Journal of Applied Fluid Mechanics, Vol. 8, No. 4, pp. 845-854, 2015.
Available online at www.jafmonline.net, ISSN 1735-3572, EISSN 1735-3645.
DOI: 10.18869/acadpub.jafm.67.223.23265

Unsteady Free Convection Flow past a Vertical Plate with Heat and Mass Fluxes in the Presence of Thermal Radiation

S. Das[1], R. N. Jana[2] and A. J. Chamkha[3†]

[1]*Department of Mathematics, University of Gour Banga, Malda 732 103, India*
[2]*Department of Applied Mathematics, Vidyasagar University, Midnapore 721 102, India*
[3]*Mechanical Engineering Department, Prince Mohammad Bin Fahd University, Al-Khobar 31952, Saudi Arabia*

†Corresponding Author Email:*achamkha@pmu.edu.sa*

(Received May 31, 2014; accepted November 1, 2014)

ABSTRACT

The problem of unsteady free convection flow past an infinite vertical plate with heat and mass fluxes in the presence of thermal radiation is studied. The dimensionless coupled linear partial differential equations governing the flow are solved by employing the Laplace transform technique. Exact solutions have been obtained for the fluid velocity, temperature and mass concentration for the cases of both uniform heat flux (UHF) and uniform wall temperature (UWT). The numerical results for the fluid velocity, temperature and mass concentration are presented graphically for various pertinent flow parameters and discussed in detail.

Keywords: Free convection; Heat and mass fluxes; Vertical plate; Thermal radiation; UHF and UWT.

1. INTRODUCTION

Free convection arises in the fluid when temperature changes cause density variation leading to buoyancy forces acting on the fluid elements. The most common example of free convection is the atmospheric flow which is driven by temperature differences. Natural convection has been analyzed extensively by many investigators. Sometimes along with the free convection currents caused by difference in temperature the flow is also affected by the differences in concentration or material constitution. There are many situations where convection heat transfer phenomena are accompanied by mass transfer also. When mass transfer takes place in a fluid at rest, the mass is transferred purely by molecular diffusion resulting from concentration gradients. For low concentration of the mass in the fluid and low mass transfer rates, the convective heat and mass transfer processes are similar in nature. A number of investigations have already been carried out with combined heat and mass transfer under the assumption of different physical situations. The illustrative examples of mass transfer can be found in the book of Cussler (1998). Combined heat and mass transfer flow past a surface have been analyzed by Das et al. (1996), Chaudhary and Arpita (2007), Muthucumaraswamy et al. (2008, 2009), Das and Jana (2010) and Rajput and Kumar (2011) with different physical conditions. Juncu (2005) pioneered to study the unsteady heat and mass transfer flow past a surface by numerical method.

Natural convection resulting from the combined thermal and mass buoyancy forces has received considerable attention of engineers and scientists due to many applications in engineering and technological processes. Noticeable applications include the thermal processes in vertical mounting board electronic components, channel-chimney systems, thermal comfort dynamics in building services, nuclear reactor thermal hydraulics and frost formation. Theoretical investigation of unsteady free convection flow due to heat and mass transfer near a vertical plate received less attention than numerical investigations. The results of theoretical analysis can serve as a guide for both experimental and numerical investigations. It is well known that the effects of radiation on free convection flow problems has become industrially more important. Many engineering processes occur at high temperatures, the knowledge of radiative heat transfer plays significant role in the design of equipment. Nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering processes. At high operating temperature, the radiation effect can be quite significant. The study of free convection flow with radiative heat and mass transfer also plays an important role in biological sciences. Effects of various parameters on human body can be studied and appropriate suggestions can be given to the persons working in hazardous areas having noticeable effects of magnetism and heat variation.

It is found from the literature that several investigations on free convective flows are available with different thermal conditions at the bounding plate which are continuous and well-defined at the wall. However, most of the practical problems appear with non-uniform or arbitrary conditions at the wall. To study such problems, it is useful to investigate them under non-uniform change in wall temperature and concentration. Ahmed and Sarmah (2009) have analyzed the thermal radiation effect on a transient MHD flow with mass transfer past an impulsively started infinite vertical plate. Sangapatnam et al. (2009) have studied the radiation and mass transfer effects on MHD free convective flow past an impulsively started isothermal vertical plate with dissipation. The unsteady MHD flow past a vertical oscillating plate with thermal radiation and variable mass diffusion has been studied by Deka and Neog (2009). Rajesh and Varma (2009) have studied the radiation and mass transfer effects on MHD free convection flow past an exponentially accelerated vertical plate with variable temperature. Reddy et al. (2010) have investigated unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical porous plate with variable viscosity and thermal conductivity. The thermal radiation effect on the flow past a vertical plate with mass transfer has been examined by Muralidharan and Muthucumaraswamy (2010) and Rajput and Kumar (2012). Natural convective flow past an oscillating plate with constant mass flux in the presence of radiation has been studied by Chaudhary and Jain (2007). Deka and Deka (2011) have discussed the radiation effects on MHD flows past an infinite vertical plate with variable temperature and uniform mass diffusion. The radiation effects on free convection flow near a moving vertical plate with Newtonian heating have been studied by Narahari and Ishaq (2011). Narahari and Nayan (2012) have presented the free convection flow past an impulsively started infinite vertical plate with Newtonian heating in the presence of thermal radiation and mass diffusion. Rajesh (2011) has studied the chemical reaction and radiation effects on the transient MHD free convection flow of dissipative fluid past an infinite vertical porous plate with ramped wall temperature. Effects of thermal radiation and mass diffusion on free convection flow near a vertical plate with Newtonian heating have been analyzed by Narahari and Dutta (2012). The radiation effects on MHD flow past an impulsively started vertical plate with variable heat and mass transfer have been analyzed by Rajput and Kumar (2012). Uwanta and Sani (2012) have presented the transient convection fluid flow with heat flux past an infinite vertical plate with chemical mass transfer. Effects of thermal radiation and mass diffusion on free convection flow near a vertical plate with Newtonian heating have been examined by Narahari and Dutta (2012). Hussanan et al. (2013) have obtained an exact solution of heat and mass transfer past a vertical plate with Newtonian heating. Jain (2013) has examined the radiation and chemical reaction effects on unsteady double diffusive convective flow past an oscillating surface with constant heat flux. Khan et al. (2014) have discussed the effects of wall shear stress on unsteady MHD conjugate flow in a porous medium with ramped wall temperature. Chamkha and Khaled (1999) studied nonsimilar hydromagnetic simultaneous heat and mass transfer by mixed convection from a vertical plate embedded in a uniform porous medium. Takhar et al. (1999) analyzed unsteady flow and heat transfer on a semi-infinite flat plate with an aligned magnetic field. Al-Mudhaf and Chamkha (2005) reported similarity solutions for MHD thermosolutal Marangoni convection over a flat surface in the presence of heat generation or absorption effects. Magyari and Chamkha (2008) reported exact analytical results for the thermosolutal MHD Marangoni boundary layers. Similarity solutions for unsteady heat and mass transfer from a stretching surface embedded in a porous medium with suction/injection and chemical reaction effects by Chamkha et al. (2010).

The main purpose of the present investigation is to study the unsteady free convection flow of an optically dense viscous incompressible fluid past a vertical plate with heat and mass fluxes in the presence of thermal radiation. Assume that the flow is laminar and the fluid is gray absorbing-emitting radiation but no scattering medium. It is considered that the fluid to be optically thick instead of optically thin in this problem. The Rosseland diffusion approximation is used to describe the radiative heat flux in the energy equation. Closed form solutions of the initial and boundary value problems that govern the flow are obtained by means of the Laplace transform technique. The effects of pertinent flow parameters on the fluid velocity, temperature and mass concentration profiles are illustrated graphically and the physical aspects of the problem are discussed.

2. MATHEMATICAL FORMULATION AND ITS SOLUTION

Consider the unsteady free convection flow of a viscous incompressible fluid past an infinite vertical plate with heat and mass fluxes in the presence of thermal radiation. Choose a cartesian coordinates system with the x- axis along the plate in the vertically upward direction, the y- axis perpendicular to the plate (see Fig. 1). Initially, at time $t \le 0$, the plate and the fluid are assumed to be at the same temperature T_∞ and concentration C_∞ and stationary. At time $t > 0$, the temperature of the plate is raised or lowered to T_w (for UWT case) and the heat transfer rate at the plate is constant (for UHF case). Also, the mass transfer rate at the plate is proportional to the concentration. The flow is considered to be laminar without any pressure gradient in the flow direction. It is also assumed that the radiative heat flux in the x - direction is negligible as compared to that in the y - direction. Fluid property variations have been considered only to the extent of a density variation which provides a buoyancy force in the equations of motion. This approximation is exact enough for both dropping liquid and gases at small values of the temperature and diffusion differences. As the plate are infinite long, the velocity and temperature fields are functions of y and t only.

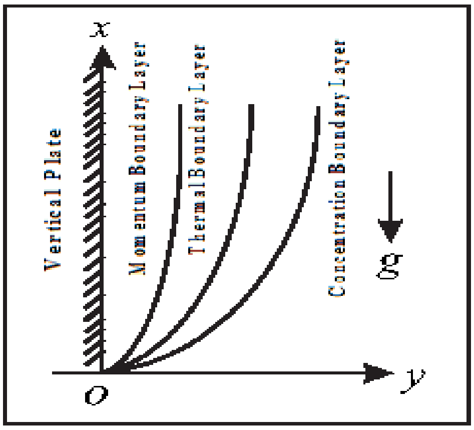

Fig. 1. Geometry of the problem.

It should be noted that the temperature and mass concentration differences are small enough in the analysis of heat and mass transfer process. Therefore use of the Boussinesq approximation and analogy between heat and mass transfer processes here is pertinent. The unsteady natural convection flow of a radiating fluid, under usual Boussinesq approximation, is governed by the following equations

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty), \quad (1)$$

$$\rho c_p \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y}, \quad (2)$$

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial y^2}, \quad (3)$$

where u is the fluid velocity in the x -direction, T the temperature of the fluid, g the acceleration due to gravity, β the coefficient of thermal expansion, β^* coefficient of volumetric expansion due to concentration change, ν the kinematic viscosity, ρ the fluid density, k the thermal conductivity, c_p the specific heat at constant pressure, mass diffusivity D and q_r the radiative heat flux. The viscous dissipation has been neglected in the energy Eq. (2) because of small velocity usually encountered in free convection flows.

The concentration of the species at the plate surface is higher than the solubility of species in the fluid far away from the plate i.e. free stream concentration. The study of combined heat and mass transfer problems are of important in many processes such as drying, evaporation at the surface of water body, energy transfer in a wet cooling tower, solidification of binary alloy, dispersion of dissolved materials, drying and dehydration operations in chemical food processing plants, and combustion of atomized liquid fuels.

The initial and boundary conditions are

$$t \le 0 : u = 0,\ T = T_\infty,\ C = C_\infty \ \text{ for all } \ y \ge 0$$

$$t > 0 : u = 0,\ \frac{\partial T}{\partial y} = -\frac{q}{k} \ \text{(UHF)},\ T = T_w \ \text{(UWT)},$$

$$\frac{\partial C}{\partial y} = -\frac{q^*}{D^*}C \ \text{ at } \ y = 0, \quad (4)$$

$$t > 0 : u \to 0,\ T \to T_\infty,\ C \to C_\infty \ \text{ as } \ y \to \infty,$$

where q is the mass flux and q^* mass flux at the plate.

In order to simplify the physical problem, the optically thick radiation limit is considered in the present analysis. The radiative heat flux for an optically thick fluid can be found from Rosseland approximation (1936) and its formula is derived from the diffusion concept of radiative heat transfer in the following way

$$q_r = -\frac{4\sigma}{3k^*}\frac{\partial T^4}{\partial y}, \quad (5)$$

where σ is the Stefan-Boltzman constant and k^* the spectral mean absorption coefficient of the medium. It should be noted that by using the Rosseland approximation we limit our analysis to optically thick fluids. If the temperature differences within the flow are sufficiently small, then the Eq. (5) can be linearized by expanding T^4 into the Taylor series about the free stream temperature T_∞ and neglecting higher order terms to give

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4. \quad (6)$$

On the use of (5) and (6), Eq.(2) becomes

$$\rho c_p \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial y^2} + \frac{16\sigma T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2}. \quad (7)$$

Introducing the dimensionless variables

$$\eta = \frac{y q^*}{D^*},\ \tau = \frac{t\nu q^{*2}}{D^{*2}},\ u_1 = \frac{u D^*}{q^*\nu},\ \theta = \frac{T - T_\infty}{T_w - T_\infty} \ \text{(UWT)},$$

$$\theta = \frac{(T - T_\infty) q^* K}{q D^*} \text{(UHF)},\ \ \phi = \frac{C - C_\infty}{C_\infty}, \quad (8)$$

Eqs. (1), (7) and (3) become

$$\frac{\partial u_1}{\partial \tau} = \frac{\partial^2 u_1}{\partial \eta^2} + \mathrm{Gr}\,\theta + \mathrm{Gc}\,\phi, \quad (9)$$

$$\alpha\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \eta^2}, \quad (10)$$

$$\mathrm{Sc}\frac{\partial \phi}{\partial \tau} = \frac{\partial^2 \phi}{\partial \eta^2}, \quad (11)$$

where $R = \dfrac{k k^*}{4\sigma T_\infty^3}$ is the radiation parameter, $\mathrm{Pr} = \dfrac{\rho\nu c_p}{k}$ the Prandtl number, $\mathrm{Gr} = \dfrac{g\beta q D^{*4}}{\nu^2 k q^{*4}}$ the thermal Grashof number (for UHF),

$\mathrm{Gr} = \dfrac{g\beta(T - T_\infty)D^{*3}}{\nu^2 q^{*2}}$ the thermal Grashof number (for UWT), $\mathrm{Gc} = \dfrac{g\,\beta^* D^{*3}\,C_\infty}{\nu^2 q^{*3}}$ the mass Grashof number, $\mathrm{Sc} = \dfrac{\nu}{D}$ Schmidt number and $\alpha = \dfrac{3\Pr R}{3R+4}$.

The corresponding initial and the boundary conditions are

$$\tau \le 0 : u_1 = 0,\ \theta = 0,\ \phi = 0,\ \text{for all}\ \eta \ge 0$$

$$\tau > 0 : u_1 = 0,\ \frac{\partial\theta}{\partial\eta} = -1\ \text{(UHF)},\ \theta = 1\ \text{(UWT)},$$

$$\frac{\partial\phi}{\partial\eta} = -(1+\phi)\ \text{at}\ \eta = 0, \tag{12}$$

$$\tau > 0 : u_1 \to 0, \theta \to 0,\ \phi \to 0\ \text{as}\ \eta \to \infty.$$

On the use of Laplace transformation, Eqs. (9) - (11) become

$$s\bar{u}_1 = \frac{\partial^2 \bar{u}_1}{\partial\eta^2} + \mathrm{Gr}\,\bar{\theta} + \mathrm{Gc}\bar{\phi}, \tag{13}$$

$$s\alpha\bar{\theta} = \frac{\partial^2\bar{\theta}}{\partial\eta^2}, \tag{14}$$

$$sSc\bar{\phi} = \frac{\partial^2\bar{\phi}}{\partial\eta^2}, \tag{15}$$

where

$$\bar{u}_1(\eta, s) = \int_0^\infty u_1(\eta,\tau)e^{-s\tau}\,d\tau,$$

$$\bar{\theta}(\eta, s) = \int_0^\infty \theta(\eta,\tau)e^{-s\tau}\,d\tau, \tag{16}$$

$$\bar{\phi}(\eta, s) = \int_0^\infty \phi(\eta,\tau)e^{-s\tau}\,d\tau$$

The corresponding boundary conditions for $\bar{u}_1$, $\bar{\theta}$ and $\bar{\phi}$ are

$$\bar{u}_1 = 0,\ \frac{\partial\bar{\theta}}{\partial\eta} = -\frac{1}{s}\ \text{(UHF)},\ \bar{\theta} = \frac{1}{s}\ \text{(UWT)},$$

$$\frac{\partial\bar{\phi}}{\partial\eta} = -\left(\frac{1}{s} + \bar{\phi}\right)\ \text{at}\ \eta = 0,$$

$$\bar{u}_1 \to 0,\ \bar{\theta} \to 0,\ \bar{\phi} \to 0\ \text{as}\ \eta \to \infty. \tag{17}$$

2.1 Solution for Uniform Heat Flux (UHF)

Solution of Eqs. (13)-(15) subject to the boundary conditions (17) are easily obtained and the inverse Laplace transform give the expression for the velocity, temperature and concentration distributions as

$$\theta(\eta,\tau) = \frac{1}{\sqrt{\alpha}} f_2(\eta\sqrt{\alpha}, \tau), \tag{18}$$

$$\phi(\eta,\tau) = f_4(\eta\sqrt{\mathrm{Sc}}, \tau) - f_1(\eta\sqrt{\mathrm{Sc}}, \tau), \tag{19}$$

$$\begin{aligned} u_1(\eta,\tau) = {} & \frac{Gr}{(\alpha-1)\sqrt{\alpha}}\left[f_5(\eta,\tau) - f_5(\eta\sqrt{\alpha},\tau)\right] \\ & + \frac{\mathrm{Sc}\,\mathrm{Gc}}{\mathrm{Sc}-1}\Big[f_1(\eta\sqrt{\mathrm{Sc}},\tau) - f_1(\eta,\tau) \\ & - f_4(\eta\sqrt{\mathrm{Sc}},\tau) + f_4(\eta,\tau)\Big] \\ & + \frac{\sqrt{\mathrm{Sc}}\,\mathrm{Gc}}{\mathrm{Sc}-1}\left[f_2(\eta\sqrt{\mathrm{Sc}},\tau) - f_2(\eta,\tau)\right] \\ & + \frac{\mathrm{Gc}}{\mathrm{Sc}-1}\left[f_3(\eta\sqrt{\mathrm{Sc}},\tau) - f_3(\eta,\tau)\right], \end{aligned} \tag{20}$$

where

$$f_1(\xi,\tau) = \operatorname{erfc}\left(\frac{\xi}{2\sqrt{\tau}}\right),$$

$$f_2(\xi,\tau) = 2\sqrt{\frac{\tau}{\pi}}\,e^{-\frac{\xi^2}{4\tau}} - \xi\operatorname{erfc}\left(\frac{\xi}{2\sqrt{\tau}}\right),$$

$$f_3(\xi,\tau) = \left(\tau + \frac{1}{2}\xi^2\right)\operatorname{erfc}\left(\frac{\xi}{2\sqrt{\tau}}\right) - \xi\sqrt{\frac{\tau}{\pi}}\,e^{-\frac{\xi^2}{4\tau}},$$

$$f_4(\xi,\tau) = e^{\frac{\tau}{Sc} - \frac{\xi}{\sqrt{Sc}}}\operatorname{erfc}\left(\frac{\xi}{2\sqrt{\tau}} - \sqrt{\frac{\tau}{Sc}}\right),$$

$$\begin{aligned} f_5(\xi,\tau) = {} & \frac{1}{3}\sqrt{\frac{\tau}{\pi}}\left(4\tau + \xi^2\right)e^{-\frac{\xi^2}{4\tau}} \\ & - \xi\left(\tau + \frac{1}{6}\xi^2\right)\operatorname{erfc}\left(\frac{\xi}{2\sqrt{\tau}}\right), \end{aligned} \tag{21}$$

and ξ is a dummy variable and f_1, f_2, f_3, f_4 and f_5 are the dummy functions and $\operatorname{erfc}(.)$ being the complementary error function.

2.2 Solution for Uniform Wall Temperature (UWT)

Solution of Eqs. (13) and (15) subject to the boundary conditions (17) are easily obtained and the inverse Laplace transform give the expression for the velocity and temperature fields as

$$\theta(\eta,\tau) = f_1(\eta\sqrt{\alpha},\tau), \tag{22}$$

$$\begin{aligned} u_1(\eta,\tau) = {} & \frac{\mathrm{Gr}}{(\alpha-1)}\left[f_3(\eta,\tau) - f_3(\eta\sqrt{\alpha},\tau)\right] \\ & + \frac{\mathrm{Sc}\,\mathrm{Gc}}{\mathrm{Sc}-1}\Big[f_1(\eta\sqrt{\mathrm{Sc}},\tau) - f_1(\eta,\tau) \\ & - f_4(\eta\sqrt{\mathrm{Sc}},\tau) + f_4(\eta,\tau)\Big] \\ & + \frac{\sqrt{\mathrm{Sc}}\,\mathrm{Gc}}{\mathrm{Sc}-1}\left[f_2(\eta\sqrt{\mathrm{Sc}},\tau) - f_2(\eta,\tau)\right] \\ & + \frac{\mathrm{Gc}}{\mathrm{Sc}-1}\left[f_3(\eta\sqrt{\mathrm{Sc}},\tau) - f_3(\eta,\tau)\right], \end{aligned} \tag{23}$$

and f_1, f_2, f_3 and f_4 are given by (21).

For constant mass diffusion at the plate, the present problem reduces to the problem studied by Narahari and Dutta (2012).

2.3 Solution for Pure Convection

In the absence of thermal radiation, i.e. if pure convection prevails (corresponds to $R \to \infty$), it is observed that $\alpha = \Pr$ and the solution for the temperature given by Eq. (18) is valid for all values of $\Pr$, but the solution for velocity given by Eq. (20) is not valid for $\Pr = 1$ or/and $\mathrm{Sc} = 1$. Since the Prandtl number is a measure of the relative importance of the viscosity and thermal conductivity of the fluid, the case $\Pr = 1$ corresponds to those fluids whose momentum and thermal boundary layer thicknesses are of the same order of magnitude. Therefore, the solution for the velocity field in the absence of thermal radiation effects when $\Pr = 1$ or/and $\mathrm{Sc} = 1$ has to be obtained separately from Eqs. (9)-(11) subject to the initial and boundary conditions (12) and given by

(i) For uniform heat flux temperature (UHF)

$$u_1(\eta,\tau) = \begin{cases} \frac{1}{2}\mathrm{Gr}\,\eta\, f_3(\eta,\tau) \\ +\frac{\mathrm{Sc\,Gc}}{\mathrm{Sc}-1}\Big[f_1(\eta\sqrt{\mathrm{Sc}},\tau) - f_1(\eta,\tau) \\ - f_4(\eta\sqrt{\mathrm{Sc}},\tau) + f_4(\eta,\tau)\Big] \\ +\frac{\sqrt{\mathrm{Sc}}\,\mathrm{Gc}}{\mathrm{Sc}-1}\Big[f_2(\eta\sqrt{\mathrm{Sc}},\tau) - f_2(\eta,\tau)\Big] \\ +\frac{\mathrm{Gc}}{\mathrm{Sc}-1}\Big[f_3(\eta\sqrt{\mathrm{Sc}},\tau) - f_3(\eta,\tau)\Big] \\ \qquad \text{for } \Pr = 1,\ \mathrm{Sc} \neq 1 \\ \frac{1}{2}\eta\Big[\mathrm{Gr}\, f_3(\eta,\tau) + \mathrm{Gc}\{ f_6(\eta,\tau) \\ - f_1(\eta,\tau) - f_2(\eta,\tau)\}\Big] \quad \text{for } \Pr = 1,\ \mathrm{Sc} = 1 \end{cases} \tag{24}$$

where

$$f_6(\xi,\tau) = e^{\tau-\xi}\,\mathrm{erfc}\left(\frac{\xi}{2\sqrt{\tau}} - \sqrt{\tau}\right) \tag{25}$$

and f_6 is the dummy function.

(ii) For uniform wall temperature (UWT)

$$u_1(\eta,\tau) = \begin{cases} \frac{1}{2}\mathrm{Gr}\,\eta\, f_2(\eta,\tau) \\ +\frac{\mathrm{ScGc}}{\mathrm{Sc}-1}\Big[f_1(\eta\sqrt{\mathrm{Sc}},\tau) - f_1(\eta,\tau) \\ - f_4(\eta\sqrt{\mathrm{Sc}},\tau) + f_4(\eta,\tau)\Big] \\ +\frac{\sqrt{\mathrm{Sc}}\,\mathrm{Gc}}{\mathrm{Sc}-1}\Big[f_2(\eta\sqrt{\mathrm{Sc}},\tau) - f_2(\eta,\tau)\Big] \\ +\frac{\mathrm{Gc}}{\mathrm{Sc}-1}\Big[f_3(\eta\sqrt{\mathrm{Sc}},\tau) - f_3(\eta,\tau)\Big] \\ \qquad \text{for } \Pr = 1,\ \mathrm{Sc} \neq 1 \\ \frac{1}{2}\eta\Big[\mathrm{Gr}\, f_2(\eta,\tau) + \mathrm{Gc}\{ f_6(\eta,\tau) \\ - f_1(\eta,\tau) - f_2(\eta,\tau)\}\Big] \\ \qquad \text{for } \Pr = 1,\ \mathrm{Sc} = 1 \end{cases} \tag{26}$$

and f_1, f_2, f_3, f_4 are given by (21) and f_6 by (23).

3. RESULT AND DISCUSSION

In order to get physical insight into the problem, a parametric study is performed and the obtained numerical results are displayed with the help of graphical illustrations. We have presented the non-dimensional fluid velocity u_1 and the fluid temperature θ for several values of radiation parameter R, Grashof number Gr, mass Grashof number Gc, Prandtl number $\Pr$, Schmidt number Sc and time τ in Figs. 2-11.The values of Schmidt number are chosen to represent the presence of species by rare-gas (0.45), water vapour (0.62), ammonia (0.78) and Propyl Benzene (2.62) at 25^o C temperature and 1 atmospheric pressure. The values of $\Pr$ are chosen 0.72 and 7.1 which represent air and water, respectively, at 20^o C temperature and 1 atmospheric pressure. The values of radiation parameter and chemical reaction parameter are chosen arbitrarily. It is revealed from Fig. 2 that the fluid velocity u_1 decreases with an increase in radiation parameter R for both UHF and UWT cases. This means that the radiation decelerates the fluid velocity, but it is slightly greater in the case of UHF than that of UWT. Such an effect may also be expected, as increasing radiation parameter R makes the fluid thick and ultimately causes the velocity and the momentum boundary layer thickness to decrease.

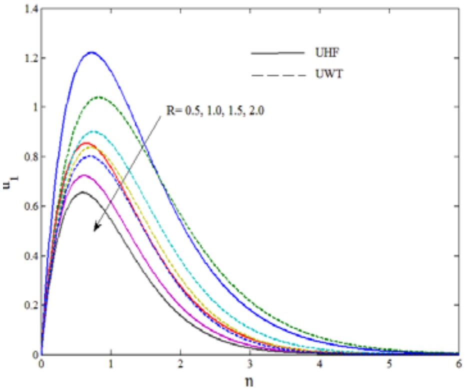

Fig. 2. Velocity profiles u_1 for different R when $\mathrm{Gr} = 5$, $\mathrm{Gc} = 3$, $\Pr = 0.72$, $\mathrm{Sc} = 2.62$ and $\tau = 0.5$.

The influence of thermal Grashof number Gr on the fluid velocity u_1 is elucidated from Fig. 3. It can be observed that the fluid velocity u_1 increases for the increasing values of Gr for both UHF and UWT cases.

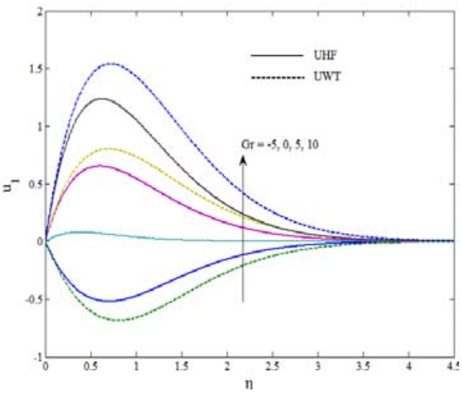

Fig. 3. Velocity profiles u_1 for different Gr when $R = 2$, $\mathrm{Gc} = 3$, $\Pr = 0.72$, $Sc = 2.62$ and $\tau = 0.5$.

It is true physically as the thermal Grashof number Gr describes the ratio of bouncy forces to viscous forces. Therefore, an increase in the values of Gr leads to increase in buoyancy forces, consequently velocity increases. Here the thermal Grashof number represents the effect of free convection currents. Physically, $\mathrm{Gr} > 0$ means heating of the fluid of cooling of the boundary surface, $\mathrm{Gr} < 0$ means cooling of the fluid of heating of the boundary surface and $\mathrm{Gr} = 0$ corresponds the absence of free convection current. It is also noticed from Fig. 3 that there exists a reverse type of flow near the plate for $\mathrm{Gr} < 0$. Fig. 4 shows that the fluid velocity u_1 increases with an increase in mass Grashof number Gc. This is due to larger buoyancy force caused by the concentration difference near the plate. The mass Grashof number $\mathrm{Gc} > 0$ indicates that the chemical species concentration in the free stream region is less than the concentration at the boundary surface. The cooling problem is often encountered in engineering applications.

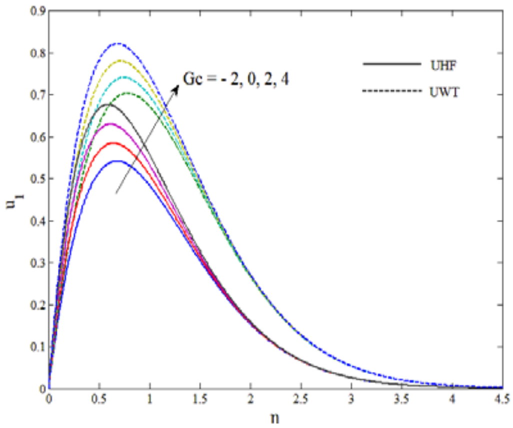

Fig. 4. Velocity profiles u_1 for different Gc when $\mathrm{Gr} = 5$ **,** $R = 2$ **,** $\mathrm{Pr} = 0.72$ **,** $\mathrm{Sc} = 2.62$ **and** $\tau = 0.5$ **.**

Fig. 5 displays that the fluid velocity u_1 decreases with an increase in Prandtl number Pr for both UHF and UWT cases. Physically, it is true as the Prandtl number describes the ratio between momentum diffusivity and thermal diffusivity and hence controls the relative thickness of the momentum and thermal boundary layers. As Pr increases the viscous forces (momentum diffusivity) dominate the thermal diffusivity and consequently decreases the velocity. The values of Schmidt number Sc were chosen to be $\mathrm{Sc} = 0.45$, 0.62, 0.78 and 2.62 representing diffusing chemical species of most common interest. It is seen from Fig. 6 that the fluid velocity u_1 decreases with an increase in Schimdt number Sc for both UHF and UWT cases. The physics behind this observation is that the increased Schmidt number decreases chemical species molecular diffusivity, which ultimately reduces the fluid velocity. Also, an increase of Sc (a predominance of diffusive transport of momentum over that of mass) represents increase in the momentum boundary layer thickness with a fixed species diffusivity and this causes the decrease in velocity.

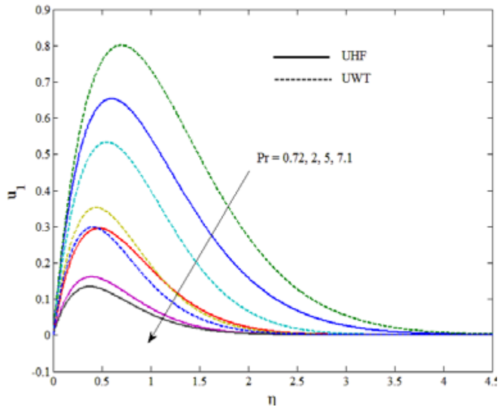

Fig. 5. Velocity profiles u_1 for different Pr when $\mathrm{Gr} = 5$ **,** $\mathrm{Gc} = 3$ **,** $R = 2$ **,** $\mathrm{Sc} = 2.62$ **and** $\tau = 0.5$ **.**

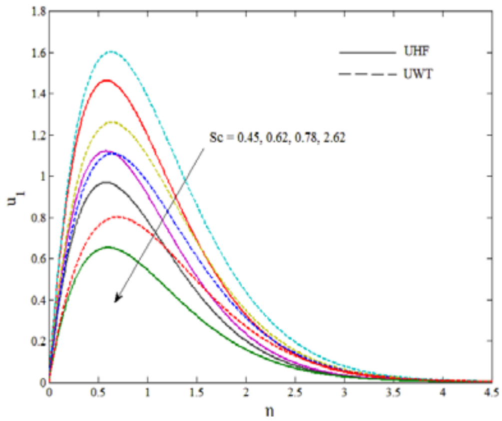

Fig. 6. Velocity profiles u_1 for different Sc when $\mathrm{Gr} = 5$ $\mathrm{Gc} = 3$ **,** $\mathrm{Pr} = 0.72$ **,** $R = 2$ **and** $\tau = 0.5$ **.**

Fig. 7 reveals that the fluid velocity u_1 increases near the plate and it oscillates away from the plate with an increase in time τ. It is worth mentioning form Figs. 3-7 that the fluid velocity is slightly greater in the case of UWT than that of UHF.

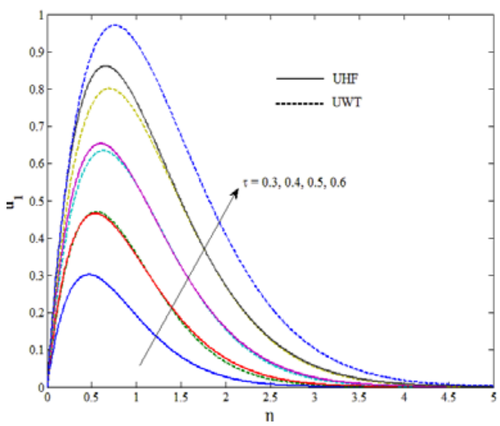

Fig. 7. Velocity profiles u_1 for different τ when $\mathrm{Gr} = 5$ **,** $\mathrm{Gc} = 3$ **,** $\mathrm{Pr} = 0.72$ **,** $\mathrm{Sc} = 2.62$ **and** $R = 2$ **.**

It is observed from Fig. 8 that the fluid temperature θ decreases with an increase in radiation parameter R for both UHF and UWT cases. In the presence of radiation, the thermal boundary layer always found to thicken which implies that the radiation provides an additional means to diffuse energy. This means that the thermal boundary layer decreases and more uniform temperature distribution across the boundary layer.

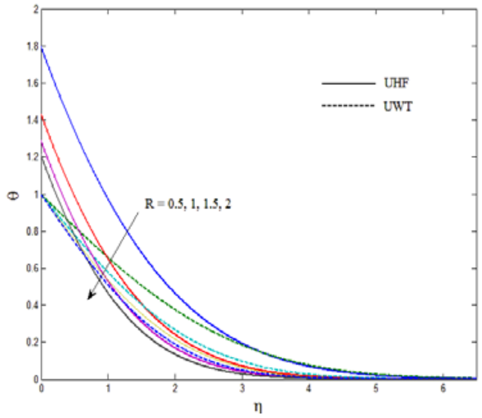

Fig. 8. Temperature profiles for different R when $\mathrm{Pr} = 0.72$ and $\tau = 0.5$.

Fig. 9 shows that the fluid temperature θ decreases with an increase in Prandtl number Pr for both UHF and UWT cases. The reason is that smaller values of Pr are equivalent to increase in the thermal conductivity of the fluid and therefore heat is able to diffuse away from the heated surface more rapidly for higher values of Pr .

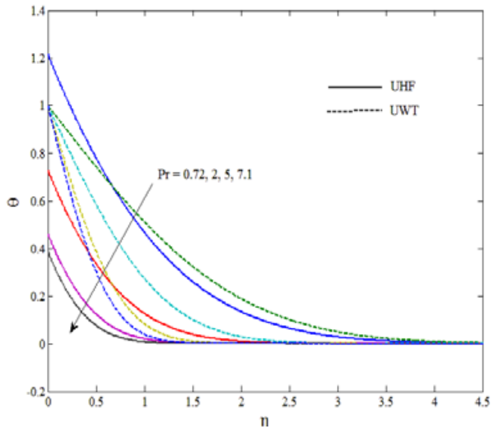

Fig. 9. Temperature profiles for different Pr when $R = 2$ and $\tau = 0.5$.

Hence in the case of smaller Prandtl numbers the thermal boundary layer is thicker and the heat transfer is reduced. The temperature profiles for different values of time τ are presented in Fig. 10. It is observed that these profiles increase with increasing time τ for both UHF and UWT cases. The temperature is highest near the plate surface and decreases asymptotically to the free stream zero value far away from the plate. It is seen from Fig. 11 that the concentration profiles increase with an increase in Schimdt number Sc . As expected, the mass transfer decreases as Sc increases by keeping all other physical parameter fixed, i.e. an increase in the value of the Schimdt number Sc is associated with the reduction in the concentration profiles.

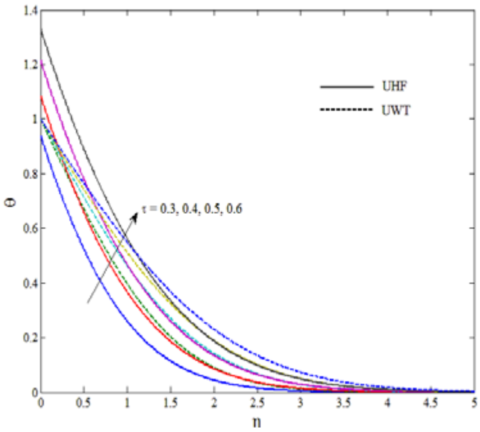

Fig. 10. Temperature profiles for different τ when $\mathrm{Pr} = 0.72$, $\mathrm{Sc} = 2.62$ and $R = 2$.

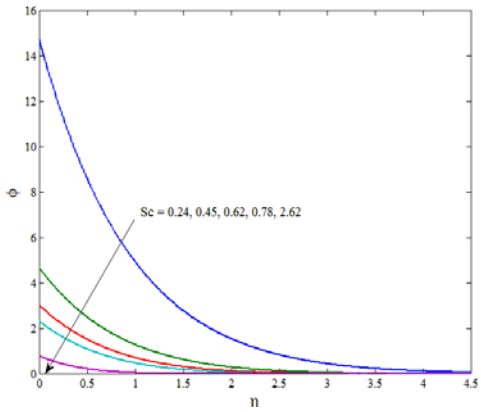

Fig. 11. Concentration profiles for different Sc when $\tau = 0.5$.

Further, it may also be observed from this figure that the effect of Schmidt number Sc on concentration distribution slowly decreases in the concentration boundary layer for higher values of Sc . The mass concentration profiles for different values of time τ are presented in Fig. 12. It is observed that the mass concentration profiles increase with an increase in time τ . The mass concentration are highest near the plate surface and decreases asymptotically to the free stream zero value far away from the plate.

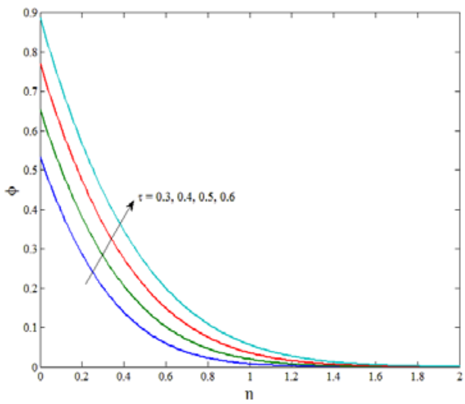

Fig. 12. Concentration profiles for different τ when $\mathrm{Pr} = 0.72$, $\mathrm{Sc} = 2.62$ and $R = 2$.

From the engineering point of view, the most important characteristic of the flow is the shear stress at the plate $\eta = 0$ which is given

$$\tau_x = \left(\frac{\partial u_1}{\partial \eta}\right)_{\eta=0}$$

$$= \begin{cases} \dfrac{\mathrm{Gr}\,\tau}{\sqrt{\alpha}(\sqrt{\alpha}+1)} + \dfrac{\mathrm{Gc}\sqrt{\mathrm{Sc}}}{(\sqrt{\mathrm{Sc}}+1)}, \\ \times\left[e^{\frac{\tau}{\mathrm{Sc}}}\left\{1+\mathrm{erf}\left(\sqrt{\dfrac{\tau}{\pi}}\right)\right\} - 2\sqrt{\dfrac{\tau}{\pi \mathrm{Sc}}} - 1\right] & \text{for UHF} \\ \dfrac{\mathrm{Gr}\sqrt{\tau}}{\sqrt{\pi}(\sqrt{\alpha}+1)} + \dfrac{\mathrm{Gc}\sqrt{\mathrm{Sc}}}{(\sqrt{\mathrm{Sc}}+1)} \\ \times\left[e^{\frac{\tau}{\mathrm{Sc}}}\left\{1+\mathrm{erf}\left(\sqrt{\dfrac{\tau}{\pi}}\right)\right\} - 2\sqrt{\dfrac{\tau}{\pi \mathrm{Sc}}} - 1\right] & \text{for UWT} \end{cases} \tag{27}$$

Numerical values of the non-dimensional shear stress at the plate $\eta = 0$ are presented in Figs. 13-17 for several values of thermal Grashof number Gr, mass Grashof number Gc, radiation parameter R, Prandtl number Pr and time τ. Figs. 13-14 shows that the shear stress τ_x at the plate $\eta = 0$ increases with an increase in either thermal Grashof number Gr or mass Grashof number Gc for both UHF and UWT cases.

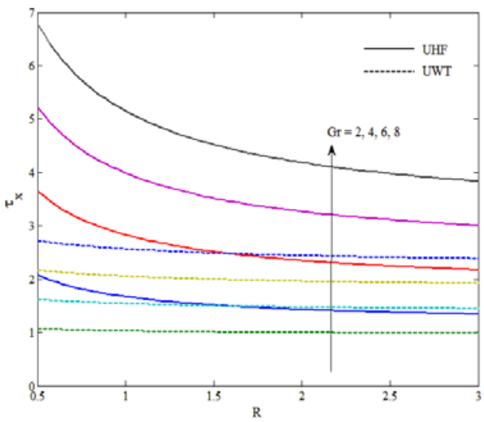

Fig. 13. Shear stress τ_x for *Gr* when $\mathrm{Gc} = 3$**,** $\mathrm{Sc} = 2.62$**,** $\mathrm{Pr} = 0.71$ **and** $\tau = 0.5$**.**

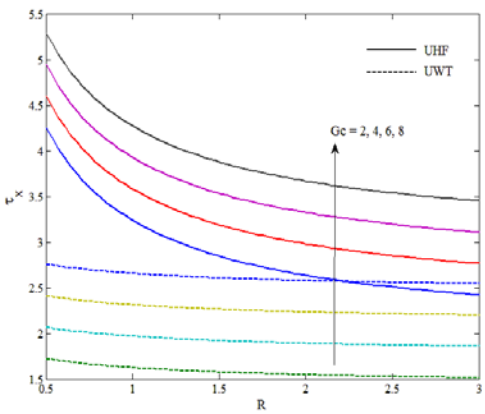

Fig. 14. Shear stress τ_x for *Gc* when $\mathrm{Gr} = 5$**,** $\mathrm{Sc} = 2.62$**,** $\mathrm{Pr} = 0.71$ **and** $\tau = 0.5$**.**

Fig. 15 reveals that the shear stress τ_x at the plate $\eta = 0$ reduces for increasing values of Schmidt number *Sc* for both UHF and UWT cases. Physically, it is correct since an increase in Sc serves to decrease momentum boundary layer thickness.

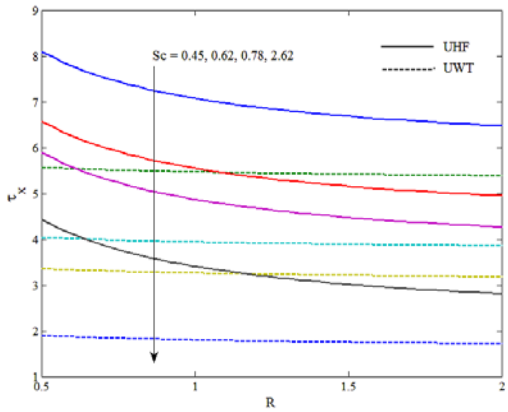

Fig. 15. Shear stress τ_x for Sc **when** $\mathrm{Gr} = 5$**,** $\mathrm{Gc} = 3$**,** $\mathrm{Pr} = 0.71$ **and** $\tau = 0.5$**.**

It is seen from Fig. 16 that the shear stress τ_x reduces with an increase in Prandtl number Pr for both UHF and UWT cases. It is observed that an increase in the Prandtl number results a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer.

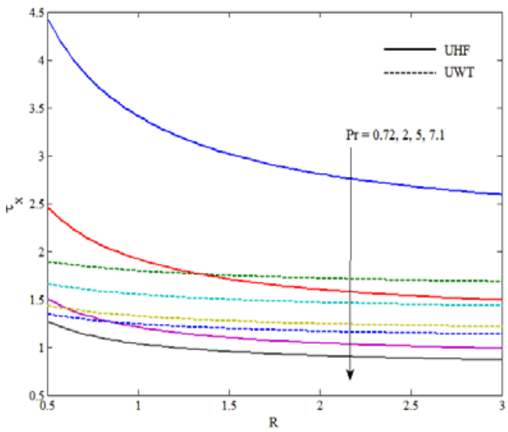

Fig. 16. Shear stress τ_x for Pr **when** $\mathrm{Gr} = 5$**,** $\mathrm{Sc} = 2.62$**,** $\mathrm{Gc} = 3$ **and** $\tau = 0.5$**.**

The reason is that smaller values of Pr are equivalent to increase in the thermal conductivity of the fluid and therefore, heat is able to diffuse away from the heated surface more rapidly for higher values of Pr. Hence, in the case of smaller Prandtl number as the thermal boundary layer is thicker and the shear stress at the plate is reduced. Fig. 17 reveals that the shear stress τ_x increases when time τ progresses for both UHF and UWT cases. This is due to increase the momentum boundary layer thickness as time progresses . It is also seen that the shear stress τ_x reduces for increasing values of radiation parameter R for both UHF and UWT cases. It is noted form Figs. 13-17 that the shear stress at the plate is greater in the case of UHF than that of UWT.

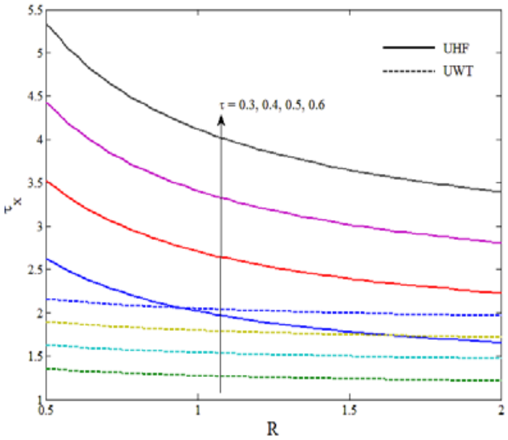

Fig. 17. Shear stress τ_x for time τ when $\mathrm{Gr} = 5$ **,** $\mathrm{Sc} = 2.62$ **,** $\mathrm{Pr} = 0.71$ **and** $\mathrm{Gc} = 3$ **.**

4. CONCLUSION

Exact solutions for the unsteady free convection flow of an optically dense incompressible viscous fluid near an infinite vertical plate with heat and mass transfer in the presence of thermal radiation have been obtained using the Laplace transform technique. The effects of the system parameters such as the radiation parameter, buoyancy forces, Schmidt number and time on the fluid velocity and temperature fields, mass concentration and shear stress at the plate have been studied in detail. The study reveals that these parameters have significant influence on the velocity, temperature, mass concentration and shear stress at the plate. The following conclusions are extracted from this study:

- An increase in the radiation parameter leads to a decrease the fluid velocity as well as the temperature. The fluid velocity in the case of UHF is slightly higher than the case of UWT for small values of the radiation parameter.
- The fluid velocity and temperature decrease with an increase in the Prandtl number.
- The fluid velocity increases with increases in the Grashof numbers and an increase in time leads to increase in the fluid velocity, temperature and concentration for both UHF and UWT cases.
- The thermal boundary layer thickness increases with increasing the radiation parameter and the Prandtl number.
- The concentration boundary layer thickness decreases with increasing values of the Schmidt number whereas it increases when time progresses.
- The shear stress at the plate is slightly higher in the case of UHF than that of UWT for all pertinent parameters.

It is expected that the results of this study will serve as a foundation for more complex and physically more realistic cases of free convective flows resulting from combined heat and mass transfer past a vertical surface in the presence of thermal radiation.

REFERENCES

Ahmed, N. and H. K.Sarmah (2009). Thermal radiation effect on a transient MHD flow with mass transfer past an impulsively started infinite vertical plate. *Int. J. Appl. Math. Mech.* 5(5), 87- 98.

Al-Mudhaf, A. and A. J. Chamkha (2005). Similarity solutions for MHD thermosolutal Marangoni convection over a flat surface in the presence of heat generation or absorption effects. *Heat and Mass Transfer 42(2), 112-121.*

Chamkha, A. J., A. M. Aly and M. A. Mansour (2010). Similarity solution for unsteady heat and mass transfer from a stretching surface embedded in a porous medium with suction/injection and chemical reaction effects. *Chemical Engineering Communications 197 (6), 846-858.*

Chamkha, A. J. and A. R. A. Khaled (1999). Nonsimilar hydromagnetic simultaneous heat and mass transfer by mixed convection from a vertical plate embedded in a uniform porous medium. *Numerical Heat Transfer: Part A: Applications 36(3), 327-344.*

Chaudhary, R. C. and J. Arpita (2007). Combined heat and mass transfer effect on MHD free convection flow past an oscillating plate embedded in porous medium. *Romanian J. Physics* 52, 505-524.

Chaudhary, R. C. and A. Jain (2007). Unsteady free convection flow past an oscillating plate with constant mass flux in the presence of radiation. *Acta Technica CSAV* 52(1), 93-108.

Cussler, E. L. (1998). Diffusion Mass Transfer in Fluid Systems, Cambridge University Press, Cambridge, UK.

Das, K. and S. Jana (2010). Heat and mass transfer effects on unsteady MHD free convection flow near a moving vertical plate in porous medium. *Bull. Soc.Mathematicians* 17, 15–32.

Das, U. N., S. N. Ray and V. M. Soundalgekar (1996). Mass transfer effects on flow past an impulsively started infinite vertical plate with constant mass flux- an exact solution. *Heat and Mass Transfer* 31(3), 163-167.

Deka, D. and R. K. Deka (2011). Radiation effects on MHD flows past an infinite vertical plate with variable temperature and uniform mass diffusion. *JP J. Math. Sci.* 1(1), 15-29.

Deka, R. K. and B. C. Neog (2009). Unsteady MHD flow past a vertical oscillating plate with thermal radiation and variable mass diffusion. *Chamchuri J. Mathematics* 1(2), 79-92.

Hussanan, A., I. Khan and S. Sharidan (2013). An exact analysis of heat and mass transfer past a vertical plate with Newtonian heating. *J. Appl. Math.*

Jain, A. (2013). Radiation and chemical reaction effects on unsteady double diffusive convective flow past an oscillating surface with constant heat flux. *ISRN Chemical Engineering* .

Juncu, G. (2005). Unsteady forced convection heat/mass transfer from a flat plate. *Heat and Mass Transfer/Waermeund Stoffuebertragung* 41(12), 1095-1102.

Khan, A., I. Khan, F. Ali, S. Ulhaq and S. Shafie (2014). Effects of wall shear stress on unsteady MHD conjugate flow in a porous medium with ramped wall temperature. *PLOS ONE*, 9(3), 1-12.

Magyari, E. and A.J. Chamkha (2008). Exact analytical results for the thermosolutal MHD Marangoni boundary layers. *International Journal of Thermal Sciences 47 (7), 848-857.*

Muralidharan, M. and R. Muthucumaraswamy (2010). Thermal radiation on linearly accelerated vertical plate with variable temperature and uniform mass flux. *Ind. J. Sci. Tech*. 3, 398-401.

Muthucumaraswamy, R. and A. Vijayalakshmi (2008). Effects of heat and mass transfer on flow past an oscillating vertical plate with variable temperature. *Int. J. Appl. Math. Mech.* 4, 59-65.

Muthucumaraswamy, R., M. R. Sundar and V. S. A. Subramanian (2009). Unsteady flow past an accelerated infinite vertical plate with variable temperature and uniform mass diffusion. *Int. J. Appl. Math. Mech.* 5, 51-56.

Narahari, M. and B. K. Dutta (2012). Effects of thermal radiation and mass diffusion on free convection flow near a vertical plate with Newtonian heating. *Chem. Eng. Comm*. 199(5), 628-643.

Narahari, M. and B.K. Dutta (2012). Effects of thermal radiation and mass diffusion on free convection flow near a vertical plate with Newtonian heating. *Chem. Eng. Commun*. 199, 628–643.

Narahari, M. and A. Ishaq (2011). Radiation effects on free convection flow near a moving vertical plate with Newtonian heating. *J. Appl. Sci*. 11, 1096-1104.

Narahari, M. and M.Y. Nayan (2011). Free convection flow past an impulsively started infinite vertical plate with Newtonian heating in the presence of thermal radiation and mass diffusion, *Turkish J. Eng. Env. Sci.* 35, 187-198.

Rajesh, V. (2011). Chemical reaction and radiation effects on the transient MHD free convection flow of dissipative fluid past an inflnite vertical porous plate with ramped wall temperature. *Chem. Ind. Chem. Eng. Quar*. 17, 189–198.

Rajesh, V. and S. V. K. Varma (2009). Radiation and mass transfer effects on MHD free convection flow past an exponentially accelerated vertical plate with variable temperature. *ARPN J. Engin. Appl. Sci*. 4(6), 20-26.

Rajput, U. S. and S. Kumar (2012). Radiation effects on MHD flow past an impulsively started vertical plate with variable heat and mass transfer. *Int. J. Applied Math. Mech.* 8, 66-85.

Rajput, U. S. and S. Kumar (2011). MHD flow past an impulsively started vertical plate with variable temperature and mass diffusion. *Appl. Math. Sci*., 5(1-4), 149-157.

Rajput, U. S. and S. Kumar (2012). Radiation effects on MHD flow past an impulsively started vertical plate with variable heat and mass transfer. *Int. J. Appl. Math. Mech.* 8(1), 66-85.

Reddy, P., B. Anki and N. B. Reddy (2010). Radiation effects on MHD combined convection and mass transfer flow past a vertical porous plate embedded in a porous medium with heat generation. *Int. J. Appl. Math. Mech.* 6(18), 33 -49.

Rosseland, S. (1936). Theoretical Astrophysics, Oxford University, New York, NY, USA.

Sangapatnam, S., B. R. Nandanoor and R. P. Vallampati (2009). Radiation and mass transfer effects on MHD free convection flow past an impulsively started isothermal vertical plate with dissipation. *Thermal Science* 13(2), 171-181.

Takhar, H. S., A. J. Chamkha and G. Nath (1999). Unsteady flow and heat transfer on a semi-infinite flat plate with an aligned magnetic field. *International Journal of Engineering Science 37 (13), 1723-1736.*

Uwanta, I. J. and M. Sani (2012). Transient convection fluid flow with heat flux in an infinite vertical plate with chemical mass transfer. *Int. J. Appl. Inf. Sys.* 4(7), 21-27.