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Numerical Solutions of Free Convective Flow from a Vertical Cone with Mass Transfer under the Influence of Chemical Reaction and Heat Generation/Absorption in the Presence of UWT/UWC

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ABSTRACT

The purpose of this paper is to present a mathematical model for the combined effects of chemical reaction and heat generation/absorption on unsteady laminar free convective flow with heat and mass transfer over an incompressible viscous fluid past a vertical permeable cone with uniform wall temperature and concentration (UWT/UWC).The dimensionless governing boundary layer equations of the flow that are transient, coupled and non-linear partial differential equations are solved by an efficient, accurate and unconditionally stable finite difference scheme of Crank-Nicholson type. The velocity, temperature, and concentration profiles have been studied for various parameters viz., chemical reaction parameter λ, the heat generation and absorption parameter Δ, Schmidt number Sc, Prandtl number $\Pr$, buoyancy ratio parameter N. The local as well as average skin friction, Nusselt number, Sherwood number, are discussed and analyzed graphically. The present results are compared with available results in open literature and are found to be in excellent agreement

Keywords: Chemical reaction; Finite difference method; Free convection; Heat generation/ absorption; Non-uniform surface temperature; Non-uniform surface concentration; Unsteady; Vertical cone.

NOMENCLATURE

a	constant
b	constant
C_P	specific heat at constant pressure
D	mass diffusivity m^2s^{-1}
Gr_L	thermal Grashof number
Gr^*	mass Grashof number
g	acceleration due to gravity ms^{-2}
k	thermal conductivity $Wm^{-1}K^{-1}$
k_1	dimensional chemical reaction parameter J
L	reference length m
N	dimensionless buoyancy ratio
Nu_x	local Nusselt number
$\overline{Nu_L}$	average Nusselt number
Nu_x	non dimensional local Nusselt number
$\overline{Nu}$	non-dimensional average Nusselt number
$\Pr$	Prandtl number
Q_o	dimensional heat generation/absorption coefficient Wm^{-3}
R	dimensionless local radius
r	local radius of the cone m
Sc	Schmidt number
T'	temperature K^0
T	dimensionless temperature
t'	time s
t	dimensionless time
U	dimensionless velocity in-X direction
u	velocity component in x direction ms^{-1}
V	dimensionless velocity in-Y direction
v	velocity component in– y direction ms^{-1}

$\overline{Nu}$	component of the resultant pressure force
X	dimensionless spatial co-ordinate along the cone generator
x	Spatial co-ordinate along the cone generator m
Y	dimensionless spatial co-ordinate along the normal to the cone generator
y	Spatial co-ordinate along the normal to the cone generator m
α	thermal diffusivity $m^2 s^{-1}$
β	volumetric thermal expansion $^0K^{-1}$
Δ	dimensionless heat generation and absorption parameter
λ	non-dimensional chemical reaction parameter
ρ	density $kg\ m^{-3}$
Δt	dimensionless time step
ΔX in X	dimensionless finite difference grid size direction
ΔY in Y	dimensionless finite difference grid size direction
μ	dynamic viscosity $kg\,m^{-1}s^{-1}$
ν	kinematic viscosity $m^2 s^{-1}$
τ_x	local skin friction
τ_X	dimensionless local skin friction
$I\overline{\tau_L}$	average skin friction
$\bar{\tau}$	dimensionless average skin friction
w	condition on the wall
$\gamma\infty$	free stream condition

1. INTRODUCTION

The problem of two-dimensional axi-symmetric free convective flow past a vertical cone with different boundary conditions has attracted the attention of many researchers in recent years. Simultaneous heat and mass transfer in natural convection flows on a vertical cone has a wide range of applications in the field of science and technology. The flow of a fluid is caused not only by the temperature differences, but also by concentration differences. These concentration differences also affect the flow and temperature near the surface of a body embedded in a fluid. In engineering applications, the concentration differences are created by either injecting the foreign gases or by coating the surface with evaporating material which evaporates due to the heat of the surface. These mass transfer differences do affect the rate of heat transfer. Also it plays an important role in manufacturing industries for the design of reliable equipment for nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles. Free convection flows under the influence of gravitational force have been studied in detail because they occur frequently in nature as well as in science and engineering applications such as nuclear reactor safety, solar energy plants, metallurgy, dispersion of dissolved materials, dying and dehydration process in chemical and food process, design of space crafts, power transformers, steam generators etc. when a heated surface is in contact with the fluid, the result of temperature difference causes buoyancy force, which induces the natural convection. The presence of foreign masses in air and water causes some kind of chemical reaction. During a chemical reaction heat also generated. A common example of heat and mass transfer is the evaporation of lake water into the wind flowing over it. In some cases, mass transfer is predominant and heat transfer may be negligible; in other cases, both remain equally predominant. Mass transfer proceeds as long as there is a difference in concentrations of some chemical species in the mixture. Hence, the concentration gradient acts as a driving potential in mass transfer, just as the temperature gradient does in heat transfer.

Since 1953 several authors have developed similarity/non-similarity solutions for two-dimensional axi-symmetrical problems for natural convection laminar flow over vertical cone in steady state(Merk and Prins (1953, 1954); Hering and Grosh (1962); Hering (1965)). Kafoussias (1992) analyzed the effects of mass transfer on a free convective flow past a vertical cone surface embedded in an infinite, incompressible and viscous fluid. Yih (1999, 1999 a) studied in saturated porous media the combined heat and mass transfer effects over a full cone with uniform wall temperature/concentration or heat/mass flux and for truncated cone with non-uniform wall temperature/variable wall concentration or variable heat/mass flux using the Keller box implicit difference method. Chamkha (2001) considered the problem of steady-state laminar heat and mass transfer by natural convection boundary layer flow around a permeable truncated cone in the presence of magnetic field and thermal radiation effects, non-similar solutions were obtained and solved numerically by an implicit finite-difference methodology. Later Chamkha and Quadri (2002) solved the problem of combined heat and mass transfer by hydro magnetic natural convection over a cone embedded in a non-Darcian porous medium with heat generation/absorption effects, anon similar form of the solution was solved numerically by an implicit, iterative, finite-difference method. Afify (2004) studied the effects of radiation and chemical reaction on a steady free convective flow and mass transfer of an optically dense viscous, incompressible and electrically conducting fluid past a vertical isothermal cone in the presence of a magnetic field, the resulting similarity equations were solved numerically using a fourth-order Runge-Kutta scheme with the shooting technique. Chamkha and Al-Mudhaf (2005) focused on the study of unsteady heat and mass transfer by mixed convection flow over a vertical permeable cone rotating in an ambient fluid with a time-dependent angular velocity in the presence of a magnetic field and heat generation or absorption effects with the cone surface is maintained at variable temperature and concentration and obtained numerical solutions by solving the governing partial differential equations using an implicit, iterative finite difference scheme. Chamkha et al. (2006) studied the effects of coupled heat and mass transfer by boundary-layer free convection over a vertical flat plate embedded in a fluid-saturated porous medium in the presence of thermophoretic particle deposition and heat generation or absorption effects, the governing partial differential equations are transformed into ordinary differential equations by using special transformations and the resulting similarity equations are solved numerically by an efficient implicit tri-diagonal finite-difference method. EL-Kabeir et al. (2006) used perturbation method to study the effect of heat and mass transfer on free convection flow with a uniform suction and injection over a cone in a micro polar fluid. EL-Kabeir and Abdou (2007) studied the effects of chemical reaction, heat and mass transfer on MHD flow over a vertical isothermal cone surface in micro polar fluids with heat generation/absorption effects and obtained numerical solutions by using the fourth-order Runge-Kutta method with shooting technique. Also El- Kabeir et al. (2007 a) discussed the linear transformation group approach to simulate the problem of heat and mass transfer in steady, two-dimensional, laminar, boundary-layer flow of a viscous, incompressible and electrically conducting fluid over a vertical permeable cone surface saturated porous medium in the presence of a uniform transverse magnetic field and thermal radiation effects. Cheng (2009) presented a boundary-layer analysis about the natural convection heat and mass transfer near a vertical cone with variable wall temperature and concentration in a porous medium saturated with non-Newtonian power-law fluids, coordinate transform is used to obtain the non similar governing equations, and the transformed boundary-layer equations are solved by the cubic spline collocation method. Cheng (2009 a, 2010) analyzed the Soret and Dufour effects on the boundary layer flow due to natural convection heat and mass transfer over a downward-pointing vertical cone and truncated cone in a porous medium saturated with Newtonian fluids with constant wall temperature and concentration, similarity analysis is performed, and similarity equations are solved by cubic spline collocation method. Murti et al. (2010) discussed the radiation and chemical reaction effects on heat and mass transfer in non-Darcy non-Newtonian fluid over a vertical surface, the governing boundary layer equations and boundary conditions are simplified by using similarity transformations and are solved numerically by means of fourth-order Runge-Kutta method coupled with double-shooting technique. Kishore etal. (2010) studied Viscoelastic buoyancy driven MHD free convective heat and mass transfer past a vertical cone with thermal radiation and viscous dissipation and obtained numerical solutions for the governing equations using Crank-Nicholson method. Mahdy (2010) focused on the study of combined heat and mass transfer on double-diffusive convection near a vertical truncated cone in a fluid-saturated porous medium in the presence of a first-order chemical reaction and heat generation or absorption with variable viscosity. Viscosity of the fluid is assumed to be an inverse linear function of the temperature; the non dimensional non-similar governing equations are solved numerically using the fourth-order Runge–Kutta integration scheme with Newton–Raphson shooting technique.

GouseMohideen et al. (2010, 2010a) discussed the combined effects of thermal radiation and viscous dissipation on unsteady, laminar, free convective flow with heat and mass transfer over an incompressible viscous fluid past vertical cone with variable surface temperature and concentration in the presence of a transverse magnetic field applied normal to the surface, heat and mass transfer in a Walters-B viscoelastic fluid along a vertical cone using Crank Nicholson finite difference scheme. Patil and Pop (2011) considered the unsteady mixed convection boundary layer flow over a vertical cone to investigate the combined effects of the buoyancy force, thermal and mass diffusion in the presence of the first order chemical reaction and surface mass transfer. The governing boundary layer equations are transformed into a non-dimensional form by a group of non-similar transformations and the resulting system of coupled non-linear partial differential equations are solved numerically by the combination of quasi-linearization technique and an implicit finite difference scheme. Recently El-Kabeir and El-Sayed (2012) studied the problem of heat and mass transfer by free convection of a viscoelastic fluid past a vertical isothermal cone surface in the presence of transverse uniform magnetic field, and chemical reaction effect taking into account the effects of viscous dissipation, Joule heating and thermal radiation. The cone surface is maintained at constant temperature and constant species concentration. The governing partial differential equations are transferred into a system of ordinary differential equations, which are solved numerically using a fourth order Runge-Kutta scheme with the shooting method. Awad et al. (2011) studied the Soret and Dufour effects on the skin-friction coefficient, the heat and the mass transfer from an inverted cone in a porous medium. Numerical solutions for the governing momentum, energy and concentration equations were found using a shooting method together with a sixth order Runge–Kutta method. The results were validated by using a linearization method. Also Mahesha Narayana et al. (2013) studied the Soret and Dufour effects on free magneto hydrodynamic convection from a vertical spinning cone. They discussed two different types of boundary heating, namely linear surface temperature (LST) where the surface of the cone is maintained at a temperature that varies linearly with the distance from origin, and linear surface heat flux (LSHF). The nonlinear coupled governing equations were solved using a shooting technique together with a Runge–Kutta method of four slopes.

The objective of the present investigation, namely numerical solutions of free convective flow from a vertical cone with heat generation, absorption, and chemical reaction in the presence of uniform wall temperature and concentration has not received any attention in literature. Hence, the present work studies and deals with unsteady free convective flow from a vertical cone with the above said effects. The governing boundary layer equations are solved by an implicit finite difference scheme of Crank-Nicolson type for various values of parameters λ, Δ, Sc, Pr and N. In order to check the accuracy of the numerical results, the present results are compared with the available results of Chamkha (2001) and they are found to be in excellent agreement.

2. MATHEMATICAL FORMULATION

An axi-symmetric unsteady, laminar free convection flow of a viscous incompressible fluid past a vertical cone uniform surface temperature and concentration under the influence of chemical reaction, heat generation/absorption is considered. It is assumed that the effects of viscous dissipation and pressure gradient along the boundary layer are negligible. It is also assumed that there exists first order chemical reaction between the fluid and the species concentration. The concentration C' of the diffusing species is assumed to be very small in comparison to the other chemical species far away from the surface of the cone C'_∞.

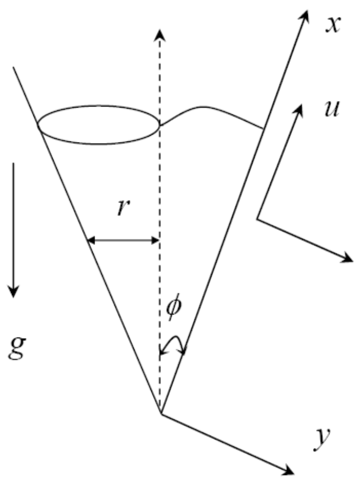

Fig. 1. Physical mode and co-ordinate system.

Hence the Soret and Dufour effects are neglected. It is also assumed that the cone surface and the surrounding fluid which is at rest are at the same temperature T'_∞ and concentration C'_∞. Then at time $t' > 0$, the temperature of the cone surface is suddenly raised to T'_w and the concentration near the cone surface is also raised to C'_w and both are maintained at the same level. The co- ordinate system is chosen (as shown in Fig. 1) such that x measures the distance along surface of the cone from the apex (x = 0) and y measures the distance normally outward. The fluid properties are assumed to be constant except the density variations causing a body force in the momentum equation. The governing boundary layer equations of continuity, momentum, energy and concentration under Boussinesq approximation are as follows:

Equation of continuity:

$$\frac{\partial}{\partial x}(ru)+\frac{\partial}{\partial y}(rv)=0 \tag{1}$$

Equation of momentum:

$$\frac{\partial u}{\partial t'}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta(T'-T'_\infty)\cos\phi$$

$$+\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta_c(C'-C'_\infty)\cos\phi \tag{2}$$

Equation of energy:

$$\frac{\partial T'}{\partial t'}+u\frac{\partial T'}{\partial x}+v\frac{\partial T'}{\partial y}=\alpha\frac{\partial^2 T'}{\partial y^2}+\frac{Q_o}{\rho c_p}(T'-T_\infty) \tag{3}$$

Equation of concentration:

$$\frac{\partial C'}{\partial t'}+u\frac{\partial C'}{\partial x}+v\frac{\partial C'}{\partial y}=D\frac{\partial^2 C'}{\partial y^2}-k_1(C'-C_\infty) \tag{4}$$

The initial and boundary conditions are

$$t'\le 0:\ u=0,\qquad v=0,\qquad T'=T'_\infty,\qquad C'=C'_\infty$$

for all *x* and *y*,

$$t'>0:\ u=0,\quad v=0,\quad T'=T'_w,\ C'=C'_w\ \ at\ \ y=0\ ,$$

$$u=0,\qquad T'=T'\quad C'=C'_\infty\qquad at\quad x=0\ , \tag{5}$$

$$u\to 0,\quad T'\to T'_\infty,\qquad C'\to C'_\infty\qquad as\quad y\to\infty\ .$$

Local skin-friction, local Nusselt number and local Sherwood number are given by

$$\tau_x=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{6}$$

$$Nu_x=\frac{-x\left(\frac{\partial T'}{\partial y}\right)_{y=0}}{T'_w-T'_\infty} \tag{7}$$

$$Sh_x=\frac{-x\left(\frac{\partial C'}{\partial y}\right)_{y=0}}{C'_w-C'_\infty} \tag{8}$$

Using the following non-dimensional quantities:

$$X=\frac{x}{L},\ \ Y=\frac{y}{L}\left(Gr_L\right)^{\frac{1}{4}},\ \ R=\frac{r}{L},\ \text{where } r=x\sin\phi,$$

$$V=\frac{vL}{\upsilon}\left(Gr_L\right)^{\frac{-1}{4}},\ U=\frac{uL}{\upsilon}\left(Gr_L\right)^{\frac{-1}{2}},\ \ t=\frac{\upsilon t'}{L^2}\left(Gr_L\right)^{\frac{1}{2}},$$

$$T = \frac{(T' - T'_\infty)}{(T_w' - T'_\infty)},\quad Gr_L = \frac{g\beta(T_w' - T'_\infty)L^3\cos\phi}{\upsilon^2},$$

$$\Pr = \frac{\upsilon}{\alpha},$$

$$T = \frac{(C' - C'_\infty)}{(C_w' - C'_\infty)},\quad Gr^* = \frac{g\beta_c(C_w' - C'_\infty)L^3\cos\phi}{\upsilon^2},$$

$$Sc = \frac{\upsilon}{D},$$

$$N = \frac{Gr^*}{Gr_L},\ \Delta = \frac{Q_o L^2}{C_p\mu}\left(Gr_L\right)^{\frac{-1}{2}},\ \lambda = \frac{k_1 L^2}{\upsilon}\left(Gr_L\right)^{\frac{-1}{2}} \tag{9}$$

Equations (1), (2), (3), (4) and (5) can then be written in the following non-dimensional form:

$$\frac{\partial(UR)}{\partial X} + \frac{\partial(VR)}{\partial Y} = 0 \tag{10}$$

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = T + NC + \frac{\partial^2 U}{\partial Y^2} \tag{11}$$

$$\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial Y^2} + \Delta T \tag{12}$$

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial X} + V\frac{\partial C}{\partial Y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} - \lambda C \tag{13}$$

The corresponding non-dimensional initial and boundary conditions are

$$t \le 0: U = 0,\ V = 0,\ T = 0,\ C = 0 \text{ for all X and Y}$$

$$t > 0: U = 0,\ V = 0,\ T = 1 \text{ and } C = 1 \text{ at Y = 0}$$

$$U = 0,\ T = 0,\ C = 0 \text{ at X = 0}$$

$$U \to 0,\ T \to 0,\ C \to 0 \text{ as } Y \to \infty \tag{14}$$

Local Skin-friction, local Nusselt number and local Sherwood number in non-dimensional quantities are

$$\tau_X = Gr_L^{\frac{3}{4}}\left(\frac{\partial U}{\partial Y}\right)_{Y=0} \tag{15}$$

$$Nu_X = \frac{X}{T_{Y=0}}\left(\frac{-\partial T}{\partial Y}\right)_{Y=0} Gr_L^{\frac{1}{4}}. \tag{16}$$

$$Sh_X = \frac{X}{C_{Y=0}}\left(\frac{-\partial C}{\partial Y}\right)_{Y=0} Gr_L^{\frac{1}{4}}. \tag{17}$$

Average skin-friction, average Nusselt number and average Sherwood number in non-dimensional quantities are

$$\bar{\tau} = 2Gr_L^{\frac{3}{4}}\int_0^1 X\left(\frac{\partial U}{\partial Y}\right)_{Y=0} dX, \tag{18}$$

$$\overline{Nu} = 2Gr_L^{\frac{1}{4}}\int_0^1 \frac{X}{T_{Y=0}}\left(\frac{-\partial T}{\partial Y}\right)_{Y=0} dX. \tag{19}$$

$$\overline{Sh} = 2Gr_L^{\frac{1}{4}}\int_0^1 \frac{X}{C_{Y=0}}\left(\frac{-\partial C}{\partial Y}\right)_{Y=0} dX. \tag{20}$$

3. SOLUTION PROCEDURE

The unsteady non-linear coupled partial differential equations (10)-(13) with the initial and boundary conditions (14) are solved by employing a finite difference scheme of Crank-Nicholson type which is rapidly convergent and unconditionally stable as discussed by Soundalgekar and Ganesan (1981), Bapuji et al. (2007), Muthucumaraswamy and Ganesan (1998, 2002). The region of integration is considered as a rectangle with $X_{\max}$ (= 1) and $Y_{\max}$ (= 20) where $Y_{\max}$ corresponds to Y= ∞ which lies very well outside both the momentum and thermal boundary layers. The maximum of Y was chosen as 20, after some preliminary investigation so that the last two boundary conditions of (14) are satisfied within the tolerance limit of 10^{-5}. The mesh sizes have been fixed as ΔX = 0.05, ΔY = 0.05 with time step Δt = 0.01. The computations are carried out first by reducing the spatial mesh sizes by 50 % in one direction, and later in both directions by 50 %. The results are compared. It is observed in all cases, that the results differ only in the fifth decimal place. Hence, the choice of the mesh sizes seems to be appropriate. The scheme is unconditionally stable as described by Bapujiet al. (2008). The local truncation error is $O(\Delta t^2 + \Delta Y^2 + \Delta X)$ and it tends to zero as $\Delta t, \Delta Y$ *and* ΔX tend to zero. Hence, the scheme is compatible. Stability and compatibility ensures the convergence.

4. RESULT AND DISCUSSION

In order to prove the accuracy of our numerical results, the present results for the steady-state flow at X = 1.0 are compared with available solutions from the open literature. The numerical values of the local skin friction τ_X and the local Nusselt number Nu_X for different values of the Prandtl number with M = 0, N = 0 and R_d =0 are compared with the results of Chamkha (2001) in Table. 1, where $f''(\infty,0)$ and $-\theta'(\infty,0)$ are the steady state local skin friction and the local Nusselt number for a full cone. It is observed that the results are in good agreement with each other.

Temperature, Velocity and concentration profiles are shown at the upper edge of the cone i.e., at X=1.0 for different values of Prandtl number $\Pr$, heat generation and absorption parameter Δ, are shown in Fig. 1((a)–(c)) respectively. The positive

Table 1 Comparison of steady_ state local skin friction and local nusselt number valees at x= 1.0 with those of chamkha (2001) for full cone, for various values of Pr when n=0, M=0, N=0 and R_d=0

	Local skin friction		Local Nusselt number	
	Chamkha (2001)	Present values	Chamkha (2001)	Present results
Pr	$f''(\infty, 0)$	$\tau_X / Gr_L^{\frac{3}{4}}$	$-\theta'(\infty, 0)$	$Nu_X / Gr_L^{\frac{1}{4}}$
0.001	1.5135	1.4149	0.0245	0.0294
0.01	1.3549	1.3356	0.0751	0.0797
0.1	1.0962	1.0911	0.2116	0.2115
1	0.7697	0.7688	0.5111	0.5125
10	0.4877	0.4856	1.0342	1.0356
100	0.2895	0.2879	1.9230	1.9316
1000	0.1661	0.1637	3.4700	3.5186

values of Δ represents the presence of heat generation and the negative values corresponds to heat absorption. Figure 1(a) shows that the temperature increases for higher values of Δ and lower values of Pr and the thermal boundary layer decreases for the larger value of Pr. It is also noted that as Δ increases from negative to positive values, the temperature as well as thermal boundary layer thickness increases. The effect of the heat generation parameter Δ on the velocity distribution is seen from Fig. 1(b) it shows that, when heat is generated the buoyancy force increases, which induces the flow rate to increase, giving rise to the increase in the velocity profiles. i.e. the fluid velocity increases for higher values of Δ and lower values of Pr. Figure 1(c) shows that the concentration decreases while Δ and Pr increases. For increasing Δ, Pr the time taken to reach the steady state is also increased.

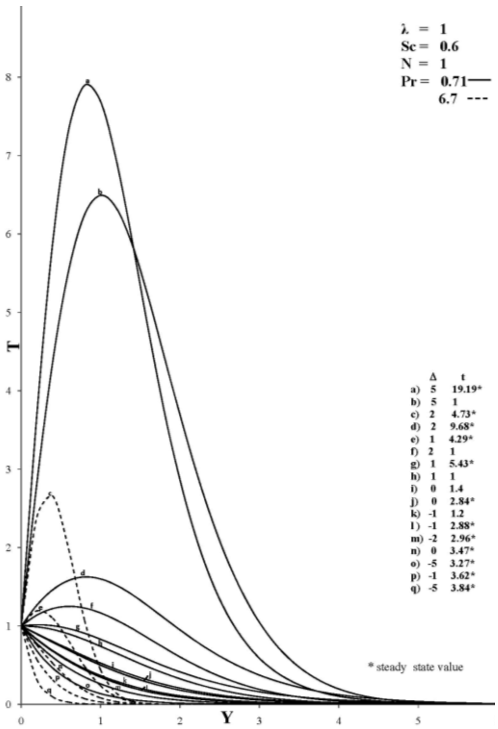

Fig. 1a. Transient Temperatur profilrs at X=1.0 for different values for Pr & Δ.

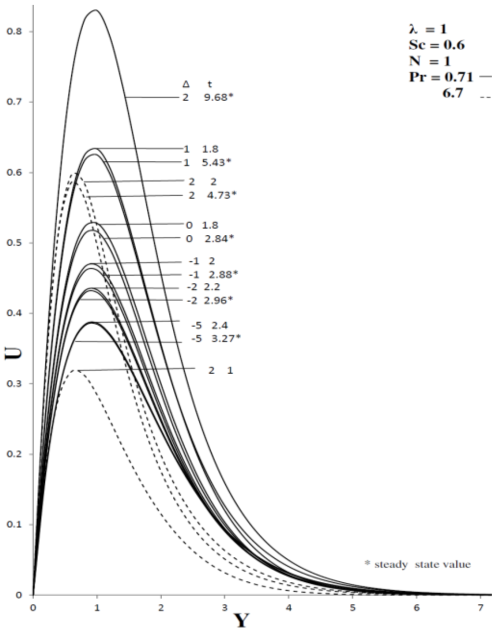

Fig. 1b. Transient Velocitu profile at X= 1.0 for different values for Pr & Δ.

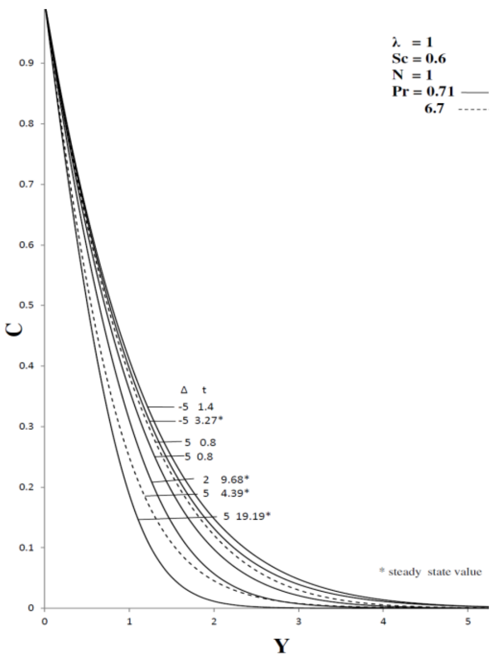

Fig. 1c. Transient concentration profile at X= 1.0 for different values for Pr & Δ.

Figure 2 ((a)-(c)) shows the transient temperature, velocity, and concentration profiles for various values of chemical reaction parameter λ, and Schmidt number Sc. The constructive chemical reaction parameter $\lambda > 0$ reduce the concentration distribution. The Schmidt number quantifies the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic (velocity) and concentration (species) boundary layers. As the Schmidt number increases the concentration decreases. This causes the concentration buoyancy effects to decrease. Also, the boundary layer thickness decreases with an increase in the chemical reaction parameter λ, where as the temperature increases for larger values of Sc and smaller values of λ (Fig.2(a)).The velocity decreases with an increase in the chemical reaction parameter λ, and Sc is seen from Fig 2(b). From Fig. 2(c) we infer that the concentration decreases for lower values of λ and higher values of Sc. Also a rise in Sc strongly suppresses concentration levels in the boundary layer regime. With increasing Sc the velocity, is depressed through the boundary layer i.e. the flow is retarded. For $Sc > 1$, momentum will diffuse faster than species causing progressively lower concentration values. For $Sc < 1$, species will diffuse much faster than momentum so that maximum concentrations will be generated in the boundary layer regime. Higher Sc values will physically correspond to a decrease of molecular diffusivity of the primary fluid causing a decrease in the rate of species diffusion. Lower Sc values will exert the reverse influence since they correspond to higher molecular diffusivities. Concentration boundary layer thickness is therefore considerably greater for Sc = 0.1 than for Sc = 5.0.

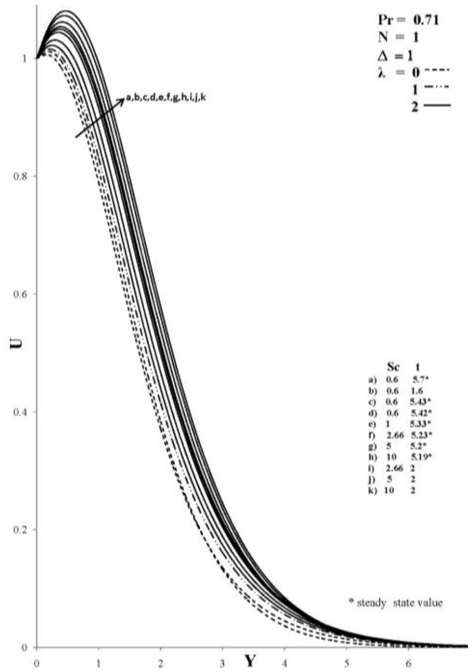

Fig. 2a. Transient temperatur profiles at X=1.0 for different values for Sc & λ.

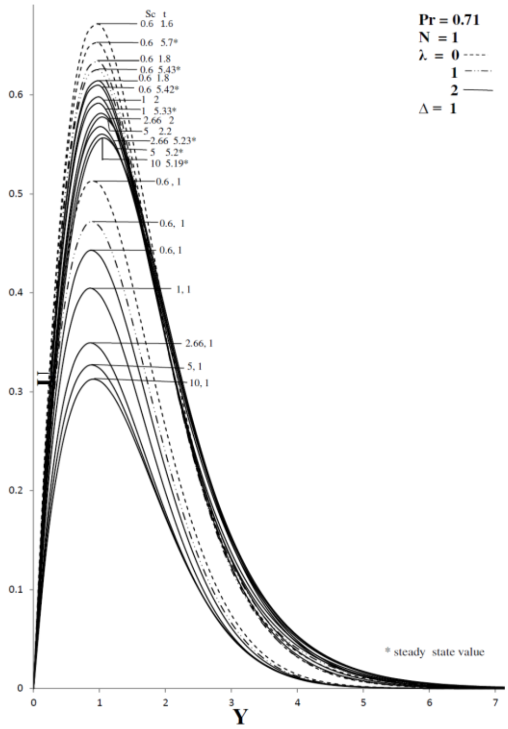

Fig. 2b. Transient t Velocity profiles at X=1.0 for different values for Sc & λ.

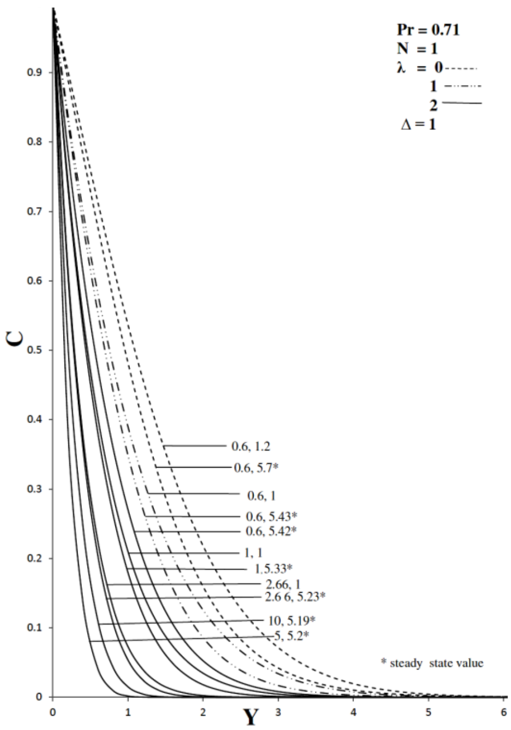

Fig. 2c. Transient concentration profile at X= 1.0 for different values for Sc & Δ.

The effects of the buoyancy ratio parameter N on the transient temperature, velocity and concentration profiles are shown in Fig. 3((a)-(c)) respectively. As we move away from the surface of the cone, the temperature decreases for all the values of N (Fig. 3(a)).The velocity increases steadily with time reaches a temporal maximum and consequently it reaches the steady state. However, time required to reach the steady state depends upon buoyancy ratio parameter N. An increase in N leads to an increase the velocity, i.e., as N increases, the combined buoyancy force also increases; therefore, the velocity increases near the surface of the cone (Fig. 3(b)). Thus for higher value of buoyancy ratio parameter N the fluid cools rapidly and concentration field decreases with increasing value of buoyancy ratio parameter N (Fig. 3(c)).

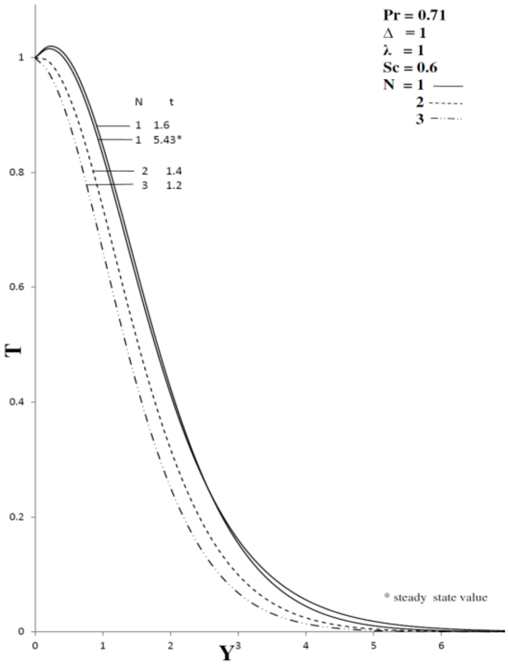

Fig. 3a. Transient temperature profiles at X=1.0 for different values for N.

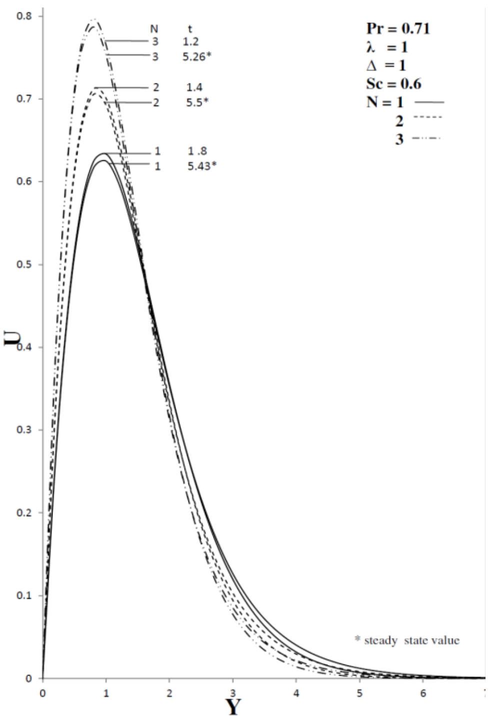

Fig. 3b. Transient velocity profiles at X=1.0 for different values for N.

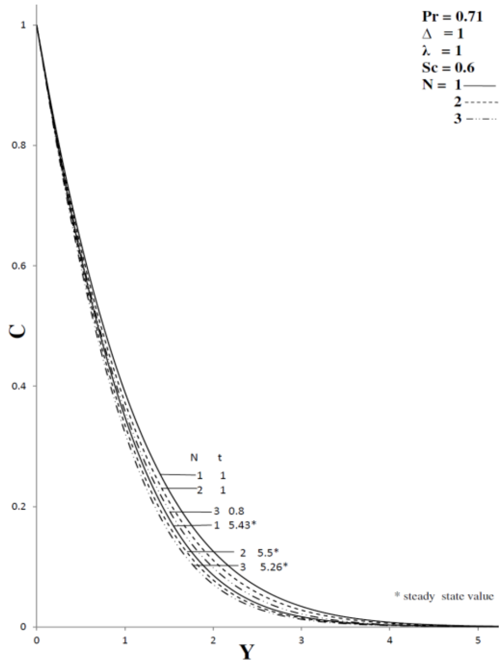

Fig. 3c. Transient concentration profiles at X=1.0 for different N.

Local values of skin friction τ_X, Nusselt number Nu_X and Sherwood number Sh_X for different parameters Pr, Δ, λ, Sc and N are plotted through Figs. 4((a)-(c)) through 6((a)-(c)). Figure 4((a)-(c)) indicates the local skin friction τ_X increases for larger values of Δ and Pr (Fig 4(a)).

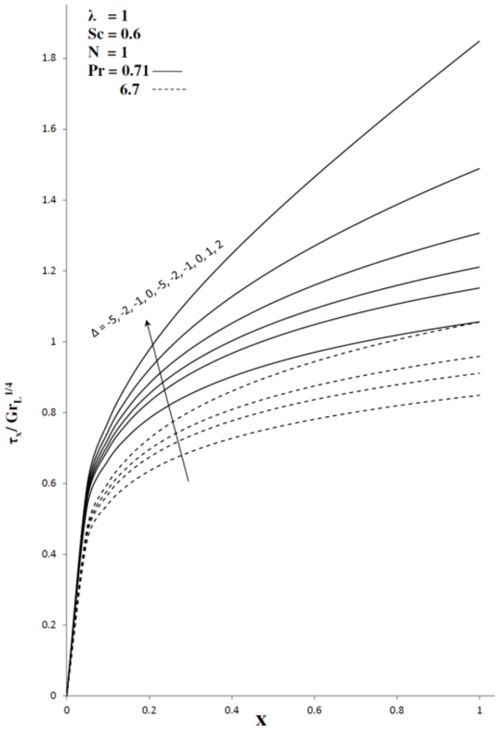

Fig. 4a. Local Skin Friction different values Pr & Δ in transient state.

The local Nusselt number Nu_X increases for smaller values of Δ and larger values of $\Pr$ i.e. the heat generation/absorption parameter Δ has the tendency to increase the magnitude of the local Nusselt number for $\Delta < 0$ (Fig. 4(b)). Whereas the local Sherwood number Sh_X increases for larger values of Δ and smaller values of $\Pr$ (Fig. 4(c)). Whereas the local Sherwood number Sh_X increases for larger values of Δ and smaller values of $\Pr$ (Fig. 4(c)).

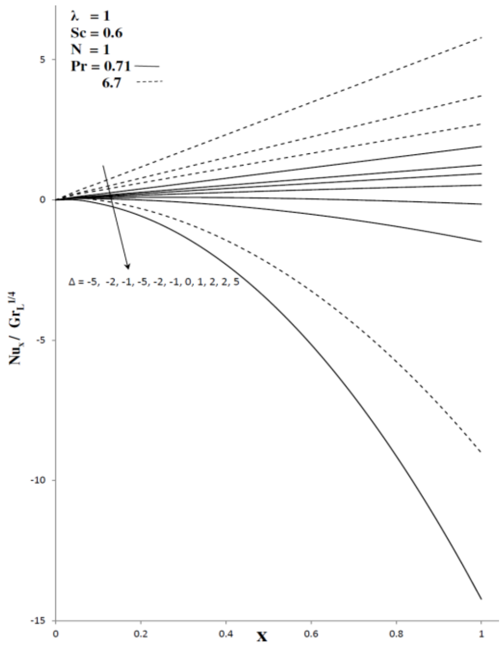

Fig. 4b. Local Nusselt Number for different values Pr & Δ in transient state.

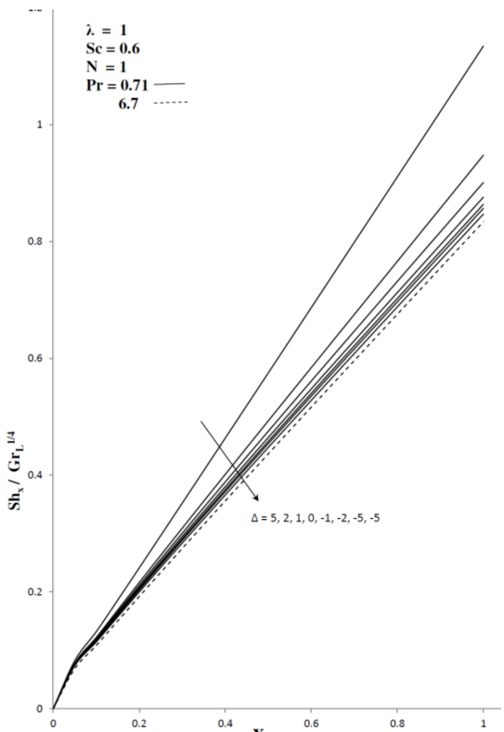

Fig. 4c. Local Sherwood Number for different values Pr & Δ in transient state.

Figure 5((a)-(c)) depicts the effects of chemical reaction parameter λ, and Schmidt number Sc on local skin friction τ_X, local Nusselt number Nu_X, and local Sherwood number Sh_X. The local skin friction and local Nusselt number Nu_X increases for smaller values of λ and Sc. Increasing λ and Sc clearly boosts the wall skin friction τ_X (Fig. 5(a)), which grows strongly from the leading edge downstream along the cone surface.

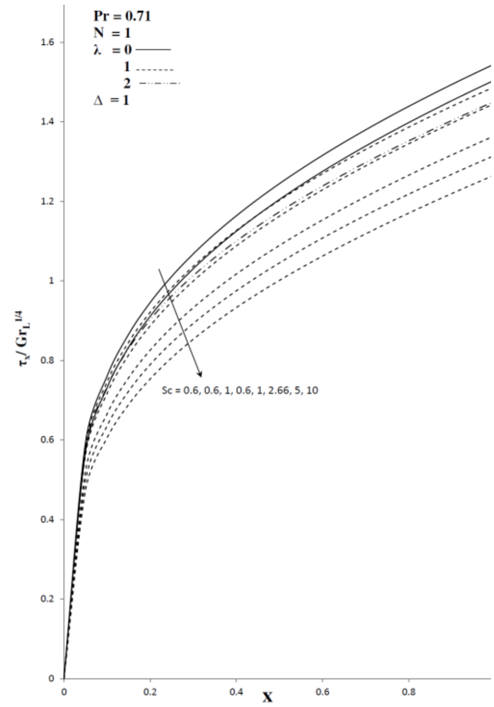

Fig. 5a. Local Skin Friction for different values Sc & λ in transient state.

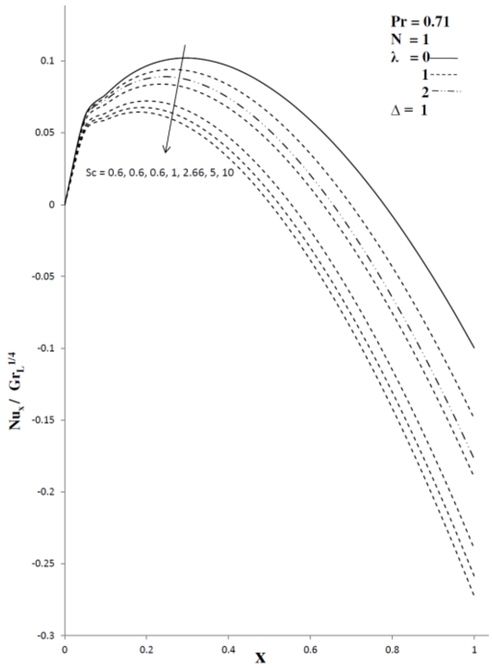

Fig. 5b. Local Nusselt Number for different values Sc & λ in transient state.

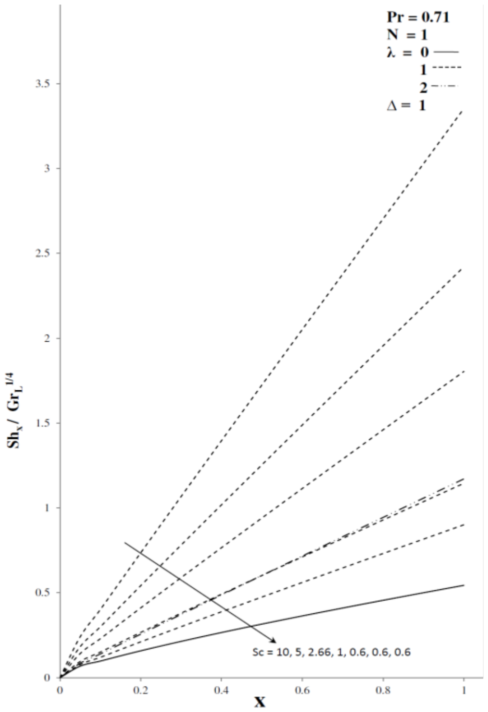

Fig. 5c. Local Sherwood Number for different values Sc & λ in transient state.

With increasing Sc , local Nusselt number Nu_X (Fig. 5(b)) is consistently reduced. The surface species gradient i.e. mass transfer rate at the cone surface is strongly elevated with a rise in λ and Sc is observed from Fig. 5(c). We study from Fig. 6((a)-(c)) the effect of buoyancy ratio parameter N on the local skin friction τ_X , local Nusselt number Nu_X , and local Sherwood number Sh_X .

Figure 6(a) illustrates that a rise in N accompanying a stronger increase in assisted buoyancy force, it strongly accelerates the flow i.e. enhances shear stress. The time required to attain the steady state is decreased with this increase in N . Inspection of Figs. 6(b) and 6(c) shows that an increase in N strongly boosts both Nu_X and Sh_X i.e.it enhances heat transfer gradient and mass transfer gradient at the cone surface. Further the Sherwood number Sh_X increases with the increase of ratio of the buoyancy forces parameter *N*. The physical reason is that positive force produced remarkable overshoot near the surface within the boundary layer for low Prandtl number fluid (Pr = 0.71) but for high Prandtl number fluid (Pr = 6.7) the velocity overshoot is not significant. Also, the buoyancy force enhanced the skin friction coefficient τ_X as well as the local Nusselt number Nu_X or local heat transfer rate. Simultaneously the time required to attain the steady state is reduced with an increase in *N* for both Nu_X and Sh_X . Time dependences of the average values of skin friction $\bar{\tau}$, Nusselt number $\overline{Nu}$, and Sherwood number $\overline{Sh}$ for various parameters $\Pr$, Δ , λ , Sc and N are plotted through Figs.7((a)-(c)) through 9((a)-(c)).From Fig.7((a)-(c))noticed that the effects of heat generation or absorption parameter Δ and $\Pr$, the average skin friction $\bar{\tau}$ increases for larger values of Δ and $\Pr$. Whereas average Nusselt number $\overline{Nu}$ increases for smaller values of Δ and higher values of $\Pr$. The average Sherwood number $\overline{Sh}$ increases for larger values of Δ and smaller values of $\Pr$.

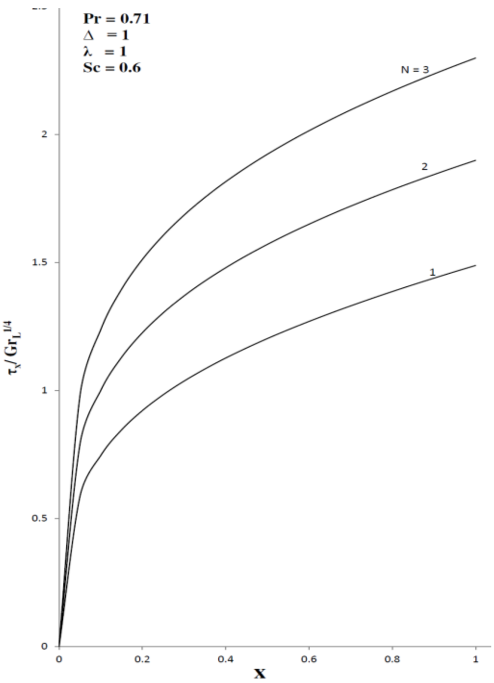

Fig. 6a. Local Friction for different values of N in transient period.

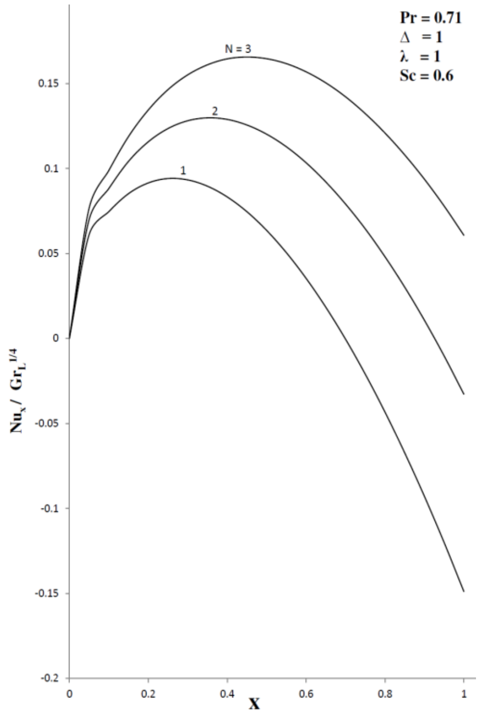

Fig. 6b. Nusselt Number for different values of N in transient period.

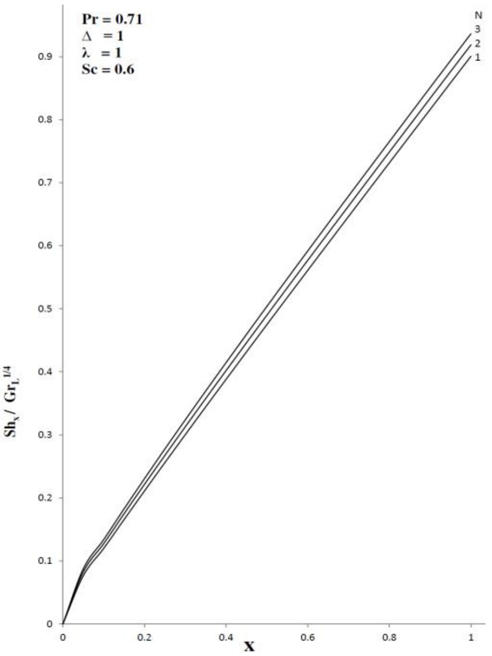

Fig. 6c. Local Sherwood number for different values of N in transient period.

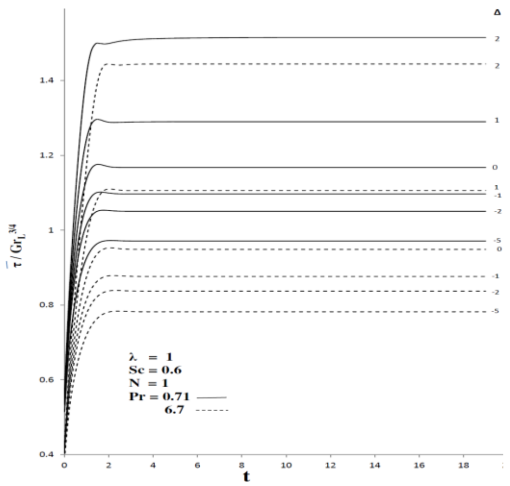

Fig. 7a. Average Skin Friction for different values of Pr &Δ in transient period.

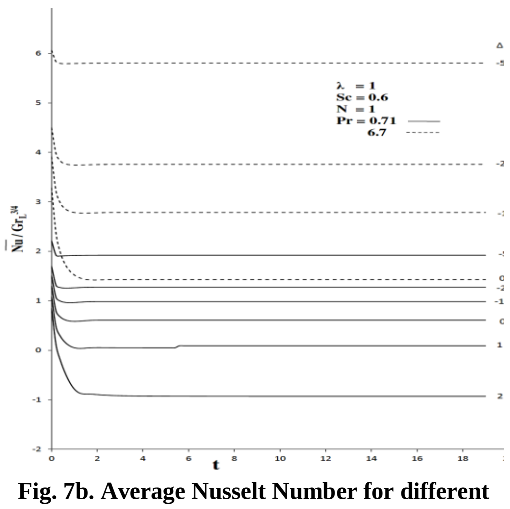

Fig. 7b. Average Nusselt Number for different values of Pr &D & m in transient period.

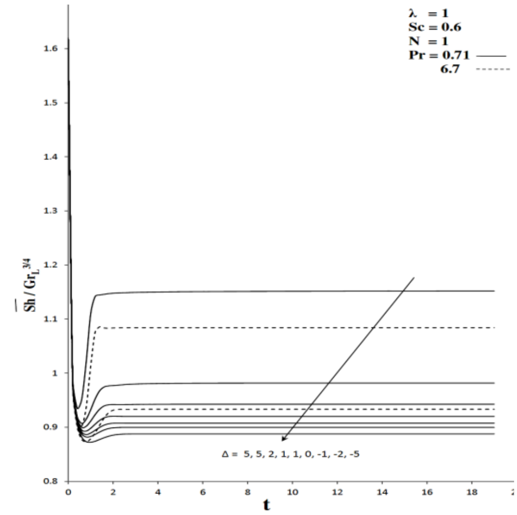

Fig. 7c. Average Sherwood Number for different values of Pr &Δ in transient period.

Figure 8 ((a)-(c)) depicts the effects of chemical reaction parameter λ, and Schmidt number Sc on average skin friction $\bar{\tau}$, average Nusselt number $\overline{Nu}$, and average Sherwood number $\overline{Sh}$. The average skin friction $\bar{\tau}$, and Nusselt number $\overline{Nu}$ increases for smaller values of λ and Sc, but the average Sherwood number $\overline{Sh}$ increases for larger values of λ and Sc.

It is clear from Fig. 9 ((a)-(c))average skin friction $\bar{\tau}$, average Nusselt number $\overline{Nu}$, and average Sherwood number $\overline{Sh}$ increases for higher values of N. Hence the average skin friction increases at small values of *t*, whereas at large *t* it is independent of *t*.

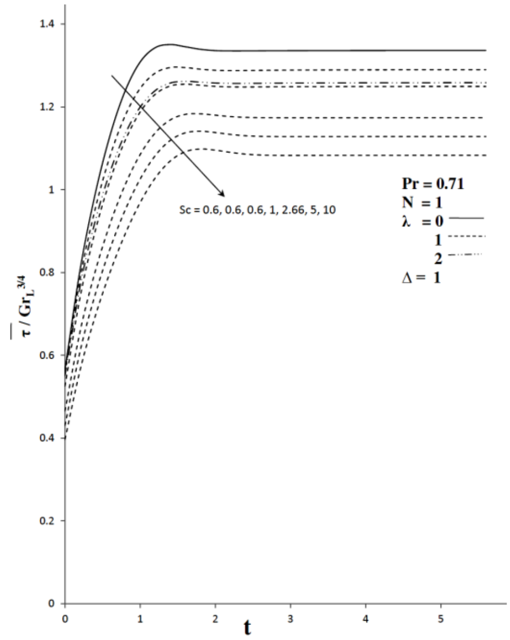

Fig. 8a. Average Skin Friction for different values of Sc & λ in transient period.

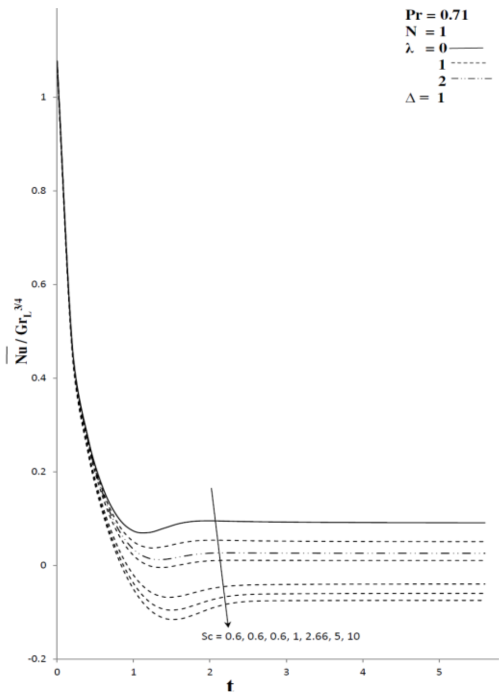

Fig. 8b. Average Nusselt Number for different values of Sc & λ in transient period.

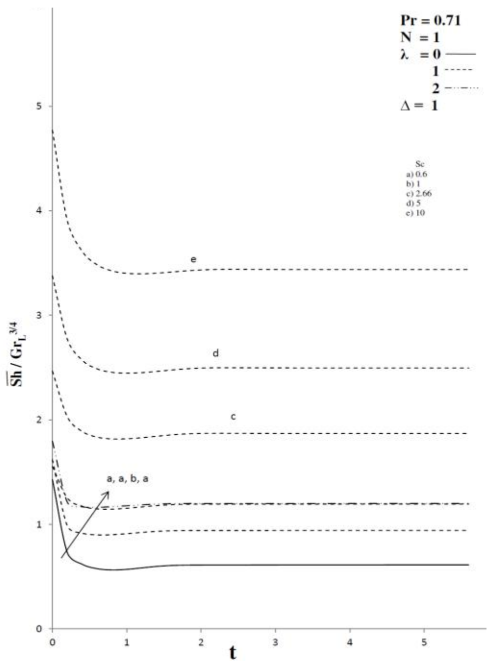

Fig. 8c. Average Sherwood number for different values of Sc & λ in transient period.

5. Conclusions

A mathematical model has been presented for the free convection flow from a vertical cone with heat generation/absorption, and chemical reaction**.** The family of governing partial differential equations is solved by an implicit finite difference scheme of Crank-Nicholson type. A parametric study is performed to illustrate the influence of thermo physical parameters on the velocity, temperature and concentration profiles. It has been shown that:

1. The time taken to reach steady state increases with increasing Δ, $\Pr$, λ, Sc and N
2. The fluid velocity increases for higher values of Δ, N and lower values of $\Pr$, λ and Sc. Temperature increases, for larger values of Δ, Sc smaller values of $\Pr$, λ and N, while the concentration of species decreases for smaller values of λ and larger values of Δ, $\Pr$, Sc and N.
3. The local skin friction increases for higher values of $\Pr$, Δ and N and for lower values of λ and Sc. The local Nusselt number increases for higher values of $\Pr$ and N, and lower values of λ, Sc and Δ. The local Sherwood number increases for higher values of λ, Sc, Δ and N and smaller values of values of $\Pr$.
4. The average skin friction increases for larger values of Δ, $\Pr$ and N and smaller values of λ, Sc. The average Nusselt number increases for higher values of $\Pr$, N and lower values of λ, Sc and Δ. The average Sherwood number increases for higher values of λ, Sc Δ, N and lower values of $\Pr$.
5. Momentum boundary layers become thick for higher values of Sc, Δ, λ and N and lower values $\Pr$. The thermal boundary layer becomes thick for higher values of, Sc, Δ and lower values of $\Pr$, λ, and N, the concentration boundary layer becomes thick for larger values of λ and smaller values of $\Pr$, Sc, Δ and N.

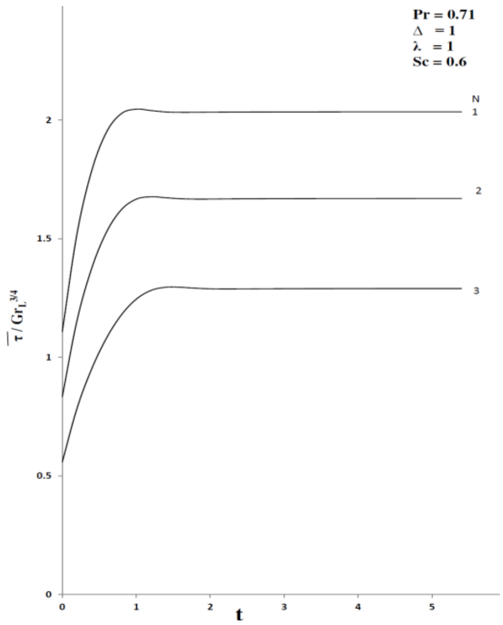

Fig. 9a. Average Skin Friction for different values of N in transient period.

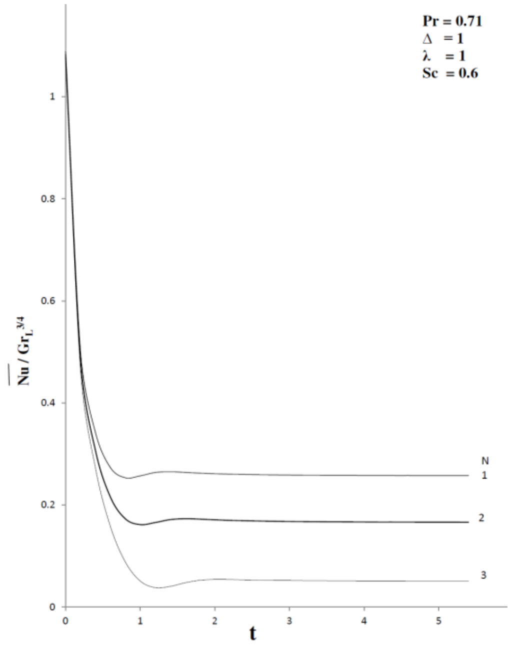

Fig. 9b. Average Nusselt Number for different values of N in transient period.

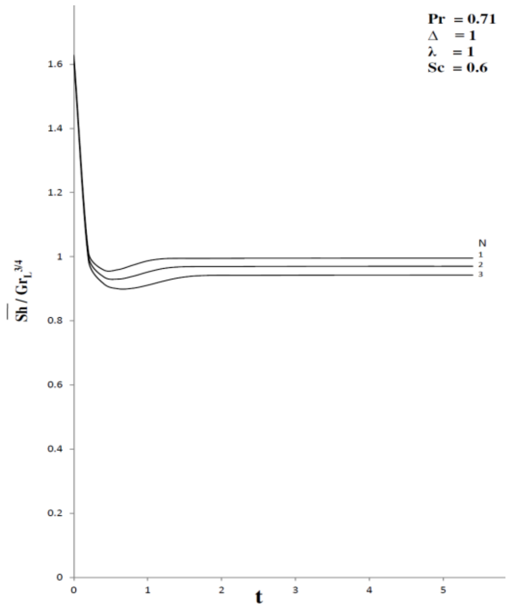

Fig. 9c. Average Sherwood number for different values of N in transient period.

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