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Electrohydrodynamic Instability in a Heat Generating Porous Layer Saturated by a Dielectric Nanofluid

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ABSTRACT

In this paper, a theoretical investigation has been carried out to study the combined effect of AC electric field and temperature depended internal heat source on the onset of convection in a porous medium layer saturated by a dielectric nanofluid. The model used for nanofluid incorporates the combined effect of Brownian diffusion, thermophoresis and electrophoresis, while for porous medium Darcy model is employed. The flux of volume fraction of a nanoparticle with the effect of thermophoresis is taken to be zero on the boundaries and the eigenvalue problem is solved using the Galerkin method. Principle of exchange of stabilities is found to be valid and subcritical instability is ruled out. The results show that increase in the internal heat source parameter H_S, AC electric Rayleigh-Darcy number R_e, the Lewis number L_e, the modified diffusivity ratio N_A and the concentration Rayleigh-Darcy number R_n are to hasten the onset of convection. The size of convection cells decreases with increasing the internal heat source parameter H_S and the AC electric Rayleigh-Darcy number R_e.

Keywords: Electrohydrodynamic instability; Nanofluids; Internal heat source; Porous medium.

1. INTRODUCTION

Nanofluids have an important impact on heat transfer enhancement in modern years. They have been utilized in various technologies including cooling systems of electronic devices (Wong and Leon 2010; Saidur *et al*. 2011), porous media solar collectors (Bég *et al*. 2012), geothermics (Sheikholeslami and Ganji 2014), automotive applications (Srinivas *et al*. 2016) and chemical engineering coating processes (Malvandi *et al*. 2015). Further, remarkable implementations of nanofluids include crystal growth (Jayaraman *et al*. 2009), materials processing (Muthtamilselvan and Rakkiyappan 2011), energy storage (Uddin *et al*. 2013) and automotive engine cooling (Leong *et al*. 2010). Nanofluids represent a significant class of heat transfer fluids obtained by dispersing very small amount of nanoparticles (<100 nanometers in diameter) in conventional poor thermal conductivity base fluids. Nanoparticles used in nanofluid are typically made of oxide ceramics (Al_2O_3, CuO), metal carbides (SiC) or metals (Al, Cu) and base fluids are water, oil, bio-fluids, polymer solutions and other common fluids. The presence of the nanoparticles in the fluid increased the effective thermal conductivity of the fluid and consequently enhanced the heat transfer characteristics. There are some review papers that show detailed characteristic feature of nanofluids (Wang 2007; Murshed *et al*. 2008; Özerinç *et al*. 2010).

A nanofluid modelling was made by Buongiorno (2006) who clarified that the key mechanisms contributing to thermal enhancement are Brownian diffusion and thermophoresis. This model was applied to study the thermal instability problem by Tzou (2008 a, b) and observed that nanofluid is less stable than regular fluid. Later, this problem was revisited by Nield and Kuznetsov (2010) by taking different types of non-dimensional parameters. Thermal instability problem for rotating nanofluid layer was studied by Chand and Rana (2012), Yadav *et al*. (2011, 2013a, 2014a) and Rana *et al*. (2014). Thermal conductivity and viscosity variation on the onset of convection in nanofluid was studied by Nield and Kuznetsov (2012), Yadav *et al*. (2013b,2014b,2015a) and Umavathi *et al*. (2015). They obtained that the consequence of these factors increase the critical Rayleigh number.

In recent years, the study of the interaction of electromagnetic fields with fluids saturated porous medium started gaining attention with the promise of applications in many fields of science and engineering including nuclear fusion, chemical engineering and medicine (Allen and Karayiannis

1995; Yabe *et al*. 1996; Moreno *et al*. 1996; Lai and Lai 2002). Depending on the nature of fluids, the effects of magnetic and electric fields become important. The magnetic field effects are dominant if the fluid is highly electrically conducting. To the opposing, the electric forces play a key role in driving the motion if the fluid is dielectric with low electrical conductivity. The investigation of convective instability together with the electrical and magnetic forces in nanofluids is of practical importance. A systematic study through a proper theory is essential to understand the physics of the complex flow behaviour of nanofluids and also to obtain invaluable scaled up information for industrial applications.

Thermal convection in a dielectric fluid layer in the presence of an AC and/or DC electric field is known as electrothermal convection (ETC). Rudraiah and Gayathri (2009) investigated the effect of vertical electric field on the electroconvection in a horizontal dielectric fluid saturated densely packed porous layer and discussed the importance of ETC in porous media. El-Sayed (2008) analyzed the problem of electrohyrodynamic instability in a horizontal layer of the Oldroydian viscoelastic dielectric liquid through Brinkman porous medium under the simultaneous action of a vertical ac electric field and a vertical temperature gradient. Using linear stability theory, both stationary and oscillatory instabilities are discussed when the liquid layer is heated from below or above. It is shown that the electrical force is the sole agency causing instability. The effect of vertical AC electric field and boundaries on the onset of ETC in sparse porous layer heated from below or above was studied by Shivakumara *et al*. (2011a). Later, the onset of ETC in a rotating Brinkman porous layer was investigated by Shivakumara *et al*. (2011b). An extensive review on this topic has been made by Nield and Bejan (2013).

In some convective flow situations, one may encounter heat generated due to radioactive decay or weak exothermic reactions and nuclear reaction, which leads to change in the energy and thus the temperature profile of the system, like in the case of celestial bodies. Therefore, the role of internal heat generation becomes very important in several applications including food engineering, nuclear heat cores, nuclear energy, nuclear waste disposals, electrical equipment, oil extractions and crystal growth. Despite the importance of internal heat generation in nanofluids, there are few investigations of its effect. The previous works have only considered the much simpler case of uniform heat generation. For example, Hamad and Pop (2011), Yadav *et al*. (2012) and Nield and Kuznetsov (2013a,b) carried out numerical investigations and found that the basic temperature distribution and the basic volumetric fraction of nanoparticle distribution deviate from linear to non-linear in the presence of internal heating, and the critical Rayleigh number decreases with an increase in the internal heat source strength.

Under the circumstances, the study of electric field with non-uniform internal heat source on the onset of dielectric nanofluid convection in a porous medium seems to be significance in electrical equipment such as distribution transformers, regulating transformers and shunt reactors (Sheikhbahai *et al*. 2012; Asadzadeh *et al*. 2012; El-Genk 2012), and has not been given any attention in the literature. Therefore, the purpose of the research reported here is to examine theoretically the combined effect of vertical AC electric field and temperature dependent internal heat source on the criterion for the onset of convection in a dielectric nanofluid saturated horizontal layer of porous medium. We used the Horton–Rogers–Lapwood model with effects due to the electric field, the internal heat generation and nanoparticles. The temperature is considered to be fixed at the boundaries, while the nanoparticles flux under the thermophoretic effects to be zero thereat (Nield and Kuznetsov 2014).

2. ANALYSIS

Consider an infinite horizontal layer of incompressible dielectric nanofluid-saturated porous layer of thickness d with internal heat generation and heated from below. A Cartesian co-ordinate system (x, y, z) is chosen in which z axis is taken at right angle to the boundaries. The nanofluid is confined between two parallel plates $z = 0$ and $z = d$, where the temperatures at the lower and upper boundaries are taken to be T_0 and T_1, respectively, T_0 being greater than T_1. Nanofluid layer is subjected to a uniform vertical AC electric field applied across the layer; lower surface is grounded and upper surface is kept at an alternating potential whose root mean square is ψ_1. For simplicity, Darcy's law is assumed to hold and the Oberbeck--Boussinesq approximation is employed. Homogeneity and local thermal equilibrium in the porous medium is also assumed. According to the works of Buongiorno (2006), the governing equations under this model are:

The continuity equation for the nanofluid is

$$\nabla \cdot \vec{\mathbf{v}} = 0, \tag{1}$$

where $\vec{\mathbf{v}} = (u, v, w)$ is the Darcy velocity.

If one introduces a buoyancy force and a force of electrical origin, the momentum equation in a Darcy-porous medium can be written as

$$0 = -\nabla p - \frac{\mu}{K}\vec{\mathbf{v}} + \rho\vec{\mathbf{g}} + \vec{\mathbf{f}}_e, \tag{2}$$

where p is the pressure, K is the permeability of the porous medium, $\vec{\mathbf{g}}$ is the gravitational acceleration, $\vec{\mathbf{f}}_e$ is the force of electrical origin, μ and ρ are the viscosity and the mass density of the nanofluid, respectively. The mass density (ρ) of nanofluid can be written as:

$$\rho = \phi\rho_p + (1-\phi)\rho_f \,, \tag{3}$$

where ϕ is volumetric fraction of nanoparticle, ρ_p and ρ_f are the density of the nanoparticle and the base fluid, respectively.

Under the Boussinesq approximation, the density of base fluid can be approximated as

$$\rho_f = \rho_{f0}\left\{1-\beta(T-T_1)\right\}, \tag{4}$$

Where ρ_{f0} is the density of the base fluid at reference temperature T_1 and β is the thermal expansion coefficient. The density of the nanofluid can be approximated by that of base fluid, i.e. $\rho_0 \cong \rho_{f0}$, since the volumetric fraction of nanoparticles is only a few percent. Hence, with the help of equations (3) and (4), the buoyancy term can be approximated as

$$\rho\vec{\mathbf{g}} \cong \left[\phi\rho_p + \rho_0(1-\phi)\left\{1-\beta(T-T_1)\right\}\right]\vec{\mathbf{g}}\,. \tag{5}$$

On using equation (5), the momentum equation (2) can be written as

$$\nabla p + \frac{\mu}{K}\vec{\mathbf{v}} = \left[\phi\rho_p + \rho_0(1-\phi)\left\{1-\beta(T-T_1)\right\}\right]\vec{\mathbf{g}} + \vec{\mathbf{f}}_e. \tag{6}$$

The conservation equation for the nanoparticle is

$$\left[\frac{\partial}{\partial t} + \frac{1}{\varphi}(\vec{\mathbf{v}}\cdot\nabla)\right]\phi = -\frac{1}{\rho_p}\nabla\cdot\vec{\mathbf{j}}_p, \tag{7}$$

where φ is the porosity of the porous medium and $\vec{\mathbf{j}}_p$ is the diffusion mass flux for the nanoparticle, given as the sum of two diffusion terms (Brownian motion and thermophoresis) by

$$\vec{\mathbf{j}}_p = -\rho_p D_B\nabla\phi - \rho_p\left(\frac{D_T}{T_1}\right)\nabla T\,, \tag{8}$$

where D_B and D_T are the Brownian and the thermophoresis diffusion coefficients of the nanoparticle, respectively. According to Einstein formula (Einstein 1905), D_B and D_T are given as:

$$D_B = \frac{k_B T}{3\pi\mu d_p}\,,\ D_T = \left(\frac{\mu}{\rho}\right)\left(\frac{0.26k}{2k+k_p}\right)\phi, \tag{9}$$

where T is the temperature, k_B is the Boltzmann's constant, d_p is the diameter of nanoparticle, k and k_p are the thermal conductivity of the fluid and the nanoparticle, respectively.

Substituting the expression of $\vec{\mathbf{j}}_p$ from the equation (8) into the equation (7), the conservation equation for the nanoparticle (7) becomes

$$\left[\frac{\partial}{\partial t} + \frac{1}{\varphi}(\vec{\mathbf{v}}.\nabla)\right]\phi = D_B\nabla^2\phi + \frac{D_T}{T_1}\nabla^2 T\,, \tag{10}$$

The energy equation is

$$\left[(\rho c)_m\frac{\partial}{\partial t} + (\rho c)(\vec{\mathbf{v}}\cdot\nabla)\right]T = -\nabla\cdot\vec{\mathbf{q}} + \varphi h_p\nabla\cdot\vec{\mathbf{j}}_p + Q(T-T_1), \tag{11}$$

where $Q(T-T_1)$ is the strength of internal heat generation, (ρc) is the heat capacity of nanofluid, $\vec{\mathbf{q}}$ is the energy flux relative to nanofluid velocity and $h_p\nabla\cdot\vec{\mathbf{j}}_p$ is the addition flow work due to the Brownian and the thermophoresis motions of nanoparticle relative to the flow velocities. The energy flux $\vec{\mathbf{q}}$ can be written as the sum of the conduction heat flux and the heat flux due to nanoparticle diffusion as

$$\vec{\mathbf{q}} = -k_m\nabla T + \varphi h_p\vec{\mathbf{j}}_p, \tag{12}$$

where k_m is the effective thermal conductivity and h_p is the specific enthalpy. Equation (12) gives

$$\begin{aligned}\nabla\cdot\vec{\mathbf{q}} &= -k_m\nabla^2 T + \nabla\cdot\left(\varphi h_p\vec{\mathbf{j}}_p\right)\\ &= -k_m\nabla^2 T + \varphi\left(h_p\nabla\cdot\vec{\mathbf{j}}_p + \vec{\mathbf{j}}_p\cdot\nabla h_p\right)\\ &= -k_m\nabla^2 T + \varphi\left\{h_p\nabla\cdot\vec{\mathbf{j}}_p + \vec{\mathbf{j}}_p\cdot\left(c_p\nabla T\right)\right\},\end{aligned} \tag{13}$$

where ∇h_p is equal to $c_p\nabla T$ and c_p is the specific heat of the material constituting the nanoparticles. Substituting the expression for $\nabla\cdot\vec{\mathbf{q}}$ from equation (13) and $\vec{\mathbf{j}}_p$ from equation (8), the energy equation (11) becomes as

$$\begin{aligned}&\left[(\rho c)_m\frac{\partial}{\partial t} + (\rho c)(\vec{\mathbf{v}}\cdot\nabla)\right]T = k_m\nabla^2 T\\ &+\varphi(\rho c)_p\left[D_B\nabla\phi\cdot\nabla T + \left(\frac{D_T}{T_1}\right)\nabla T\cdot\nabla T\right] + Q(T-T_1)\end{aligned}. \tag{14}$$

The force of electrical origin $\vec{\mathbf{f}}_e$ in equation (6) can be expressed by Landau and Lifshitz (1960) for incompressible nanofluid as

$$\vec{\mathbf{f}}_e = \rho_e\vec{\mathbf{E}} - \frac{1}{2}\left(\vec{\mathbf{E}}\cdot\vec{\mathbf{E}}\right)\nabla\in, \tag{15}$$

where $\vec{\mathbf{E}}$ is the root mean square value of the electric field, ρ_e is the charge density and $\in$ is the dielectric constant. The first term on the right hand side is the Coulomb force due to a free charge and the second term depends on the gradient of $\in$. If an AC electric field is applied at a frequency much higher than the reciprocal of the electrical relaxation time, the free charge does not have time to accumulate. Moreover, the electrical relaxation times of most dielectric liquids appear to be sufficiently long to prevent the buildup of free charge at standard power line frequencies. At the same time, dielectric loss at these frequencies is so low that it makes no significant contribution to the temperature field. Under the circumstances, only the force induced by non-uniformity of the dielectric constant is considered. Furthermore, since the second term in the above equation depends on $\left(\vec{\mathbf{E}}\cdot\vec{\mathbf{E}}\right)$ rather than $\vec{\mathbf{E}}$ and the variation of $\vec{\mathbf{E}}$ is very rapid, the root mean square value of $\vec{\mathbf{E}}$ can be assumed as the effective value. In other words, we can treat the AC electric field as the DC electric field whose strength is equal to the root mean square value of the AC electric field. Assuming the free charge density is negligibly small, the relevant Maxwell equations are:

$$\nabla\times\vec{\mathbf{E}} = 0, \tag{16}$$

$$\nabla\cdot\left(\in\vec{\mathbf{E}}\right) = 0. \tag{17}$$

In view of equation (16), $\vec{\mathbf{E}}$ can be expressed as

$$\vec{\mathbf{E}} = -\nabla\psi, \tag{18}$$

where ψ is the root mean square value of the electric potential.

The dielectric constant is assumed to be a linear function of temperature in the form

$$\in=\in_0\left[1-\gamma\left(T-T_1\right)\right]=0, \tag{19}$$

where γ(>0) is the thermal expansion coefficient of dielectric constant and is assumed to be small.

We assume that the temperature is constant and nanoparticles flux including the effect of thermophoresis is zero on the boundaries. Thus, boundary conditions are:

$$w=0,\ T=T_0,\ D_B\frac{d\phi}{dz}+\frac{D_T}{T_1}\frac{dT}{dz}=0 \text{ at } z=0, \tag{20a}$$

$$w=0,\ T=T_1,\ D_B\frac{d\phi}{dz}+\frac{D_T}{T_1}\frac{dT}{dz}=0 \text{ at } z=d. \tag{20b}$$

2.1. Basic State

Assuming the basic state to be quiescent, the quantities at the basic state are given by:

$$\vec{\mathbf{v}}=0,\ T=T_b(z),\ p=p_b(z),\ \phi=\phi_b(z),$$

$$\in=\in_b(z),\ \psi=\psi_b(z),\ E=E_b(z). \tag{21}$$

The solution of the basic flows is:

$$T_b=T_1+\Delta T\frac{\sin\left[\sqrt{\frac{Q}{k_m}}\left(d-z\right)\right]}{\sin\left[d\sqrt{\frac{Q}{k_m}}\right]},$$

$$\phi_b=\left(-\frac{D_T}{T_1 D_B}\right)T_b+\phi_0,\ E_b=\frac{E_0}{1+\gamma\Delta Tz/d}\hat{\mathbf{k}},$$

$$\psi_b(z)=-\frac{E_0 d}{\gamma\Delta T}\log\left(1+\frac{\gamma\Delta T}{d}\right)\hat{\mathbf{k}},$$

$$\in_b=\in_0\left(1+\frac{\gamma\Delta T}{d}z\right)\hat{\mathbf{k}},$$

where subscript b denote the steady state, ϕ_0 is a reference scale for the nanoparticle fraction, $\Delta T=\left(T_0-T_1\right)$ and $E_0=-\frac{\psi_1\gamma\Delta T/d}{\log\left(1+\gamma\Delta T\right)}$ is the root mean square value of the electric field at $z=0$.

2.2. PERTURBATION SOLUTION

Let the initial basic state as described by equation be slightly perturbed so that the perturbed state is given by:

$$\mathbf{v}=\mathbf{v}',\ p=p_b(z)+p',\ T=T_b(z)+T',$$

$$\phi=\phi_b(z)+\phi',\ \in=\in_b+\in',\ \vec{\mathbf{E}}=\vec{\mathbf{E}}_b+\vec{\mathbf{E}}',$$

$$\psi=\psi_b+\psi', \tag{22}$$

where the prime denote the perturbed quantities. On substituting the equation (22) into the equations (1), (6), (10), (14) -(20) and linearizing by neglecting the products of primed quantities, eliminating the pressure term from the momentum equation by operating curl twice, and retaining the vertical component and converting the resulting equations to non-dimensional form by introducing the following dimensionless variables:

$$(x'',y'',z'')=\left(\frac{x',y',z'}{d}\right),\ t''=\frac{\alpha_m}{\sigma d^2}t',$$

$$(u'',v'',w'')=\left(\frac{u',v',w'}{\alpha_m}\right)d,\ T''=\frac{T'-T_1}{\Delta T},$$

$\phi''=\frac{\phi'-\phi_0}{\phi_0}$, $\psi''=\frac{\psi'}{\gamma E_0\Delta T d}$, where ϕ_0 is a reference scale for the nanoparticle fraction and $\alpha_m=\frac{k_m}{(\rho c)}$, $\sigma=\frac{(\rho c)_m}{(\rho c)}$, we obtain the linear stability equations (dropping the dashes (") for simplicity) in non dimensional form as:

$$\nabla^2 w=R_D\nabla_H^2 T-R_n\nabla_H^2\phi+R_e\nabla_H^2\left(T-\frac{\partial\psi}{\partial z}\right), \tag{23}$$

$$\frac{\partial \mathrm{T}}{\partial \mathrm{t}}+wf(z)=\left(\nabla^2+H_S\right)\mathrm{T}+\frac{\mathrm{N_A N_B}}{\mathrm{L_e}}f(z)\frac{\partial\mathrm{T}}{\partial \mathrm{z}} +\frac{\mathrm{N_B}}{\mathrm{L_e}}f(z)\frac{\partial\phi}{\partial \mathrm{z}}, \tag{24}$$

$$\frac{1}{\sigma}\frac{\partial\phi}{\partial t}-\frac{N_A}{\varphi}wf(z)=\frac{1}{L_e}\nabla^2\phi+\frac{N_A}{L_e}\nabla^2 T, \tag{25}$$

$$\nabla^2\psi=\frac{\partial T}{\partial z}, \tag{26}$$

where $f(z)=\frac{-\sqrt{\mathrm{H_S}}\cos\left[\sqrt{\mathrm{H_S}}\left(z-1\right)\right]}{\sin\left[\sqrt{\mathrm{H_S}}\right]}$.

In the above equations the following non-dimensional parameters are given as:

$L_e = \dfrac{\alpha_m}{D_B}$ is the Lewis number, $R_D = \dfrac{\rho_0 g \beta \Delta T K d}{\mu \alpha_m}$ is the thermal Rayleigh-Darcy number, $R_n = \dfrac{(\rho_p - \rho_0)\phi_0 g K d}{\mu \alpha_m}$ is the nanoparticle Rayleigh-Darcy number, $R_e = \dfrac{\gamma^2 \in E_0^2 (\Delta T)^2 K}{\mu \alpha_m}$ is the AC electric Rayleigh-Darcy number, $\mathrm{H_S} = \dfrac{Q d^2}{k_m}$ is the internal heat source parameter, $N_A = \dfrac{D_T \Delta T}{D_B T_1 \phi_0}$ is the modified diffusivity ratio, $N_B = \varphi \dfrac{(\rho c)_p \phi_0}{(\rho c)}$ is the modified particle-density increment.

In non- dimensional form, the boundary conditions become:

$$w = \frac{\partial \psi}{\partial z} = T = 0, \frac{\partial \phi}{\partial z} + N_A \frac{\partial T}{\partial z} \quad \text{at } z = 0,1 \tag{27}$$

Analyzing the disturbances into the normal modes and assuming that the perturbed quantities are of the form:

$$\begin{aligned}[w, T, \phi, \psi] &= [W(z), \Theta(z), \Phi(z), \Psi(z)] \\ &\times \exp(ik_x x + ik_y y + nt),\end{aligned} \tag{28}$$

where, n is the growth rate of disturbances, k_x and k_y are wave numbers in x and y directions, respectively.

On using equation (28), into equations (23) - (26), we have:

$$\begin{aligned}&(D^2 - a^2)W + R_D a^2 \Theta - R_n a^2 \Phi \\ &+ R_e a^2 \left(\Theta - \frac{\partial \Psi}{\partial z} \right) = 0,\end{aligned} \tag{29}$$

$$\begin{aligned}&-Wf(z) + \left[D^2 - a^2 + H_S - n + \frac{N_A N_B f(z)}{L_e} D \right] \Theta \\ &+ \frac{N_B f(z)}{L_e} D\Phi = 0,\end{aligned} \tag{30}$$

$$\begin{aligned}&\frac{N_A}{\varphi} W f(z) + \left[\frac{1}{L_e}(D^2 - a^2) - \frac{n}{\sigma} \right] \Phi \\ &+ \frac{N_A}{L_e}(D^2 - a^2)\Theta = 0,\end{aligned} \tag{31}$$

$$(D^2 - a^2)\Psi - D\Theta = 0, \tag{32}$$

where, $D = \dfrac{d}{dz}$, $f(z) = \dfrac{-\sqrt{\mathrm{H_S}} \cos\left[\sqrt{\mathrm{H_S}}(z-1) \right]}{\sin\left[\sqrt{\mathrm{H_S}} \right]}$

and $a = \sqrt{k_x^2 + k_y^2}$ is the resultant dimensionless wave number. The boundary conditions in view of normal mode analysis are:

$$\begin{aligned}&W = 0, \ \Theta = 0, \ D\Psi = 0, \\ &D\Phi + N_A D\Theta = 0 \quad \text{at } z = 0,1.\end{aligned} \tag{33}$$

The growth rate n is in general a complex quantity such that $n = \omega_r + i\omega_i$, the system with $\omega_r < 0$ is always stable, while for $\omega_r > 0$ it will become unstable. For neutral stability, the real part of ω is zero. Hence, we write $n = i\omega_i$, where ω_i is real dimensionless frequency.

The Galerkin weighted residuals method is used to obtain an analytical solution to the system of equations (29) - (32). Accordingly, the base functions W, Θ, Φ and Ψ are taken in the following way:

$$\begin{aligned}&W = \sum_{P=1}^{N} A_p W_p, \quad \Theta = \sum_{P=1}^{N} B_p \Theta_p, \\ &\Phi = \sum_{P=1}^{N} C_p \Phi_p, \quad \Psi = \sum_{P=1}^{N} D_p \Psi_p,\end{aligned} \tag{34}$$

where $W_p = \Theta_p = \sin p\pi z$, $\Phi_p = -N_A \sin p\pi z$, $\Psi_p = \cos p\pi z$, (satisfying the corresponding boundary conditions), A_p, B_p, C_p and D_p are unknown coefficients, and $p = 1,2,3,...,N$. On using above expression for W, Θ, Φ and Ψ into equations (29)-(32) and multiplying the resulting first equation by W_p second equation by Θ_p, third equations by Φ_p and fourth equation by Ψ_p and integrating in the limits from zero to unity, we obtained a system of $4N$ linear algebraic equations in the $4N$ unknowns A_p, B_p, C_p and D_p, $p = 1,2,3,...,N$. For the existence of non trivial solution, the determinant of coefficients matrix must vanish, which gives the characteristic equation for the system, with the thermal Rayleigh- Darcy number R_D as the eigenvalue of the characteristic equation. For a first approximation, we take $N = 1$; this produces the result

$$\begin{vmatrix} \dfrac{-J}{2}, & \dfrac{a^2(R_D + R_e)}{2}, & \dfrac{a^2 N_A R_n}{2}, & \dfrac{a^2 \pi R_e}{2} \\ -F(z), & \dfrac{\left[-J - i\omega_i + H_S - 2G(z) \right]}{2}, & G(z), & 0 \\ \dfrac{-N_A^2 F(z)}{\varphi}, & \dfrac{N_A^2 J}{2L_e}, & \dfrac{N_A^2(-i\omega_i L_e - J\sigma)}{2L_e \sigma}, & 0 \\ 0, & \dfrac{-\pi}{2}, & 0, & \dfrac{-J}{2} \end{vmatrix} = 0, \tag{35}$$

Where $J = \left(a^2 + \pi^2\right)$,

$$F(z) = \int_0^1 f(z)\sin^2 \pi z \; dz = \frac{2\pi^2}{\mathrm{H_S} - 4\pi^2},$$

$$G(z) = \frac{\sqrt{\mathrm{H_S}} N_A N_B \pi^2 \tan\left[\sqrt{\mathrm{H_S}}/2\right]}{L_e\left(\mathrm{H_S} - 4\pi^2\right)}$$

Generally when we employ a single-term Galerkin approximation in this situation we get a value overestimate by about 3%. But in the present case, the single-term Galerkin approximation gives the exact result. Because the terms containing N_B involves as a function of N_B/L_e and the value of N_B/L_e is too small of order $10^{-2} \sim 10^{-5}$, pointing to the zero contribution of the nanoparticle flux in the thermal energy conversation.

3. RESULTS AND DISCUSSION

3.1 Stationary Convection

First, consider the case of stationary convection, i.e. $\omega_i = 0$. Then, from equation (35), the expression of thermal Rayleigh- Darcy number R_D for the stationary convection can be obtained as

$$R_D = \frac{\varphi\left(H_S - J\right)J^2}{2a^2F(z)\left[\varphi J + 2L_eG(z)\right]} - \frac{a^2R_e}{J} - \frac{N_A R_n\left[\varphi J + \left\{2G(z) - H_S + J\right\}L_e\right]}{\varphi J + 2L_eG(z)}. \tag{36}$$

It is obvious from equation (36) that the thermal Rayleigh-Darcy number R_D decreases with increasing the internal heat source parameter H_S, AC electric Rayleigh-Darcy number R_e and nanofluid parameters while increases with porosity parameter φ.

When $H_S = 0$ and $R_e = 0$, i.e. in the absence of internal heat source and electric field, therefore equation (36) becomes

$$R_D = \frac{\left(a^2 + \pi^2\right)^2}{a^2} - N_A R_n\left(1 + \frac{L_e}{\varphi}\right). \tag{37}$$

This result is identical with the result of Yadav and Lee (2015) for a thermal equilibrium case with $D_a = T_D = 0$.

In the absence of nanoparticles and electric field, i.e. $R_n = N_B = \mathrm{R_e} = 0$, equation (36) gives

$$R_D = \frac{\left(H_S - a^2 - \pi^2\right)\left(a^2 + \pi^2\right)}{2a^2F(z)}. \tag{38}$$

The above is the same result obtained by Bhadauria (2012) for homogeneous single component convection and absence of electric field in a porous layer.

Now rescaling the parameter as $\chi = a_c^2/\pi^2$ and putting into the equation (36) and then taking $\partial R_D/\partial\chi = 0$, the expression for critical thermal Rayleigh-Darcy number $R_{D,c}$ is obtained, corresponding to the critical value of the wave number $\chi = \chi_c$ which satisfies the following equation

$$\begin{aligned}
&4F(z)G(z)L_e^2\chi^2\left\{2G(z)R_e + N_A\pi^2R_n(1+\chi)^2\right\} \\
&+\varphi^2\pi^4(1+\chi)^2\left\{2F(z)R_e\chi^2 + H_S(1+\chi)^2\right. \\
&\left.+\pi^2(-1+\chi)(1+\chi)^3\right\} + 2\varphi L_e\pi^2(1+\chi) \\
&\times\left[F(z)H_SN_AR_n\chi^2(1+\chi)\right. \\
&+G(z)\left\{4F(z)R_e\chi^2 - H_S(-1+\chi)(1+\chi)^2\right. \\
&\left.\left.+\pi^2(1+\chi)^3(-1+2\chi)\right\}\right] = 0.
\end{aligned} \tag{39}$$

Here χ refer to critical value.

In the absence of nanoparticles and electric field, i.e. $R_n = N_B = \mathrm{R_e} = 0$, equation (39) gives:

$$\pi^2\chi^2 - \pi^2 + H_S = 0. \tag{40}$$

Equation (40) coincides with that of Bhadauria (2012) for homogeneous single component convection and absence of electric field in a porous layer.

3.2 OSCILLATORY CONVECTION

Oscillatory convection is possible only if some supplementary constraints, such as rotation, magnetic field, salinity gradient etc., are present in the system. For oscillatory convection $\omega_i \neq 0$, the real and imaginary parts of equation (35) yield:

$$\begin{aligned}
&2a^2F(z)L_e\left[J\left(-H_S + J\right)N_AR_n + 2G(z)\right. \\
&\left.\times\left\{-\pi^2\mathrm{R_e} + J\left(R_{D,Osc} + \mathrm{R_e} + N_AR_n\right)\right\}\right]\sigma \\
&+\varphi J\left[-H_SJ^2\sigma + J^3\sigma - 2a^2F(z)\pi^2\right. \\
&\times\mathrm{R_e}\sigma + J\left\{-L_e\omega_i^2 + 2a^2F(z)\right. \\
&\left.\left.\times\left(R_{D,Osc} + \mathrm{R_e} + N_AR_n\right)\sigma\right\}\right] = 0,
\end{aligned} \tag{41}$$

$$\begin{aligned}
&\varphi L_e\left[J\left\{J\left\{2G(z) - H_S + J\right\} + 2a^2F(z)R_{D,Osc}\right\}\right. \\
&\left.+2a^2F(z)a^2R_e\right] + \varphi J^3\sigma + 2a^2F(z)JL_eN_AR_n\sigma = 0.
\end{aligned} \tag{42}$$

Equations (41) and (42) give the following expressions for the thermal Rayleigh- Darcy number $R_{D,Osc}$ and the frequency of oscillation ω_i:

Table 1 Comparison of critical thermal Rayleigh-Darcy number $\mathrm{R}_{D,c}$ and critical wave number a_c with different values of internal heat source parameter $\mathrm{H_S}$ and AC electric Rayleigh-Darcy number $\mathrm{R_e}$ for regular fluid

$\mathrm{H_S}$	$\mathrm{R_e}$	$\mathrm{R}_{D,c}$	a_c	$\mathrm{H_S}$	$\mathrm{R_e}$	$\mathrm{R}_{D,c}$	a_c
0	0	39.48	3.14	1.0	0	36.50	3.06
	10	34.32	3.35		10	31.47	3.27
	20	28.85	3.55		20	26.09	3.50
	30	23.10	3.76		30	20.41	3.72
	40	17.08	3.96		40	14.44	3.93
	50	10.83	4.15		50	8.22	4.13
0.5	0	37.98	3.10	2.0	0	33.57	2.97
	10	32.89	3.31		10	28.67	3.20
	20	27.47	3.53		20	23.40	3.44
	30	21.74	3.74		30	17.79	3.67
	40	15.75	3.94		40	11.87	3.90
	50	9.52	4.14		50	5.68	4.11

Table 2 Comparison of critical thermal Rayleigh-Darcy number $\mathrm{R}_{D,c}$ and critical wave number a_c with different values of internal heat source parameter $\mathrm{H_S}$ and AC electric Rayleigh-Darcy number $\mathrm{R_e}$ for nanofluid at $N_A = 2,\ L_e = 5,\ N_B = 0.01,\ \varphi = 0.7,\ R_n = 0.5$

$\mathrm{H_S}$	$\mathrm{R_e}$	$\mathrm{R}_{D,c}$	a_c	$\mathrm{H_S}$	$\mathrm{R_e}$	$\mathrm{R}_{D,c}$	a_c
0	0	31.34	3.14	1.0	0	28.74	3.07
	10	26.18	3.35		10	23.68	3.29
	20	20.71	3.55		20	18.28	3.51
	30	14.96	3.76		30	12.58	3.73
	40	8.94	3.96		40	6.59	3.94
	50	2.69	4.15		50	0.35	4.14
0.5	0	30.03	3.11	2.0	0	26.22	3.00
	10	24.92	3.32		10	21.26	3.23
	20	19.49	3.53		20	15.94	3.47
	30	13.76	3.75		30	10.28	3.70
	40	7.75	3.95		40	4.32	3.93
	50	1.51	4.15		50	-1.91	4.14

$$R_{D,Osc} = \frac{-J\left\{2G(z) - H_S + J\right\}}{2a^2F(z)} - \frac{a^2R_e}{J} - \frac{N_A R_n \sigma}{\varphi} - \frac{J^2\sigma}{2a^2L_eF(z)}. \quad (43)$$

$$\omega_i^2 = \frac{\sigma}{L_e^2\varphi^2J}\left[-2\varphi Le\left[\varphi J + \left\{2G(z) - H_S + J\right\}L_e\right] \times\left\{G(z)J - a^2F(z)N_AR_n\right\} - \left\{\varphi J + 2G(z)L_e\right\} \times\left\{\varphi J^2 + 2a^2F(z)L_eN_AR_n\right\}\sigma\right]. \quad (44)$$

From equation (44), it is interesting to note that the vertical AC electric field does not influence the existence of oscillatory convection. Following Buongiorno (2006), Nield and Kuznetsov (2013a,b) and Yadav (2014) for most of nanofluids, the Lewis number L_e is on the order of $10^1 \sim 10^3$, N_A is on the order of $1 \sim 10$, the nanoparticle Rayleigh-Darcy number R_n and σ are on the order of $1 \sim 10$, and Hence from equation (44), the value of ω_i^2 will be always negative. Since ω_i is real for oscillatory convection, therefore oscillatory convection cannot occur and the principle of the exchange of stability is valid for the case of nanofluid.

The results which are given in the equations (36) and (39) are presented graphically in the Figs.1-12 and also tabled in the Tables 1-6 for fixed values of $H_S = 1,\ N_A = 2,\ L_e = 5,\ N_B = 0.01,\ \varphi = 0.7,\ R_n = 0.5$ except the varying parameters. The range of parameters fall in these figures is taken from the available literature (Buongiorno 2006; Nield and Kuznetsov 2013a,b; Yadav 2014; Kefayati 2013; Kuznetsov 2012, Yadav *et al*. 2015c, 2016a,b,c,d,e,f,g).

The linear stability theory expresses the criteria of stability in terms of the critical thermal Rayleigh-Darcy number $R_{D,c}$, below which the system is

Table 3 Comparison of critical thermal Rayleigh-Darcy number $\mathrm{R}_{D,c}$ and critical wave number a_c with different values of nanoparticles Rayleigh-Darcy number R_n and AC electric Rayleigh-Darcy number R_e for nanofluid at $\mathrm{H_S}=1,\ N_A=2,\ L_e=5,\ N_B=0.01,\ \varphi=0.7$

R_n	R_e	$\mathrm{R}_{D,c}$	a_c	R_n	R_e	$\mathrm{R}_{D,c}$	a_c
0.2	0	33.41	3.07	0.6	0	27.19	3.08
	10	28.36	3.28		10	22.12	3.29
	20	22.98	3.50		20	16.72	3.52
	30	17.28	3.72		30	11.01	3.74
	40	11.30	3.94		40	5.01	3.95
	50	5.08	4.13		50	-1.23	4.15
0.4	0	30.30	3.07	1.0	0	20.97	3.09
	10	25.24	3.29		10	15.88	3.31
	20	19.85	3.51		20	10.46	3.53
	30	14.14	3.73		30	4.73	3.75
	40	8.16	3.94		40	-1.28	3.96
	50	1.93	4.14		50	-7.53	4.16

Table 4 Comparison of critical thermal Rayleigh-Darcy number $\mathrm{R}_{D,c}$ and critical wave number a_c with different values of Lewis number L_e and AC electric Rayleigh-Darcy number R_e for nanofluid at $\mathrm{H_S}=1,\ N_A=2,\ N_B=0.01,\ \varphi=0.7,\ R_n=0.5$

L_e	R_e	$\mathrm{R}_{D,c}$	a_c	L_e	R_e	$\mathrm{R}_{D,c}$	a_c
2	0	32.81	3.07	8	0	24.68	3.08
	10	27.76	3.28		10	19.60	3.30
	20	22.38	3.50		20	14.19	3.52
	30	16.68	3.72		30	8.47	3.74
	40	10.70	3.94		40	2.47	3.95
	50	4.48	4.13		50	-3.78	4.15
5	0	28.74	3.07	11	0	20.61	3.09
	10	23.68	3.29		10	15.52	3.31
	20	18.28	3.51		20	10.09	3.53
	30	12.58	3.73		30	4.36	3.75
	40	6.59	3.94		40	-1.65	3.96
	50	0.35	4.14		50	-7.90	4.16

stable, while above which it is unstable. To determine the accuracy of the numerical procedure used to obtain the critical stability parameters, first the test computations are carried out for different values of internal heat source parameter $\mathrm{H_S}$ under the limiting case of nanoparticle and electric field i.e. $R_n=\mathrm{R}_e=0$ and results are tabled in Table 1. From the Table 1, we identify that in the absence of nanoparticles, internal heat source and electric field, we recover the exactly well-known result that the critical Rayleigh-Darcy number $\mathrm{R}_{D,c}$ is equal to $4\pi^2$ and the corresponding wave number a_c is π. This verifies the accuracy of the numerical method used.

The critical thermal Rayleigh-Darcy number $R_{D,c}$ and the corresponding critical wave number a_c as a function of AC electric Rayleigh-Darcy number R_e for different values of internal heat source parameter $\mathrm{H_S}$ are shown in Figs. 1 and 2, respectively. From Fig. 1, it is observed that on increasing the value of internal heat source parameter $\mathrm{H_S}$, the critical value of thermal Rayleigh-Darcy number decreases; thus the effect of $\mathrm{H_S}$ is to advance the onset of convection. This is because increase in $\mathrm{H_S}$, amounts to increase in energy supply to the system which in turn improve the disturbances in the fluid layer and thus system is more unstable. From Fig.1, it is also found that the critical thermal Rayleigh-Darcy number $\mathrm{R}_{D,c}$ decreases with the AC electric Rayleigh-Darcy number R_e.

Table 5 Comparison of critical thermal Rayleigh-Darcy number $\mathrm{R}_{D,c}$ and critical wave number a_c with different values of modified diffusivity ratio N_A and AC electric Rayleigh-Darcy number R_e for nanofluid at $\mathrm{H_S}=1,\ L_e=5,\ N_B=0.01,\ \varphi=0.7,\ R_n=0.5$

N_A	R_e	$\mathrm{R}_{D,c}$	a_c	N_A	R_e	$\mathrm{R}_{D,c}$	a_c
1	0	32.62	3.07	3	0	24.86	3.08
	10	27.57	3.28		10	19.79	3.30
	20	22.19	3.51		20	14.38	3.52
	30	16.49	3.73		30	8.66	3.74
	40	10.51	3.94		40	2.66	3.95
	50	4.29	4.14		50	-3.59	4.15
2	0	28.74	3.07	4	0	20.98	3.09
	10	23.68	3.29		10	15.89	3.31
	20	18.28	3.51		20	10.47	3.53
	30	12.58	3.73		30	4.74	3.75
	40	6.59	3.94		40	-1.27	3.96
	50	0.35	4.14		50	-7.52	4.16

Table 6 Comparison of critical thermal Rayleigh-Darcy number $\mathrm{R}_{D,c}$ and critical wave number a_c with different values of porosity parameter φ and AC electric Rayleigh-Darcy number R_e for nanofluid at $\mathrm{H_S}=1,\ N_A=2,\ L_e=5,\ N_B=0.01,\ \varphi=0.7,\ R_n=0.5$

φ	R_e	$\mathrm{R}_{D,c}$	a_c	φ	R_e	$\mathrm{R}_{D,c}$	a_c
0.1	0	-11.97	3.17	0.7	0	28.74	3.07
	10	-17.18	3.39		10	23.68	3.29
	20	-22.71	3.61		20	18.28	3.51
	30	-28.54	3.82		30	12.58	3.73
	40	-34.64	4.03		40	6.59	3.94
	50	-40.98	4.22		50	0.35	4.14
0.4	0	23.67	3.09	1.0	0	30.77	3.07
	10	18.59	3.30		10	25.72	3.29
	20	13.17	3.53		20	20.33	3.51
	30	7.45	3.74		30	14.63	3.73
	40	1.44	3.96		40	8.64	3.94
	50	-4.80	4.15		50	2.41	4.14

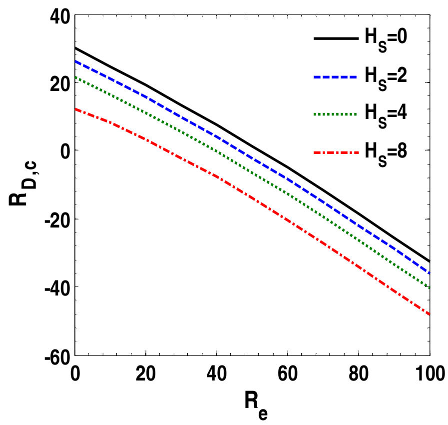

Fig. 1. Effect of AC electric Rayleigh-Darcy number R_e on the critical thermal Rayleigh-Darcy number $\mathrm{R}_{D,c}$ for different values of internal heat source parameter $\mathrm{H_S}$ at $N_A=2,\ L_e=5,\ N_B=0.01,\ \ \varphi=0.7,\ \ R_n=0.5$.

Therefore, the effect of imposed AC electric field results in the reduction of stability of the system. Fig 2 shows that increase in the values of internal heat source parameter $\mathrm{H_S}$ and AC electric Rayleigh-Darcy number R_e tend to increase a_c and thus its effect is to decrease the size of convection cells. Also, it is interesting to note that the effect of internal heat source parameter $\mathrm{H_S}$ on the critical wave number a_c is reversed for higher values of AC electric Rayleigh-Darcy number $\mathrm{R}_e\,(>80)$.

Figurs 3and 4 indicate the influence of nanoparticles Rayleigh-Darcy number R_n on the stability characteristics.

From Fig. 3, it is observed that the critical thermal Rayleigh-Darcy number $R_{D,c}$ decreases as nanoparticles Rayleigh-Darcy number R_n increases. Therefore, nanoparticles Rayleigh-Darcy number R_n has a destabilizing effect on the stability of the system. This is because as an increase in nanoparticles Rayleigh-Darcy number R_n, increases the Brownian motion of the nanoparticles and thus system is more unstable. The corresponding critical wave number a_c is plotted in Fig. 4 and found that the nanoparticles Rayleigh-Darcy number R_n has no significant effect on the critical wave number.

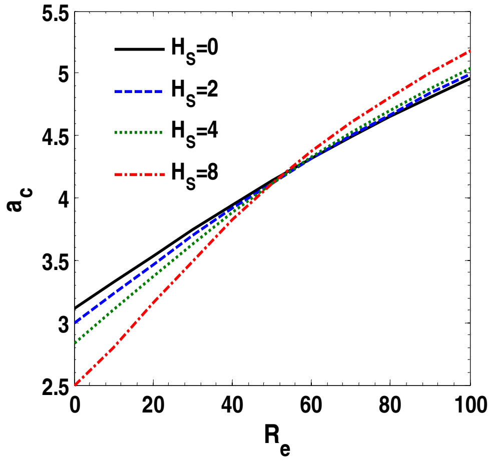

Fig. 2. Effect of AC electric Rayleigh-Darcy number R_e on the critical wave number a_c for different values of internal heat source parameter H_S at $N_A = 2,\ L_e = 5,\ N_B = 0.01,\ \varphi = 0.7,\ R_n = 0.5.$

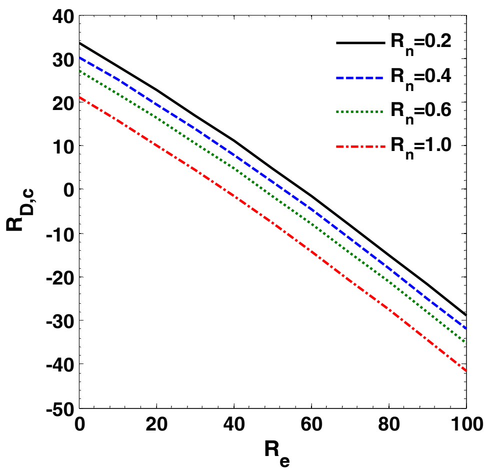

Fig. 3. Effect of AC electric Rayleigh-Darcy number R_e on the critical thermal Rayleigh-Darcy number $\mathrm{R}_{D,c}$ for different values of concentration Rayleigh-Darcy number R_n at $N_A = 2,\ L_e = 5,\ N_B = 0.01,\ \varphi = 0.7,\ H_S = 1.$

Figs. 5-10 show the effect of Lewis number L_e, the modified diffusivity ratio N_A and porosity parameter φ on the stability of the system. From Figs. 5, 7 and 9, we found that the Lewis number L_e and the modified diffusivity ratio N_A accelerate the onset of convection, while porosity parameter φ delays the convection in a nanofluid layer. It may be happened because the thermophoresis at a higher value of thermophoretic diffusivity is more supportable to the disturbance in nanofluids, while both thermophoresis and Brownian motion are driving forces in favour of the motion of nanoparticles. From Figs. 6, 8 and 10, we found that the Lewis number L_e, the modified diffusivity ratio N_A and porosity parameter φ have no significant effect on the critical wave number a_c.

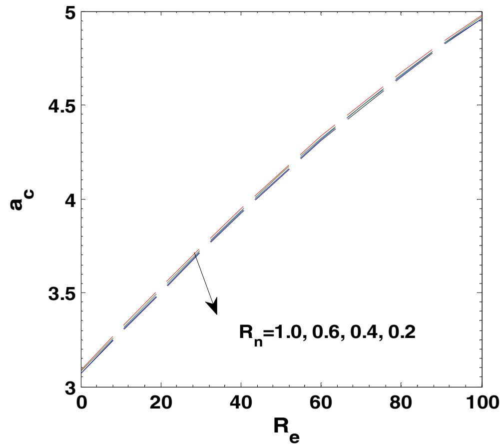

Fig. 4. Effect of AC electric Rayleigh-Darcy number R_e on the critical wave number a_c for different values of concentration Rayleigh-Darcy number R_n at $N_A = 2,\ L_e = 5,\ N_B = 0.01,\ \varphi = 0.7,\ H_S = 1.$

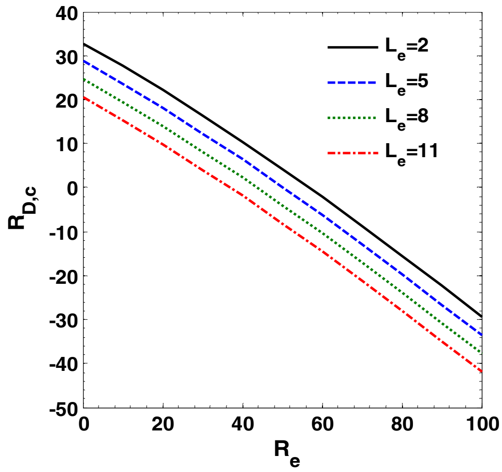

Fig. 5. Effect of AC electric Rayleigh-Darcy number R_e on the critical thermal Rayleigh-Darcy number $\mathrm{R}_{D,c}$ for different values of Lewis number L_e at $N_A = 2,\ N_B = 0.01,\ \varphi = 0.7,\ H_S = 1,\ \mathrm{R}_n = 0.5.$

The effect of modified specific heat increment N_B on the onset of convection is shown in Figs. 11 and 12. It is found that the modified specific heat increment N_B has no significant effect on the stability of system. This is occurred due to the terms containing N_B involves as a function of N_B/L_e and the value of N_B/L_e is too small of order $10^{-2} \sim 10^{-5}$, pointing to the zero contribution of the nanoparticle flux in the thermal energy conversation.

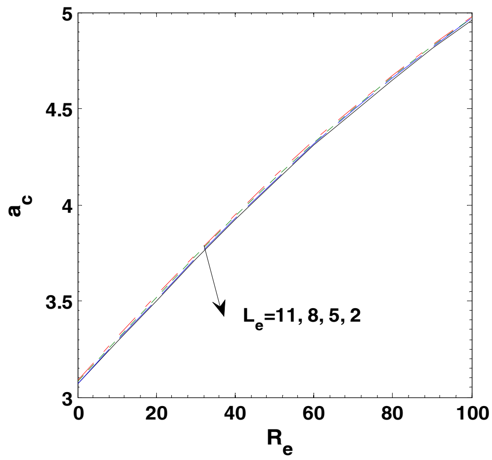

Fig. 6. Effect of AC electric Rayleigh-Darcy number R_e on the critical wave number a_c for different values of Lewis number L_e at $N_A = 2,\ N_B = 0.01,\ \varphi = 0.7,\ H_S = 1,\ R_n = 0.5.$

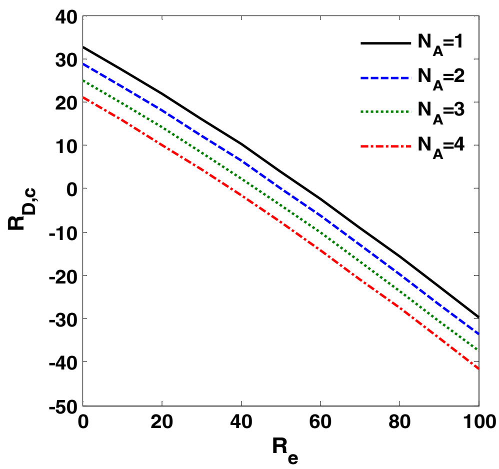

Fig. 7. Effect of AC electric Rayleigh-Darcy number R_e on the critical thermal Rayleigh-Darcy number $R_{D,c}$ for different values of modified diffusivity ratio N_A at $L_e = 5,\ N_B = 0.01,\ \varphi = 0.7,\ H_S = 1,\ R_n = 0.5.$

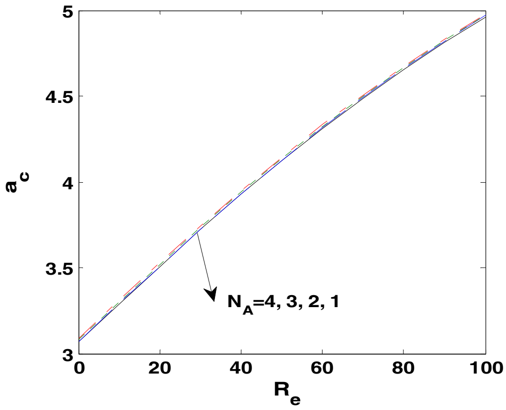

Fig.8. Effect of AC electric Rayleigh-Darcy number R_e on the critical wave number a_c for different values of modified diffusivity ratio N_A at $L_e = 5,\ N_B = 0.01,\ \varphi = 0.7,\ H_S = 1,\ R_n = 0.5.$

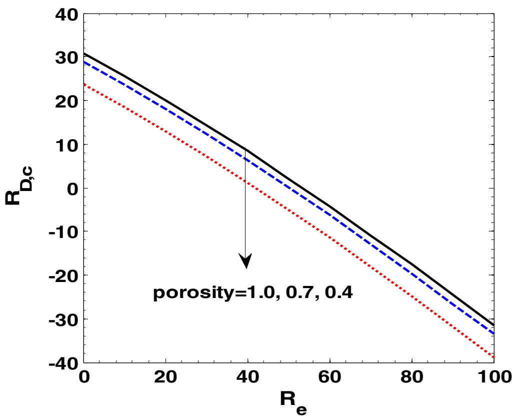

Fig. 9. Effect of AC electric Rayleigh-Darcy number R_e on the critical thermal Rayleigh-Darcy number $R_{D,c}$ for different values of porosity φ at $N_A = 2,\ L_e = 5,\ N_B = 0.01,\ R_n = 0.5,\ H_S = 1.$

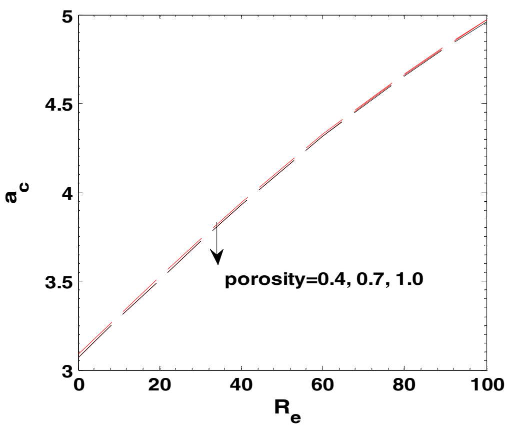

Fig. 10. Effect of AC electric Rayleigh-Darcy number R_e on the critical wave number a_c for different values of porosity φ at $N_A = 2,\ L_e = 5,\ N_B = 0.01,\ R_n = 0.5,\ H_S = 1.$

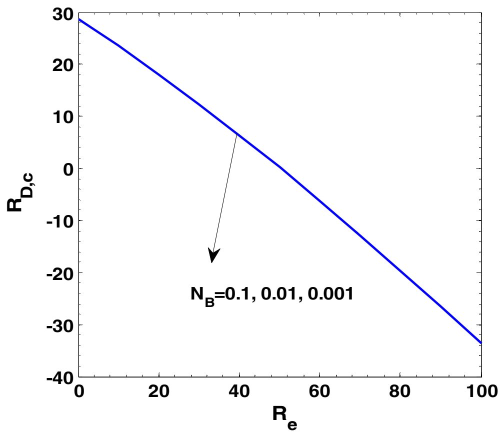

Fig. 11. Effect of AC electric Rayleigh-Darcy number R_e on the critical thermal Rayleigh-Darcy number $R_{D,c}$ for different values of modified specific heat increment N_B at $N_A = 2,\ L_e = 5,\ \varphi = 0.7,\ R_n = 0.5,\ H_S = 1.$

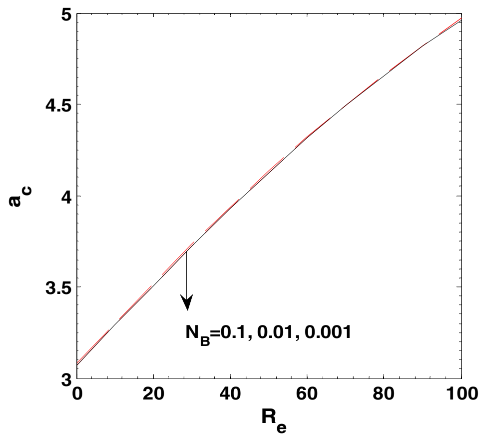

Fig. 12. Effect of AC electric Rayleigh-Darcy number R_e on the critical wave number a_c for different values of modified specific heat increment N_B at $N_A = 2,\ L_e = 5,\ \varphi = 0.7,$ $R_n = 0.5,\ H_S = 1.$

4. CONCLUSIONS

The combined effect of vertical AC electric field and temperature depended internal heat source on the onset of convection in a nanofluid-saturated porous layer has been studied analytically with the assumption that the nanoparticle flux is zero under the thermophoresis at the boundaries. The results have been obtained in terms of the critical thermal Rayleigh-Darcy number $R_{D,c}$. We have the following observations:

1) The instability of the nanofluid is reinforced with an increase in the value of internal heat source H_S, AC electric Rayleigh-Darcy number R_e, the Lewis number L_e, the modified diffusivity ratio N_A and the concentration Rayleigh-Darcy number R_n.

2) The size of convection cells decreases with increasing the AC electric Rayleigh-Darcy number R_e.

3) For lower values of AC electric Rayleigh-Darcy number $R_e\,(<80)$, the size of convection cells decreases with increasing the internal heat source H_S, while reverse for higher values of AC electric Rayleigh-Darcy number R_e.

4) The vertical AC electric field does not influence the existence of oscillatory convection and it is ruled out for nanofluid convection due to absence of the two opposing agencies who affect the instability.

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