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Effect of Horizontal Magnetic Field and Horizontal Rotation on Thermosolutal Stability of a Dusty Couple-Stress Fluid through a Porous Medium: a Brinkman Model

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ABSTRACT

The problem of the onset of double diffusive convection in a couple-stress fluid saturated with a porous medium is studied under the effects of magnetic field, rotation and suspended dust particles. Linear stability analysis based on the method of perturbations of infinitesimal amplitude is performed using the normal mode technique for the case of free-free boundaries. The governing hydrodynamic and hydromagnetic equations of fluid flow are governed by the Brinkman model. The stability analysis examines the effects of various embedded parameters for the stationary mode both analytically and graphically. The principle of exchange of stabilities holds good in the absence of solute gradient parameter. Also, the sufficient conditions responsible for the existence or non-existence of overstability are obtained.

Keywords: Couple-stress fluid; Magnetic field; Rotation; Dust particles; Brinkman porous medium.

NOMENCLATURE

t	time coordinate,
d	thickness of porous fluid layer,
$\mathbf{q}$	fluid velocity vector having components
$\mathbf{q_d}$	suspended particle velocity having components
p	pressure
T_0	reference temperature
S_0	reference concentration
T	temperature
S	concentration
K'	stokes' drag coefficient
N_0	number density of dust particles
k_1	Darcy-brinkman medium permeability
k_T	coefficient of heat conduction
k_S	coefficient of solute concentration
D	differential operator
m	mass of suspended particles
n	frequency of the harmonic disturbance
$\mathbf{X_i}$	gravitational acceleration vector
c_s	heat capacity of solid material
c_v	specific heat of the fluid at constant volume
$\mathbf{w}$	vertical fluid velocity
$\mathbf{s}$	vertical particle velocity
$\mathbf{w}^*$	complex conjugate of
$\mathbf{H}$	horizontal magnetic field having components
$\mathbf{h}$	perturbation in magnetic field strength
$\mathbf{X}$	vertical component of current density after applying normal mode method
$\mathbf{W}$	vertical component of fluid velocity after applying normal mode method
$\mathbf{K}$	vertical component of magnetic field after applying normal mode method
$\mathbf{Z}$	vertical component of vorticity after applying normal mode method
k_x	wave number in x direction
k_y	wave number in y direction
k	Resultant wave number

∇p	pressure gradient term
ρ_0	density of fluid
ρ_s	density of solid material
μ	fluid viscosity
μ'	couple-stress fluid viscosity
$\tilde{\mu}_{ef}$	effective viscosity
$\tilde{\upsilon}_{ef}$	effective kinematic viscosity
μ_e	magnetic permeability
α_T	thermal expansion coefficient
α_S	solute expansion coefficient
β_T	adverse temperature gradient
β_S	solute concentration gradient
η	electrical resistivity
Θ	temperature component after applying normal mode method
Φ	solute component after applying normal mode method
δp	perturbation in fluid pressure p
$\delta\rho$	perturbation in fluid density ρ
υ	kinematic viscosity
υ'	kinematic viscoelasticity
κ_T	thermal diffusivity
κ_S	solute diffusivity
ζ	z-component of vorticity
ξ	z-component of current density
$\mathbf{\Omega}$	horizontal rotational vector having components
∇^2	3-dimensional Laplacian operator
σ	growth rate of harmonic disturbance after applying normal mode method
θ	perturbation in temperature T
γ	perturbation in solute concentration S
δ	Suspended particles radius
λ_i	vertical unit vector

Non-dimensional Parameters

a	dimensionless wave number
P_l	dimensionless medium permeability
p_1	thermal Prandtl number
p_2	magnetic Prandtl number
q_1	Schmidt number
B	suspended particle parameter
Q_1	modified Chandrasekhar's number
D_{A_1}	modified Darcy-Brinkman number
R_1	modified Darcy-brinkman thermal Rayleigh number
S_1	modified solute Rayleigh number
T_{A_1}	modified Taylor's number
$\in$	Darcy-Brinkman medium porosity
Υ_1	modified couple-stress parameter

1. INTRODUCTION

The study of fluid dynamics is of great importance for scientific researchers and engineers to understand various significant and fascinating applications of fluid mechanical phenomena such as calculating forces and moments on aircrafts, determining the rate of mass flow in oil industry through pipelines, forecasting weather patterns, understanding nebulae in interstellar space and in traffic engineering by considering traffic as a continuously distributed fluid. Copious literatures (Lin, 1955; Batchelor 1967; Rajagopal, 1978; Drazin and Reid, 1981; Bansal, 2004; Gupta and Gupta, 2013) are available that provide a basic theoretical and experimental support for various fluid dynamical phenomena as well as hydrodynamic stability of Newtonian and non-Newtonian fluids. The influence of magnetic field and rotation on thermal instability of an incompressible Newtonian fluid layer has been the subject matter of great interest over the years since the pioneering work by Chandrasekhar (1981) within the framework of linear stability theory. In double diffusive (when the fluid layer is simultaneously heated and soluted from underside) phenomena, convection process is induced by the combined effects of temperature difference and concentration difference which have different diffusion rates. Double diffusive (thermosolutal) convective phenomena through a porous medium frequently occur in seawater flow, mantle flow in earth's crust, ground water hydrology (underground disposal of nuclear wastes), astrophysics (helium acts like a salt in raising the density in stellar case), oceanography, soil science (transportation of soil, solid waste and mud particles into the rivers and lakes), solar system (where heat and helium diffuse at differing rates), atmospheric science and limnology.

The presence of waste matter in water or air is responsible for certain chemical reactions helpful to understand several mass transport processes such as groundwater pollution and transportation of nuclear wastes, food processing industry, contaminated air transport in atmosphere (presence of dust particles in atmosphere), manufacturing of glassware and in polymer production. Nearly all fluid flow mechanisms happening in the universe involve circulation or rotational effects which have applications to a greater or lesser extent in a number of processes which include large scale circulations in the atmosphere and oceans, motion of a hurricane, tornado and tsunami and in many small scale flows like stirring of tea in a cup. Thermo-convective phenomenon in a rotating system is of practical significance and finds its applications in rotating machinery, crystal growth, food processing industry, centrifugal casting of metals and in thermal power plants (to generate electricity by the rotation of turbine blades). Greenspan (1969) studied the theory of rotating fluids which has applications in various technological situations which are governed by the action of coriolis force. The double diffusive convective problems of couple-stress fluid through an anisotropic porous medium considering rotational effect have been discussed in the references by Malashetty and Kollur (2011) and Malashetty *et al.* (2011). A major part of the universe is filled with charged particles and magnetic field is present in and around the heavenly bodies. Magnetic field plays a dominant role in several clinical purposes such as in neurology and orthopaedics for examining the internal organs of the body in various diseases like tumours detection, heart and brain diseases, stroke damage etc. The theory of magneto-hydrodynamics (MHD) has several scientific and practical applications in geophysics (in the study of earth's core), atmospheric science (solar wind is governed by MHD), astrophysics, plasma physics, space sciences etc. Thermal instability problem of an electrically conducting couple-stress fluid heated from below through a porous medium in the presence of a uniform magnetic field has been investigated by Sharma and Thakur (2000). Sharma and Sharma (2004) have considered the effect of suspended particles on couple-stress fluid heated from below in the presence of vertical rotation and vertical magnetic field and noted that the effect of rotation is to stabilize the system, whereas suspended particles have destabilizing effects. Thermosolutal convective problem for a couple-stress fluid through a porous medium under the influences of uniform vertical magnetic field and uniform rotation has been studied by Singh and Kumar (2011), whereas the effect of suspended particles on couple-stress fluid heated and soluted from below through a porous layer has been noted by Sunil *et al.* (2004).

Couple-stress theory due to Stokes (1966) is of vital importance to understand different aspects including the lubrication mechanism and functioning of synovial joints during human locomotion and opened new vistas in several areas of scientific and technical research. Shivakumara *et al.* (2011) illustrated the linear and nonlinear stability problem of double diffusive convective phenomena for couple-stress fluid through a porous layer. Kumar *et al.* (2015a, b, 2016) considered, theoretically, thermal convection problems for both couple-stress fluid and ferrofluid to include the effects due to magnetic field, rotation, compressibility, variable gravity and heat source strength through Darcy as well as Brinkman porous medium. Thermal as well as thermosolutal convective instability problems through a porous medium have extensive attention over the years and also recognized as the problems of fundamental importance in solidification and chemical processing industry, bio-medical sciences, geophysical fluid dynamics, soil sciences, petroleum industry and filtering technology. Extensive reviews related to thermal and thermosolutal convective problems through a porous medium have been covered in the books by Ingham and Pop (2005), Vafai (2000, 2005), Nield and Bejan (2006) and Vadasz (2008). McDonnel (1978) has conducted extensive and systematic studies regarding the impact of porosity in astrophysical situations because a greater part of the universe is filled with fine dust particles. It is well known that the Darcy equation fails to give satisfactory results for high permeability porous medium. So, the consideration of non-Darcy (Brinkman) model, which take care of boundary and inertia effects, is of practical interest for high permeability porous medium. Kuznetsov and Nield (2010) analyzed the thermal instability problem in a porous layer saturated by a nanofluid employing the Brinkman model for the porous medium. Mahajan and Sharma (2012) studied the asymptotic stability and also determine the stability bounds for both equilibrium and arbitrary flows of a couple-stress fluid in a Brinkman flow. Recently, the linear and nonlinear stability analysis of double-diffusive reaction convection in an anisotropic Darcy-Brinkman porous layer is performed by Gaikwad and Dhanraj (2016).

Therefore, the Brinkman model, which is physically more realistic than the other ones, is considered for the porous medium to include effects due to horizontal magnetic field and horizontal rotation in a dusty couple-stress double-diffusive convection. Some previous published works by Sharma and Thakur (2000), Sharma and Sharma (2004), Sunil *et al.* (2004) and Singh and Kumar (2011) can be recovered from the present investigation.

2. Mathematical Formulation

Consider an infinite, horizontal porous layer of thickness d of an incompressible couple-stress fluid with dust particles heated and soluted from below through a porous medium of porosity $\in$ and permeability k_1. The fluid layer is acted upon by a uniform horizontal magnetic field $\mathbf{H}=(H,0,0)$ and a uniform horizontal rotation $\mathbf{\Omega}=(\Omega,0,0)$. The fluid layer is heated and soluted from below such that an adverse temperature gradient $\left(\beta_T=\left|\frac{dT}{dz}\right|\right)$ and concentration gradient $\left(\beta_S=\left|\frac{dS}{dz}\right|\right)$ are maintained. The coriolis effect has been taken into account by including the coriolis force term $2(q\times\Omega)$ in the momentum equation, whereas the Centrifugal force term $\left(-\frac{1}{2}\text{grad}\left|\Omega\times r\right|^2\right)$ can be realized as a gradient of a scalar and, therefore, has been absorbed into the pressure term $p=\left(p_f-\frac{1}{2}\rho\left|\Omega\times r\right|^2\right)$. The term p_f stands for the fluid pressure and Ω denotes the angular velocity of rotation. The Brinkman model is employed for the porous medium and the fluid density variation is based on Boussinesq approximation (1903). The boundaries are taken to be free maintained at uniform temperatures and the porous layer

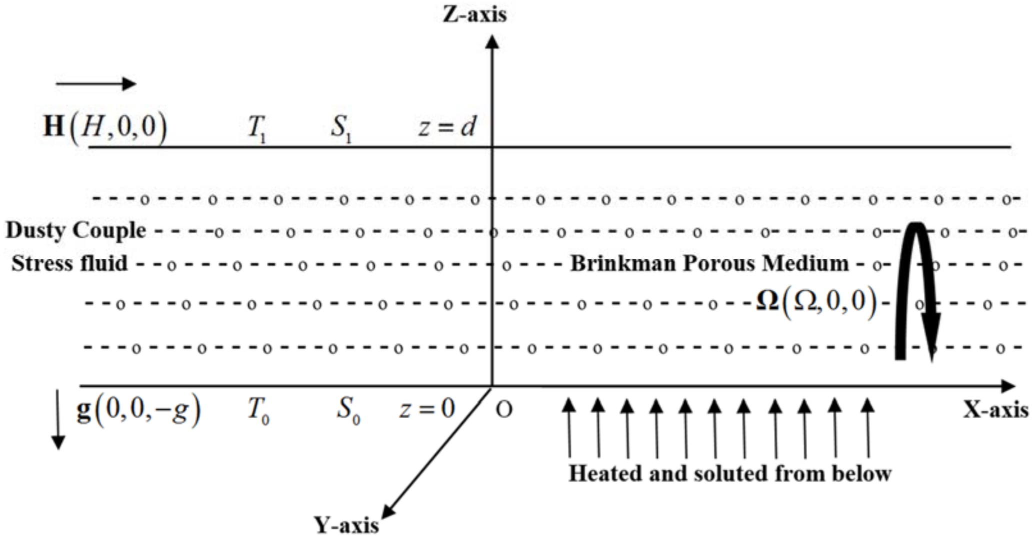

Fig. a. Geometrical Sketch of the Physical Problem.

is extended infinitely in x and y directions and the z axis is taken vertically upward with the origin at the lower boundary. The pressure p, density ρ, viscosity μ and viscoelasticity μ' depend upon the vertical co-ordinate z-only.

The basic equations in a double diffusive convection for an incompressible couple-stress fluid saturating a Brinkman (1947a, b) porous medium under Boussinesq approximation are defined as

The density equation of state is given by

$$\rho = \rho_0\left[1+\alpha_T\,(T_0-T\,)+\alpha_S\,(S\,-S_0)\right] \tag{1}$$

Equations of momentum and mass conservation are defined as

$$\frac{\rho_0}{\in}\left[\frac{\partial \mathbf{q}}{\partial t}+\frac{1}{\in}(\mathbf{q}.\nabla)\mathbf{q}\right]=\begin{bmatrix}-\nabla p+\rho_0\mathbf{X_i}-\frac{1}{k_1}\left(\mu-\mu'\nabla^2\right)\mathbf{q}+\\ \frac{2\rho_0}{\in}(\mathbf{q}\times\mathbf{\Omega})+\left(\frac{\tilde{\mu}_{ef}}{\in}\right)\nabla^2\mathbf{q}+\frac{K'N_0}{\in}\\ (\mathbf{q_d}-\mathbf{q})+\frac{\mu_e}{4\pi}(\nabla\times\mathbf{H})\times\mathbf{H}\end{bmatrix} \tag{2}$$

$$\nabla.\mathbf{q}=0 \tag{3}$$

Neglecting the variations in pressure force, Darcian force, magnetic field and buoyancy force (due to gravity) on the particles, the equations of motion and continuity for the particles are as

$$mN_0\left[\frac{\partial \mathbf{q_d}}{\partial t}+\frac{1}{\in}(\mathbf{q_d}.\nabla)\mathbf{q_d}\right]=K'N_0(\mathbf{q}-\mathbf{q_d}) \tag{4}$$

$$\in\frac{\partial N_0}{\partial t}+\nabla.(N_0\mathbf{q_d})=0 \tag{5}$$

Assuming that the suspended particles and the fluid particles are in thermal and solute equilibrium, the equations for temperature and solute concentration can be presented as

$$\begin{aligned}&\left[\in\rho_0 c_v+\rho_s c_s(1-\in)\right]\frac{\partial T}{\partial t}+\rho_0 c_v\,(\mathbf{q}.\nabla)T\\&+mN_0 c_{pt}\left[\in\frac{\partial}{\partial t}+(\mathbf{q_d}.\nabla)\right]T\;=k_T\nabla^2 T\end{aligned} \tag{6}$$

$$\begin{aligned}&\left[\in\rho_0 c'_v+\rho_s c'_s(1-\in)\right]\frac{\partial S}{\partial t}+\rho_0 c'_v\,(\mathbf{q}.\nabla)S\\&+mN_0 c'_{pt}\left[\in\frac{\partial}{\partial t}+(\mathbf{q_d}.\nabla)\right]S\;=k_S\nabla^2 S\end{aligned} \tag{7}$$

where, c'_v, c'_s, c'_{pt} and k_S denote the analogous solute coefficients.

The Maxwell's (1866) electromagnetic equations yield

$$\in\frac{\partial \mathbf{H}}{\partial t}=\nabla\times(\mathbf{q}\times\mathbf{H})+\in\eta\nabla^2\mathbf{H} \tag{8}$$

and $\nabla.\mathbf{H}=0$ (9)

3. PERTURBATION TECHNIQUE AND LINEAR STABILITY ANALYSIS

Here, the stability of the basic state of system has been examined by using the perturbation technique. Let the perturbations in the basic variables such as fluid velocity $q(0,0,0)$, particle velocity $q_d\,(0,0,0)$, temperature T, pressure p, density ρ, particle number density N_0 and magnetic field H

be denoted by $q\left(u,v,w\right)$, $q_d\left(l,r,s\right)$, θ, δp, $\delta\rho$, N, $h\left(h_x,h_y,h_z\right)$, respectively.

After linearizing the system of Eqs. (1) – (9) and following Boussinesq approximation, the perturbation equations obtained after eliminating the pressure gradient term are as follows

$$\delta\rho=-\left(\alpha_T\theta-\alpha_S\gamma\right)\rho_0 \tag{10}$$

$$\frac{1}{\in}\left\{\frac{\partial}{\partial t}\left(\nabla^2\mathbf{w}\right)\right\}=\begin{bmatrix}\mathbf{g}\left\{\alpha_T\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\theta-\alpha_S\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\gamma\right\}\\ -\left(\frac{2\Omega}{\in}\right)\left(\frac{\partial\zeta}{\partial x}\right)+\left(\frac{\tilde{\upsilon}_{ef}}{\in}\right)\left(\nabla^4\mathbf{w}\right)-\\ \left\{\frac{mN_0}{\rho_0\in\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)}\right\}\left\{\frac{\partial}{\partial t}\left(\nabla^2\mathbf{w}\right)\right\}+\frac{\mu_e\mathbf{H}}{4\pi\rho_0}\\ \left\{\frac{\partial}{\partial x}\left(\nabla^2\mathbf{h_z}\right)\right\}-\frac{1}{k_1}\left(\upsilon-\upsilon'\nabla^2\right)\left(\nabla^2\mathbf{w}\right)\end{bmatrix} \tag{11}$$

$$\frac{1}{\in}\left\{\frac{\partial\zeta}{\partial t}\right\}=\begin{bmatrix}\left(\frac{2\Omega}{\in}\right)\left(\frac{\partial\mathbf{w}}{\partial x}\right)+\left(\frac{\tilde{\upsilon}_{ef}}{\in}\right)\left(\nabla^2\zeta\right)-\\ \left\{\frac{mN_0}{\rho_0\in\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)}\right\}\left\{\frac{\partial\zeta}{\partial t}\right\}+\\ \frac{\mu_e\mathbf{H}}{4\pi\rho_0}\left\{\frac{\partial\xi}{\partial x}\right\}-\frac{1}{k_1}\left(\upsilon-\upsilon'\nabla^2\right)\zeta\end{bmatrix} \tag{12}$$

$$\in\left(\frac{\partial\mathbf{h_z}}{\partial t}\right)=\mathbf{H}\left(\frac{\partial\mathbf{w}}{\partial x}\right)+\in\eta\left(\nabla^2\mathbf{h_z}\right) \tag{13}$$

$$\in\left(\frac{\partial\xi}{\partial t}\right)=\mathbf{H}\left(\frac{\partial\zeta}{\partial x}\right)+\in\eta\left(\nabla^2\xi\right) \tag{14}$$

$$\left[\left(E+b\in\right)\frac{\partial}{\partial t}-\kappa_T\nabla^2\right]\theta=\beta_T\left(\mathbf{w}+b\mathbf{s}\right) \tag{15}$$

$$\left[\left(E'+b'\in\right)\frac{\partial}{\partial t}-\kappa_S\nabla^2\right]\gamma=\beta_S\left(\mathbf{w}+b'\mathbf{s}\right) \tag{16}$$

where,

$$E=\in+\left(1-\in\right)\left(\frac{\rho_s c_s}{\rho_0 c_v}\right),\ E'=\in+\left(1-\in\right)\left(\frac{\rho_s c_s'}{\rho_0 c_v'}\right),$$
$$b=\left(\frac{mN_0 c_{pt}}{\rho_0 c_v}\right)\text{ and }b'=\left(\frac{mN_0 c_{pt}'}{\rho_0 c_v'}\right).$$

Differential Eqs. (11) – (16) can be solved using the method of normal modes. Suppose that the perturbations in various physical quantities have a solution with a dependence on *x, y* and *t* of the form

$$\left[\mathbf{w},\theta,\zeta,\mathbf{h_z},\xi,\gamma\right]=\begin{bmatrix}\mathbf{W}(\mathbf{z}),\Theta(z),\mathbf{Z}(\mathbf{z})\\ ,\mathbf{K}(\mathbf{z}),\mathbf{X}(\mathbf{z}),\Phi(z)\end{bmatrix}\exp\left(ik_x x+ik_y y+nt\right) \tag{17}$$

Using expression (17), the Eq. (11) – (16) into non-dimensional form reduce to (after dropping the asterisk for convenience)

$$\left[\frac{\sigma}{\in}\left\{1+\frac{f}{\left(\tau_1\sigma+1\right)}\right\}+\frac{1}{P_l}\left\{1-\Upsilon\left(D^2-a^2\right)\right\}-\frac{D_A}{P_l\in}\left(D^2-a^2\right)\right]$$
$$\left(D^2-a^2\right)\mathbf{W}(\mathbf{z})+\frac{ga^2d^2}{\upsilon}\left[\alpha_T\Theta(z)-\alpha_S\Phi(z)\right]+\frac{2\mathbf{\Omega}ik_xd^4}{\in\upsilon}$$
$$\mathbf{Z}(\mathbf{z})-\frac{\mu_e\mathbf{H}ik_xd^2}{4\pi\rho_0\upsilon}\left(D^2-a^2\right)\mathbf{K}(\mathbf{z})=0 \tag{18}$$

$$\left[\frac{\sigma}{\in}\left\{1+\frac{f}{\left(\tau_1\sigma+1\right)}\right\}+\frac{1}{P_l}\left\{1-\Upsilon\left(D^2-a^2\right)\right\}-\frac{D_A}{P_l\in}\left(D^2-a^2\right)\right]$$
$$\mathbf{Z}(\mathbf{z})=\frac{2\mathbf{\Omega}ik_xd^2}{\in\upsilon}\mathbf{W}(\mathbf{z})+\frac{\mu_e\mathbf{H}ik_xd^2}{4\pi\rho_0\upsilon}\mathbf{X}(\mathbf{z}) \tag{19}$$

$$\left[p_2\sigma-\left(D^2-a^2\right)\right]\in\mathbf{K}(\mathbf{z})=\left(\frac{\mathbf{H}ik_xd^2}{\eta}\right)\mathbf{W}(\mathbf{z}) \tag{20}$$

$$\left[p_2\sigma-\left(D^2-a^2\right)\right]\in\mathbf{X}(\mathbf{z})=\left(\frac{\mathbf{H}ik_xd^2}{\eta}\right)\mathbf{Z}(\mathbf{z}) \tag{21}$$

$$\left[\left(D^2-a^2\right)-p_1E_1\sigma\right]\Theta(z)=-\left(\frac{\beta_Td^2}{\kappa_T}\right)\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)\mathbf{W}(\mathbf{z}) \tag{22}$$

$$\left[\left(D^2-a^2\right)-q_1E_2\sigma\right]\Phi(z)=-\left(\frac{\beta_Sd^2}{\kappa_S}\right)\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)\mathbf{W}(\mathbf{z}) \tag{23}$$

where, the following non-dimensional quantities with the scaling have been used in Eqs. (18) – (23)

$$z^*=\left(\frac{z}{d}\right),\ k=\left(\frac{a}{d}\right),\ \sigma=\frac{nd^2}{\upsilon},P_l=\frac{k_1}{d^2},p_1=\frac{\upsilon}{\kappa_T},$$
$$q_1=\frac{\upsilon}{\kappa_S},p_2=\frac{\upsilon}{\eta},\ \tau_1=\frac{\tau\upsilon}{d^2},\tau=\frac{m}{K'},N_0=\frac{\rho_0 f}{m},B=1+b,$$
$$E_1=E+b\in,\ E_2=E'+b'\in,\ \Upsilon=\frac{\upsilon'}{\upsilon d^2},\ D_A=\left(\frac{\tilde{\mu}_{ef}P_l}{\mu}\right).$$

The boundary conditions appropriate for the case of two free boundaries are defined as

$$\left.\begin{array}{l}W=D^2W=X=DZ=\Theta=\Phi=0,at\ z=0\text{ and }1.\\ DX=K=0,\text{ on a perfect conducting boundary}\end{array}\right\} \tag{24}$$

Eliminating $\Theta(z)$, $\Phi(\mathbf{z})$, $\mathbf{Z}(\mathbf{z})$, $\mathbf{X}(\mathbf{z})$ and $\mathbf{K}(\mathbf{z})$ from Eqs. (18)–(23) and considering an appropriate solution for W (vertical component of fluid velocity) of the form

$$W=W_0\sin l\pi z, \tag{25}$$

a dispersion relation is obtained as follows

$$A_1A_2A_3A_4\left(1+x\right)+\left\{\frac{T_{A_1}A_2A_3A_4{}^2x\cos^2\theta}{\in\left(A_1A_4\in+Q_1x\cos^2\theta\right)}\right\}+\left(\frac{B+i\tau_2\sigma_1}{1+i\tau_2\sigma_1}\right)$$
$$xA_4\left(S_1A_2-R_1A_3\right)+\left\{\frac{Q_1A_2A_3x\left(1+x\right)\cos^2\theta}{\in}\right\}=0 \tag{26}$$

where,

$$R_1=\frac{R}{l^4\pi^4}, x=\frac{a^2}{l^2\pi^2}, i\sigma_1=\frac{\sigma}{l^2\pi^2}, P_l=\frac{P}{l^2\pi^2}, k^2=\frac{l^2\pi^2 x}{d^2},$$

$$Q_1=\frac{Q}{l^2\pi^2}, \Upsilon=\frac{\Upsilon_1}{l^2\pi^2}, T_{A_1}=\frac{T_A}{l^4\pi^4}, \tau_1=\frac{\tau_2}{l^2\pi^2}, S_1=\frac{S}{l^2\pi^2},$$

$$D_A=\frac{D_{A_1}}{l^2\pi^2},\ k_x=k\cos\theta, A_2=\left[(1+x)+i\sigma_1 p_1 E_1\right],$$

$$A_1=\left[\frac{i\sigma_1}{\in}\left\{1+\frac{f}{(1+i\tau_2\sigma_1)}\right\}+\frac{1}{P}\left\{1+\Upsilon_1(1+x)\right\}+\frac{D_{A_1}}{P\in}(1+x)\right],$$

$$A_3=\left[(1+x)+i\sigma_1 q_1 E_2\right],\quad A_4=\left[(1+x)+i\sigma_1 p_2\right],$$

$R=\dfrac{g\alpha_T\beta_T d^4}{\upsilon\kappa_T}$ (Darcy-Brinkman thermal Rayleigh number)

$S=\dfrac{g\alpha_S\beta_S d^4}{\upsilon\kappa_S}$ (Solute Rayleigh number)

$T_A=\dfrac{4\mathbf{\Omega}^2 d^4}{\upsilon^2}$ (Taylor's number)

$Q=\dfrac{\mu_e\mathbf{H}^2 d^2}{4\pi\rho_0\upsilon\eta}$ (Chandrasekhar's number)

Equation (26) is the required dispersion relation accounting the effects of suspended particles, horizontal magnetic field, horizontal rotation, medium permeability and medium porosity on thermosolutal instability of a couple-stress fluid saturating a Brinkman porous medium.

4. THE STATIONARY STATE

In a stationary convection, the marginal state occurs when $\sigma=0$ (i.e. growth rate vanishes). Substituting $\sigma=0$ in Eq. (26), the stationary Rayleigh number R_1 in terms of various parameters and the square of wave number x is obtained as

$$R_1=\frac{1}{Bx}\left[\begin{array}{l}\dfrac{(1+x)^2}{P}\left\{1+\left(\Upsilon_1+\dfrac{D_{A_1}}{\in}\right)(1+x)\right\}+\dfrac{Q_1x(1+x)\cos^2\theta}{\in}\\+S_1xB\\+\dfrac{T_{A_1}x(1+x)^2\cos^2\theta}{\in\left[\dfrac{\in(1+x)}{P}\left\{1+\left(\Upsilon_1+\dfrac{D_{A_1}}{\in}\right)(1+x)\right\}+Q_1x\cos^2\theta\right]}\end{array}\right] \tag{27}$$

The effect of various parameters on thermosolutal convection can be realized analytically by evaluating the following derivatives $\dfrac{dR_1}{dT_{A_1}},\dfrac{dR_1}{dS_1},\dfrac{dR_1}{d\in},\dfrac{dR_1}{dB},$. $\dfrac{dR_1}{dQ_1},\dfrac{dR_1}{d\Upsilon_1},\dfrac{dR_1}{dD_{A_1}}$ and $\dfrac{dR_1}{dP}$

Equation (27) yields

$$\frac{dR_1}{dT_{A_1}}=\left[\frac{(1+x)^2P\cos^2\theta}{B\in\left\{\in(1+x)G+Q_1Px\cos^2\theta\right\}}\right] \tag{28}$$

$$\frac{dR_1}{dS_1}=1 \tag{29}$$

$$\frac{dR_1}{d\in}=-\frac{(1+x)\cos^2\theta}{B\in}\left[\begin{array}{l}\dfrac{Q_1}{\in}+\dfrac{T_{A_1}P(1+x)^2G}{\left\{\in(1+x)G+Q_1Px\cos^2\theta\right\}^2}\\+\dfrac{T_{A_1}P(1+x)}{\in\left\{\in(1+x)G+Q_1Px\cos^2\theta\right\}}\end{array}\right] \tag{30}$$

$$\frac{dR_1}{dB}=-\frac{1}{B^2x}\left[\begin{array}{l}\dfrac{(1+x)^2G}{P}+\dfrac{Q_1x(1+x)\cos^2\theta}{\in}\\+\dfrac{T_{A_1}x(1+x)^2\cos^2\theta}{\in\left\{\dfrac{\in(1+x)G}{P}+Q_1x\cos^2\theta\right\}}\end{array}\right] \tag{31}$$

$$\frac{dR_1}{dQ_1}=\frac{(1+x)\cos^2\theta}{B\in}\left[1-\frac{T_{A_1}P^2x(1+x)\cos^2\theta}{\left\{\in(1+x)G+Q_1Px\cos^2\theta\right\}^2}\right] \tag{32}$$

$$\frac{dR_1}{d\Upsilon_1}=\frac{(1+x)^3}{BPx}\left[1-\frac{T_{A_1}P^2x(1+x)\cos^2\theta}{\left\{\in(1+x)G+Q_1Px\cos^2\theta\right\}^2}\right] \tag{33}$$

$$\frac{dR_1}{dD_{A_1}}=\frac{(1+x)^3}{BP\in x}\left[1-\frac{T_{A_1}P^2x(1+x)\cos^2\theta}{\left\{\in(1+x)G+Q_1Px\cos^2\theta\right\}^2}\right] \tag{34}$$

$$\frac{dR_1}{dP}=\frac{(1+x)^2G}{Bx}\left[\frac{T_{A_1}x(1+x)G\cos^2\theta}{\left\{\in(1+x)G+Q_1Px\cos^2\theta\right\}^2}-\frac{1}{P^2}\right] \tag{35}$$

where, $G=\left\{1+\left(\Upsilon_1+\dfrac{D_{A_1}}{\in}\right)(1+x)\right\}.$

From the derivatives (28) - (35), it is clear that the Taylor's number and solute gradient rule out the possibility of the onset of convection, whereas suspended particles and medium porosity accelerate the onset of convection. The magnetic field, couple-stress and Darcy-Brinkman parameter have stabilizing (or destabilizing) effect and the medium permeability has a destabilizing (or stabilizing) on thermosolutal instability if $\left[\in(1+x)G+Q_1Px\cos^2\theta\right]^2>(or<)T_{A_1}P^2x(1+x)\cos^2\theta$ respectively. In the absence of rotation $(i.e. T_{A_1}=0)$, magnetic field, couple-stress and Darcy-Brinkman parameters always postpone the onset of convection, whereas medium permeability assures the destabilizing effect on the system. The variation in Rayleigh number R_1 for the stationary state against various values of parameters such as rotation, solute gradient, medium porosity, suspended particles, magnetic field, couple-stress, Brinkman number and medium permeability on double-diffusive convection saturating a Brinkman porous medium are depicted graphically in Figs. 1-8, respectively for $\theta = 45^\circ$.

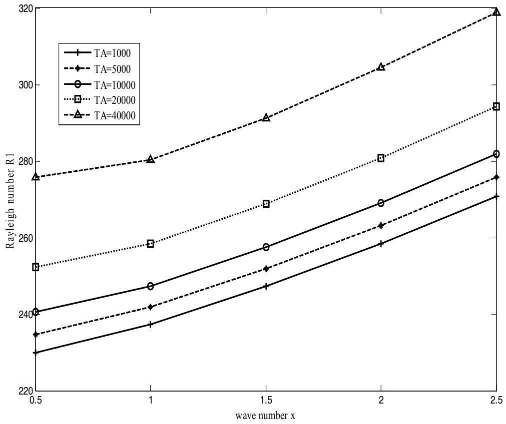

Fig. 1. Variations of R_1 with X for various values of rotation parameter $\mathbf{T_{A_1}}$ (1000,5000,10000,20000, 40000) and
$B=5, S_1=200, \in=2, P=3, D_{A_1}=20, \Upsilon_1=20,$
$Q_1=200, \theta=45^\circ.$

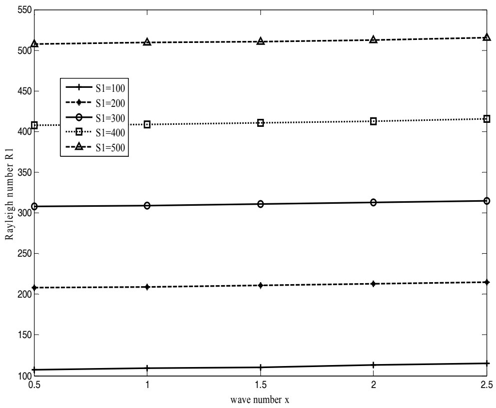

Fig. 2. Variations of R_1 with X for various values of solute gradient parameter $S_1=(100,200,300,400,500)$ and
$B=10, P=10, \in=5, D_{A_1}=25, \Upsilon_1=25, Q_1=200,$
$T_{A_1}=10{,}000, \theta=45^\circ.$

5. PRINCIPLE OF EXCHANGE OF STABILITIES

Here, the conditions have been derived, if any, under which principle of exchange of stabilities (PES) holds true and also the possibility of oscillatory modes.

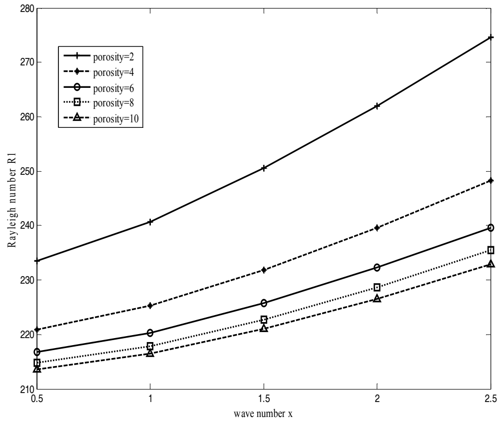

Fig. 3. Variations of R_1 with X for various values of medium porosity $\in=(2,4,6,8,10)$ and
$B=5, P=3, S_1=200, Q_1=200, D_{A_1}=20, \Upsilon_1=20,$
$T_{A_1}=4000, \theta=45^\circ.$

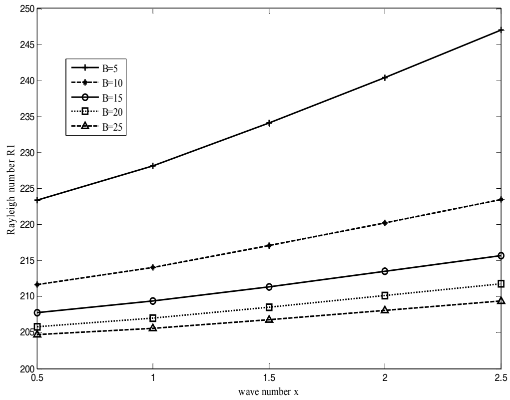

Fig. 4. Variations of R_1 with X for various values of suspended particles $B=(5,10,15,20,25)$ and $P=20, \in=5, D_{A_1}=25, \Upsilon_1=25, Q_1=500, T_{A_1}=20,$
$000, S_1=200, \theta=45^\circ.$

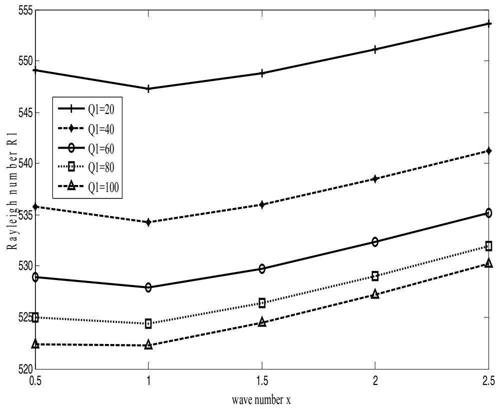

Fig. 5. Variations of R_1 with X for various values of magnetic field $Q_1=(20,40,60,80,100)$ and
$B=10, \in=2, P=20, D_{A_1}=20, \Upsilon_1=20, T_{A_1}=10,$
$000, S_1=500, \theta=45^\circ.$

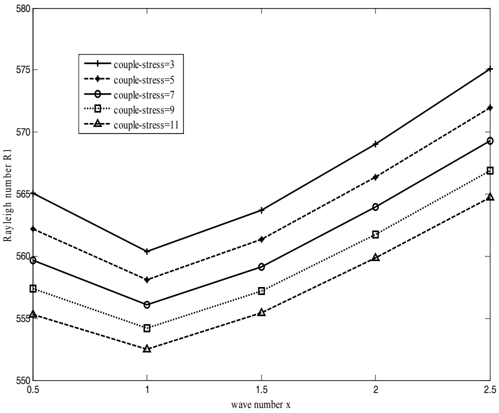

Fig. 6. Variations of R_1 with X for various values of couple-stress parameter $\Upsilon_1(3,5,7,9,11)$ and $B=5, \in=5, P=15, D_{A_1}=10, T_{A_1}=40{,}000, S_1=500, Q_1=100,\ \theta=45°.$

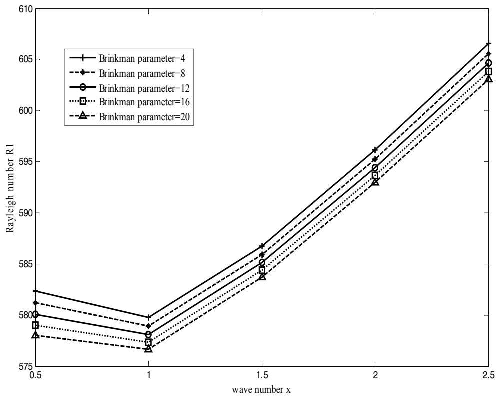

Fig. 7. Variations of R_1 with X for various values of Darcy-Brinkman parameter $D_{A_1}(4,8,12,16,20)$ and $B=3, \in=2, P=15, \Upsilon_1=5, T_{A_1}=20{,}000, S_1=500, Q_1=100,\ \theta=45°.$

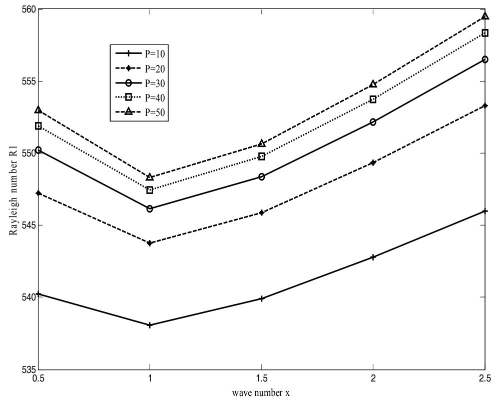

Fig. 8. Variations of R_1 with X for various values of permeability $P=(10,20,30,40,50)$ and $B=10, \in=2, D_{A_1}=10, \Upsilon_1=5, Q_1=40, T_{A_1}=10{,}000, S_1=500,\ \theta=45°.$

For this purpose, multiplying Eq. (18) by W^* and then integrated over the range of z. Using Eqs. (19)-(23) with the help of boundary conditions (24) gives

$$\left[\left\{\frac{\sigma}{\in}\left(1+\frac{f}{(1+\tau_1\sigma)}\right)+\frac{1}{P_l}\right\}\right]I_1 + \left\{\frac{(\in\Upsilon+D_A)}{P_l\in}\right\}I_2 - \frac{g\alpha_T a^2}{\beta_T p_1}\left(\frac{1+\tau_1\sigma^*}{B+\tau_1\sigma^*}\right)\left(I_3+\sigma^* p_1 E_1 I_4\right) + \frac{g\alpha_S a^2}{\beta_S q_1}\left(\frac{1+\tau_1\sigma^*}{B+\tau_1\sigma^*}\right)\left(I_5+\sigma^* q_1 E_2 I_6\right) - d^2\left[\left\{\frac{\sigma^*}{\in}\left(1+\frac{f}{(1+\tau_1\sigma)}\right)+\frac{1}{P_l}\right\}I_7+\left\{\frac{(\in\Upsilon+D_A)}{P_l\in}\right\}I_8\right] - \frac{\mu_e\in}{4\pi\rho_0 p_2}\left(p_2 I_9\sigma^*+I_{10}\right)+\frac{\mu_e\in d^2}{4\pi\rho_0 p_2}\left(p_2 I_{11}\sigma+I_{12}\right)=0 \tag{36}$$

Substituting, $\sigma=\sigma_r+i\sigma_i$ in Eq. (36) and equating real and imaginary parts leads to

$$\sigma_r\left\{\begin{array}{l}\dfrac{1}{\in}\left(1+\dfrac{f\,(1+\tau_1\sigma_r)}{\left\{(1+\tau_1\sigma_r)^2+(\tau_1\sigma_i)^2\right\}}\right)\left(I_1-d^2I_7\right)-\dfrac{g\alpha_T a^2}{\beta_T p_1}\\ \left(\dfrac{\left\{(1+\tau_1\sigma_r)(B+\tau_1\sigma_r)+\tau_1^2\sigma_i^2\right\}p_1E_1I_4}{\left\{(B+\tau_1\sigma_r)^2+\tau_1^2\sigma_i^2\right\}}\right)+\dfrac{g\alpha_S a^2}{\beta_S q_1}\\ \left(\dfrac{\left\{(1+\tau_1\sigma_r)(B+\tau_1\sigma_r)+\tau_1^2\sigma_i^2\right\}q_1E_2I_6}{\left\{(B+\tau_1\sigma_r)^2+\tau_1^2\sigma_i^2\right\}}\right)-\dfrac{\mu_e\in}{4\pi\rho_0}\\ \left(I_9-d^2I_{11}\right)\end{array}\right\}$$

$$\left\{\begin{array}{l}\dfrac{\left(I_1-d^2I_7\right)}{P_l}+\left(\dfrac{f\,\tau_1\sigma_i^2\left(I_1+d^2I_7\right)}{\in\left\{(1+\tau_1\sigma_r)^2+(\tau_1\sigma_i)^2\right\}}\right)\\ +\dfrac{(\in\Upsilon+D_A)(I_2+I_8)}{\in P_l}-\dfrac{g\alpha_T a^2}{\beta_T p_1}\\ +\left\{\begin{array}{l}\left\{(1+\tau_1\sigma_r)(B+\tau_1\sigma_r)+\tau_1^2\sigma_i^2\right\}I_3+\\ \dfrac{\tau_1\sigma_i^2(1-B)p_1E_1I_4}{\left\{(B+\tau_1\sigma_r)^2+\tau_1^2\sigma_i^2\right\}}\end{array}\right\}\\ +\left(\begin{array}{l}\left\{(1+\tau_1\sigma_r)(B+\tau_1\sigma_r)+\tau_1^2\sigma_i^2\right\}I_5+\\ \dfrac{\tau_1\sigma_i^2(1-B)q_1E_2I_6}{\left\{(B+\tau_1\sigma_r)^2+\tau_1^2\sigma_i^2\right\}}\end{array}\right)\\ +\dfrac{g\alpha_S a^2}{\beta_S q_1}+\dfrac{\mu_e\in d^2}{4\pi\rho_0 p_2}\left(I_{12}-I_{10}\right)\end{array}\right\}=0 \tag{37}$$

and

$$\sigma_i\left\{\begin{aligned}&\left[\frac{1}{\in}\left(1+\frac{f}{\left\{(1+\tau_1\sigma_r)^2+(\tau_1\sigma_i)^2\right\}}\right)I_1+\frac{g\alpha_T a^2}{\beta_T p_1}\left(\frac{\left\{(1+\tau_1\sigma_r)(B+\tau_1\sigma_r)+\tau_1^2\sigma_i^2\right\}p_1E_1I_4+\tau_1(B-1)(I_3+\sigma_r p_1E_1I_4)}{\left\{(B+\tau_1\sigma_r)^2+\tau_1^2\sigma_i^2\right\}}\right)\right.\\&\frac{g\alpha_S a^2}{\beta_S q_1}\left(\frac{\left\{(1+\tau_1\sigma_r)(B+\tau_1\sigma_r)+\tau_1^2\sigma_i^2\right\}q_1E_2I_6+\tau_1(B-1)(I_5+\sigma_r q_1E_2I_6)}{\left\{(B+\tau_1\sigma_r)^2+\tau_1^2\sigma_i^2\right\}}\right)+\\&\left.\frac{d^2}{\in}\left(1+\frac{f\,(1+2\tau_1\sigma_r)}{\left\{(1+\tau_1\sigma_r)^2+(\tau_1\sigma_i)^2\right\}}\right)I_7+\frac{\mu_e\in\left(I_9+d^2I_{11}\right)}{4\pi\rho_0}\right]\end{aligned}\right\}=0 \tag{38}$$

where, the positive defined integrals I_1-I_{12} are defined below as

$$I_1=\int_0^1\left(|DW|^2+a^2|W|^2\right)dz,$$

$$I_2=\int_0^1\left(\left|D^2W\right|^2+a^4|W|^2+2a^2|DW|^2\right)dz,$$

$$I_3=\int_0^1\left(|D\Theta|^2+a^2|\Theta|^2\right)dz,\ I_4=\int_0^1\left(|\Theta|^2\right)dz,$$

$$I_5=\int_0^1\left(|D\Phi|^2+a^2|\Phi|^2\right)dz,\ I_6=\int_0^1\left(|\Phi|^2\right)dz,$$

$$I_7=\int_0^1\left(|Z|^2\right)dz,\ I_8=\int_0^1\left(|DZ|^2+a^2|Z|^2\right)dz,$$

$$I_9=\int_0^1\left(|DK|^2+a^2|K|^2\right)dz,$$

$$I_{10}=\int_0^1\left(\left|D^2K\right|^2+a^4|K|^2+2a^2|DK|^2\right)dz,$$

$$I_{11}=\int_0^1\left(|X|^2\right)dz,\ I_{12}=\int_0^1\left(|DX|^2+a^2|X|^2\right)dz.$$

Equation (37) implies that either $\sigma_r>0$ or $\sigma_r<0$, meaning that the modes of the system may be unstable or stable, respectively. Thus, it is concluded that the modes may be oscillatory or non-oscillatory. Equation (38) implies that σ_i may be either zero or non-zero, which signifies that the modes may be non-oscillatory or oscillatory, respectively.

In the absence of solute concentration $(i.e.\ \alpha_S=0)$, Eq. (38) gives Eq. (39).

As the terms inside the bracket in Eq. (39) are positive. So, $\sigma_i=0$ which assures that the oscillatory modes are not allowed and also confirms the validity of the PES in the absence of solute concentration gradient with the condition that $B>1$.

Hence, the oscillatory modes are dominant due to the presence of solute concentration gradient α_S only.

$$\sigma_i\left\{\begin{aligned}&\left[\frac{1}{\in}\left(1+\frac{f}{\left\{(1+\tau_1\sigma_r)^2+(\tau_1\sigma_i)^2\right\}}\right)I_1+\frac{g\alpha_T a^2}{\beta_T p_1}\left(\frac{\left\{(1+\tau_1\sigma_r)(B+\tau_1\sigma_r)+\tau_1^2\sigma_i^2\right\}p_1E_1I_4+\tau_1(B-1)(I_3+\sigma_r p_1E_1I_4)}{\left\{(B+\tau_1\sigma_r)^2+\tau_1^2\sigma_i^2\right\}}\right)+\right.\\&\left.\frac{d^2}{\in}\left(1+\frac{f\,(1+2\tau_1\sigma_r)}{\left\{(1+\tau_1\sigma_r)^2+(\tau_1\sigma_i)^2\right\}}\right)I_7+\frac{\mu_e\in}{4\pi\rho_0}\left(I_9+d^2I_{11}\right)\right]\end{aligned}\right\}=0 \tag{39}$$

6. OVERSTABILITY CASE

Here, the possibility whether the observed instability may actually be overstability has been examined.

Rewriting Eq. (26) in the following form

$$\begin{aligned}&\left[\left(\frac{i\sigma_1}{\in}\left\{1+\frac{f}{(1+i\tau_2\sigma_1)}\right\}+G^\dagger\right)^2\{(1+x)+i\sigma_1p_1E_1\}\{(1+x)+i\sigma_1q_1E_2\}\{(1+x)+i\sigma_1p_2\}\right]\\&\in(1+x)+\left[\left(\frac{i\sigma_1}{\in}\left\{1+\frac{f}{(1+i\tau_2\sigma_1)}\right\}+G^\dagger\right)\{(1+x)+i\sigma_1p_2\}\{(1+x)+i\sigma_1q_1E_2\}\{(1+x)+i\sigma_1p_1E_1\}\right]Q_1x\\&(1+x)\cos^2\theta-\left[\left(\frac{i\sigma_1}{\in}\left\{1+\frac{f}{(1+i\tau_2\sigma_1)}\right\}+G^\dagger\right)\{(1+x)+i\sigma_1q_1E_2\}\{(1+x)+i\sigma_1p_2\}^2\right]\\&R_1x\in\left(\frac{B+i\tau_2\sigma_1}{1+i\tau_2\sigma_1}\right)-Q_1R_1x^2\cos^2\theta\left(\frac{B+i\tau_2\sigma_1}{1+i\tau_2\sigma_1}\right)\\&\left[\{(1+x)+i\sigma_1q_1E_2\}\{(1+x)+i\sigma_1p_2\}\right]+S_1x\in\\&\left[\left(\frac{i\sigma_1}{\in}\left\{1+\frac{f}{(1+i\tau_2\sigma_1)}\right\}+G^\dagger\right)\{(1+x)+i\sigma_1p_1E_1\}\{(1+x)+i\sigma_1p_2\}^2\right]\left(\frac{B+i\tau_2\sigma_1}{1+i\tau_2\sigma_1}\right)+\\&Q_1S_1x^2\cos^2\theta\left[\{(1+x)+i\sigma_1p_1E_1\}\{(1+x)+i\sigma_1p_2\}\right]\\&\left(\frac{B+i\tau_2\sigma_1}{1+i\tau_2\sigma_1}\right)+\left[\left(\frac{i\sigma_1}{\in}\left\{1+\frac{f}{(1+i\tau_2\sigma_1)}\right\}+G^\dagger\right)\{(1+x)+i\sigma_1p_2\}\{(1+x)+i\sigma_1p_1E_1\}\{(1+x)+i\sigma_1q_1E_2\}\right]\\&Q_1x(1+x)\cos^2\theta+\left(\frac{Q_1^2x^2(1+x)\cos^4\theta}{\in}\right)\\&\left[\{(1+x)+i\sigma_1p_1E_1\}\{(1+x)+i\sigma_1q_1E_2\}\right]+\left(\frac{T_{A_1}x\cos^2\theta}{\in}\right)\\&\left[\{(1+x)+i\sigma_1p_1E_1\}\{(1+x)+i\sigma_1q_1E_2\}\{(1+x)+i\sigma_1p_2\}^2\right]=0\end{aligned} \tag{40}$$

Equating the real and imaginary parts of Eq. (40) gives

$$\left[\left(G^{\dagger}\in+\sigma_1^2L_4\right)^2-\sigma_1^2\left\{\tau_2^2\sigma_1^2+(1+f)\right\}^2\right]\left[\left\{(1+x)^3-\sigma_1^2L_1\right\}\right]\in$$
$$(1+x)-2\sigma_1\left[\left\{(1+f)\in G^{\dagger}+\sigma_1^2L_5+\sigma_1^4L_6\right\}\right]\left[\sigma_1L_2-\sigma_1^3L_3\right]\in$$
$$(1+x)+\left[G^{\dagger}\in+\sigma_1^2L_7+\sigma_1^4L_6\right]\left[\left\{(1+x)^3-\sigma_1^2L_1\right\}\right]Q_1x(1+x)\in$$
$$\cos^2\theta-Q_1x(1+x)\in\cos^2\theta\left[\sigma_1(1+f)+\sigma_1^3\tau_2^2(2+f)+\sigma_1^5\tau_2^4\right]$$
$$\left[\sigma_1L_2-\sigma_1^3L_3\right]-R_1x\in^2\left[G^{\dagger}\in+\sigma_1^2L_4\right]\left[B(1+x)^3-\sigma_1^2L_8-\sigma_1^4L_9\right]$$
$$+R_1x\in^2\left[\sigma_1(1+f)+\sigma_1^3\tau_2^2\right]\left[\sigma_1L_{10}-\sigma_1^3L_{11}-\sigma_1^5\left(\tau_2^2p_2^2q_1E_2\right)\right]-$$
$$Q_1R_1x^2\in^2\cos^2\theta\left(1+\sigma_1^2\tau_2^2\right)\left[B(1+x)^2-\sigma_1^2L_{12}-\sigma_1^4\left(\tau_2^2p_2q_1E_2\right)\right]$$
$$+S_1x\in^2\left[G^{\dagger}\in+\sigma_1^2L_4\right]\left[B(1+x)^3-\sigma_1^2L_{13}-\sigma_1^4L_{14}\right]-S_1x\in^2$$
$$\left[\sigma_1(1+f)+\sigma_1^3\tau_2^2\right]\left[\sigma_1L_{15}-\sigma_1^3L_{16}-\sigma_1^5\left(\tau_2^2p_1p_2^2E_1\right)\right]+Q_1S_1x^2\in^2$$
$$\cos^2\theta\left(1+\sigma_1^2\tau_2^2\right)\left[B(1+x)^2-\sigma_1^2L_{17}-\sigma_1^4\left(\tau_2^2p_1p_2E_1\right)\right]+Q_1x(1+x)$$
$$\in\cos^2\theta\left[G^{\dagger}\in+\sigma_1^2L_{18}+\sigma_1^4L_{19}\right]\left[\left\{(1+x)^3-\sigma_1^2L_1\right\}\right]-Q_1x(1+x)\in$$
$$\cos^2\theta\left[\sigma_1(1+f)+\sigma_1^3\tau_2^2(2+f)+\sigma_1^5\tau_2^4\right]\left[\sigma_1L_2-\sigma_1^3L_3\right]+Q_1^2x^2$$
$$(1+x)\in\cos^4\theta\left[1+2\sigma_1^2\tau_2^2+\sigma_1^4\tau_2^4\right]\left[\left\{(1+x)^2-\sigma_1^2\left(p_1q_1E_1E_2\right)\right\}\right]+$$
$$T_Ax\in\cos^2\theta\left[1+2\sigma_1^2\tau_2^2+\sigma_1^4\tau_2^4\right]\left[\left\{(1+x)^4-\sigma_1^2L_{20}+\sigma_1^4L_{21}\right\}\right]=0$$

(41)

and

$$\left[\left(G^{\dagger}\in+\sigma_1^2L_4\right)^2-\sigma_1^2\left\{\tau_2^2\sigma_1^2+(1+f)\right\}^2\right]\left[\sigma_1L_2-\sigma_1^3L_3\right]\in(1+x)$$
$$+2\sigma_1\left[\left\{(1+f)\in G^{\dagger}+\sigma_1^2L_5+\sigma_1^4L_6\right\}\right]\left[\left\{(1+x)^3-\sigma_1^2L_1\right\}\right]\in(1+x)$$
$$+Q_1x(1+x)\in\cos^2\theta\left[G^{\dagger}\in+\sigma_1^2L_7+\sigma_1^4L_6\right]\left[\sigma_1L_2-\sigma_1^3L_3\right]+Q_1x$$
$$\left[\sigma_1(1+f)+\sigma_1^3\tau_2^2(2+f)+\sigma_1^5\tau_2^4\right]\left[\left\{(1+x)^3-\sigma_1^2L_1\right\}\right](1+x)\in$$
$$\cos^2\theta-R_1x\in^2\left[G^{\dagger}\in+\sigma_1^2L_4\right]\left[\sigma_1L_{10}-\sigma_1^3L_{11}-\sigma_1^5\left(\tau_2^2p_2^2q_1E_2\right)\right]$$
$$-R_1x\in^2\left[(1+f)+\sigma_1^3\tau_2^2\right]\left[B(1+x)^3-\sigma_1^2L_8-\sigma_1^4L_9\right]-Q_1R_1x^2\in^2$$
$$\cos^2\theta\left(1+\sigma_1^2\tau_2^2\right)\left[\sigma_1L_{22}+\sigma_1^3L_{23}-\sigma_1^5L_{24}\right]+S_1x\in^2\left[G^{\dagger}\in+\sigma_1^2L_4\right]$$
$$\left[\sigma_1L_{15}-\sigma_1^3L_{16}-\sigma_1^5\left(\tau_2^2p_1p_2^2E_1\right)\right]+S_1x\in^2\left[\sigma_1(1+f)+\sigma_1^3\tau_2^2\right]$$
$$\left[B(1+x)^3-\sigma_1^2L_{25}-\sigma_1^4L_{26}\right]+\left(1+\sigma_1^2\tau_2^2\right)\left[\sigma_1L_{27}+\sigma_1^3L_{28}\right]Q_1S_1x^2$$
$$\in^2\cos^2\theta+Q_1x(1+x)\left[G^{\dagger}\in+\sigma_1^2L_{18}+\sigma_1^4L_{19}\right]\left[\sigma_1L_2-\sigma_1^3L_3\right]\in$$
$$\cos^2\theta+Q_1x(1+x)\in\cos^2\theta\left[\sigma_1(1+f)+\sigma_1^3\tau_2^2(2+f)+\sigma_1^5\tau_2^4\right]$$
$$\left[\left\{(1+x)^3-\sigma_1^2L_1\right\}\right]+Q_1^2x^2(1+x)\in\cos^4\theta\left[1+2\sigma_1^2\tau_2^2+\sigma_1^4\tau_2^4\right]$$
$$\left(\sigma_1L_{29}\right)+T_Ax\in\cos^2\theta\left[1+2\sigma_1^2\tau_2^2+\sigma_1^4\tau_2^4\right]\left[\sigma_1L_{30}-\sigma_1^3L_{31}\right]=0$$

(42)

where, the symbols $G^{\dagger}$ and L_1-L_{31} are defined as

$$G^{\dagger}=\left(\frac{G}{P}\right)=\frac{1}{P}\left\{1+\left(\Upsilon_1+\frac{D_{A_1}}{\in}\right)(1+x)\right\},$$

$$L_1=\left\{(1+x)\left(p_2q_1E_2+p_1q_1E_1E_2+p_1p_2E_1\right)\right\},$$

$$L_2=\left\{(1+x)^2\left(p_2+q_1E_2+p_1E_1\right)\right\},L_3=\left(p_1p_2q_1E_1E_2\right),$$

$$L_4=\left(\tau_2^2\in G^{\dagger}+f\,\tau_2\right),L_5=\begin{bmatrix}\left(\tau_2^2\in G^{\dagger}+f\,\tau_2^2\right)\\ (1+f)+\left(\tau_2^2\in G^{\dagger}\right)\end{bmatrix},$$

$$L_6=\left(\tau_2^4\in G^{\dagger}+f\,\tau_2^4\right),L_7=\left(2\tau_2^2\in G^{\dagger}+f\,\tau_2^2\right),$$

$$L_8=\begin{bmatrix}p_2B(1+x)\{p_2+2q_1E_2\}-\left\{(1+x)^3\tau_2^2\right\}\\ -\tau_2(1+x)^2(B-1)\{2p_2+q_1E_2\}\end{bmatrix},$$

$$L_9=\left[\tau_2^2p_2(1+x)\{p_2+2q_1E_2\}+\left\{p_2^2q_1E_2\tau_2(B-1)\right\}\right],$$

$$L_{10}=\left[(1+x)^2\left\{\tau_2(1+x)(1-B)+B\left(2p_2+q_1E_2\right)\right\}\right],$$

$$L_{11}=\begin{bmatrix}(1+x)\left\{\tau_2p_2^2(1-B)+2\tau_2p_2q_1E_2(1-B)\right\}\\ +\left(p_2^2q_1E_2B\right)-\tau_2^2(1+x)^2\{2p_2+q_1E_2\}\end{bmatrix},$$

$$L_{12}=\begin{bmatrix}\left(p_2q_1E_2B\right)-\left\{\tau_2^2(1+x)^2\right\}-\\ \tau_2(B-1)(1+x)\{p_2+q_1E_2\}\end{bmatrix},$$

$$L_{13}=\begin{bmatrix}p_2B(1+x)\{p_2+2p_1E_1\}-\left\{(1+x)^3\tau_2^2\right\}\\ -\tau_2(1+x)^2(B-1)\{2p_2+p_1E_1\}\end{bmatrix},$$

$$L_{14}=\left[\tau_2^2p_2(1+x)\{p_2+2p_1E_1\}+\left\{\tau_2p_1p_2^2E_1(B-1)\right\}\right],$$

$$L_{15}=\left[(1+x)^2\left\{\tau_2(1+x)(1-B)+B\left(2p_2+p_1E_1\right)\right\}\right]$$

$$L_{16}=\begin{bmatrix}\tau_2p_2(1+x)(1-B)\{p_2+2p_1E_1\}+\\ \left(p_1p_2^2E_1B\right)-\tau_2^2(1+x)^2\{2p_2+p_1E_1\}\end{bmatrix},$$

$$L_{17}=\begin{bmatrix}\left(p_1p_2E_1B\right)-\left\{\tau_2^2(1+x)^2\right\}-\\ \tau_2(B-1)(1+x)\{p_2+p_1E_1\}\end{bmatrix},$$

$$L_{18}=\left(2\tau_2^2\in G^{\dagger}+f\,\tau_2\right),L_{19}=\left(\tau_2^4\in G^{\dagger}+f\,\tau_2^3\right),$$

$$L_{20}=\left[(1+x)^2\left(p_2^2+p_1q_1E_1E_2+2p_1p_2E_1+2p_2q_1E_2\right)\right],$$

$$L_{21}=\left(p_1p_2^2q_1E_1E_2\right),L_{22}=\left[B(1+x)\begin{Bmatrix}q_1E_2+p_2+\tau_2\\ (1+x)(1-B)\end{Bmatrix}\right],$$

$$L_{23}=\begin{bmatrix}\tau_2^2(1+x)\{p_2+q_1E_2\}+\tau_2(1-B)\\ \left\{(1+x)^2\tau_2^2-p_2q_1E_2B\right\}\end{bmatrix},$$

$$L_{24}=\left\{\tau_2^3(1-B)p_2q_1E_2\right\},$$

$$L_{25}=\begin{bmatrix}p_2B(1+x)\{p_2+2p_1E_1\}-\left\{(1+x)^3\tau_2^2\right\}\\ -\tau_2(1+x)^2(B-1)\{2p_2+p_1E_1\}\end{bmatrix},$$

$$L_{26}=\left[\tau_2^2p_2(1+x)\{p_2+2p_1E_1\}+\left\{p_2^2p_1E_1\tau_2(B-1)\right\}\right],$$

$$L_{27}=\left[B(1+x)\left\{p_1E_1+p_2+\tau_2(1+x)(1-B)\right\}\right],$$

$$L_{28}=\left[\tau_2^2(1+x)\{p_2+p_1E_1\}-\left\{p_1p_2E_1\tau_2(1-B)\right\}\right],$$

$$L_{29}=\left[(1+x)\left(q_1E_2+p_1E_1\right)\right],L_{30}=\left[(1+x)^3\begin{Bmatrix}2p_2+q_1\\ E_2+p_1E_1\end{Bmatrix}\right],$$

$$L_{31}=\left[(1+x)\left\{2p_1p_2q_1E_1E_2+p_1p_2^2E_1+q_1p_2^2E_2\right\}\right].$$

(43)

Eliminating R_1 from Eqs. (41) and (42) and assuming $\sigma_1{}^2 = z$, an eight degree polynomial in z is obtained as

$$\alpha_0 z^8 + \alpha_1 z^7 + \alpha_2 z^6 + \alpha_3 z^5 + \alpha_4 z^4 + \alpha_5 z^3 + \alpha_6 z^2 + \alpha_7 z + \alpha_8 = 0 \tag{44}$$

where,

$$\alpha_0 = \left\{ \tau_2{}^8 p_1 p_2{}^3 q_1{}^2 E_1 E_2{}^2 \in^3 x\left(1+x\right) \right\} \tag{45}$$

and

$$\begin{aligned}
\alpha_8 = & \left[\left\{ G^{\dagger} \in \left(1+x\right) + Q_1 B x \cos^2\theta \right\} \tau_2 \in^2 x\left(1+x\right)^2 \left(B-1\right) \right] \\
& \left[\left\{ \begin{array}{l} G^{\dagger 2} \in^2 \left(1+x\right) + Q_1 \in x\cos^2\theta\left(1+x\right)\left(1+G^{\dagger}\right) \\ + Q_1{}^2 x^2 \cos^4\theta + T_A x\left(1+x\right)\cos^2\theta \end{array} \right\} \in \left(1+x\right)^3 \right] \\
& + \left[\begin{array}{l}
G^{\dagger 3} \in^6 x\left(1+x\right)^6 B\left(p_1E_1 - p_2\right) + G^{\dagger 2} \in^5 x\left(1+x\right)^7 B\left(1+f\right) \\
+ 2G^{\dagger} \in^5 x^2\left(1+x\right)^6 Q_1 B\cos^2\theta\left(G^{\dagger}p_1E_1 - p_2\right) + G^{\dagger} \in^5 x^2 \\
\left(1+x\right)^6 Q_1 q_1 E_2 B\cos^2\theta\left(G^{\dagger} - 1\right) + Q_1 \in^4 x^2\left(1+x\right)^7 B\left(1+f\right) \\
\cos^2\theta\left(G^{\dagger} - 1\right) + G^{\dagger 2} S_1 \in^6 x^2\left(1+x\right)^5 B^2\left(p_1E_1 - q_1E_2\right) + 2 \\
G^{\dagger} Q_1 S_1 \in^5 x^3\left(1+x\right)^4 B^2\cos^2\theta\left(p_1E_1 - q_1E_2\right) + T_A Q_1 \in^3 x^3 \\
\left(1+x\right)^5 B\cos^4\theta\left(p_1E_1 + p_2\right) + G^{\dagger} \in^4 x^3\left(1+x\right)^5 Q_1{}^2 B\cos^4\theta \\
\left(2p_1E_1 - p_2\right) + Q_1{}^2 \in^4 x^3\left(1+x\right)^5\left(G^{\dagger}p_1E_1 - p_2\right) B\cos^4\theta + \\
Q_1{}^2 \in^4 x^3\left(1+x\right)^5 q_1E_2 B\cos^4\theta\left(G^{\dagger} - 1\right) + G^{\dagger} T_A \in^4 x^2\left(1+x\right)^6 \\
p_1E_1 B\cos^2\theta + G^{\dagger 2} Q_1 \in^5 x^2\left(1+x\right)^5 p_1E_1 B\cos^2\theta + Q_1{}^2 S_1 \in^4 \\
x^4\left(1+x\right)^3\left(p_1E_1 - q_1E_2\right) B^2\cos^4\theta + Q_1{}^2 \in^3 x^3\left(1+x\right)^6\left(1+f\right) \\
B\cos^4\theta + Q_1{}^3 \in^3 x^4\left(1+x\right)^4 B\cos^6\theta\left(p_1E_1 - p_2\right) + \in^3 x^2 \\
\left(1+x\right)^6\left(1+f\right) B\cos^2\theta\left\{ 2G^{\dagger} \in Q_1 - T_A\left(1+x\right) \right\}
\end{array} \right]
\end{aligned} \tag{46}$$

The coefficients $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6$ and α_7 are quite large and also of trivial importance in determining the overstability of the system. Since σ_1 should be real for overstability to occur, therefore, all the roots of Eq. (44) should be positive.

From Eq. (44), the product of roots $= \left(\dfrac{\alpha_8}{\alpha_0}\right)$ i.e. positive and this has to be negative for the non-happening of overstability. Since α_0 is always positive as obvious from Eq. (45) and α_8 will be negative if

$$\begin{aligned} & B < 1,\ G^{\dagger} < 1,\ p_1E_1 < p_2,\ G^{\dagger} p_1E_1 < p_2, \\ & p_1E_1 < q_1E_2,\ 2G^{\dagger} \in Q_1 < T_A\left(1+x\right). \end{aligned} \tag{47}$$

Thus, the aforementioned inequalities are the sufficient conditions for the non-occurrence of overstability, the violation of which does not assure the possibility of overstable modes.

7. CONCLUSION

The onset of double-diffusive convection in a couple-stress porous fluid layer with the presence of magnetic field, rotation and suspended particles was analyzed analytically using the linear stability theory. The Brinkman model is employed for the momentum equation. For the stationary state, it is found that the stable solute gradient and rotational parameter rule out the possibility of the onset of convection, whereas suspended particles and medium porosity accelerate the onset of convection. For a rotating medium, the magnetic field, medium permeability, couple-stress and Darcy-Brinkman parameter have dual effects, whereas for a non-rotating system, the parameters like magnetic field, couple-stress and Darcy-Brinkman have stabilizing effects and medium permeability has a destabilizing effect and these effects are also depicted both analytically and graphically. The principle of exchange of stabilities is found to hold true in the absence of solute gradient parameter for $B > 1$. Hence, the oscillatory modes are dominant due to the presence of stable solute gradient parameter $\left(\alpha_S\right)$ only. Also, the sufficient conditions for the non-happening of overstability are obtained.

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