MHD Stagnation Flow of a Newtonian Fluid towards a Uniformly Heated and Moving Vertical Plate

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ABSTRACT

Stagnation flow of an electrically conducting incompressible viscous fluid towards a moving vertical plate in the presence of a constant magnetic field is investigated. By using the appropriate transformations for the velocity components and temperature, the partial differential equations governing flow and heat transfer are reduced to a set of nonlinear ordinary differential equations. These equations are solved approximately using a numerical technique for the following two problems: (i) two-dimensional stagnation-point flow on a moving vertical plate, (ii) axisymmetric stagnation-point flow on a moving vertical plate. The effects of non-dimensional parameters on the velocity components, wall shear stresses, temperature and heat transfer are examined carefully.

Keywords: Stagnation point; Newtonian fluid; Magnetohydrodynamics (MHD); Similarity transformation; Numerical solution.

NOMENCLATURE

a	physical constant	T_∞	fluid temperature at infinity
B	applied magnetic field vector	t_{xz}, t_{yz}	shear stress components
B_0	applied magnetic field strength	u, v, w	components of the velocity vector
c_p	specific heat at constant pressure	$u_\infty, v_\infty, w_\infty$	velocity components at infinity
F	electromagnetic body force per unit volume	V_0	translation velocity of plate
F_x, F_y, F_z	body force components	(x, y, z)	cartesian coordinates
F,H,M	dimensionless similarity functions	Γ	dimensionless heat generation parameter
f,h,m	similarity functions		
g	acceleration due to gravity	s	coefficient of thermal expansion
Ha	dimensionless magnetic parameter	y	dimensionless similarity variable
k	thermal conductivity	}	dimensionless convection parameter
p	pressure	~	coefficient of viscosity
Pr	Prandtl number	"	dimensionless temperature function
Q_0	heat generation constant	...	density
q_w	heat flux at the wall	†$_0$	electrical conductivity
T	fluid temperature	‡$_{wx}$, ‡$_{wy}$	dimensionless shear stress components
T_w	wall temperature		

1. INTRODUCTION

Flow and heat transfer phenomena over a moving flat surface have become an active area of academic research in recent years due to their importance in various branches of science, engineering, and technology. These analyses find applications in many manufacturing processes such as the boundary layer along material handling conveyers, the aerodynamic extrusion of plastic sheet, and the cooling of an infinite metallic plate in a cooling bath.

The classical two-dimensional stagnation point flow towards a rigid horizontal plane was first studied by Hiemenz (1911). Goldstein (1938) reported the corresponding temperature distribution. The axisymmetric case was investigated by Homann (1936). Sibulkin (1952) derived a semi-analytical relation for laminar heat transfer of impinging flow to a body of revolution. Both two-dimensional and axisymmetric flows were extended to three dimensions by Howarth (1951) and Davey (1961), respectively. Stagnation point flows on moving plates were considered by Rott (1956), Glauert (1956), Wang (1973), Libby (1974), and Weidman and Mahalingam (1997). Wang (2008) reviewed the existing steady similarity stagnation-point flow solutions and discussed a new area of research on the stagnation point flow with slip. The most general solution of the Navier-Stokes equations and energy equation for non-axisymmetric three-dimensional stagnation-point flow and heat transfer on a flat plate was presented by Abbasi and Rahimi (2009).

Convection in a boundary layer flow was first considered by Sparrow *et al.* (1959). They obtained similarity solutions for the combined forced and free convection flow and heat transfer about a non-isothermal body subjected to a non-uniform free stream velocity. Lloyd and Sparrow (1970) investigated mixed convection flow along an isothermal vertical surface with the method of local similarity. Wilks (1973) studied the same problem under the condition of uniform surface heat flux from the plate. Wang (1987) was the first to investigate stagnation point flow and heat transfer problem towards a vertical plate. He obtained similarity solutions for the axisymmetric stagnation point flow towards an isothermal vertical plate. Wang (1988) considered the same problem for the case of tilted two-dimensional stagnation point flow. Mixed convection in two-dimensional stagnation point flow was studied by Ramachandran *et al.* (1988). They obtained similarity solutions under the conditions of an arbitrary wall temperature and arbitrary surface heat flux. Lin and Chen (1988) solved the non-similar boundary layer equations numerically for the mixed convection flow problem on an isothermal vertical plate. The effects of uniform suction or injection on the two-dimensional stagnation point flow towards a stretching horizontal plate with heat generation were studied by Attia and Seddeek (2007). Recently, Wang and Ng (2013) have presented the solutions for the two-dimensional and axisymmetric stagnation flow towards a heated vertical plate with Navier's slip condition.

Another situation commonly observed in industrial applications is the hydromagnetic stagnation-point flow with thermal effects, for instance, cooling of electronic devices by fans, heat exchanger design, MHD accelerators, and many others. In this type of problems, there exists the motion of an electrically conducting fluid towards a moving plate in the presence of an applied electromagnetic field. The study of hydromagnetic interaction of an electrically conducting viscous fluid with an applied magnetic field in stagnation-point flow was initiated by Neuringer and Mcllroy (1958a). In their subsequent study (Neuringer and Mcllroy 1958b), they considered the heat transfer aspect of the same problem. Ariel (1994) reexamined the Hiemenz flow in hydromagnetics in general, and the numerical procedure given by Na (1979) in particular. The problem of steady forced convection flow of an electrically conducting and heat generating/absorbing fluid near a stagnation point was solved numerically by Chamkha (1998a). In a following paper Chamkha (1998b) obtained non-similar solution with the finite difference method for the problem of mhd mixed convection flow along a semi-infinite vertical plate embedded in a uniform porous medium with heat generation and magnetic dissipation. Hiemenz and Homann magnetic flows and heat transfer problems on a permeable surface were considered by Attia (2003a, b) in the presence of uniform suction or injection. Attia (2007) concerned with the axisymmetric stagnation point flow towards a stretching surface in the presence of uniform magnetic field with heat generation. Two-dimensional MHD stagnation point flow towards a stretching sheet with variable surface temperature was studied by Ishak *et al.* (2009). Javed *et al.* (2009) investigated the development of two-dimensional or axisymmetric stagnation flow of an incompressible viscous fluid over a moving plate with partial slip. Abbasbandy and Hayat (2009) developed the homotopy analysis solution for the problem considered by Chamkha (1998a). Rashidi and Keimanesh (2010) constructed the analytical approximate solutions of the MHD flow in a laminar liquid film from a horizontal stretching surface using the DTM-Pade technique. Reza and Gupta (2012) extended the problem studied by Wang (1985) to the case when an electrically conducting incompressible viscous fluid impinges orthogonally on the surface of another quiescent heavier incompressible viscous electrically conducting fluid. Rashidi and Erfani (2012) investigated the thermal-diffusion and diffusion-thermo effects on combined heat and mass transfer of a steady MHD convective and slip flow due to a rotating disk. They found the approximate analytic solutions by the DTM-Pade technique. Borelli *et al.* (2013) studied numerically the steady three-dimensional stagnation-point flow of an incompressible, homogenous, electrically conducting Newtonian fluid over a flat plate. The recent paper by Rashidi *et al.* (2014) has analysed the MHD and slip flow over a rotating infinite disk with variable properties of the fluid.

In this study, the hydromagnetic viscous flow and heat transfer in the vicinity of a stagnation-point on a moving vertical plate is investigated. Our motivation in this study is to generalize the stagnation point flow and heat transfer problem of a viscous fluid on an infinite vertical plate by letting this plate move laterally with constant velocity in the presence of a uniform magnetic field and heat generation. A study of the problem under discussion for a fixed vertical plate was conducted by Wang and Ng (2013) in the absence of a magnetic field and heat generation. The governing equations are
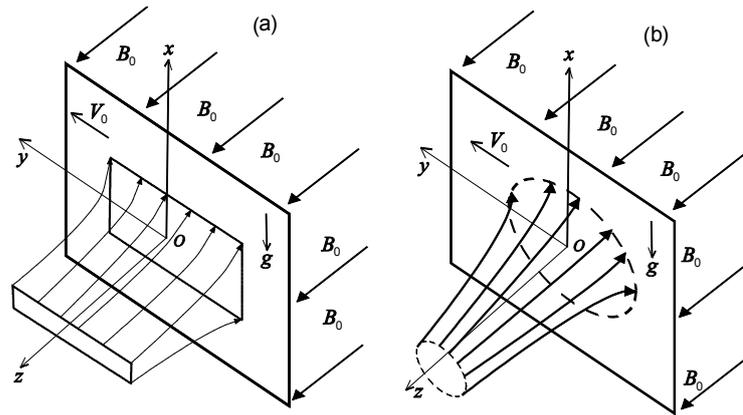
Fig. 1. Physical model and coordinate system for (a) two-dimensional stagnation-point flow on a moving vertical plate (b) axisymmetric stagnation-point flow on a moving vertical plate.

transformed into a system of nonlinear ordinary differential equations by means of similarity functions. The resulting equations, together with their corresponding boundary conditions, are solved numerically using the Matlab routine bvp4c. We compute the velocity components, temperature field, shear stresses and heat transfer on a moving vertical plate by assigning some specific values to the parameters entering into the problem. The effects of these parameters on the above fields are examined carefully. Particular cases of our results are compared with existing results of Wang and Ng (2013) and the agreement is found to be excellent.

2. TWO DIMENSIONAL STAGNATION-POINT FLOW TOWARDS A UNIFORMLY HEATED AND MOVING VERTICAL PLATE

The orthogonal two-dimensional stagnation-point flow in the x-z plane against an infinite vertical flat plate at $z = 0$ moving with constant velocity V_0 in the y- direction is illustrated in Fig. 1a. A Newtonian fluid flowing in the direction of the negative z- axis approaches a moving vertical plane at $z = 0$, and divides into streams proceeding away from the stagnation-point at the origin. An external uniform magnetic field B_0 is applied in the z-direction. The velocity components corresponding to the x-, y- and z- directions are denoted by u, v and w, respectively. Far from the plate, as z tends to infinity, the velocity distribution in the frictionless potential flow is given by

$$u_\infty = ax, \quad v_\infty = 0, \quad w_\infty = -az \tag{1}$$

where a is a physical constant, depending on the velocity in potential motion. Since the velocity field given in Eq. (1) does not satisfy the no-slip conditions at the plate, it is not an acceptable solution of the equations of viscous fluid flow. The problem is to obtain a solution that satisfies the no-slip boundary conditions and agrees with the outer solution far from the stagnation point. We shall seek a similarity solution compatible with the continuity

equation through the variable

$$u = m(z) + x f'(z)$$
$$v = h(z)$$
$$w = -f(z) \tag{2}$$
$$T = T_\infty + (T_w - T_\infty)_n(z)$$

where the prime denotes the differentiation with respect to z. It is important to note that the function $h(z)$ represents the velocity profile due to the translation of the plate at $z = 0$.

The three-dimensional Navier-Stokes equations with the Boussinesq term and the energy equation governing such type of flow are written as

$$\dots(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}) = -\frac{\partial p}{\partial x}$$
$$+\sim(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}) + g s (T - T_\infty) + F_x \tag{3}$$

$$\dots(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}) = -\frac{\partial p}{\partial y}$$
$$+\sim(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}) + F_y \tag{4}$$

$$\dots(u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}) = -\frac{\partial p}{\partial z}$$
$$+\sim(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}) + F_z \tag{5}$$

$$\dots c_p(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z})$$
$$= k(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}) + Q_0(T - T_\infty) \tag{6}$$

where $\dots$ is the density, p the pressure, $\sim$ the dynamic viscosity, T the temperature, c_p the specific heat at constant pressure, k the thermal conductivity, Q_0 the volumetric rate of heat generation, T_w the temperature of the plate, T_∞ the temperature far from the surface, g the gravitational acceleration, s the coefficient of

thermal expansion and (F_x, F_y, F_z) are the components of the source term **F** due to the imposed magnetic field. The electromagnetic body force per unit volume can be simplified as

$$\mathbf{F} = \dagger_0 (\mathbf{v} \times \mathbf{B}) \times \mathbf{B} \tag{7}$$

where $\dagger_0$ is the electrical conductivity and $\mathbf{B} = (0,0,B_0)$ is the constant uniform magnetic field applied to the fluid.

The boundary conditions for the velocity and temperature field are

$$z = 0: \quad u = 0, \quad v = V_0, \quad w = 0, \quad T = T_w$$
$$z \to \infty: \quad u \to u_\infty = ax, \quad v \to v_\infty = 0, \tag{8}$$
$$w \to w_\infty = -az, \quad T \to T_\infty.$$

Our investigation is restricted to the following assumptions:

(i) All the fluid properties are constant.

(ii) The flow is steady and laminar.

(iii) The plate is electrically non-conducting.

(iv) The magnetic Reynolds number is so small that the induced magnetic field produced by motion of fluid can be ignored in comparison to the applied one. In addition, the imposed and induced electrical fields are assumed to be negligible.

(v) The effects of viscous dissipation, Ohmic heating and Hall current are not included in the analysis, since they are generally small in the stagnation-point region. Also, the radiant heating is neglected.

Substituting Eq. (2) into Eqs.(3)-(6), and eliminating the pressure term from these equations, we arrive at the following equations:

$$\frac{\tilde{}}{...} f''' + f f'' - (f')^2 - \frac{\dagger_0 B_0^2}{...} f' + a^2 + \frac{\dagger_0 B_0^2}{...} a = 0 \tag{9}$$

$$\frac{\tilde{}}{...} m'' + f m' - m f' - \frac{\dagger_0 B_0^2}{...} m$$
$$+ g s (T_w - T_\infty)_{,,} = 0 \tag{10}$$

$$\frac{\tilde{}}{...} h'' + f h' - \frac{\dagger_0 B_0^2}{...} h = 0 \tag{11}$$

$$k_{,,}'' + ... c_p f_{,,}' + Q_{0,,} = 0 \tag{12}$$

The corresponding boundary conditions for the velocity and temperature field are re-written as

$$z = 0: \quad f(0) = 0, \quad f'(0) = 0, \quad h(0) = V_0,$$
$$m(0) = 0, \quad _{,,}(0) = 1$$
$$z \to \infty: \quad f'(\infty) \to a, \quad h(\infty) \to 0, \tag{13}$$
$$m(\infty) \to 0, \quad _{,,}(\infty) \to 0.$$

To write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are invoked

$$y = \sqrt{\frac{...a}{\tilde{}}} z, \quad F(y) = \sqrt{\frac{...}{\tilde{} a}} f(z),$$
$$H(y) = \frac{1}{V_0} h(z), \quad M(y) = \sqrt{\frac{...}{\tilde{} a}} m(z) \tag{14}$$

With help of the above quantities, the governing equations reduce to the following dimensionless form:

$$F''' + FF'' - (F')^2 + Ha(1 - F') + 1 = 0 \tag{15}$$

$$H'' + FH' - Ha\,H = 0 \tag{16}$$

$$M'' + FM' - MF' - Ha\,M + \}_{,,} = 0 \tag{17}$$

$$_{,,}'' + Pr F_{,,}' + r Pr_{,,} = 0 \tag{18}$$

Similarly, the transformed boundary conditions are given by

$$y = 0: \quad F(0) = 0, \quad F'(0) = 0, \quad H(0) = 1,$$
$$M(0) = 0, \quad _{,,}(0) = 1$$
$$y \to \infty: \quad F'(\infty) = 1, \quad H(\infty) = 0, \tag{19}$$
$$M(\infty) = 0, \quad _{,,}(\infty) \to 0.$$

where the prime denotes the differentiation with respect to y. Ha is the non-dimensional magnetic parameter, $\}$ is the non-dimensional convection parameter, Pr is the Prandtl number and r is the non-dimensional heat generation parameter and they are defined as

$$Ha = \frac{\dagger_0 B_0^2}{... a}, \quad \} = \frac{g s (T_w - T_\infty)\sqrt{...}}{a\sqrt{\tilde{} a}},$$
$$Pr = \frac{\tilde{} c_p}{k}, \quad r = \frac{Q_0}{... c_p a}. \tag{20}$$

The dimensionless expressions for the velocity components are given through the following equations:

$$\frac{u}{ax} = \frac{1}{x}\sqrt{\frac{\tilde{}}{...a}} M(y) + F'(y),$$
$$\frac{v}{V_0} = H(y), \quad \frac{w}{-\sqrt{\tilde{} a/...}} = F(y) \tag{21}$$

For the problem under consideration, it is important to find the shear stress on the plate in the x and y directions. From the constitutive equation of a Newtonian fluid, we obtain

$$\ddagger_{wx} = -\frac{1}{\tilde{} a} t_{xz}\Big|_{z=0} = -M'(0) - x\sqrt{\frac{...a}{\tilde{}}} F''(0),$$
$$\ddagger_{wy} = -\frac{1}{V_0\sqrt{\tilde{}...a}} t_{yz}\Big|_{z=0} = -H'(0) \tag{22}$$

where t_{xz} and t_{yz} are the shear stress components of a Newtonian fluid and they are defined as

$$t_{xz} = \tilde{}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right), \quad t_{yz} = \tilde{}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \tag{23}$$

The heat transfer rate per unit area on the plate can be written by Fourier's law as follows:

$$q_w = -k \frac{\partial T}{\partial z}\Big|_{z=0} = -k\sqrt{\frac{...a}{\sim}}(T_w - T_\infty)_{\prime\prime}{'}(0). \qquad (24)$$

3. AXISYMMETRIC STAGNATION-POINT FLOW TOWARDS A UNIFORMLY HEATED AND MOVING VERTICAL PLATE

Figure 1b shows an orthogonal axisymmetric stagnation point flow against an infinite vertical plate at $z = 0$ moving with constant velocity V_0 in the y-direction in the presence of a uniform magnetic field in the z- direction. Let the Cartesian velocity components at infinity be

$$u_\infty = ax, \quad v_\infty = ay, \quad w_\infty = -2az \qquad (25)$$

which represents a rotational inviscid flow.

We look for a solution of the form

$$\begin{aligned} u &= m(z) + xf'(z) \\ v &= h(z) + yf'(z) \\ w &= -2f(z) \\ T &= T_\infty + (T_w - T_\infty)_{\prime\prime}(z). \end{aligned} \qquad (26)$$

By using equations of motion, energy equation, non-dimensional quantities given in Eq. (14) and above similarity transformations, it can be shown that the governing equations for the problem under discussion are

$$F''' + 2FF'' - (F')^2 + Ha(1 - F') + 1 = 0 \qquad (27)$$

$$H'' + 2FH' - F'H - HaH = 0 \qquad (28)$$

$$M'' + 2FM' - MF' - HaM + \}_{\prime\prime} = 0 \qquad (29)$$

$$_{\prime\prime}'' + 2Pr F_{\prime\prime}' + \Gamma Pr_{\prime\prime} = 0 \qquad (30)$$

The boundary conditions for the above equations are as follows

$$\begin{aligned} &F(0) = 0, \quad F'(0) = 0, \quad F'(\infty) \to 1, \\ &H(0) = 1, \quad H(\infty) \to 0, \quad M(0) = 0, \\ &M(\infty) \to 0, \quad _{\prime\prime}(0) = 1, \quad _{\prime\prime}(\infty) \to 0 \end{aligned} \qquad (31)$$

The dimensionless expressions for the velocity components and the shear stresses on the plate are given through the following equations, respectively:

$$\begin{aligned} \frac{u}{ax} &= \frac{1}{x}\sqrt{\frac{\sim}{...a}}M(y) + F'(y), \\ \frac{v}{V_0} &= H(y) + \frac{ay}{V_0}F'(y), \\ \frac{w}{-2\sqrt{\sim a/...}} &= F(y) \end{aligned} \qquad (32)$$

$$\begin{aligned} \ddagger_{wx} &= -M'(0) - x\sqrt{\frac{...a}{\sim}}F''(0), \\ \ddagger_{wy} &= -H'(0) - \frac{ay}{V_0}F''(0) \end{aligned} \qquad (33)$$

4. RESULTS AND DISCUSSION

In the presence of a constant magnetic field, the flow and heat transfer problems involving two-dimensional and axisymmetric three-dimensional stagnation-point flows on a moving vertical plate are governed by the similarity equations and boundary conditions given in Eqs.(15)-(19) and Eqs. (27)-(31), respectively. The equations are nonlinear and have no analytical solutions. Therefore, they must be solved numerically. These numerical solutions were obtained using the Matlab solver boundary value problem (bvp4c) designed for the solution of two point boundary value problems. The code is based on a collocation formula. An error estimate for the global error of the approximate solution is also provided. Mesh selection and error control are based on the residual of continuous solution. We set the relative and absolute tolerance equal to 10^{-7}. We refer the reader to the book by Shampine *et al.* (2003) for details about how to solve boundary value problems with bvp4c.

The above-mentioned differential equations were integrated from $y = 0$ to $y = y_\infty$, where y_∞ is a sufficiently large number. In practice, setting y_∞ as low as 12 yields satisfactory accuracy for the problems under discussion. To validate the accuracy of our numerical solutions, we have compared our results for the values of $F''(0)$, $M'(0)$ and $_{\prime\prime}'(0)$ with those of Wang and Ng (2013) in Table 1 for the special case of $Ha = 0$, $\Gamma = 0$ and $\} = 1$. This table shows an excellent agreement with the existing results in Wang and Ng (2013). Hence, it is concluded that the Matlab solver bvp4c is very powerful and efficient in finding the numerical solutions of the boundary value problems discussed in this research.

Figures 2 to 4 show the velocity profiles for the two dimensional and axisymmetric cases, respectively. It is clear from Figures 2 and 3 that the main effect of the magnetic field on the flow is to increase the velocity components in the x- and z- directions, whereas it is to decrease the velocity component in the y- direction. Furthermore, for both stagnation-point flows, the tangential velocity component approaches the free stream values more quickly as Ha increases. In other words, the velocity boundary layer is thicker in the absence of a magnetic field. From Figures 2b and 3b, we observe that the velocity component in the x- direction increases with the convection parameter $\}$. This behavior is a consequence of the fact that the buoyancy increases with the increase in $\}$. To investigate the effect of Prandtl number on the velocity component in the x- direction, we have plotted the dimensionless velocity component u/ax against y in Figure 4. It is evident from the figure that an increase in the Prandtl number leads to a decrease in the upward velocity component. Note that the dependence of velocity components in the y- and z- directions on $\}$, Pr and Γ is insignificant, so they are not presented here.

Table 1 Comparison of the numerical results with those of Wang and Ng (2013)

		Wang and Ng (2013)		Present Work ($Ha = 0$, $r = 0$, $\} = 1$)	
		Pr=0.7	Pr=7	Pr=0.7	Pr=7
Axisymmetric case	$F''(0)$	1.31194	1.31194	1.311938	1.311938
	$M'(0)$	0.4955	0.2849	0.495440	0.284883
	$_\pi'(0)$	-0.6654	-1.5458	-0.665378	-1.545779
Two dimensional case	$F''(0)$	1.23259	1.23259	1.232588	1.232588
	$M'(0)$	0.6155	0.3619	0.615467	0.361859
	$_\pi'(0)$	-0.4959	-1.1784	-0.495866	-1.178375
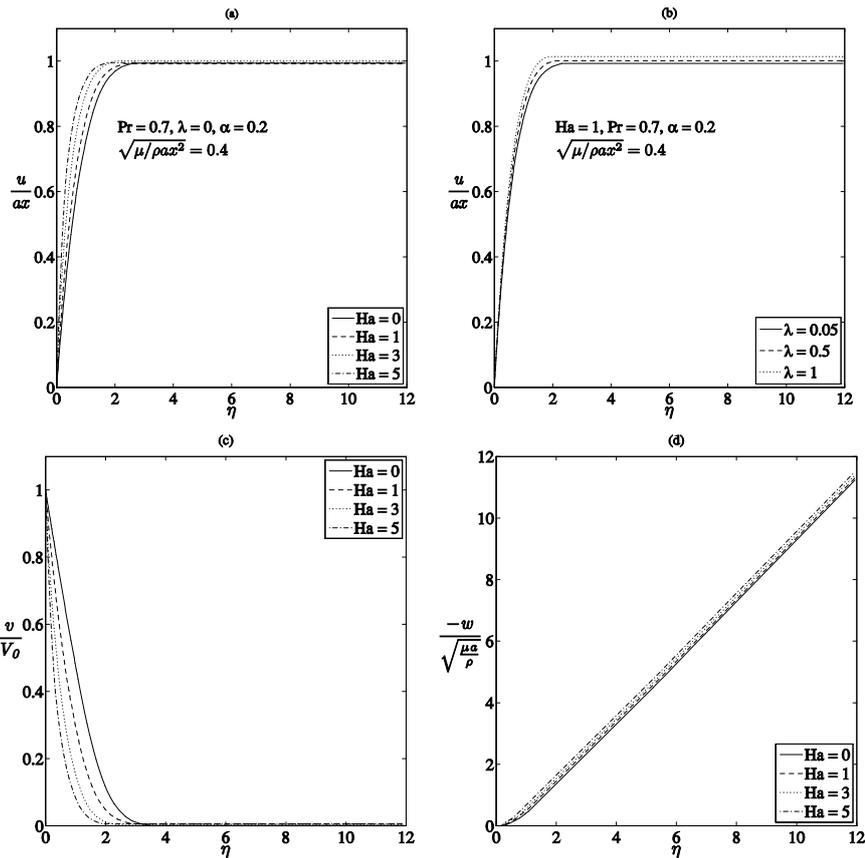
Fig. 2. Velocity profiles for the two dimensional case.

Figures 5 and 6 illustrate the temperature profiles for different values of the non-dimensional parameters. The numerical results show that the effect of increasing values of Prandtl number results in a decrease in the thermal boundary layer thickness. The reason is that smaller values of Pr are equivalent to increasing the thermal conductivities, and therefore heat is able to diffuse more rapidly. Hence, in the case of small Prandtl numbers the thermal boundary layer is thicker. It is apparent from Figures 5 and 6 that the presence of the heat generation parameter r leads to an increase in temperature. This result qualitatively agrees with the expectation, since the effect of internal heat generation is to increase the rate of energy transport to the fluid, thereby increasing the temperature of fluid. Again from Figures 5 and 6, we observe that the temperature decreases with the magnetic parameter Ha.

The values of tangential shear stresses on the plate are tabulated in Tables 2 to 4 for different values of the non-dimensional parameters. We conclude from these tables that the tangential shear stresses increase with the increase in the magnetic parameter Ha. This can be explained physically as follows. We observe from figures related to the velocity components in the x- and y- directions that when the magnetic parameter increases, the velocity gradients at the moving plate increases. This causes the tangential shear stresses to increase, and hence the force necessary to move the plate in the y- direction is greater. Similarly, this change in tangential shear stresses on the plate is more pronounced for the case of large heat generation and convection parameters. Note that the effects of $\}$, Pr and r on the tangential shear stress $\ddagger_{wy}$ on the plate is insignificant, so they are not presented here.
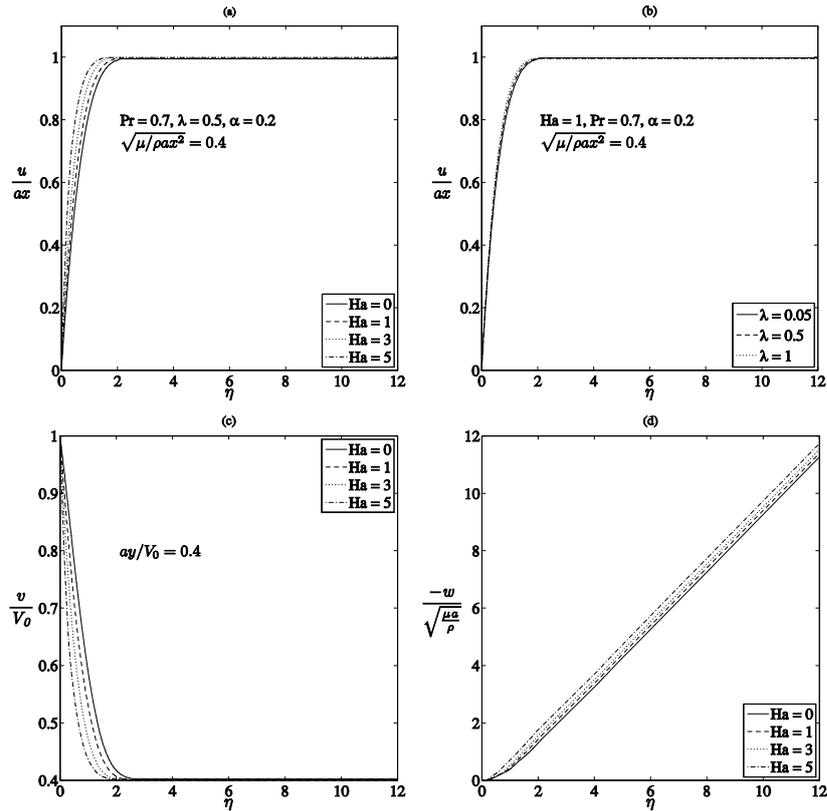
Fig. 3. Velocity profiles for the axisymmetric case.
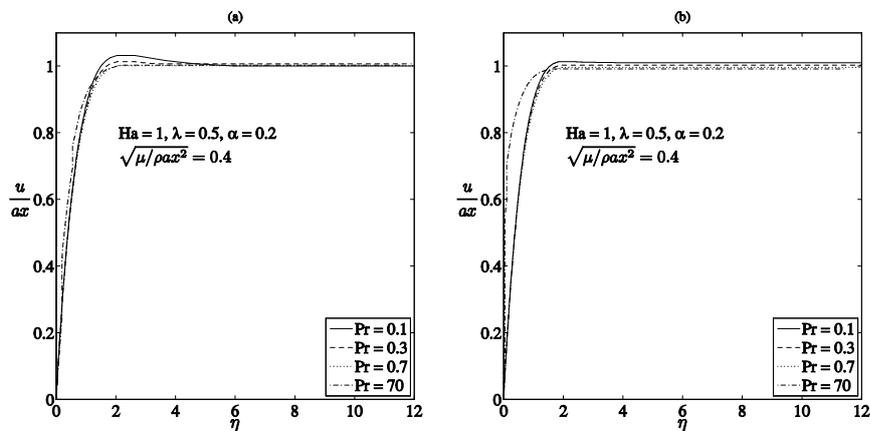
Fig. 4. Variation of the non-dimensional velocity component in the x-direction with Pr **(a) two dimensional case (b)axisymmetric case.**

The values of temperature gradient $-_n{}'(0)$ on the plate are listed in Tables 5 and 6 for a selection of values of the non-dimensional parameters. The magnitude of the wall temperature gradient increases as the magnetic parameter Ha increases. Also, in the case of small Prandtl numbers the rate of heat transfer is reduced as the thermal boundary layer is thicker. Again from these tables, we observe that the magnitude of the wall temperature gradient decreases with the heat generation parameter Γ. Finally, the wall temperature gradient $-_n{}'(0)$ changes its sign from positive to negative for some values of the parameters. This implies that

heat flows from the fluid to the plate. From a physical point of view, this follows from the fact that in the presence of significant internal heat generation, the local temperature in the neighborhood of $y = 0$ may be larger than T_w so that heat may flow from the fluid to the plate.

5. CONCLUSION

In this paper, the hydromagnetic stagnation-point flows on moving vertical plate were studied theoretically. By means of appropriate similarity transformations, the governing equations were

Table 2 Tangential shear stress τ_{wx} **on the plate for the two dimensional case** $\sqrt{\mu a x^2/\nu} = 0.4$

Ha	λ	α	τ_{wx}			
			Pr = 0.1	Pr = 0.7	Pr = 7	Pr = 70
		0	0.535254	0.523808	0.511128	0.502474
	0.05	0.2	0.536610	0.525657	0.513586	0.505910
		0.4	0.538232	0.528002	0.517403	0.517640
		0	0.915220	0.800768	0.673964	0.587428
0	0.5	0.2	0.928784	0.819256	0.698549	0.621781
		0.4	0.945000	0.842702	0.736714	0.739086
		0	1.337405	1.108502	0.854894	0.681822
	1	0.2	1.364534	1.145477	0.904064	0.750528
		0.4	1.396964	1.192368	0.980394	0.985136
		0	0.665884	0.658898	0.649614	0.642505
	0.05	0.4	0.666665	0.660121	0.651359	0.644925
		0.6	0.667588	0.661642	0.653917	0.651182
		0	0.951646	0.881794	0.788947	0.717855
1	0.5	0.2	0.959463	0.894020	0.806396	0.742059
		0.4	0.968690	0.909225	0.831980	0.804627
		0	1.269160	1.129456	0.943762	0.801578
	1	0.2	1.284793	1.153908	0.978660	0.849985
		0.4	1.303247	1.184317	1.029827	0.975121
		0	0.872410	0.868517	0.862178	0.856571
	0.05	0.2	0.872822	0.869261	0.863345	0.858203
		0.4	0.873302	0.870166	0.864963	0.861603
		0	1.080433	1.041508	0.978115	0.922042
3	0.5	0.2	1.084552	1.048941	0.989780	0.938368
		0.4	1.089352	1.057998	1.005965	0.972369
		0	1.311569	1.233719	1.106935	0.994787
	1	0.2	1.319808	1.248587	1.130263	1.027440
		0.4	1.329408	1.266699	1.162634	1.095443
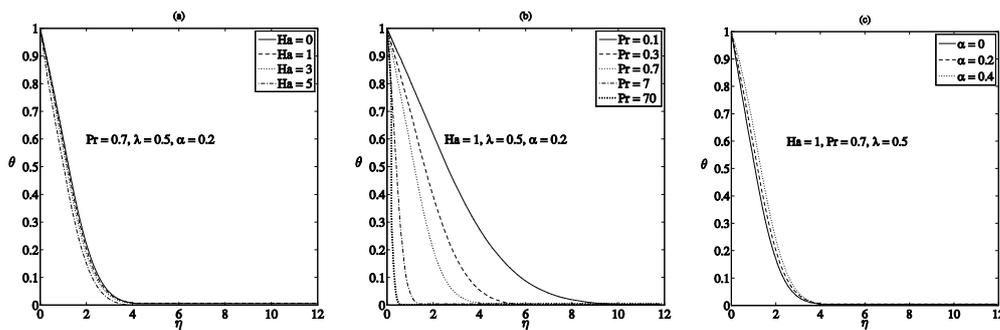
Fig. 5. Temperature profiles for the two dimensional case.

reduced to a set of ordinary differential equations. Numerical solutions to these equations were obtained by employing a Matlab routine built on the bvp4c boundary value problem solver. Solutions were calculated over the domain $0 \leq y \leq y_\infty$, where y_∞ was chosen to be sufficiently large that results were insensitive to changes in the integration length. The graphical and tabular presentation of the results revealed the effects of the relevant parameters on the velocity components, temperature distribution, tangential shear stresses and heat transfer. It is hoped that the results obtained in this work be of use for understanding of more complicated problems involving stagnation-point flows.

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Table 3 Tangential shear stress -τ_{wx} on the plate for the axisymmetric case $(\sqrt{\nu/\omega}\, ax^2 = 0.4)$

Ha	λ	r	τ_{wx}			
			Pr = 0.1	Pr = 0.7	Pr = 7	Pr = 70
		0	0.560137	0.549547	0.539019	0.532148
	0.05	0.2	0.560785	0.550363	0.540060	0.533463
		0.4	0.561493	0.551282	0.541361	0.535681
		0	0.878392	0.772495	0.667217	0.598501
0	0.5	0.2	0.884872	0.780654	0.677628	0.611658
		0.4	0.891950	0.789845	0.690633	0.633832
		0	1.232010	1.020216	0.809658	0.672227
	1	0.2	1.244968	1.036534	0.830480	0.698540
		0.4	1.259125	1.054916	0.856492	0.742889
		0	0.686696	0.679203	0.670748	0.664811
	0.05	0.2	0.687131	0.679803	0.671542	0.665813
		0.4	0.687604	0.680474	0.672511	0.667373
		0	0.943801	0.868871	0.784318	0.724947
1	0.5	0.2	0.948150	0.874873	0.792263	0.734973
		0.4	0.952878	0.881578	0.801951	0.750572
		0	1.229473	1.079614	0.910506	0.791766
	1	0.2	1.238171	1.091617	0.926398	0.811817
		0.4	1.247628	1.105027	0.945774	0.843015
		0	0.888355	0.883668	0.877372	0.872460
	0.05	0.2	0.888611	0.884069	0.877938	0.873181
		0.4	0.888887	0.884513	0.878610	0.874219
		0	1.084622	1.037758	0.974797	0.925676
3	0.5	0.2	1.087179	1.041767	0.980455	0.932885
		0.4	1.089942	1.046202	0.987176	0.943260
		0	1.302696	1.208968	1.083047	0.984804
	1	0.2	1.307811	1.216986	1.094362	0.999223
		0.4	1.313336	1.225857	1.107805	1.019973

Table 4 Tangential shear stress τ_{wy} on the plate (Pr = 7, r = 0.4, λ = 1)

Ha	τ_{wy}	
	Two dimensional case	Axisymmetric case ($ay/V_0 = 0.4$)
0	0.570465	0.413956
0.5	0.874875	0.560884
1	1.112340	0.685898
1.5	1.311997	0.795906
2	1.486990	0.894928
2.5	1.644414	0.985515
3	1.788563	1.069392
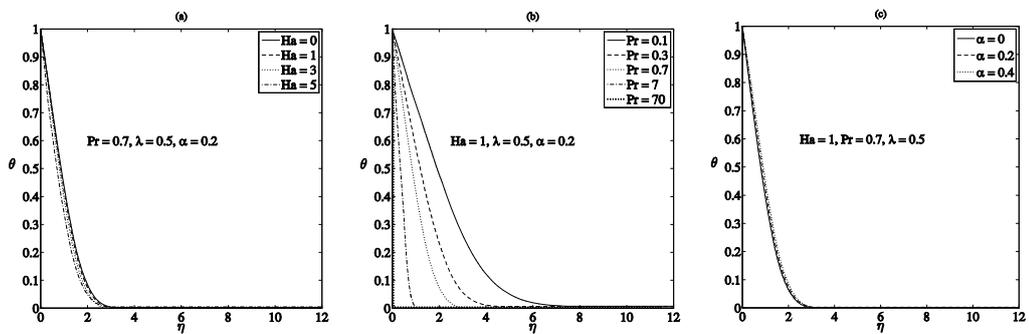
Fig. 6. Temperature profiles for the axisymmetric case.

Table 5 Temperature gradient on the plate for the two-dimensional case ($\gamma = 1$)

Ha	r	$-\theta\,'(0)$			
		Pr = 0.1	Pr = 0.7	Pr = 7	Pr = 70
	0	0.219566	0.495866	1.178375	2.642045
0	0.2	0.180129	0.377157	0.668846	0.047644
	0.4	0.135247	0.237625	-0.017283	-6.494990
	0	0.224177	0.515632	1.251033	2.839845
1	0.2	0.185234	0.400810	0.774613	0.519358
	0.4	0.141024	0.267068	0.157239	-4.113287
	0	0.229270	0.538476	1.339427	3.086727
3	0.2	0.190830	0.427653	0.896983	1.025959
	0.4	0.147300	0.299721	0.343883	-2.437674

Table 6 Temperature gradient on the plate for the axisymmetric case ($\gamma = 1$)

Ha	r	$-\theta\,'(0)$			
		Pr = 0.1	Pr = 0.7	Pr = 7	Pr = 70
	0	0.301472	0.665378	1.545779	3.427570
0	0.2	0.273881	0.580779	1.179949	1.666793
	0.4	0.244558	0.489312	0.757530	-0.837951
	0	0.307786	0.690931	1.634221	3.661087
1	0.2	0.280545	0.608957	1.288815	2.038886
	0.4	0.251629	0.520695	0.896079	-0.139310
	0	0.315367	0.722735	1.748164	3.966509
3	0.2	0.288519	0.643721	1.425596	2.492850
	0.4	0.260057	0.559026	1.064681	0.619589

comments and suggestions regarding an earlier version of this paper.

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