Journal of Applied Fluid Mechanics, Vol. 8, No. 2, pp. 265-272, 2015.
Available online at www.jafmonline.net, ISSN 1735-3572, EISSN 1735-3645.
DOI: 10.18869/acadpub.jafm.67.221.22830

On the Onsetof Thermal Instability in a Low Prandtl Number Nanofluid Layer in a Porous Medium

R. Chand[1†], G. C. Rana[2] and A. K. Hussein[3]

[†1]*Department of Mathematics, Government College Dhaliara, Himachal Pradesh, 177103, India*
[2]*Department of Mathematics, Government College, Nadaun, Himachal Pradesh, 177033, India*
[3]*College of Engineering, Mechanical Engineering Department, Babylon University- Babylon City, Iraq*

†*Corresponding Author Email: rameshnahan@yahoo.com*

(Received March 13, 2014; accepted May 21, 2014)

ABSTRACT

Thermal instability in a low Prandtl number nanofluid in a porous medium is investigated by using Galerkin weighted residuals method for free-free boundaries. For porous medium, Brinkman-Darcy modelis applied. The model used for the nanofluid describes the effects of Brownian motion and thermophoresis. Linear stability theory based upon normal mode analysis is employed to find the expression for stationary and oscillatory convection. The effects of Prandtl- number, Darcy number, Lewis number and modified diffusivity ratio on the stationary convection are investigated both analytically and graphically. The results indicated that the Prandtl and Darcy numbers have a destabilizing effect while the Lewis number and modified diffusivity ratio have a stabilizing effect for the stationary convection.

Keywords: Nanofluid, Grashof number, Low Prandtl- number, Darcy number, Galerkin method, Porous medium.

NOMENCLATURE

a	dimensionless resultant wave number
d	thickness of nanofluid layer
D_B	Brownian diffusion coefficient
D_T	thermophoretic diffusion coefficient
Da	Darcy number
g	acceleration due to gravity
k_1	medium permeability
k_m	thermal conductivity
Le	Lewis number
N_A	modified diffusivity ratio
N_B	modified particle -density increment
n	growth rate of disturbances
Pr	Prandtl number
p	hydrostatic pressure
q	Darcy velocity vector
Ga	Grashof number
Ga_c	critical Grashof number
Gm	density Grashof number
Gn	concentration Grashof number
T	temperature
T_1	reference temperature
t	time
u,v,w	velocity components
(x,y,z)	space co-ordinates

Greek symbols

α	thermal expansion coefficient
μ	viscosity
ε	porosity
ρ	density of the nanofluid
$(\rho c)_m$	heat capacity in porous medium
$(\rho c)_p$	heat capacity of nanoparticles
φ	volume fraction of the nanoparticles
ρ_p	density of nano particles
ρ_f	density of base fluid
κ	thermal diffusivity
ω	dimensionless frequency

Superscripts

'	non-dimensional variables
''	perturbed quantities

Subscripts

p	particle
f	fluid
0	lower boundary
1	upper boundary
H	horizontal plane

1. INTRODUCTION

When a small amount of nano-sized particles are added to the base fluid, the thermal conductivity of the fluid is enhanced and such a fluid is called nanofluid which was first coined by Choi (1995).Due to this property of the nanofluid, they have wide range of industrial applications especially in the process where cooling is of primary interest. Buongiorno (2006) observed that the nanoparticles absolute velocity can be viewed as the sum of the base fluid velocity and a relative (slip) velocity. Alloui *et al.* (2010) studied the natural convection of a nano fluid in a shallow cavity heated from below. The Bénard problem (the onset of convection in a horizontal layer which was uniformly heated from below) for a nanofluid was studied by Tzou (2008a, b) on the basis of Buongiorno's model. Effect of axial conduction and variable properties on two-dimensional conjugate heat transfer of nanofluid in microchannel is studied by Rramiar *et al.* (2012) while Analytical study on boundary layer flow and heat transfer of nanofluid induced by a non-linearly stretching sheet is studied by Malvandi *et al.* (2014). Nield and Kuznetsov (2010a and 2011a) studied the convection in a nanofluid layer of finite depth and the double-diffusive convection in a nanofluid layer. Thermal instability in a porous medium has many technological applications in geophysics, food processing, oil reservoir modeling, petroleum industry, bio-mechanics, building of thermal insulations and nuclear reactors. Many researchers had investigated thermal instability problems by taking different types of fluids. Lapwood (1948) had studied the convective flow in a porous medium using linearized stability theory. The Rayleigh instability of a thermal boundary layer in the flow through a porous medium had been considered by Wooding (1960). A good account of convection problems in a porous medium are given by Vafai and Hadim (2000), Ingham and Pop (1981) and Nield and Bejan (2006) respectively.

Owing, applications of the nanofluid and porous media theory in chemical engineering to study theory in drying and freezing of food, cooling of microchips in computers by using of metal foams and their use in heat pipesetc. Therefore, the study of nanofluid in a porous medium turns to be important to researchers. Recently, the thermal instability of a nanofluid in a porous medium had been investigated by Nield and Kuznetsov (2009), Nield and Kuznetsov (2010b, 2011b), Kuznetsov and Nield (2010 a, b, c), Sheu (2011), Chand and Rana (2012a, b, c), Chand (2013a, b) and Chand *et al.* (2013a, b).

The problem of thermal convection of a Boussinesq fluid in ahorizontal layer heated from below is characterized by two non-dimensional parameters, the Rayleigh number and the Prandtl number. Since the Rayleigh number Rais defined in such a way that the onset of convection occurs at a value Ra**,** independent of the Prandtl number Pr, the latter usually plays a secondary role in the study of convection. But the influence of the Prandtl number on nonlinear properties such as the heat transport is significant and not yet well understood. In particular, in the limit of low Prandtl number, large discrepancies exist between various theoretical predictions for the convective heat transport. Low-Prandtl-number problems are important many engineering applications; most studies of low-Prandtl-number convection have been motivated by astrophysical applications. Since Prandtl numbers in stars may be as low as 10^{-8}, as pointed as noted by Clever and Busse (1981). Low-Prandtl-number flows are especially important to the dynamics of the outer planets and their moons. The surface of Jupiter is formed by convection in a fluid with a Prandtl number of around 0.01 [Zhang and Schubert (2000)]. The Galileo missions have confirmed the existence of convecting oceans on the Jovian moons Europa and Callisto [Khurana, 1998]. Low-Prandtl-number convection at high Rayleigh numbers will allow us to better model heat flux and surface deformation on the Jovian moons. Thermal convection in Rayleigh-Bénard problem is important in the flow of liquid metal which has been used for rapid cooling of nuclear reactors where Prandtl number is very-very small [Steward and Weinberg (1972), Mohamad and Viskanta (1991)]. Cookey (2009) studied the thermal convection of low Prandtl number in a horizontal fluid layer through a porous medium and found that low Prandtl number destabilize the fluid layer.One of the main motivations for the current research on Rayleigh-Benard convection in a layer of nanofluid is to theoretically understand turbulent thermal convection for very large Rayleigh number Ra, far beyond the range after onset of convection. In the post onset range still spatially coherent patterns prevail.

In present paper, an attempt is made to investigate the thermal instability in a layer nanofluid having a Prandtl number of the range $0.1 < Pr < 0.0001$ in a porous medium

2. MATHEMATICAL FORMULATION AND PERTURBATION EQUATIONS.

Consider an infinite horizontal layer of a nanofluid of thickness 'd' bounded by planes $z = 0$ and $z = d$, heated from below in a porous medium of porosity (ε) and medium permeability (k_1) as shown in Fig.1. It is assumed that nanoparticles are suspended in the nanofluid using either surfactant or surface charge technology. This prevents particles from agglomeration and deposition on the porous matrix. Each boundary wall is assumed to be impermeable and perfectly thermal conducting. Fluid layer is acted upon by gravity force **g** (0, 0, -g). The temperature (T) and volumetric fraction (φ) of nanoparticles at $z = 0$ are taken to be T_0, φ_0 and T_1, φ_1 at $z = d$, where ($T_0 > T_1$). The reference temperature is taken to be T_1. Thermo physical properties of the nanofluid are assumed to be constant for the analytical formulation, but these properties are not constant and strongly depend upon volume fraction of nanoparticles. We are aware that thermal lagging between particles and the fluid has been proposed as an explanation of the increased thermal conductivity that has been observed in the nanofluid [Vadasz (2006)].

According to works of Tzou(2008a, b), Buongiorno (2006), Wooding (1960) and Chandrasekhar (1961),equations of continuity and motion for nanofluid layer in a porous medium are written as

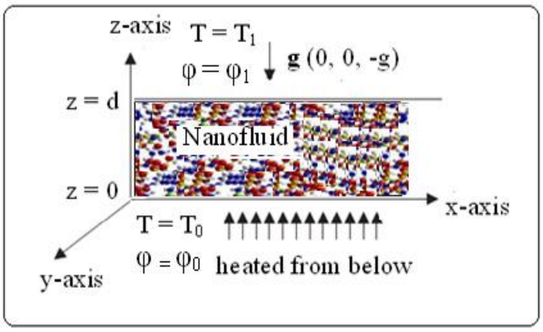

Fig. 1. Physical configuration of the problem

$$\nabla \cdot \mathbf{q} = 0, \qquad (1)$$

$$\frac{\rho}{\varepsilon}\frac{d\mathbf{q}}{dt} = -\nabla p + \left(\varphi\rho_p + (1-\varphi)\{\rho(1-\alpha(T-T_0))\}\right)\mathbf{g} - \frac{\mu}{k_1}\mathbf{q}, \qquad (2)$$

Where **q (u, v, w)** is the Darcy velocity vector, p is the hydrostatic pressure, μ is the viscosity, α is the coefficient of thermal expansion, φ is the volume fraction of the nanoparticles, ρ_p density of nanoparticles, ρ_f density of base fluid and $\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{1}{\varepsilon}(\mathrm{q} \cdot \nabla)$ represents the convection derivative.

The equation of energy for nanofluid in porous medium is given by

$$(\rho c)_m \frac{\partial T}{\partial t} + (\rho c)_f \, \mathrm{q}.\nabla T = k_m \nabla^2 T$$
$$+\varepsilon(\rho c)_p\left(D_B \nabla\varphi.\nabla T + \frac{D_T}{T_1}\nabla T.\nabla T\right), \qquad (3)$$

where $(\rho c)_m$ is the effective heat capacity of fluid, $(\rho c)_p$ is the heat capacity of nanoparticles and k_m is the effective thermal conductivity of the porous medium.

The equation of continuity for nanoparticles is given by

$$\frac{\partial\varphi}{\partial t} + \frac{1}{\varepsilon}\mathbf{q}\cdot\nabla\varphi = D_B\nabla^2\varphi + \frac{D_T}{T_1}\nabla^2 T, \qquad (4)$$

where D_B is the Brownian diffusion coefficient, given by Einstein-Stokes equation while D_T is the thermoporetic diffusion coefficient of nanoparticles.

Since, the fluid under consideration is confined between two horizontal planes z = 0 and z = d. Therefore, on these two planes certain boundary conditions must be satisfied. We take the case of free-free surface and assume that temperature and volumetric fraction of the nanoparticles are constant, so the boundary conditions [Chandrasekhar (1961) and Nield and Bejan (2006)]are given by

$$w = 0, \quad T = T_{0,} \quad \varphi = \varphi_0 \ \text{at} \quad z = 0 \ \text{and}$$

$$w = 0, \quad T = T_{1,} \quad \varphi = \varphi_1 \ \text{at} \quad z = d. \qquad (5)$$

Introducing non-dimensional variables as

$$(x', y', z') = \left(\frac{x, y, z}{d}\right), \quad (u', v', w') = \left(\frac{u, v, w}{\nu}\right)d, \ t' = \frac{t\nu}{\sigma d^2}, \ p' = \frac{p}{\mu\nu}d^2,$$

$$\varphi' = \frac{(\varphi - \varphi_0)}{(\varphi_1 - \varphi_0)}, \ T' = \frac{(T - T_1)}{(T_0 - T_1)},$$

where $\sigma = \frac{(\rho c)_m}{(\rho c)_f}$.

Eqs. (1) - (4) in non-dimensional form can be written as

$$\nabla.q' = 0, \qquad (6)$$

$$\frac{1}{\varepsilon}\frac{\partial q'}{\partial t} = -\nabla p' - \frac{1}{Da}q' - Gm\hat{e}_z + GaT'\hat{e}_z - Gn\varphi'\hat{e}_z, \qquad (7)$$

$$\frac{\partial T'}{\partial t} + q'.\nabla T' = \frac{1}{\Pr}\nabla^2 T' + \frac{N_B}{LePr}\nabla\varphi'.\nabla T'$$
$$+\frac{N_A N_B}{LePr}\nabla T'.\nabla T', \qquad (8)$$

$$\frac{1}{\sigma}\frac{\partial\varphi'}{\partial t} + \frac{1}{\varepsilon}q'.\nabla\varphi' = \frac{1}{LePr}\nabla^2\varphi' + \frac{N_A}{LePr}\nabla^2 T', \qquad (9)$$

where non-dimensional parameters are given as:

Prandtl number $p' = \frac{k_1}{\mu\kappa}p,$

Lewis number $Le = \frac{\kappa}{D_B};$

Darcy number $Da = \frac{k_1}{d^2};$

Grashof number $Ga = \frac{g\alpha(T_0 - T_1)k_1 d}{\mu\kappa};$

Density Grashof number $Gm = \frac{(\rho_p\varphi_0 + \rho(1-\varphi_0))gk_1 d}{\mu\kappa};$ Concentration Grashof number $Gn = \frac{(\rho_p - \rho)(\varphi_1 - \varphi_0)gk_1 d}{\mu\nu};$

Modified diffusivity ratio $N_A = \frac{D_T(T_0 - T_1)}{D_B T_1(\varphi_1 - \varphi_0)};$

Modified particle-density increment $N_B = \frac{(\rho c)_p(\varphi_1 - \varphi_0)}{(\rho c)_f}.$

The dimensionless boundary conditions are

$$w = 0, \quad T = T_{0,} \quad \varphi = \varphi_0 \ \text{at} \quad z = 0 \ \text{and}$$

$$w = 0, \quad T = T_{1,} \quad \varphi = \varphi_1 \ \text{at} \quad z = 1. \qquad (10)$$

The basic state was assumed to be quiescent and is given by

$$u = v = w = 0, \ p = p(z), \ T = T_b(z), \ \varphi = \varphi_b(z). \qquad (11)$$

where $T_b = 1 - z, \ \varphi_b = z.$

Thses solutions are same as obtained by Nield and Kuznetsov (2010b, 2011b), Chand and Rana(2012a, b, c).

To study the stability of the system, we superimposed infinitesimal perturbations on the basic state, which are of the forms

$$q'(u,v,w)=0+q''(u,v,w), T'=T_{b,}+T'',$$
$$\varphi'=\varphi_b+\varphi'',\ p'=p_b+p'', \text{with } T_b=1-z, \varphi_b=z. \quad (12)$$

(There after dropping dashes (") for simplicity)

Substituting the expression (12) in Eqs. (6) - (9) and linearize by neglecting the product of the prime quantities we obtained following perturbed equations as

$$\nabla.\mathbf{q}=0, \quad (13)$$

$$\frac{1}{\varepsilon}\frac{\partial\mathbf{q}}{\partial t}=-\nabla p-\frac{1}{Da}\mathbf{q}+GaT\hat{e}_z-Gn\varphi\hat{e}_z, \quad (14)$$

$$\frac{1}{\sigma}\frac{\partial\varphi}{\partial t}+\frac{1}{\varepsilon}w=\frac{1}{LePr}\nabla^2\varphi+\frac{N_A}{LePr}\nabla^2T, \quad (15)$$

$$\frac{\partial T}{\partial t}-w=\frac{1}{Pr}\nabla^2T+\frac{N_B}{LePr}\left(\frac{\partial T}{\partial z}-\frac{\partial\varphi}{\partial z}\right)-\frac{2N_AN_B}{LePr}\frac{\partial T}{\partial z}. \quad (16)$$

The dimensionless boundary conditions are given by

$$w=0,\quad T=1\quad \varphi=0 \text{ at}\quad z=0 \text{ and}$$
$$w=0,\quad T=0,\quad \varphi=1 \text{ at}\quad z=1. \quad (17)$$

It will be noted that the parameter Gm is not involved in these and subsequent equations. It is just a measure of the basic static pressure gradient.

The sixunknown's u, v, w, p, T and φcanbereducedtothreeby operating Eq. (14) with e_z.curl curl, we get

$$\left(\frac{1}{\varepsilon}\frac{\partial}{\partial t}+\frac{1}{Da}\right)\nabla^2w=Ga\nabla_H^2T-Gn\nabla_H^2\varphi, \quad (18)$$

where ∇_H^2, is the two-dimensional Laplacian operator on the horizontal plane.

3. NORMAL MODES AND STABILITY ANALYSIS

Analyzing the disturbances into the normal modes and assuming that the perturbed quantities are of the form

$$[w,T,\varphi]=[W(z),\Theta(z),\Phi(z)]\exp(ik_xx+ik_yy+nt), \quad (19)$$

where k_x and k_y are wave numbers in x and y directions respectively , while n is the growth rate of disturbances.

By using Eq. (19), Eqs. (15), (16)and (18) become

$$\left[\left(\frac{n}{\varepsilon}+\frac{1}{Da}\right)(D^2-a^2)\right]W+a^2Ga\Theta-a^2Gn\Phi=0, \quad (20)$$

$$\frac{W}{\varepsilon}-\frac{N_A}{LePr}(D^2-a^2)\Theta-\left(\frac{1}{LePr}(D^2-a^2)-\frac{n}{\sigma}\right)\Phi=0, \quad (21)$$

$$W+\left(\frac{1}{Pr}(D^2-a^2)-n+\frac{N_A}{LePr}D-\frac{2N_AN_B}{LePr}D\right)\Theta-\frac{N_B}{LePr}D\Phi=0. \quad (22)$$

Where $D=\frac{d}{dz}$ and $a=\sqrt{k_x^2+k_y^2}$ is dimensionless the resultant wave number.

The boundary conditions of the problem in view of normal mode analysis are

$$W=0, D^2W=0,\ \Theta=0,\ \Phi=0 \text{ at } z=0 \text{ and}$$

$$W=0, D^2W=0,\ \Theta=0,\ \Phi=0 \text{ at } z=1. \quad (23)$$

For neutral stability, the real part of n is zero. Hence we write n=iω, where ω is real and is the dimensionless frequency.

4. METHOD OF SOLUTION

The Galerkin weighted residuals method is used to obtain an approximate solution to the system of Eqs. (20) - (22) satisfying boundary conditions (23). In this method the test functions are the same as the base (trial) functions. Accordingly W, Θ and Φ are taken as

$$W=\sum_{p=1}^{n}A_pW_p,\ \Theta=\sum_{p=1}^{n}B_p\Theta_p,\ \Phi=\sum_{p=1}^{n}C_p\Phi_p \quad (24)$$

Where A_p, B_p and C_p are unknown coefficients, p =1, 2, 3... N and the base functions W_p, Θ_p and Φ_p are assumed in the following form for free-free boundaries

$$W_p=\sin p\pi z, \Theta_p=\sin p\pi z, \Phi_p=\sin p\pi z, \quad (25)$$

such that W_p, Θ_p and Φ_p satisfy the boundary conditions (23). Using expression for W, Θ and Φ in Eqs. (20) - (22) and multiplying the first equation by W_p , the second equation by Θ_p and the third equation by Φ_p and integrating in the limits from zero to unity, we obtain a set of 3N linear homogeneous equations with 3N unknown A_p, B_p and C_p; p =1,2,3,...N. For existing of non- trivial solution, the vanishing of the determinant of coefficients produces the characteristics equation of the system in term of Grashof number Ga.

5. LINEAR STABILITY ANALYSIS

For one term Galerkin approximation, we take N=1, so the trial functions are given as

$$W_p=\sin\pi z, \Theta_p=\sin\pi z, \Phi_p=\sin\pi z, \quad (26)$$

which satisfied boundary conditions (23).

(a) Stationary Convection

Consider the case of non-oscillatory convection i.e., ω = 0, using the one-term Galerkin approximation, the following expression for the Grashof number (Ga) can be written as

$$Ga=\frac{(\pi^2+a^2)^2}{a^2Da\,Pr}-\left(\frac{Le}{\varepsilon}+N_A\right)Gn. \quad (27)$$

Thus we get anexpression for the stationary Grashof number (Ga) instead of stationary Rayleigh number

(Ra). The stationary Grashof number (Ga) which is a function of Prandtl number (Pr) is useful to investigate effect of Prandtl number especially for low- Prandtl numbers.

In order to investigate effects of the Prandtl number, Darcy number, Lewis number and the modified diffusivity ratio, we examine the behavior of $\frac{\partial Ga}{\partial Pr}, \frac{\partial Ga}{\partial Da}, \frac{\partial Ga}{\partial Le}$ and $\frac{\partial Ga}{\partial N_A}$ analytically.

From Eq. (33), we have

$\frac{\partial Ga}{\partial Pr} < 0, \frac{\partial Ga}{\partial Da} < 0$ and $\frac{\partial Ga}{\partial Le} > 0, \frac{\partial Ga}{\partial N_A} > 0$ for a bottom-heavy distribution of nanoparticles i.e. for negative value of Gn.

Thus, the Prandtl number and Darcy number have a destabilizing effect while the Lewis number and the modified diffusivity ratio have a stabilizing effect on the stationary convection in a nanofluid layer.

If Da = 1, then this result reduces to the result as obtained by Chand and Rana (2012a, b, c). The interweaving behaviors' of the Brownian motion and the thermoporesis of nanoparticles evidently does not change the critical size of the Bénard cell at the onset of instability.

In the absence of nanoparticles ($Gn = Le = N_A = 0$) i.e. for ordinary fluid, we have

$$Ga = \frac{\left(\pi^2 + a^2\right)^2}{a^2 Da\, Pr}.$$

When we take Da =1, then this result reduces to the well known result derived by Chandrasekhar (1961) and Kuznetsov and Nield (2010b).

(b) Oscillatory Convection

For oscillatory convection ω # 0, by using the one-term Galerkin approximation, we obtain the eigen-value equation as follows

$$a^2 Gn\left(\frac{N_A J}{Le Pr} + \frac{1}{\varepsilon}\left(\frac{J}{Pr} + i\omega\right)\right) + \left(\frac{J}{Le Pr} + \frac{i\omega}{\sigma}\right)\left(a^2 Ga - \left(\frac{J}{Da} + \frac{i\omega}{\varepsilon}\right)\left(\frac{J}{Pr} + i\omega\right)\right) = 0, \quad (28)$$

where $J = \pi^2 + a^2$.

Equating real and imaginary parts of Eq. (28), we get

$$a^2 Ga \frac{J}{Le Pr} + a^2 Gn\left(\frac{N_A J}{Le Pr} + \frac{J}{\varepsilon Pr}\right) = \frac{J^3}{Le Pr^2 Da} - \omega^2\left(\frac{J}{\sigma\varepsilon Pr} + \frac{J}{\sigma Pr} + \frac{J}{Le Pr \varepsilon}\right) = 0, \quad (29)$$

and

$$\frac{a^2}{\sigma} Ga + a^2 Gn - \left(\frac{J^2}{\sigma Pr Da} + \frac{J^2}{Le Pr^2 \varepsilon} + \frac{J^2}{Le Pr Da} - \frac{\omega^2}{\varepsilon\sigma}\right) = 0. \quad (30)$$

Eliminating ω^2 between Eqs. (29) and (30), we get

$$a^2 Ga \frac{J}{Le Pr} + a^2 Gn\left(\frac{N_A J}{Le Pr} + \frac{J}{\varepsilon Pr}\right) = \frac{J^3}{Le Pr^2 Da} + \left(\begin{array}{l} \varepsilon a^2 Ga + \varepsilon\sigma a^2 Gn \\ -\left(\frac{\varepsilon J^2}{Pr Da} + \frac{\sigma J^2}{Le Pr^2} + \frac{\varepsilon\sigma J^2}{Le Pr Da}\right)\end{array}\right)\left(\frac{J}{\sigma\varepsilon Pr} + \frac{J}{\sigma Pr} + \frac{J}{Le Pr \varepsilon}\right), \quad (31)$$

In order to make ω real, it is necessary that

$$\left(\frac{a^2}{\sigma} Ga + a^2 Gn\right) \le \left(\frac{J^2}{\sigma Pr Da} + \frac{J^2}{Le Pr^2 \varepsilon} + \frac{J^2}{Le Pr Da}\right). \quad (32)$$

Hence Eq. (31) gives the oscillatory stability boundary when Eq. (32) satisfied. The frequency of the oscillatory mode is given by

$$\frac{\omega^2}{\varepsilon\sigma} = \left(\frac{J^2}{\sigma Pr Da} + \frac{J^2}{Le Pr^2 \varepsilon} + \frac{J^2}{Le Pr Da}\right) - \left(\frac{a^2}{\sigma} Ga + a^2 Gn\right). \quad (33)$$

6. RESULTS AND DISCUSSION

The onset of thermal instability in a low Prandtl number nanofluid layer heated which from below in a porous medium is analyzed using Galerkin weighted residuals method. The linear theory based on the usual normal mode technique is used to find the expressions for the Grashof number (Ga) instead of Rayleigh number (Ra). The Grashof number (Ga) which is a function of Prandtl number (Pr) is useful to investigate effect of Prandtl number especially low-Prandtl number (Pr). Graphs have been plotted to find out the effect of various parameters on the system.

The stationary convection curves in the (Ga -a)plane for Prandtl number, Darcy number and Lewis number are shown in Figs (2-4).

It is clear from Figs. 2(a, b) that the rise of the heat transport in the transition region becomes steeper for decreasing Prandtl number in order that the large Prandtl-number value can be approached within a finite interval of the Grashof numbers. But it is difficult to say how large this interval is, since the curves shown in Figs. 2(a, b) approach each other relatively slowly as Pr decreases. Finally, at Grashof numbers approaching 10^4**,** the heat transport becomes nearly independent of the Prandtl number as shown in Fig.2 (a). It is also found that the Grashof number (Ga) decreases as values of Prandtl number (for both low and high value of Prandtl numbers) inceases. Therefore, Prandtl number destabilizes the stationary convection.

Fig. 3 shows the variation of the Grashof number (Ga) with the wave number (a) for different values of Darcy number. It is found that the Grashof number (Ga) decreases as values of Darcy number inceases. Therefore, Darcy number destabilizes the stationary convection.

Fig. 4 shows the variation of Grashof number (Ga) with wave number (a) for different values of Lewis number.

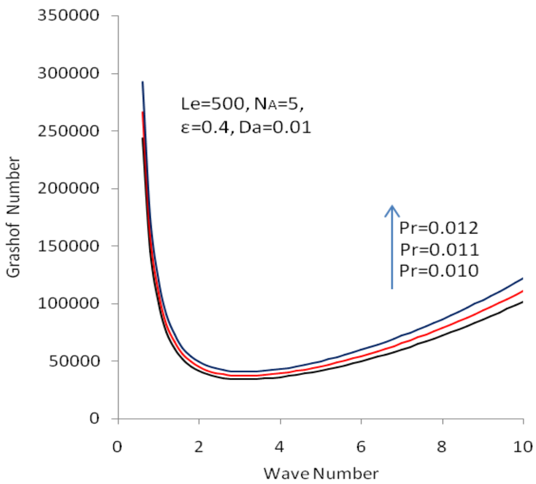

Fig. 2(a). Variation of Grashof number Ga with wave number for different values of sall value of Prandtl number Pr.

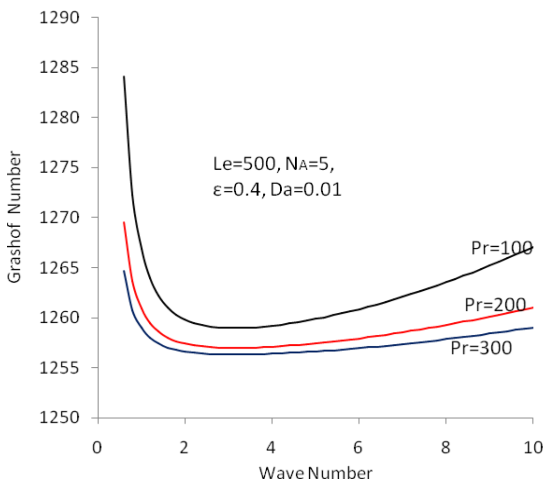

Fig. 2(b). Variation of Grashof number Ga with wave numberfor different values of Large value Prandtl number Pr.

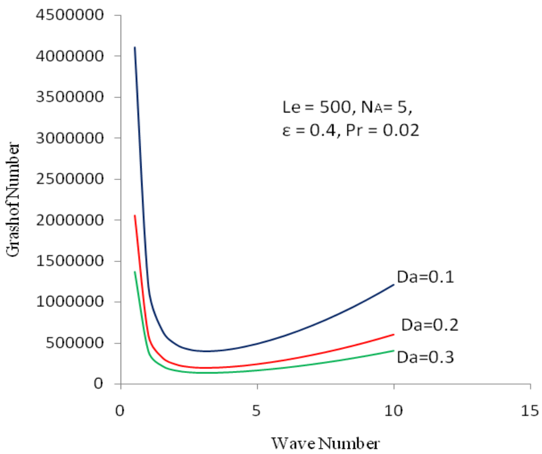

Fig. 3. Variation of Grashof number Ga with wave number for different values of Darcy Number Da

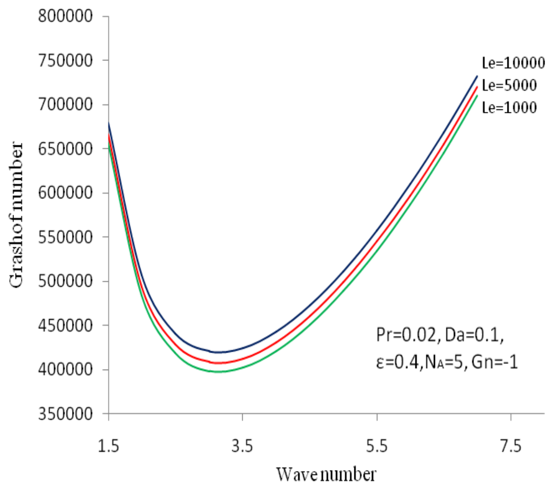

Fig. 4. Variation of Grashof number Ga with wave number for different values of Lewis number Le.

It is found that the Grashof number (Ga) increases as values of Lewis number (Le) inceases. Therefore, Lewis number stabilizes the stationary convection.

7. CONCLUSIONS

Alinear stability analysis of thermal instability in a low-Prandtl number nanofluid layer in a porous medium is investigated using Galerkin weighted residuals method. The main conclusions are summarized as follows:

(i) The critical cell size is not a function of any thermo physical properties of the nanofluid.

(ii) For large value of Grashof number the heat transport becomes nearly independent of the Prandtl number.

(iii) Prandtl number and Darcy number have a destabilizing effect on the stationary convection.

(iv) Lewis number and the modified diffusivity ratio stabilize the stationary convection.

(v) Oscillatory convection is possible if

$$\left(\frac{a^2}{\sigma}Ga+a^2Gn\right)\leq\left(\frac{J^2}{\sigma\, Pr\, Da}+\frac{J^2}{Le\, Pr^2\,\varepsilon}+\frac{J^2}{Le\, Pr\, Da}\right)$$

ACKNOWLEDGMENT

The authors are grateful to the reviewers for their valuable comments and suggestions for improvement of the paper.

REFERENCES

Alloui Z, Vasseur P, Reggio M. (2010). Natural convection of nanofluids in a shallow cavity heated from below, International Journal of Thermal Science, 50(3), 385-393.

Buongiorno, J. (2006). Convective Transport in Nanofluids, ASME Journal of Heat Transfer, 128, 240-250.

Chand R. and Rana, G.C. (2012a). Oscillating Convection of Nanofluid in Porous Medium, Transp Porous Med, 95, (2012), 269-284.

Chand R. and Rana, G.C. (2012b). On the onset of thermal convection in rotating nanofluid layersaturatinga Darcy-Brinkman porous medium, Int. J. of Heat and Mass Transfer, 55, 5417-5424.

Chand R. and Rana, G.C. (2013c).Thermal instability of Rivlin-Ericksen Elastico-Viscous nanofluid saturated by a porous medium, J. Fluids Eng., 134(12), 121203.

Chand R., Rana G.C. and Kumar, S.(2013a). Variable gravity effects on thermal instability of nanofluid in anisotropic porous medium, Int. J. of Appl. Mech. and Engg., 18, 3, 631-642.

Chand R., Rana G.C., Kumar, A. and Sharma, V. (2013b).Thermal instability in a layer of nano-fluid subjected to rotation and suspended particles, Research J. Sc.and Tech.5(1), 32-40.

Chand R.(2013a).Thermal instability of rotating nanofluid, Int. J. Appl. Math and Mech. 9(3), 70-90.

Chand R. (2013b). On the onset of Rayleigh–Bénard convection in a layer of nanofluid in Hydromagnetics, Int. J. of Nanoscience, 12(6), 1350038.

Chandrasekhar, S. (1961). Hydrodynamic and Hydromagnetic Stability, Dover Publications,New York.

Choi, S. (1995). Enhancing thermal conductivity of fluids with nanoparticles. In: Siginer D.A., Wang, H.P. (eds.) Developments and Applications of Non-Newtonian Flows, ASME FED-Vol. 231/MD-Vol. 66, 99-105.

Clever, R.M. and Busse, F.H. (1981).Low-Prandtl number convection in a layer heated from below, J.Fluid Mech., 102, 61-74.

Cookey, C. I. (2009). Radiation absorption effect on thermal convection of a low-Prandtl-number horizontal fluid layer in a porous medium heated from below, Int. J. Fluid Mech. 1, 69-79.

Ingham, D. D., Pop, L., (1981). Transport Phenomena in Porous Media, Elsvier, New York.

Khurana, K. K., M. G. Kivelson, D. J. Stevenson, G. Schubert, C. T. Russell, R. J.Walker and C. Polanskey (1998), Induced magnetic fields as evidence for subsurface oceans on Europa and Callisto, Nature, 195, 777-780.

Kuznetsov, A. V., Nield, D. A. (2010a). Effect of local Thermal Non-equilibrium on the onset of Convection in a Porous Medium Layer Saturated by a nanofluid, Transport in Porous Media 83, 425-436.

Kuznetsov, A. V., Nield, D. A. (2010b). Thermal instability in a porous medium layer saturated by a nanofluid: Brinkman Model, Transp. Porous Medium, 81, 409-422.

Kuznetsov, A. V., Nield, D. A. (2010c).The Onset of Double-Diffusive Nanofluid Convection in a Layer of a Saturated Porous Medium, *Transport in Porous Media*, 85, 3, 941-951.

Lapwood, E. R. (1948). Convection of a fluid in porous medium, Proc. Camb. Phil. Soc., 44,508-519.

Malvandi A., Hedayati, F. and Nobari, M. R. H. (2014). An Analytical Study on Boundary Layer Flow and Heat Transfer of Nanofluid Induced by a Non-Linearly Stretching Sheet, Journal of Applied Fluid Mechanics, 7(2), 375-384.

Flow and Heat Transfer of Nanofluid Induced by a Non-Linearly Stretching Sheet, Journal of Applied Fluid Mechanics, 7(2), 375-384.

Mohammad, A. A., Viskanta, R. (1991). Transient natural convection of low-Prandtl-numberfluids in differentially heated cavity, Int. J. Num. Methods Fluids 13: 61-81.

Nield, D. A., Bejan, A., (2006). Convection in Porous Medium, Springer, New York.

Nield, D.A., Kuznetsov, A.V. (2009). Thermal instability in a porous medium layer saturated by a nanofluid, Int. J. Heat Mass Transf. 52, 5796–5801.

Nield, D.A., Kuznetsov, A.V. (2010a). The onset of convection in a horizontal nanofluid layer offinite depth, European Journal of Mechanics - B/Fluids, 29, 217-223.

Nield, D.A., Kuznetsov, A.V. (2010b). The onset of convection in a layer of cellular porous material: Effect of temperature-dependent conductivity arising from radiative transfer, J. Heat Transfer,132, 7, 074503-4.

Nield, D.A., Kuznetsov, A.V.(2011a).The onset of double-diffusive convection in a nanofluid layer,International Journal of Heat and Fluid Flow 32, 771-776.

Nield, D.A., Kuznetsov, A.V. (2011b).The Effect of Vertical Through flow on Thermal Instability in a Porous Medium Layer Saturated by a Nanofluid, Transp in Porous Media 87, 765-775.

Ramiar A., Ranjbar, A.A. and Hosseinizadeh S.F. (2012).Effect of Axial Conduction and Variable Properties on Two-Dimensional Conjugate Heat Transfer of Al_2O_3-EG/Water Mixture Nanofluid in Microchannel, Journal of Applied Fluid Mechanics, 5, (3), 79-87.

Sheu, L. J. (2011). Linear stability of convection in a in a visco elastic nanofluid layer, World Acad. Sc. Engg. Tech. 58, 289-295.

Stewart, M. J., Weinberg, F (1972). Fluid flow in liquid metals, I. Theoretical analysis, J. Crystal Growth 12, 217-227.

TzouD.Y. (2008a).Thermal instability of nanofluids in natural convection, *Int. J. of Heat and Mass Transfer*, 51, 2967-2979.

TzouD.Y.(2008b). Instability of nanofluids in natural convection, *ASME Journal of Heat Transfer*, 130, 372-401.

Vadasz, P. (2006). Heat conduction in nanofluid suspensions. ASME J. Heat Transf. 128, 465-477.

Vafai, K. A., Hadim, H. A. (2000). Hand Book of Porous Media, M. Decker, New York.

Wooding, R. A. (1960). Rayleigh instability of a thermal boundary layer in flow through a porous medium, J. Fluid Mech.,9, 183-192.

Zhang, K. and G. Schubert. (2000). Teleconvection: Remotely driven thermal convection in rotating stratified spherical layers, *Science*. 290, 1944-1947.