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Development of an Acoustic Metamaterials for Aero Acoustic Noise Control

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ABSTRACT

This paper aims at proposing a novel type tunable acoustic metamaterials with complete band gap composed of piezoelectric rods (Lithium Niobate) with square array as inclusion embedded polyimide aerogel background. The plane wave expansion method and the principles of Bloch-Floquet method used to get a band frequency and study the pass band for noise control. The results of this paper provide the required guidance for designing tunable wave filters or wave guide which might be useful in high-precision mechanical systems operated in certain frequency ranges, and switches made of piezoelectric; they also propose a novel type of tunable mechanical meta composite, where is independent of wave direction and has an equal sensitivity in all directions in which reacts omnidirectional and improves the aero acoustic noise control (e.g. bladeless fans) as well as general performance of vibrating structures (e.g. wind turbine).

Keywords: Phononic crystal; Acoustic band gaps; Passive control; Piezoelectricity; Low frequency regime shielding.

NOMENCLATURE

A_G	amplitude vector of the partial waves	$U(\mathbf{r},t)$	generalized displacement or generalized stress vectors
A_c	area of the primitive unit cell	r	radius of cylindrical scattering material
c_{ijkl}	elastic coefficients tensor	σ_{ij}	cauchy stress tensor
D_i	electric displacement vector fields	ε_{ij}	linear part of the elastic strain tensor
E_i	electric filed	ρ	material density
e_{ijk}	piezoelectric coefficients	ω	frequency
f	filling fraction ratio	ϕ	electric potential
$F(\mathbf{G})$	structure function	$\alpha(\mathbf{r})$	fourier component of arbitrary material constants
$\mathbf{G}$	vectors of reciprocal lattice	$2\pi / \Lambda_{x_2}$	normalized reciprocal lattice vector along x_2
J_1	first kind Bessel function of first order		
k	"Bloch" wave vector		
u_i	displacement vector		

1. INTRODUCTION

Structures or materials that protect or isolate their payload from unwanted noise and vibrations are key and innovative elements in vibrating structures. In general, the control of a complex physical process like Blade-Vortex Interaction (BVI) requires a thorough understanding of the underlying physics. Unfortunately, due to the complexity of BVI, prior research on noise control has largely been through trial and error applications of various control strategies based on physical intuition (Collis, 2002).

Recently, a new type of fans that is called Bladeless fans, could multiply volume flow rate of its intake by sucking air from backing of the fan, as a result of its specific geometry (Jafari, 2016). Frequency spectrum over Sound Pressure Level (SPL) from bladeless fans (Jafari, 2015) shows that most of the acoustic energy is approximately concentrated in the frequency range up to 2000Hz that makes it quite annoying as the human ear is very sensitive to sound at the higher end of the frequency range. Since the produced noise is proportional to inlet and outlet flow rate, commercial development of this type of fan needs, efficient noise control.

More in detail, the idea is to improve the aero acoustic noise control and general performance of vibrating structures (e.g. wind turbine or bladeless fans) by applying a new design and control based on inherently passive technique and using a novel type of tunable acoustic metamaterials where is independent of wave direction and has an equal sensitivity in all directions in which reacts omnidirectional. The Omni shield property requires full band gap property, where the wave, independent of direction, at specific frequency range, cannot propagate through it.

The phononic crystals have analogue properties to photonic crystals (Joannopoulos, 2011), (Pennec, 2004) and (Soukoulis 2001). Hence, shortly after starting research on photonic band gaps, where the band gaps or stop bands observed for electron waves in semiconductors, the idea was extended to both electromagnetic waves in photonic crystals and elastic waves in phononic crystals (Thomas *et al.*, 2011).These types of meta materials, constituted by a periodic repetition of two different materials, can either show absolute band gaps in their transmission spectra. (Sigalas, 1993) and (Kushwaha, 1993), where the elastic waves do not propagate at some frequencies or dictate the modes, those are allowed to be propagated in the material. Moreover, these types of materials can decrease the velocity of the elastic waves and even represent negative refraction (Ding, 2007). This capability offers constructing new meta materials with special performance for vibration control (Bergamini, 2014) or acoustic shield with applications in designing elastic filters, wave guides, mirrors, and transducers. Similarly, several phenomena such as guiding (Torres, 1999); (Kafesaki 2000); (Khelif, 2003), bending (Miyashita, 2002), (Khelif, 2004), filtering (Khelif, 2002); (Khelif, 2003), demultiplexing (Pennec, 2004), and super lenses of acoustic waves (Pennec, 2005) have been predicted, so far. In addition, phononic crystals can tailor the allowed modes and their wave speeds inside the material, in such a way that the frequencies of various material loss subjected to different regimes to be matched with the density of some states and frequencies of some modes provide enhanced energy absorption (Thomas *et al*., 2011). Since mechanical waves propagate in a solid in the form of both longitudinal and transverse waves, a designated structure with complete phononic filters might have band gaps for both waves in the same frequency region (Gorishnyy, 2005).

The geometry and composition characteristics of acoustic metamaterials have essential roles in showing forbidden gaps of wave propagation in these filters, regardless of wave polarization and propagation directions. However, the larger the bandwidth for this forbidden zones is, the more the applications for phononic filters are. Actually, the bandwidth for this forbidden zone is a key factor for the targeted purposes, especially for noise control at specific frequencies. Some researchers have tried to enlarge the width of band gaps, e.g. see (Phani, 2006) they showed that the width of band gaps may be determined by the contrast of elastic constants, the inclusion (filling) volume fraction, and the lattice of the constructed parts. Changing either the geometry or the elastic characteristics of the constitutive materials through external stimuli such as wave propagation causes the band structure of phononic crystal to be adjusted to a specific range of frequencies, and thus, represents either a partial band gap or a full band gap.

The design of geometry of phononic crystals with its inclusions, and matrix with tunable band gaps is an interesting but challenging issue. Thus, it is needed to design a phononic crystal that can be tuned in a desired band gap configuration where made waveguide with selective frequency.

To capture a tunable pass band and stop band in phononic crystals with higher efficiency, some functional materials were selected to make periodic structures such as thermally activated shape memory alloy, electro-rheological material, dielectric elastomeric layer, and either magneto-elastic or magneto-electro-elastic materials (Yeh, 2007); (Robillard, 2009); (Wu, 2009); (Ruzzene, 2000);(Wang, 2008).

Representing a full band gap for phononic crystals could lead to improve the design of transducers and vibration controller.

The high electromechanical coupling factor and low wave impedance at piezoelectric materials (Zou, 2008); (Zhao, 2012) stimulate the piezoelectric-based phononic crystal developments. Moreover, piezoelectric materials have some unique properties as compared with other tunable materials such as shape memory alloys and electro-rheological materials. These unique properties are the high accuracy in control of displacement, quick response, and strong reduction of the device size, made by piezoelectric materials (Park, 1997). But, since the piezoelectric substrate is not isotropic, they allow the bulk waves to travel at different speeds in different directions.

Since, there are three polarization planes for elastic waves in piezoelectric materials and as three different body waves namely longitudinal, transverse (shear) in the plane, and transverse out of the plane are propagated in each plane, it must prohibit propagating all types of waves in all directions in order to have a structure for a full band gap (Dowling, 1998).

Propagating elastic waves in a medium is usually described by a dispersion relationship between frequency and wave vector. There are some tools for band diagram prediction, which are very complicated in inhomogeneous materials. However, there are also various methods to obtain high-quality dispersion curves, including Plane Wave Expansion Method (PWEM) (Johnson 2001).

Although this method is developed for electromagnetic wave and powerful even for calculating the dispersion relationships for both acoustic and elastic waves, it fails in some cases where the material contrast between inclusions and matrix is high. In this context, an extremely large number of Fourier's components are needed to ensure enough accuracy. The principles of Bloch-Floquet method (Collet, 2011) is carried out to calculate the wave modes and group velocities even for complex propagation constant. Recent developments in piezoelectric-based smart materials (e.g. one dimensional phononic crystal) (Wang, 2014) or piezoelectric-based vibration control (Jalili, 2009) have promised to reach vibration control with an enough large band gap through suitable rod materials, but at low frequency regime, the passive vibration control or noise control is very difficult; therefore, the aero acoustic control with complete band gap in such a way that no wave is propagated, will have excellent performance, and stimulates developments in practical purposes and easy use.

While the traditional types of vibration isolator have poor performances or some meta materials exhibit partial band gap, in this article, we propose a novel type tunable phononic crystal in the form of acoustic metamaterials with complete band gap and inherently passive behavior, which consists of cylindrical piezoelectric rods with square array as inclusion embedded in polyimide aerogel (Airloy™ X114) background. The Plane Wave Expansion Method and the principles of Bloch-Floquet method are employed to get the band structure and study the pass band or forbidden band. The results summarize a suitable concrete foundation for the development of aero acoustic noise control especially at low frequency regime for bladeless fans developments.

This paper consists of six sections as follows:

Section 2 introduces the candidate piezoelectric buffer rods and polyimide aerogel, and associated acoustic wave velocities. Section 3 introduces utilized Plane Wave Expansion Method (PWEM) associated with "Bloch-Floquet" method for acoustic pressure filed. In Section 4, the forbidden band of this acoustic metamaterials documented and Section 5 offers the analysis of the results and finally, conclusions are presented in Section 6.

2. DETERMINING THE ACOUSTIC WAVE VELOCITY AT CANDIDATE PIEZOELECTRIC

"Lithium Niobate" with its inherent high purity makes it as a selected candidate for acoustic wave applications. Due to its chemical insensitivity, the etching techniques are suitable fabrication technique to achieve the periodic structures with reasonable size and filling fraction. Furthermore, these techniques are compatible with acoustic wave guides where fabricating a two dimensional phononic crystal in which the acoustic waves are confined in three dimensions will be possible. The material properties of this buffer rod are described here.

Piezoelectricity is described mathematically within a material's constitutive equation, which defines how the piezoelectric material's stress (T), strain (S), charge-density displacement (D), and electric field (E) interact.

The piezoelectric constitutive law (in Strain-Charge form) is:

$$S = s_E\, T + d^t . \mathrm{E} \qquad (1)$$

$$D = d\, T + \varepsilon_T . E \qquad (2)$$

The matrix d contains the piezoelectric coefficients for the material, and it appears twice in the constitutive equation (the superscript t stands for matrix-transpose), where

$$s_E = \begin{pmatrix} 5.78 & -1.01 & -1.47 & -1.02 & 0 & 0 \\ -1.01 & 5.78 & -1.47 & 1.02 & 0 & 0 \\ -1.47 & -1.47 & 5.02 & 0 & 0 & 0 \\ -1.02 & 1.02 & 0 & 17 & 0 & 0 \\ 0 & 0 & 0 & 0 & 17 & -2.04 \\ 0 & 0 & 0 & 0 & -2.04 & 13.6 \end{pmatrix} *10^{-12} \frac{m^2}{N}$$

$$d = \begin{pmatrix} 0 & 0 & 0 & 0 & 68 & -42 \\ -21 & 21 & 0 & 68 & 0 & 0 \\ -1 & -1 & 6 & 0 & 0 & 0 \end{pmatrix} *10^{-12} \frac{C}{N}$$

And Relative Permittivity is:

$$\frac{\varepsilon_T}{\varepsilon_0} = \begin{pmatrix} 84 & 0 & 0 \\ 0 & 84 & 0 \\ 0 & 0 & 30 \end{pmatrix}$$

$$\varepsilon_0 = 8.854 *10^{-12} \frac{F}{m}$$

The crystal cut determines the properties of the crystal and affected mode of vibration, frequency stability, acoustic wave velocity, aging and other parameters. The cuts are labeled based on the plane to, that they are perpendicular. Dominant modes of acoustic waves in piezoelectric rods relative to the velocities that rods can support, with coupling and without coupling effects, based on Modified Christoffel’s equation, (Newnham, 2005), and for buffers of different orientation (i.e. crystal cut) of them were calculated and plotted.

To cover the full band gap, all acoustic wave velocity that selected piezoelectric can support, was considered to lead toward the full band gap. In order to increase the efficiency and performance of metamaterials, especially in specific frequency regime and reduced the phononic crystal size, the polyimide aerogel, regarded to Bragg band gap definition, was selected. Lower sound velocity in polyimide aerogel makes the materials particularly adapted to low frequency sound control where space is a concern. The acoustic wave velocity at polyimide aerogel is assumed 90 meter per second. Airloy X103 properties scale with density approximately as Table1:

Table 1 Airloy 103 properties scale with density

Density Class	Density	Compressive Strengtha	Compressive Modulusa	Ultimate Yield Strengtha	Thermal Conductivitya
H	0.4 ± 0.02 g/cc	3 MPa	113 MPa	167 MPa	32 mW/m-K

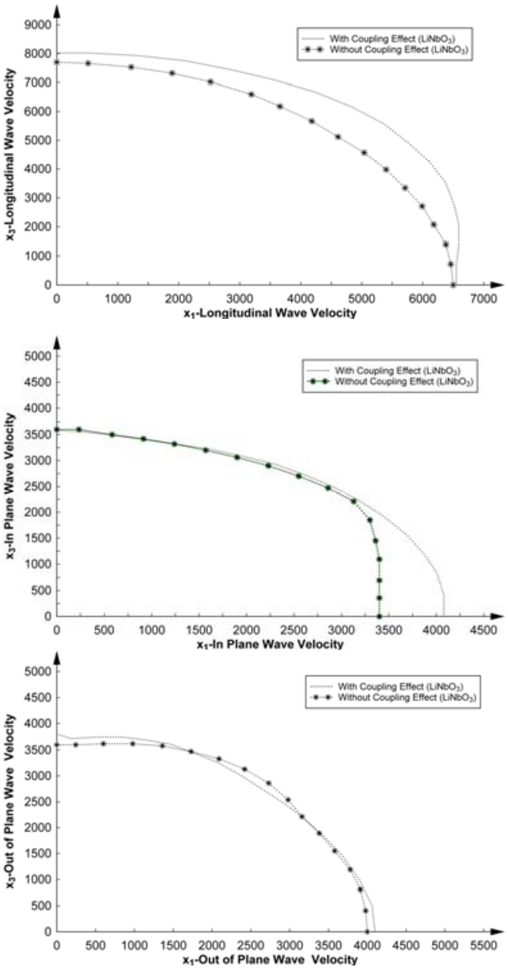

Fig. 1. (1-a), (1-b) and (1-c) represent the wave velocity of "Z -cut" wafers, denoted by longitudinal, in plane and out of the plane, for Lithium Niobate with and without coupling effect.

3. Utilizing The Plane Wave Expansion Method

Although the Plane Wave Expansion Method is powerful even for calculating the dispersion relationships for both acoustic and elastic waves, it fails in some cases where the materials densities contrast between inclusions and matrix is high. In this case, an extremely large number of Fourier's components are needed to ensure enough accuracy, so the principle of "Bloch-Floquet" method is carried out to calculate the wave modes and group velocities even for complex propagation constant, and since the acoustic pressure filed rather than displacement field, has Bloch form, this method can be applied.

3.1 Geometric Description and Definitions of Metamaterials

The model considered here is a periodic two-dimensional system of cylindrical rods that play as scattering materials (Fig. 2.), infinite long in the x_3-direction embedded in Air or Aerogel background. With this model, we are able to calculate the band structure for the propagation of plane waves traveling through this structure. The depth of the cylinders sets up a certain wavelength in such a way that any wave with wavelength smaller than the depth of cylinder sees the cylinders as infinitely long as far as the scattering process is concerned. This subject has been shown by (Meseguer, 1999). The Distance between cylindrical scattering materials is called as the lattice parameter and is denoted by Λ_{X_1} along x_1 and Λ_{X_2} long" x_2 ", while the radius of cylindrical scattering material is denoted by " r ". The lattice parameter along x_1 and x_2 is assumed equal where means $\Lambda_{X_1} = \Lambda_{X_1} = \Lambda_0$ .Filing fraction, f, is defined as the ratio of the scattered area to the area of unit cell, and it is expressed as $f = \pi(r / \Lambda_0)^2$ for circular scatters at square array (Fig. 3).

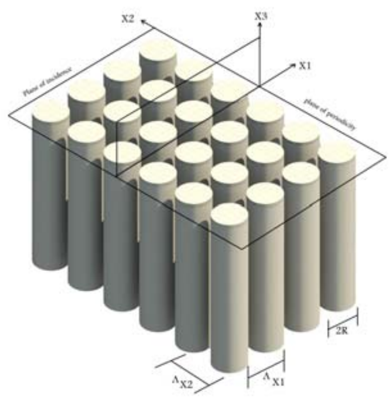

Fig. 2. A square-lattice, two-dimensional phononic crystal, consists of cylindrical piezoelectric rod embedded in Air or Aerogel background. Adopted from © 2012 Antos R, Veis M. originally published in (Antos, 2012) under CC BY 3.0 license. Available in: http://dx.doi.org/10.5772/34679.

3.2 Bloch Form of PWEM

With a Cartesian coordinate system (x_1, x_2, x_3) as reference, the constitutive equations for a piezoelectricity with the equation of motion and Poisson's condition at dielectric media are given by (Wilm, 2002)

$$\sigma_{ij} = c_{\mathrm{ijkl}}\varepsilon_{kl} + e_{kij}\frac{\partial\phi}{\partial x_k} \tag{3}$$

$$D_k = e_{kij}\varepsilon_{ij} - \varepsilon_{ki}\frac{\partial\phi}{\partial x_i} \tag{4}$$

$$\rho\frac{\partial^2 u_j}{\partial t^2} = \sigma_{ij,i} \tag{5}$$

$$D_{i,i} = 0 \tag{6}$$

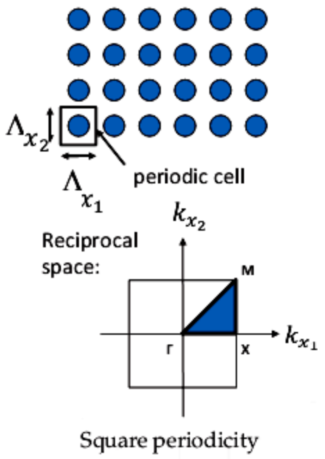

Fig. 3. 2-D periodic arrangements, and the associated first Brillouin zones. the corresponding first irreducible Brillouin zone where, ΓX and ΓM are related to the (1, 0) and the (1, 1) direction, respectively, and XM is the variation wave vector from (1, 0) to (1, 1) on the side of it. Adopted from © 2012 Antos R, Veis M. originally published in (Antos, 2012) under CC BY 3.0 license. Available in: http://dx.doi.org/10.5772/34679.

where the summation convention has been employed and $\sigma_{ij}(x_1,x_2,x_3,t)$ is the Cauchy stress tensor, $\varepsilon_{ij}(x_1,x_2,x_3,t)$ the linear part of the elastic strain tensor, and $D_i(x_1,x_2,x_3,t)$ is the electric displacement vector field. In addition, c_{ijkl} is the elastic coefficient, and e_{ijk} is the piezoelectric coefficient. In this paper, all the material coefficients are considered depend on (x_1, x_2) and independent of x_3. The elastic strain tensor, ε_{ij}, is expressed in terms of displacement vector u_i as

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right). \tag{7}$$

By the quasi-static approximation, the electric filed E_i may be expressed as the gradient of an electric potential $\phi(x_1,x_2,x_3)$ as

$$E_i = -\frac{\partial\phi}{\partial x_i} = -\phi_{,i}. \tag{8}$$

The PWEM is a commonly used numerical technique to calculate the band structures for phononic crystals (Sigalas, 1992; Kushwaha, 1994; Chen, 2001-1, and Chen, 2001-2). The PWEM can be applied to a phononic crystal with any shape of scatter; however, only infinite arrays can be modeled (Cao, 2004). According to the Bloch-Floquet theorem, all fields in a periodic solid such as displacements and stresses can be expanded as infinite series (Wilm, 2002 and Laude, 2005).

$$h(\mathbf{r},t) = \sum_{\mathbf{G}} h_{\mathbf{G}}(\omega,\mathbf{k})\exp(j(\omega t - \mathbf{k}.\mathbf{r} - \mathbf{G}.\mathbf{r})), \tag{9}$$

where h stands for either the displacements, the stresses, the electric potential, the electric displacements or the acoustic pressure field, $j = \sqrt{-1}$, $\mathbf{r} = (x_1,x_2,x_3)^T$. The upper script "T" is transposition, and the vectors of reciprocal lattice are:

$$\mathrm{G}(\Lambda_{x_1},\Lambda_{x_1}) = (2\pi/\Lambda_{x_1}, 2\pi/\Lambda_{x_2}, 0)^T,$$

which means that the set of all wave vectors G that give plane waves $e^{j\mathbf{G}\mathbf{r}}$ with the periodicity of the lattice and the G-vectors correspond to the reciprocal lattice points. Here, $2\pi/\Lambda_{x_1}$ and $2\pi/\Lambda_{x_2}$, are the normalized reciprocal lattice vectors and are assumed to be periodic with a period of Λ_{x_j} along the x_j − axes (Antos, 2012).

In this expression, k is the "Bloch" wave vector and in a special case $a_1 = a_2 = a$ where a is lattice constant.

The main part of the PWEM is to expand the system functions such as density, speeds and wave functions by plane waves exist in the wave equation in the form of Fourier series in terms of x_1, x_2 and x_3 as:

$$\alpha(\mathbf{r}) = \sum_{\mathbf{G}} \alpha_{\mathbf{G}} e^{-j\mathbf{G}.\mathbf{r}}, \tag{10}$$

Where $\alpha(\mathbf{r})$ is the Fourier component of material constants including the material density, elasticity, piezoelectric, and dielectric tensors for periodic structure which depend on position or $\alpha = \{\rho, c_{ijkl}, e_{ijk}, \varepsilon_{ij}\}$

i.e.

$$\rho(\mathbf{r}) = \sum_{\mathbf{G}} e^{j\mathbf{G}.\mathbf{r}}\rho_{\mathbf{G}} \tag{11}$$

$$c_{ijkl}(\mathbf{r}) = \sum_{\mathbf{G}} e^{j\mathbf{G}.\mathbf{r}} c_{\mathbf{G}}^{ijkl} \quad (12)$$

The Fourier harmonics $\alpha_{\mathbf{G}}$ are calculated for various scatter materials and lattice geometries (Wu, 2009); (Wilm, 2002) and (Vasseur, 1994) and, the sum is taken only over the reciprocal lattice points. One may write $\alpha_{\mathbf{G}}$ as follows

$$\begin{aligned} \alpha_{\mathbf{G}} &= \alpha_{\mathbf{G}I} f + \alpha_{\mathbf{G}M}(1-f) \quad \text{when} \quad \mathbf{G}=0 \\ \alpha_{\mathbf{G}} &= (\alpha_{\mathbf{G}I} - \alpha_{\mathbf{G}M})\mathrm{F}(\mathbf{G}) \quad \text{when} \quad \mathbf{G}\neq 0 \end{aligned} \quad (13)$$

Where $\alpha_{\mathbf{G}I}$ and $\alpha_{\mathbf{G}M}$ stand for Fourier harmonics for the inclusion (cylindrical rod) and matrix (substrate). In addition, f is the area filling fraction that is defined as the cross sectional area of a cylinder relative to a unit-cell area. As mentioned earlier the subscripts I and M represent the inclusion and matrix, respectively. $\mathrm{F}(\mathbf{G})$ in Eq. (13) is called the structure function defined as follows

$$\mathrm{F}(\mathbf{G}) = A_c^{-1} \int_{A_c} d^2\mathbf{r} e^{-j\mathbf{G}.\mathbf{r}} \quad (14)$$

i.e.

$$\rho_{\mathbf{G}} = A_c^{-1} \int_{A_c} d^2\mathbf{r}\rho(\mathbf{r}) e^{-j\mathbf{G}.\mathbf{r}} \quad (15)$$

$$c_{\mathbf{G}}^{ijkl} = A_c^{-1} \int_{A_c} d^2\mathbf{r} c^{ijkl}(\mathbf{r}) e^{-j\mathbf{G}.\mathbf{r}} \quad (16)$$

Where, A_c is the cross section area of the filling structure or the area of the primitive unit cell of a two-dimensional phononic structure. The proposed system consists of piezoelectric rods with square array and circular cross-section embedded in Air or Aerogel substrate material, which results in the following structure function factors for circular scatter, (Kushwaha, 1994)

$$F_{\mathbf{G}} = 2f\,\frac{J_1(\mathbf{G}r_0)}{\mathbf{G}r_0}, \quad 0 \le f = \pi\frac{r_0^2}{a^2} \le \frac{\pi}{4}, \quad (17)$$

Where, J_1 is the first kind Bessel function of first order. In each case, the maximum value of the filling fraction f corresponds to the close packing of the rods in the matrix. The irreducible part of the Brillouin zone of a square lattice is shown in Fig. 3.

The square lattice configuration has a reciprocal lattice vector defined in the PWE method as

$$\mathbf{G} = \left(\frac{2\pi}{a}\right)(n_1 x_1 + n_2 x_2) \quad (18)$$

For the propagation of acoustic waves in the x_1-plane normal to the axis of cylinders (i.e. the x_3 axis), the wave polarized in the x_3- direction, transverse wave is decoupled from the other two modes i.e. the other transverse wave and longitudinal modes of the waves polarized in the x_1-plane where the former mode of the wave is called the ‘‘single’’ mode and the latter two modes coupled to each other called the ‘‘mixed’’ mode. (Tanaka, 2000)

Using the Bloch theorem and expanding the unknown fields $\mathbf{u}(\mathbf{r},t)$ in Fourier series with respect to the 2D reciprocal lattice vectors (RLVs), we have as follows:

$$\mathbf{u}(\mathbf{r},t) = \sum_{\mathbf{G}} e^{jkx_1 - j\omega t}\left(e^{j\mathbf{G}x_1}\mathbf{A}_{\mathbf{G}} e^{jk_3 x_3}\right), \quad (19)$$

Here, $\mathbf{k} = (k_1, k_2)$ is the Bloch wave vector, ω is the frequency, k_3 is the wave number of the partial waves along the x_3-axis, and $\mathbf{A}_{\mathbf{G}} = (\mathrm{A}_{\mathbf{G}}^1, \mathrm{A}_{\mathbf{G}}^2, \mathrm{A}_{\mathbf{G}}^3)$ is the amplitude vector of the partial waves. If the component of the wave vector k_3 equals zero, the above equation degenerates into the vector filed of a bulk elastic wave.

One can define either a generalized displacement field $\{u_{\mathbf{G}}^{x_1}, u_{\mathbf{G}}^{x_2}, u_{\mathbf{G}}^{x_3}, \varphi_{\mathbf{G}}\}^T$ in which $\varphi_{\mathbf{G}}$ represents the electrical potential or a generalized stress vectors $\{\sigma_G^{x_1}, \sigma_G^{x_2}, \sigma_G^{x_3}, \mathrm{D}_G\}^T$ where D_G represents the electrical displacement or acoustic pressure field. If one denote either of these vector by $\mathbf{U}$, then one may write

$$\begin{aligned} \mathbf{U} &= \{u_{\mathbf{G}}^{x_1}, u_{\mathbf{G}}^{x_2}, u_{\mathbf{G}}^{x_3}, \varphi_{\mathbf{G}}\}^T \\ \mathbf{U} &= \{\sigma_{\mathbf{G}}^{x_1}, \sigma_{\mathbf{G}}^{x_2}, \sigma_{\mathbf{G}}^{x_3}, \mathrm{D}_{\mathbf{G}}\}^T \end{aligned} \quad (20)$$

On utilizing the Bloch theorem and expanding this generalized displacement or generalized stress vector, $\mathbf{U}(\mathbf{r},t)$,in Fourier series, and expanding the system functions such as density and elastic stiffness tensor and substitute both of them into the Eq. (5), we have:

$$\omega^2\mathbf{RU} = \mathbf{QU} \quad (21)$$

Where, R and Q are two different $4N \times 4N$ matrices that are functions of K, G, G′, the material constants and the filling fraction (Laude, 2005). This equation defines a generalized eigenvalue problem which can be solved for ω^2 as a function of K to obtain the band structure of elastic waves. For detailed deducing process of secular equation, one can refer to the works by (Kushwaha, 1994; Miyashita, 2005; Laude, 2005).

Since the density contrast of piezoelectric rods and Air is large, the shear stress and transverse waves inside the piezoelectric rods will not have significant contribution to the scattering of acoustic wave in the Air background, so the PWE method is acceptable, and scalar wave equation is adequate to describe the system (Elford, 2011); and thus, the acoustic wave pressure field rather than displacement is carried out to get the corresponding band gap for this system. In addition, the acoustic wave pressure field has Bloch form.

Table 2 Acoustic wave velocities of selected piezoelectric materials with certain wafers that the piezoelectric can support

Acoustic Wave Speed $m.s^{-1}$	Lithium Niobate X – CUT	Lithium Niobate Y – CUT	Lithium Niobate Z – CUT
$C_{longitudianlWave}$	6920	6964	6550
$C_{InPlaneTransverseWave}$	3990	4120	4100
$C_{OutofPlaneTransverseWave}$	4669	4954	4980

The secular equation which gives the dispersion relation between the frequency and the wave vector for acoustic pressure field by applying the "Bloch theorem" is:

$$-\det[\Gamma_{(\tilde{G}).(\tilde{G}')}] = \det[\alpha((\tilde{G})-(\tilde{G}'))((\tilde{k})+(\tilde{G})).((\tilde{k})+(\tilde{G}')) - \beta((\tilde{G})-(\tilde{G}'))\omega^2]_{(\tilde{G}),(\tilde{G}')} = 0 \quad (22)$$

Where, $(\tilde{k})$ is the Bloch wave vector, $\omega(k)$ is the frequency and $(\tilde{G})$ is the reciprocal lattice vector. $\alpha(\tilde{G})$ and $\beta(\tilde{G})$ are determined from an inverse Fourier transform and related to material property in acoustic wave equation.

When scattering occurs coherently from equally spaced layers in a phononic crystal, the band gap opens up at the border of the first Brillouin zone, although the dispersion in the vicinity of the band gap will be modified, the center frequency of the band gap is approximately given by assuming linear dispersion based on the center frequency of a Bragg band gap ($f_c \approx \frac{v}{2a}$) where the v is the velocity at host and "a" is the lattice parameter (Elford, 2011), (Kushwaha, 1994).

Based on this approximation, the acoustic wave velocity at polyimide aerogel has direct effect on center frequency of a Bragg band gap.

4. NUMERICAL ANALYSIS

In this section, to show the effect of piezoelectric rods or inclusion on longitudinal, in plan shear wave and out of plan shear wave pass bands, numerical analysis of band gap structures are presented. The acoustic wave velocity of selected piezoelectric materials, at certain wafers (because the acoustic wave velocity has directional dependency), are presented. Table2 shows the acoustic wave velocity for different rods calculated numerically based on the triple Fig. 1."Z-cut" periodically poled lithium Niobate on polyimide aerogel substrate is selected as an acoustic metamaterials.

To study the effect of the lattice parameter on band gap width, the lattice parameter was variable, and the bandwidth variation was measured. The geometrical characteristics of this type of metamaterials include lattice parameter, and inclusion ratios were chosen in order to ensure the existence and bandwidth of complete band gap at low frequencies regime.

5. RESULTS ANALYSIS AND DISCUSSION

Figure 4 demonstrate the complete band gap using the "Reduced Wave Vector over Frequency" graph, for acoustic pressure field for Z-cut periodically poled lithium Niobate on polyimide aerogel substrate where lead to dispersion relation for an acoustic metamaterials. The lattice parameter and the cylinder radius equal to 22mm and 9 mm respectively (i.e. a = 22 mm, r = 9 mm) that means filling fraction is equal to 0.5 for square lattice, and the blue line, shows the location of a band gap. The acoustic wave velocity for piezoelectric rods was calculated based on modified Christoffel's equation provided in Table 2. The acoustic wave velocity at polyimide aerogel was assumed 90 meter per second and the density for polyimide aerogel was assumed 0.4 g/cm3.

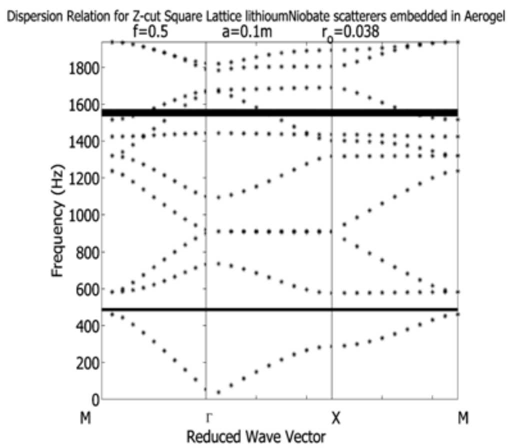

Fig. 4. Dispersion relation for an acoustic metamaterials consisting of " Z - cut" periodically poled lithium Niobate on polyimide aerogel substrate (f=0.5) for square lattice where the blue line shows the location of a band gap base on Brillouin zone.

Since, any propagated acoustic wave through the acoustic metamaterials have full band gap at 450Hz central frequency for all direction, Fig. 5. show that any type of acoustic wave at this range is shielded completely and cannot propagate through this acoustic metamaterials.

Studying the reliability, the width of band gap is investigated. Because, broad band devices that work over a large frequency domain, is a commercial interest, So, the comparison between Figs. 4 and 5 show that the width of band gap is changed by changing the lattice parameter (i.e. the filling fraction). Using the filling fractions 0.52 and 0.64 respectively, the shield range was changed from 425-475 Hz to 400-600 Hz that means the width of band gap was increased.

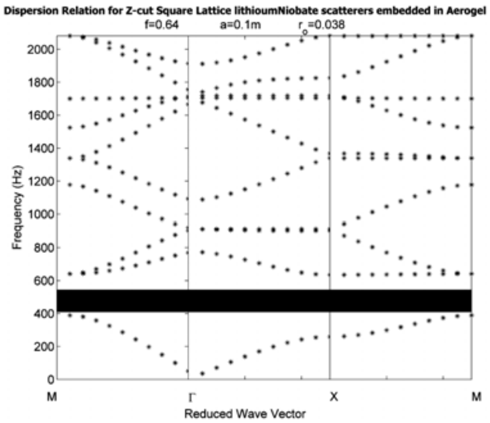

Fig. 5. Dispersion relation for square lattice of " Z -cut" periodically poled lithium Niobate on polyimide aerogel substrate with filling fraction (f=0.64).

6. CONCLUSIONS

To demonstrate the efficiency of the suggested metamaterials for aero acoustic shielding especially at low frequency regime by using the modified Christoffel's equation, the acoustic wave velocity that piezoelectric rod can support, associated with anisotropy effect was calculated. Using the utilized modified PWEM in conjunction with "Bloch-Floquet" theorem that acoustic wave equation with pressure instead of the displacement was applied; the complete band gap by the contrast between acoustic wave velocities at rods and inclusion was achieved.

The proposed low-frequency phononic crystal system showed the feasibility of using acoustic metamaterials to attenuate low-frequency noise, but at the expense of higher size. Despite of limitations in the range for increasing the size, this type of metamaterials is a suitable candidate for aero acoustic noise shielding or control especially at low frequency regime with reasonable size and complete band gap, because at low frequency regime, other technologies are too large for this type of applications.

The omnidirectional property of this type of metamaterials, where the shielding has no dependency on wave direction, is a key property for excellent performance over traditional acoustic isolator or metamaterials with partial band gap.

The dependency of band gap to lattice parameter and filling fraction means that these metamaterials are tunable, and the tune-ability is controlled by lattice parameter and filling fraction variation, or these are waveguide and have the selectivity frequency property.

The tunable property of this type promises to be adjustable in real time through filling fraction parameter and lattice structure or in an optional logic port where in turn is a suitable candidate for adaptive noise control.

Furthermore, a new shield type at low frequencies regime or an acoustic transducer with reasonable size is promised specially for aerospace applications with a new type of background material which is called "Aerogel" with ultralow sound velocity property (lower than that in the air) where will increase the effectiveness and miniaturization with wider adaptability with its inherently passive behavior.

The elastic wave attenuation features of this acoustic metamaterials and its selectivity property could apply to elastic wave shields for unwanted noise that occurs at certain frequency ranges (e.g. wind turbine).

Although the vibration properties are determined by material selection and geometry, in near future a new type of intelligent metamaterials is promised that their vibration property not only depends on geometry and material properties but also, by an optional logic port, will have adaptive vibration property, i.e. reacting to vibration and altering their property proportional to a specific frequency waves spectrum, and promising a practice toward the programmable materials commercialization.

Finally, since the plane wave expansion method, associated with "Bloch-Flouqet" theorem, relies on Fourier transform, the limitation of Fourier transform is inherited; in other words, it does not include location information that consequently has difficulty in representing transients for periodic structures, especially with large acoustic mismatch. In this case, a wavelet-based method that has location and frequency is recommended.

Fourier transform does not include location information especially with large acoustic mismatch, so to overcome this difficulty the wavelet-base method can be implemented. With expanding the elastic constants and the wave fields in the wavelet bases and by variational theory the elastic wave equations are reduced to an eigenvalue problem that can be solved by the wavelet integral technique.

Wavelets are well localized in both time and space domain, unlike the basis function in PWE method, so they can be efficient at discontinuities description in phononic crystal and spatial oscillations of the wave fields or in general phononic crystal with defect, or phononic crystal with time-dependent elasticity (i.e. viscoelastic host).

The wavelet-based method with an appropriate post processing scheme may be useful to improve the total efficiency of the inverse design and optimization of the acoustic wave band gap metamaterials.

This method in this particular case can represent the fast convergence and time saving.

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