right) \\ + 2b^2 \left(h_2^{1/2} - h_1^{1/2} \right) \end{array} \right] \tag{6}$$

where h_1 and h_2 are the initial and final height of the fluid, respectively. The time interval Δt for the draining of the volume flow between h_1 and h_2 can be derived from Eq-6.

Precise calculation of the discharge coefficient c_d is difficult and usually must be obtained empirically as it varies considerably with changes in area aspect ratio, friction force, and the Reynolds number (*ISO,* 2003; Reader-Harris, Brunton, Gibson, Hodges, and Nicholson, 2001; Shah *et al*., 2012). However, it is possible to precisely calculate the discharge coefficient by comparing the numerical and analytical evaluations.

4. NUMERICAL SIMULATION

The accuracy issues in Marsh funnel experimental studies impose a considerable amount of uncertainty. The time dependency of grouting material properties is another challenge that may considerably change the results.

Accurate and precise prediction of air-fluid two-phase flow behavior is one of the most computationally challenging subjects under investigation in hydraulic and civil engineering. An ideal numerical two-phase flow model needs to be accurate, fast, and robust in the definition of a macroscopic interface and precise enough to take into account all stresses and forces being exerted on the material. Rapid advances in computational power and capacity, as well as availability of computational fluid dynamics (CFD) codes, make it possible to simulate the fluid flow behavior. CFD, which largely reduces the number of required measurements and provides great potential for improving prediction accuracy, was employed to model three-dimensional, transient, two-phase Marsh funnel flow.

ICEM CFD ANSYS software subdivided the funnel into 0.85 million hexahedral cells (Fig. 3), and a grid independence test used three different mesh densities to ensure correct two-phase flow interface predictions. The hexahedral interface region grid ensure accurate surface tension prediction (Gupta *et al*., 2009).

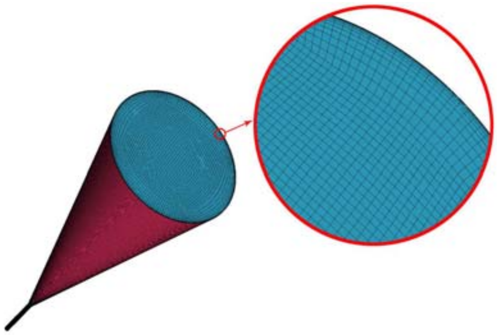

Fig. 3. Layout of hexahedral (structured) surface mesh.

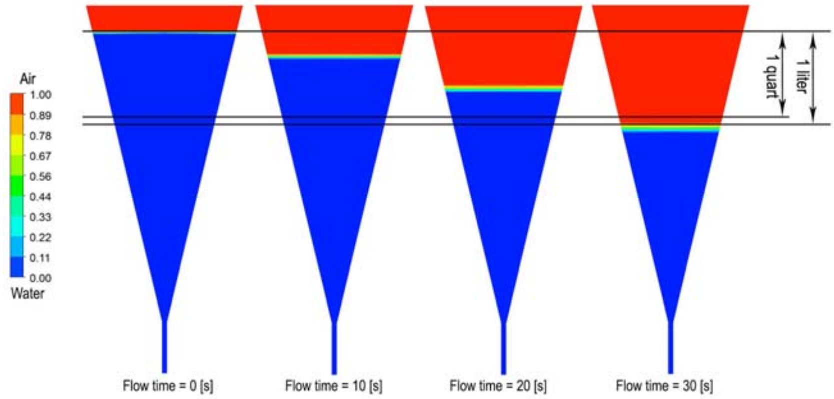

Fig. 4. Fluid volume fraction at different flow times.

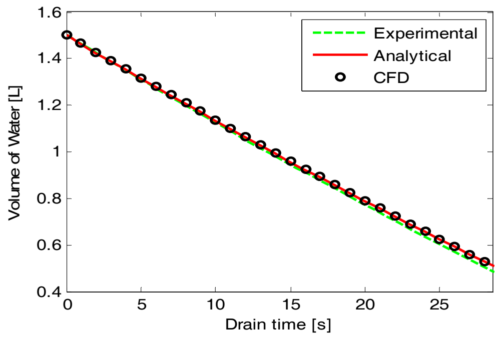

Fig. 5. Draining time as a function of water volume (discharge coefficient cd=0.895).

Though a number of methods have been previously proposed to approximate and compute complicated free boundary configurations, however, volume-of-fluid (VOF) method exhibits more flexibility and efficiency than others (Hirt and Nichols, 1981). The VOF method is a surface-tracking technique based on the Euler-Euler multiphase modeling approach that was designed to capture the multi-phase interface. This method is used to identify the air-liquid interface by solving a volume fraction equation:

$$\frac{\partial \rho}{\partial t} + \vec{v}.\nabla \alpha = 0 \tag{7}$$

where $\vec{v}$ is the velocity vector that acts as a shared velocity of the two fluids, ρ is fluid density, and α represents the phase volume fraction of the fluid in each cell. Completely filled cells with fluid will be represented by $\alpha=1$, while cells filled with air are denoted by $\alpha=0$, with the interface being localized to the cells where $0<\alpha<1$.

The VOF model solves a single momentum equation throughout the domain, and the resulting velocity field is shared among the phases. To track the interface between the gas-liquid phases, the continuity equation was solved through explicit time discretization for the volume fraction of each phase.

$$\frac{\partial}{\partial t}(\rho v) + \nabla.(\rho \vec{v}\vec{v}) + \nabla p = \nabla.\left[\mu\left(\nabla \vec{v} + \nabla \vec{v}^{T}\right)\right] + \rho \vec{g} + \vec{F} \tag{8}$$

where $\vec{g}$ is gravitational acceleration, p is pressure, ρ and μ are density and molecular viscosity respectively. $\vec{F}$ is the surface tension force approximated as a body force in the vicinity of the interface. The abovementioned momentum equation is dependent on the volume fractions of all phases through the mean properties of the phases such as density and viscosity.

In order to increase convergence reliability and speed, the steady flow solution is taken as an initial condition for time-accurate computations. The Marsh funnel fluid flow ranges from laminar (along the cone) to transitional (vicinity of the orifice) to turbulent (along the orifice), depending on the fluid level and rheology; thus it is difficult to assign a turbulence model that can cover all flow regimes. In the present study, the Marsh funnel flow was considered as laminar shear, since most satisfactory flows fall within this regime (Nguyen *et al.*, 2006).

The commercial software FLUENT 17.0 was employed to calculate the numerical prediction, with a pressure-based segregated solver and SIMPLE algorithm selected to solve the coupling between velocity and pressure. A second-order upwind scheme discretized the convection terms. Convergence criteria for the continuity and energy equations were set at 1×10^{-5}. A pressure-inlet boundary condition was applied to the funnel opening and a pressure-outlet boundary to the orifice outlet, both with a zero gauge pressure. No slip boundary condition was assigned to the funnel wall. The fluid flow interface was normal to the gravitational acceleration as dictated by the chosen contact angle of 76°. A variable time step, based upon a fixed Courant number of 0.25, was set for momentum and pressure equations.

Depending on either European or American practice, the volume measurement unit can be either

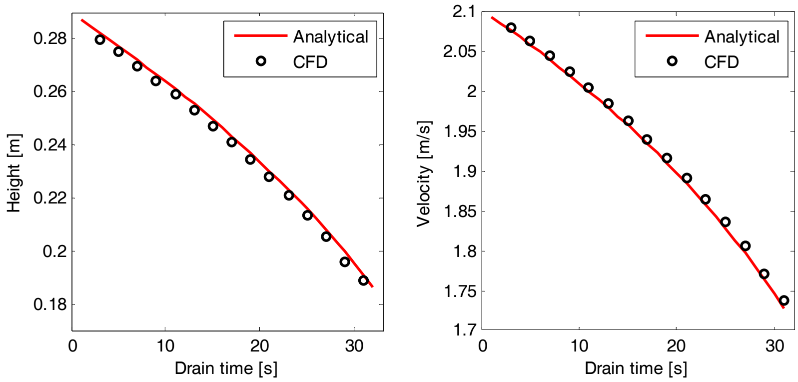

Fig. 6. Height level (left) and velocity (right) of water as a function of drain time.

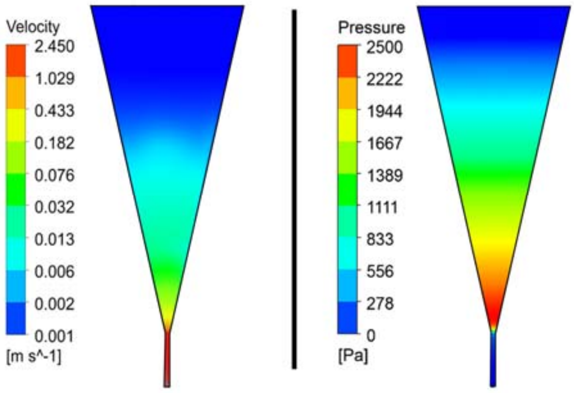

Fig. 7. Velocity magnitude and pressure contour plot along the vertical plane of the funnel (flow time=2s).

a liter or a quart. The time taken for one liter of fresh water, at a temperature of 21°C, to drain should be 28 ± 0.5 seconds; this will be reduced to 26 ± 0.5 seconds if the quart was considered (*ISO 10414-1:*2008).

Figure-4 shows the fluid volume fraction contour plot for different flow times, achieved via CFD simulation. The time taken for a liter of water to drain from the Marsh funnel is $\Delta t=28.89$ *sec*.

By implementing the calculated Δt from CFD simulation into Eq. 6, the discharge coefficient of $c_d=0.895$ can be analytically calculated.

Figure-5 shows the predicted and calculated results from CFD simulation and analytical evaluation, as well as the experimental study results. The experimental study was reiterated 10 times in order to obtain statistically reliable data.

The overall relative error between CFD, the analytical evaluation, and the experimental data is less than 5 percent. The small discrepancy between the numerical simulation and experimental data might be due to the zero wall roughness being considered in the CFD simulation. The experimental study water temperature might also affect the solution, as there is no accurate device to measure it. However, the overall agreements are within the limits required for engineering accuracy.

Figure-6-left presents the variation in fluid height (water-air interface position) versus drain time. Due to the continuous changing of the funnel cross-section, interface velocity increased (acceleration) by the time and thus pressure reduction, because of height change, is also accelerating (The greater the slope, the faster the acceleration is). Water drain velocity (Figure-6-right) however decelerated accordingly. This means that velocity reduces all along the drain time (from 2.1 down to 1.7) with a varied deceleration.

Figure-7 shows the velocity and pressure contour plot along a vertical Marsh funnel plane. The highest velocity is predicted in the orifice, while the rest of the funnel has a negligible velocity. This is in line with all theoretical assumptions that the interface velocity well above the orifice is considered to be zero.

The highest pressure within the funnel was before the orifice; however, by increasing the velocity after the orifice inlet, pressure drops drastically from 2.5 kPa to almost 0 Pa.

5. CONCLUSION

The present work was an attempt to propose a simple and quick analytical approach for evaluating fluid viscosity using the Marsh funnel. A numerical simulation based on the VOF interface capturing method-using CFD provided accurate fluid flow measures within the test apparatus. Comparing the analytical and numerical approaches with an experimental effort verified the results, leading to the conclusion that the proposed analytical method was accurate within the required engineering accuracy. However, it should be calibrated with either CFD simulation or measurement data. It was also concluded that the CFD technique was capable of predicting fluid flow through the Marsh funnel as an alternative to the experiments. Further investigation is, however, needed to expand the research area from Newtonian fluid to Non-Newtonian using all three approaches in the later stage of the project.

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