Axial Magnetic Field Effect on Taylor-Couette Flow

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ABSTRACT

This study is interested in the effect of an axial magnetic field imposed on incompressible flow of electrically conductive fluid between two horizontal coaxial cylinders. The imposed magnetic field is assumed uniform and constant. The effect of heat generation due to viscous dissipation is also taken into account. The inner and outer cylinders are maintained at different uniform temperatures. The movement of the fluid is due to rotation of the cylinder with a constant speed. An exact solution of the equations governing the flow was obtained in the form of Bessel functions. A finite difference implicit scheme was used in the numerical solution. The velocity and temperature distributions were obtained with and without the magnetic field. The results show that for different values of the Hartmann number, the velocity between the two cylinders decreases as the Hartmann number increases. Also, it is found that by increasing the Hartmann number, the average Nusselt number decreases. On the other hand, the Hartmann number does not affect the temperature.

Keywords: Rotating cylinders, viscous dissipation, heat transfer, magnetic field, Bessel function, finite difference.

NOMENCLATURE

a	thermal diffusivity	u	radial velocity
B_0	external magnetic field density	v	angular velocity
Cp	specific heat	w	axial velocity
d	width of the annular space	μ	dynamic viscosity
Ec	Eckert number	υ	fluid kinematic viscosity
Ha	Hartmann number	θ	non-dimensional temperature
Pr	Prandtl number	ρ	fluid density
r	radius	σ	electric conductivity
t	time	Ω	rotational speed
T	temperature	η	radius ratio
k	thermal conductivity	ϕ	viscous dissipation function

1. INTRODUCTION

The study of flow of electrically conductive fluids, called magnetohydrodynamic (MHD) has attracted much attention due to its various applications. In astrophysics and geophysics, it is applied to the study of stellar structures, terrestrial cores and solar plasma. In industrial processes, it finds its application in MHD pumps, nuclear reactors, the extraction of geothermal energy, metallurgical and crystal growth in the field of semiconductors, the control of the behavior of fluid flow and heat and mass transfer and the stability of convective flows. Several studies have been conducted to evaluate the effect of magnetic field on the convective flows for different conditions. Chandrasekhar (1961) has made the prediction of the linear stability of hydrodynamics and hydromagnetic Taylor-Couette flow. Tatsuo et al (1993) carried out experimental investigations about the natural convection of a magnetic fluid between two concentric cylinders and horizontal isotherms. Ben Hadid, and Henry (1996) investigated numerically the effect of a constant magnetic field on a three-dimensional buoyancy-induced flow in a cylindrical cavity, they put in light the structural changes of the flow induced by the magnetic field for each field orientation. Singh et al (1997) presented exact solutions for fully developed natural convection in open-ended vertical concentric annuli under a radial magnetic field. Bessaih et al

(1999) studied numerically MHD laminar flow of a liquid metal contained in a cylindrical enclosure having an aspect ratio equal to 1, and whose upper wall is in rotation. The assembly is subjected to a vertical external magnetic field. El Amin (2003) studied the effects of both first- and second-order resistance due to the solid matrix on forced convective flow from a horizontal circular cylinder in the presence of a magnetic field and viscous dissipation, with a variable surface temperature boundary condition. The study of the effects of the azimuthal magnetic field of an electrically conducting fluid in a rotating annulus has also been presented by Kurt et al (2004). Hayat and Kara (2006) investigated the Couette time-dependent flow of an incompressible third-grade fluid subjected to a magnetic field of variable strength analytically. Group theoretic methods were employed to analyze the nonlinear problem and a solution for the velocity field was obtained analytically. Sankar et al (2006) studied numerically a natural convection of a low Prandtl number electrically conducting fluid under the influence of either axial or radial magnetic field in a vertical cylindrical annulus. They showed that the magnetic field can be suppress the flow and heat transfer. Bessaih et al (2009) studied the MHD stability of an axisymmetric rotating flow in a cylindrical enclosure containing liquid metal (Pr = 0.015), with an aspect ratio equal to 2, and subjected to a vertical temperature gradient and an axial magnetic field. Wrobel et al (2010) presented an experimental and numerical analysis of a thermo-magnetic convective flow of paramagnetic fluid in an annular enclosure with a round rod core and a cylindrical outer wall under gravitational and magnetic environments. Azim et al (2010) studied numerically the effect of magnetic field and Joule heating on the coupling of convection flow along and conduction inside a vertical flat plate in the presence viscous dissipation and heat generation. Ellahi et al (2010) determined analytic solutions for a nonlinear problem governing the MHD flow of a third grade fluid in the annulus of rotating concentric cylinders. Makinde and Onyejekwe (2011) investigated a steady flow and heat transfer of an electrically conducting fluid with variable viscosity and electrical conductivity between two parallel plates in the presence of a transverse magnetic field. Venkatachalappa et al (2011) carried out numerical computations to investigate the effect of axial or radial magnetic field on the double-diffusive natural convection in a vertical cylindrical annular cavity. Kakarantzas et al (2011) studied numerically the combined effect of a horizontal magnetic field and volumetric heating on the natural convection flow and heat transfer of a low Prandtl number fluid in a vertical annulus. Seth et al (2011) studied the effects of rotation and magnetic field on unsteady Couette flow of a viscous incompressible electrically conducting fluid between two horizontal parallel porous plates in a rotating medium. Mozayyeni and Rahimi (2012) investigated numerically the problem of mixed convection of a fluid in the fully developed region between two horizontally concentric cylinders with infinite lengths, in the presence of a constant magnetic field with a radial MHD force direction, considering the effects of viscous heat dissipation in the fluid in both steady and unsteady states. Seth et al (2012) investigated the effects of

Hall current on unsteady hydromagnetic Couette flow of a viscous incompressible electrically conducting fluid in a rotating system in the presence of an inclined magnetic field. Seth and Singh (2013) studied theoretically the effect of Hall current and a uniform transverse magnetic field on unsteady MHD Couette flow of class-II in a rotating system. J. Prakash (2014) proved analytically that the principle of the exchange of stabilities in convection in a Rotating Ferrofluid Saturated Porous is not, in general, valid for the case of free boundaries but the study shown that a sufficient condition for the validity of this principle can be derived. Bhuiyan et al (2014) studied numerically the effects of joule heating on magneto-hydrodynamic natural convection flow in presence of viscous dissipation and pressure stress work from a horizontal circular cylinder.

Although the exact solutions for the Hartmann flow and the MHD Couette flow have been achieved for more than seventy years, the solutions for a heat transfer in flow between concentric rotating cylinders, also known as Taylor Couette flows, under external magnetic field have been restricted to high Hartmann numbers.

The aim of the present study is to examine analytically and numerically the effects of an external axial magnetic field applied to the forced convection flow of an electrically conducting fluid between two horizontal concentric cylinders, considering the effects of viscous heat dissipation in the fluid. It should be noted that the natural convection is supposed negligible in this work, which is not always the case of the vertical cylinder. The forced flow is induced by the rotating inner cylinder, in slow constant angular velocity and the other is fixed.

1. FORMULATION OF THE PROBLEM

Consider a laminar flow of a viscous incompressible electrically conductive fluid between two coaxial cylinders. The inner cylinder of radius r_1 is rotated at a constant speed Ω_1 and the outer cylinder of radius r_2 is kept fixed. The inner and outer walls are maintained at a constant and different temperatures T_1 and T_2 respectively, while the top and bottom walls are insulated. The two cylinders are electrically isolated. The flow is subjected to a constant uniform and axially magnetic field B_0. Geometry of the problem is presented in Fig.1. We assume that the magnetic Reynolds number is neglected. When the magnetic field is uniform and externally applied, its time variations can be neglected and the set of flow equations further simplified to involve only the Navier-Stokes equations and the conservation of the electric current. Also we assume that the electric field is zero. In this study the viscous dissipation term in the energy equation is considered.
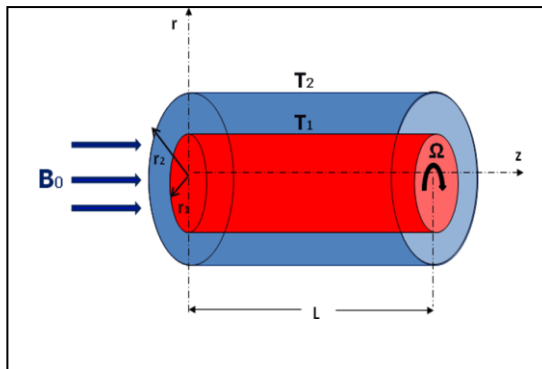
Fig. 1. Geometry of the problem

2. ANALYTICAL STUDY

The flow is assumed to be steady, laminar and unidirectional, therefore the radial and axial components of the velocity and the derivatives of the velocity with respect to θ and z are zero. Under these assumptions and in cylindrical coordinates, the governing equations for the flow following the azimuthal direction can be written as follows:

$$\upsilon\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} - \frac{v}{r^2}\right) - \frac{\sigma v B_0^2}{\rho} = 0 \tag{1}$$

$$\frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = -\mu\left(\frac{\partial v}{\partial r} - \frac{v}{r}\right)^2 \tag{2}$$

$$r = r_1 : v(r) = \Omega_1 r_1, T = T_1 \tag{3}$$

$$r = r_2 : v(r) = 0, T = T_2 \tag{4}$$

The governing equation and boundary conditions, eq.(1) to (4), witch are in non-adimensional form, become:

$$\frac{\partial^2 v^*}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial v^*}{\partial r^*} - \left(\frac{Ha^2}{(1-\eta)^2} + \frac{1}{r^{*2}}\right)v^* = 0 \tag{5}$$

$$\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial \theta}{\partial r^*}\right) = -Ec\Pr\left(\frac{\partial v^*}{\partial r^*} - \frac{v^*}{r^*}\right)^2 \tag{6}$$

$$r^* = \eta : v^*(r^*) = 1, \theta = 1 \tag{7}$$

$$r^* = 1 : v^*(r^*) = 0, \theta = 0 \tag{8}$$

Where

$$r^* = \frac{r}{r_2}, v^* = \frac{v}{\Omega_1 r_1}, \eta = \frac{r_1}{r_2}, Ha = B_0 d\sqrt{\frac{\sigma}{\rho\upsilon}}, \theta = \frac{T - T_2}{T_1 - T_2},$$

$$\Pr = \frac{\upsilon}{a}, Ec = \frac{(\Omega_1 r_1)^2}{Cp\Delta T}$$

Where, the stars are dropped for convenience.

The velocity profile in the annular space is obtained by solving the Eq. (5) as follows:

$$v(r) = C_1 I_1\left(\frac{Ha}{1-\eta}r\right) + C_2 K_1\left(\frac{Ha}{1-\eta}r\right) = 0 \tag{9}$$

Where:

$$M = \frac{Ha}{1-\eta}$$

C_1 and C_2 are the constants of integration, which are determined from the boundary conditions on the velocity.

$$C_1 = \frac{K_1(M) - bK_1(\eta M)}{I_1(\eta M)K_1(M) - K_1(\eta M)I_1(M)}$$

$$C_2 = \frac{bI_1(\eta M) - I_1(M)}{I_1(\eta M)K_1(M) - K_1(\eta M)I_1(M)}$$

I_1 is the modified Bessel function of the first kind of order 1, and K_1 is the modified Bessel function of the second kind of order 1.

To obtain the temperature field from Eq. (6), we performed calculations by using the first, second and third term used by Omid M. et al (2012) of the expansions of the modified Bessel functions $I_1(Mr)$ and $K_1(Mr)$ for small values of Ha.

3.1 Expansion with one term of Bessel modified functions

$$I_1(Mr) \cong \frac{1}{2Mr} \tag{10}$$

$$K_1(Mr) \cong \frac{1}{Mr} \tag{11}$$

By substituting the values of $I_1(Mr)$ and $K_1(Mr)$ from the above expansions in the velocity equation, Eq. (9), and using the new velocity distribution in Eq. (6) to find the temperature field.

The temperature gradient is given then by the following equation:

$$\frac{\partial \theta}{\partial r} = 2Br\frac{C_2^2}{M^2 r^3} + \frac{C_3}{r} \tag{12}$$

The temperature profile is given by:

$$\theta = C_3\ln(r) + C_4 - \frac{Ec\Pr C_2^2}{(Mr)^2} \tag{13}$$

Where
C_3 and C_4 are the constants of integration with respect to θ:

$$C_3 = \frac{\left(1 - C_4 + \frac{BrC_2^2}{M^2\eta^2}\right)}{\ln(r)}$$

$$C_4 = \frac{BrC_2^2}{M^2}$$

3.1 Expansion with two terms of modified Bessel functions

$$I_1(Mr) \cong \frac{1}{2}Mr + \frac{(Mr)^3}{16} \tag{14}$$

$$K_1(Mr) \cong \frac{1}{Mr} + \left[\frac{1}{2}\ln(\frac{Mr}{2}) - \frac{1}{4}(-2\gamma + 1)\right](Mr) \tag{15}$$

Where:
γ is Euler's constant defined by:

$$\gamma = \lim_{x \to \infty} \left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots + \frac{1}{m} - \ln(m) \right]$$

$$= 0,5772156649\ldots$$

The temperature gradient is therefore expressed as follows:

$$\frac{\partial \theta}{\partial r} = \frac{C_3}{r} - Br \left[\begin{array}{c} \dfrac{C_1^2 M^6 r^5}{384} - \dfrac{C_1 C_2 M^2 r}{4} + \dfrac{C_1 C_2 M^4 r^3}{32} \\[2mm] + \dfrac{2 C_2^2}{M^2 r^3} - \dfrac{2 C_2^2 \ln(r)}{r} + \dfrac{C_2^2 M^2 r}{8} \end{array} \right] \tag{16}$$

The temperature profile is given by:

$$\theta = C_3 \ln(R) + C_4 - Br \left[\begin{array}{c} \dfrac{C_1^2 M^6 r^6}{2304} - \dfrac{C_1 C_2 M^2 r^2}{8} + \dfrac{C_1 C_2 M^4 r^4}{128} \\[2mm] + \dfrac{C_2^2}{M^2 r^2} - C_2^2 (\ln(r))^2 + \dfrac{C_2^2 M^2 r^2}{16} \end{array} \right] \tag{17}$$

Where

C_3 and C_4 are the constants of integration, which are determined from the boundary conditions on the temperature.

$$C_3 = 1 - C_4 + Br \left[\begin{array}{c} \dfrac{C_1^2 M^6 \eta^6}{2304} - \dfrac{C_1 C_2 M^2 \eta^2}{8} + \dfrac{C_1 C_2 M^4 \eta^4}{128} \\[2mm] + \dfrac{C_2^2}{M^2 \eta^2} - C_2^2 (\ln(\eta))^2 + \dfrac{C_2^2 M^6 \eta^6}{16} \end{array} \right]$$

$$C_4 = Br \left[\dfrac{C_1^2 M^6}{2304} - \dfrac{C_1 C_2 M^2}{8} + \dfrac{C_1 C_2 M^4}{128} + \dfrac{C_2^2}{M^2} + \dfrac{C_2^2 M^2}{16} \right] \tag{18}$$

3.1 Expansion with three terms of modified Bessel functions

$$I_1(Mr) \cong \frac{1}{2} Mr + \frac{(Mr)^3}{16} + \frac{(Mr)^5}{384} \tag{19}$$

$$K_1(Mr) \cong \frac{1}{Mr} + \left[\frac{1}{2} \ln(\frac{Mr}{2}) - \frac{1}{4}(-2\gamma + 1) \right](Mr)$$
$$+ \left[\frac{1}{16} \ln(\frac{Mr}{2}) - \frac{1}{32}(\frac{5}{2} - 2\gamma) \right](Mr)^3 \tag{20}$$

The temperature gradient is given by the following equation:

$$\frac{\partial \theta}{\partial r} = \frac{C_3}{r} - \frac{Br}{r} \left[\begin{array}{c} 2 C_2^2 \ln\left(\dfrac{Mr}{2}\right) + \dfrac{2 C_2^2}{M^2 r^2} + C_5 (Mr)^6 \ln\left(\dfrac{Mr}{2}\right) + \\[2mm] C_6 (Mr)^6 - \dfrac{1}{384} C_2^2 (Mr)^6 \left(\ln\left(\dfrac{Mr}{2}\right) \right)^2 \\[2mm] + C_7 (Mr)^2 + C_8 (Mr)^4 - \dfrac{1}{32} C_2^2 (Mr)^4 \ln\left(\dfrac{Mr}{2}\right) \\[2mm] + C_9 (Mr)^8 - \dfrac{1}{3072} C_1 C_2 (Mr)^8 \ln\left(\dfrac{Mr}{2}\right) \\[2mm] - \dfrac{1}{92160} C_1^2 (Mr)^{10} \end{array} \right] \tag{21}$$

Where

The constants C_5 to C_9 are given in terms of C_1 and C_2 as follows:

$$C_5 = \frac{11}{2304} C_1^2 - \frac{1}{192} C_1 C_2 - \frac{1}{192} C_2^2 \gamma$$

$$C_6 = \frac{11}{2304} C_2^2 \gamma + \frac{7}{2304} C_1 C_2 - \frac{1}{384} C_2^2 \gamma - \frac{1}{384} C_1^2 - \frac{125}{55296} C_2^2 - \frac{1}{192} C_1 C_2 \gamma$$

$$C_7 = \frac{1}{4} C_2^2 \gamma - \frac{7}{16} C_2^2 + \frac{1}{4} C_1 C_2$$

$$C_8 = \frac{1}{32} C_2^2 - \frac{1}{32} C_2^2 \gamma - \frac{1}{48} C_1 C_2$$

$$C_9 = \frac{7}{24576} C_1 C_2 - \frac{1}{3072} C_1 C_2 \gamma - \frac{1}{3072} C_1^2$$

The solution of the energy equation is:

$$\theta = C_4 + C_3 \ln(r) + Br \left[\begin{array}{c} C_{10}(Mr)^6 + C_{11}(Mr)^8 + C_{12}(Mr)^4 + \dfrac{1}{2} C_7 (Mr)^2 - \dfrac{1}{921600} C_1^2 (Mr)^{10} \\[2mm] + C_2^2 \left(\ln\left(\dfrac{Mr}{2}\right) \right)^2 2 C_2^2 \ln\left(\dfrac{Mr}{2}\right) + \dfrac{C_2^2}{(Mr)^2} + \dfrac{1}{6} C_5 (Mr)^6 \ln\left(\dfrac{Mr}{2}\right) \\[2mm] + C_6 (Mr)^6 - \dfrac{1}{2304} C_2^2 (Mr)^6 \left(\ln\left(\dfrac{Mr}{2}\right) \right)^2 + \dfrac{1}{6912} C_2^2 (Mr)^6 \ln\left(\dfrac{Mr}{2}\right) \\[2mm] - \dfrac{1}{128} C_2^2 (Mr)^4 \ln\left(\dfrac{Mr}{2}\right) - \dfrac{1}{24576} C_1 C_2 (Mr)^8 \ln\left(\dfrac{Mr}{2}\right) \end{array} \right] \tag{22}$$

Where

The Constants C_{10}, C_{11} and C_{12} are given as follows:

$$C_{10} = \frac{1}{36} C_5 + \frac{1}{6} C_6 - \frac{1}{41472} C_2^2$$

$$C_{11} = \frac{1}{8} C_9 + \frac{1}{196608} C_1 C_2$$

$$C_{12} = \frac{1}{4} C_8 + \frac{1}{512} C_2^2$$

3. NUMERICAL STUDY

In this numerical study, we consider a two-dimensional and axisymmetric unsteady flow. We opted for the velocity - pressure formulation due to its rapidity of prediction, its lower cost, and its ability to simulate real conditions. The finite difference scheme adopted for the resolution is very similar to that used by R.Peyret (1976), A.Ghezal et al (1992) and (2011), this is a semi implicit scheme of Crank-Nicholson type. The spatial discretization using the Marker And Cell (MAC) is shown in fig.2.

The iterative procedure is assumed converged when the following test is verified

$$\max(|L_u|, |L_v|, |L_w|, |L_\theta||D|) < \varepsilon$$

where L_u, L_v, L_w, L_θ and D represents operators differences relating to system equations corresponding to the problem variables u, v, w,θ and Π respectively, ε is of the order of 10^{-4} depending on the considered case. We then proceeded to a study of the mesh sensitivity of the field of study. This study led us to retain a mesh of 336 nodes along the direction r and 48 nodes in the z direction.
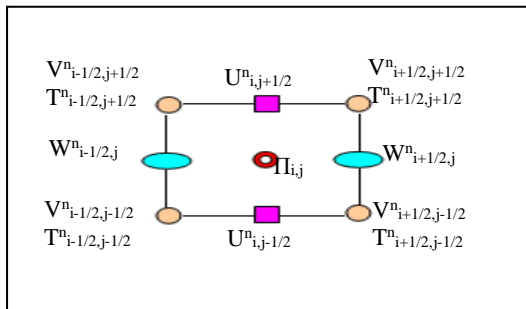
Fig. 2. M.A.C cell

4.1 Mathematical equations

Based on these dimensionless variables, the conservation equations of mass, momentum and energy are written in non rotating frame cylindrical coordinates as follows (where the stars are dropped for convenience):

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{23}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} - \frac{v^2}{r} + w\frac{\partial u}{\partial z} = -\frac{\delta\Pi}{\delta r} + \frac{1-\eta}{Ta}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} - \frac{u}{r^2}\right) - \frac{Ha^2 u}{(1-\eta)Ta} \tag{24}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \frac{vu}{r} + w\frac{\partial v}{\partial z} = \frac{1-\eta}{Ta}\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2} - \frac{v}{r^2}\right) - \frac{Ha^2 v}{(1-\eta)Ta} \tag{25}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{\partial\Pi}{\partial z} + \frac{1-\eta}{Ta}\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right) \tag{26}$$

$$\frac{\partial\theta}{\partial t} + u\frac{\partial\theta}{\partial r} + w\frac{\partial\theta}{\partial z} = \frac{1-\eta}{PrTa}\left(\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} + \frac{\partial^2\theta}{\partial z^2}\right) + \frac{(1-\eta)Ec\,\Phi}{Ta} \tag{27}$$

Where:

$Ha = Bd\sqrt{\dfrac{\sigma}{\rho\upsilon}}$ is the Hartmann number,

$Ta = \dfrac{\Omega_1 r_1 d}{\nu}$ is the Taylor number,

$d = r_1 - r_2$ is the width of the annular space,

$$\Phi = 2\left[\left(\frac{\partial u}{\partial r}\right)^2 + \left(\frac{u}{r}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right)^2$$
$$+ \left(\frac{\partial v}{\partial r} - \frac{v}{r}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2$$

is the viscous dissipation function
The rate of heat transfer in non – dimensional for the inner and outer cylinder is given by:

$$Nu_i(z) = -\xi\frac{\partial\theta}{\partial r}\bigg|_{r=\eta}$$

$$Nu_e(z) = -\xi\frac{\partial\theta}{\partial r}\bigg|_{r=1}$$

With: $\xi = 1 - \eta$

The average Nusselt number on the inner and outer cylinders is given by:

$$\overline{Nu_i} = \frac{1}{L}\int_0^z Nu_i(z)dz$$

$$\overline{Nu_e} = \frac{1}{L}\int_0^z Nu_e(z)dz$$

4.2 Initial and boundary conditions

At the time t=0:

$$u(r,z,0) = v(r,z,0) = w(r,z,0) = \Pi(r,z,0) = \theta(r,z,0) = 0 \tag{28}$$

The boundary conditions are as follows:

$$r = \eta \quad z \geq 0 : \quad u(r_1,z) = v(r_1,z) = w(r_1,z) = 0,\ \theta(r_1,z)=1 \tag{29}$$

$$r = 1 \quad z \geq 0 : \quad u(r_2,z) = v(r_2,z) = w(r_2,z) = \theta(r_2,z)=0 \tag{30}$$

$$\eta < r < 1 \quad z = 0 : \quad u = v = w = 0,\ \frac{\partial\theta}{\partial z} = 0 \tag{31}$$

$$z = L : \quad w = 0, \frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = \frac{\partial\theta}{\partial z} = 0 \tag{32}$$

4. RESULTS AND DISCUSSION

In order to understand the physical situation of the problem and the effects of the Hartmann and Eckert numbers, we have found the numerical and analytical values of the velocity, temperature, and the Nusselt number.

The results obtained through the numerical code were presented in figs. 3 and 4 were compared with those calculated using the three analytical approach for small value of Hartmann number.

It is noticed from fig. 3 that the analytical results for the three cases of the expansions with one, two and three term of modified Bessel functions coincide well with the numerical results for small Hartmann number Ha=0.5. It can also be observed that the influence of the order of development on temperature is negligible

As can be seen from fig. 4 that whether for the average Nusselt number on inner and outer surfaces versus Hartman numbers. The analytic approach corresponding to the expansion of three terms is closer to the numerical approach.

For the Hartmann number values less than Ha=0.2 the influence of the order of development in the analytical approach is insignificant.

So the next analytical results in this work are done by the expansions with three terms of modified Bessel functions.
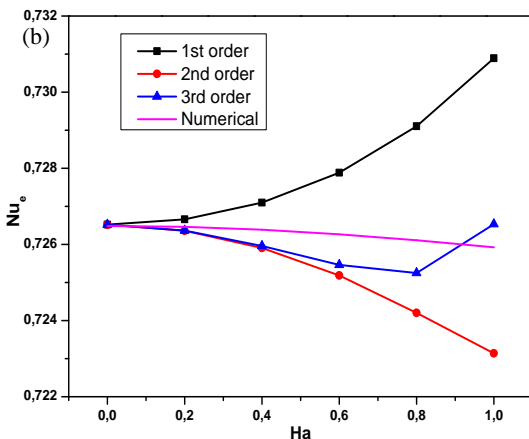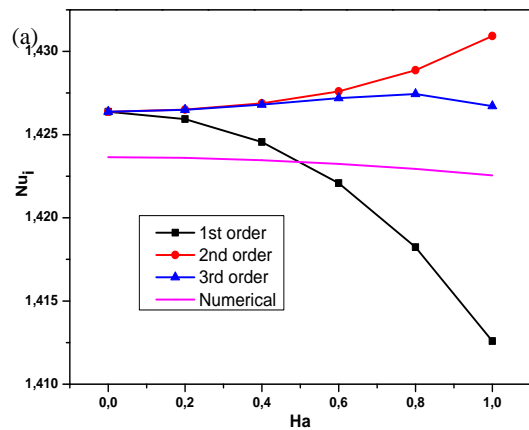
The velocity and temperature are evaluated analytically and numerically for different values of Hartmann number in figs. 5 and 6.
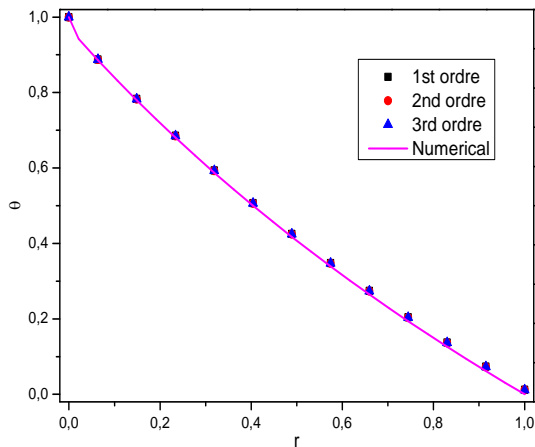
Fig. 3. Effect of the development of Bessel functions on temperature distribution, for Ha= 0.5, *η* = 0.5, Pr = 0.02, Ec=0.5.
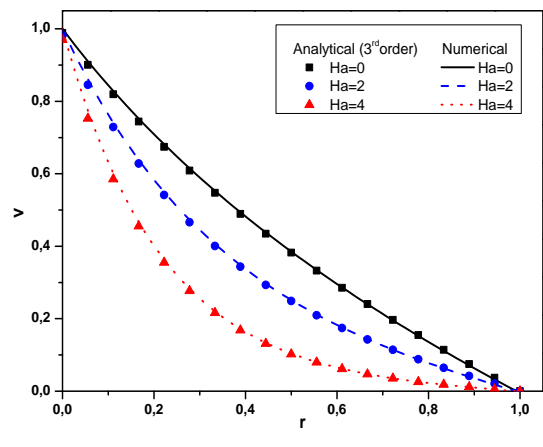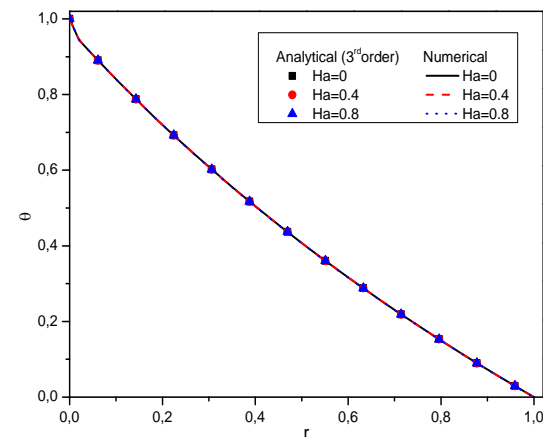

Fig. 5. Comparison of analytical and numerical results of velocity profile, for η = 0.5, Ta=20,t*=120.

Fig. 4. Effect of the development of Bessel functions on average Nusselt number on (a) inner and (b) outer surfaces of the cylinder against the Hartmann number, for *η = 0.5, Pr = 0.02, Ec=0.5*

Fig. 6. Comparison of analytical and numerical results Temperature profile, for η = 0.5, Ta=20, Pr = 0.02, Ec=0.0001, *t*=120.*

which causes a reduction of the velocity in the annular space because the centrifugal force is counter-productive and the Lorentz electromagnetic force acts as a flow damper.

It is observed from fig. 6 that the effect of weak magnetic field on the radial profile of temperature is insignificant.

It should be noted that the effect of magnetic field on the temperature distribution is insignificant, whereas the changes induced by the magnetic field on the temperature gradient and therefore on the Nusselt number is considerable.

Fig. 7 displays the effect of Hartmann number on the temperature, as shown in this figure, the temperature profile is similar to those shown in Figs. 6. It is evident that the effect of weak and strong magnetic field on the radial profile of temperature is insignificant. It is valid in the case of low and high values of Hartmann

Obviously, the velocity and temperature profiles, for various Ha obtained via these two different methods, agree with each other reasonably well. We can notice that the velocity profile without magnetic field Ha=0 is quasi-linear, and an increase in Hartman number,
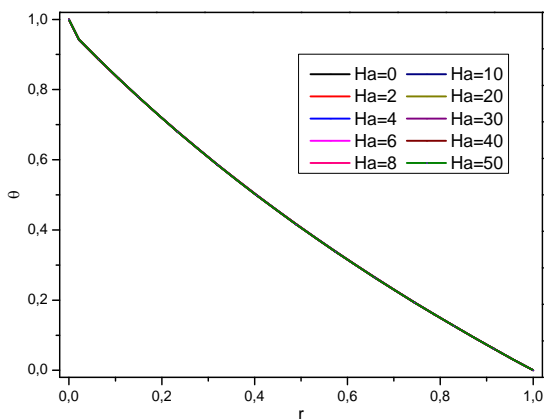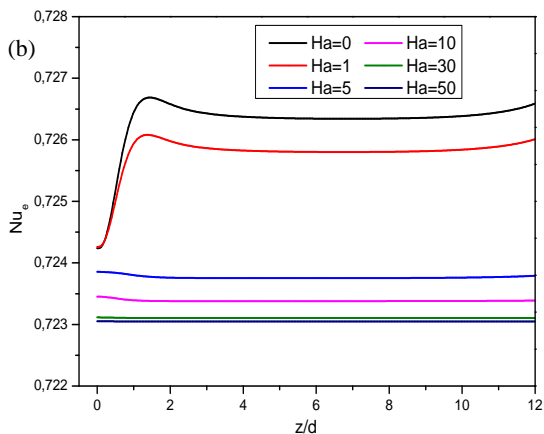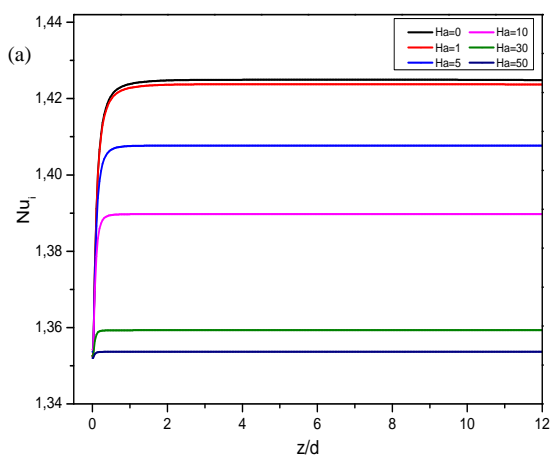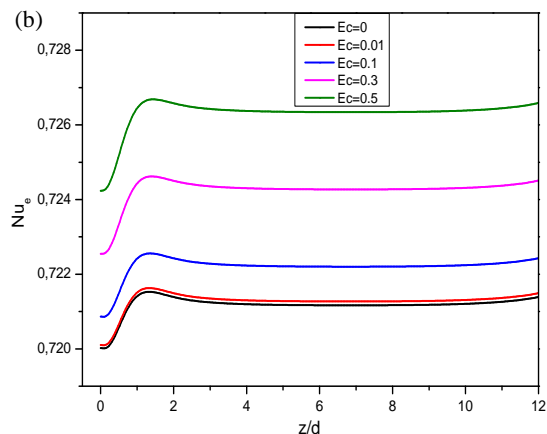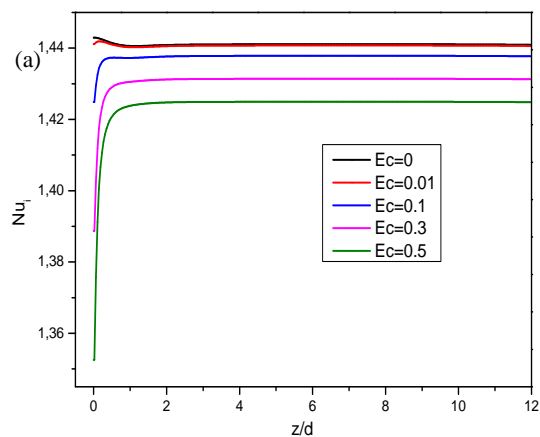
Fig. 7. Temperature profile as a function of Hartmann number, for η = 0.5, Ta=20, Pr = 0.02, Ec=0.5, t^*=120.

Fig. 8. Effect of Hartman number on local Nusslet number distribution on (a) inner and (b) outer cylinders, for η = 0.5, Pr = 0.02, Ec=0.5, t^*=120

Fig. 9. Effect of Eckert number on local Nusselet number distribution on (a) inner and (b) outer cylinders, for η = 0.5, Pr = 0.02, Ha=0, t^*=120.

In fact when the Eckert number is considerable. The heat generation in the fluid increases due to viscous dissipation. Thus the temperature of the fluid in the annular space increases causing a decrease in the temperature gradient in the vicinity of the inner cylinder and an increase of the gradient in the vicinity of the outer cylinder. A significant increase in the Hartmann number, causes a reduction of the centrifugal force, which results in a gradual decrease in the Nusselt number. The analysis of the variation of local Nusselt number on the inner and outer cylinder shows that this number tends to a limit value.

Effect of Eckert number on the distribution of local Nusselt number on the inner and outer cylinders is displayed in Fig. 9, for Ha = 0. As can be seen, with increase of Eckert number, the influence of heat transfer due to the viscous dissipation in the annular space is improved, which leads to the increase in the average temperature of the fluid at this region. The dimensionless temperatures of inner and outer cylinders are maintained at 1.0 and 0.0, respectively. It is evident that by increasing the average temperature of fluid in annular space, the rate of heat transfer between the fluid and inner cylinder decreases due to the reduction of the temperature difference between them. Secondly, the convective heat transfer between the fluid and the outer cylinder is improved because of the increase in the temperature.

number. Also we can notice that the temperature profiles don't change for Ec=0.5 and Ec=0.0001, so the Eckert number don't affect the temperature.
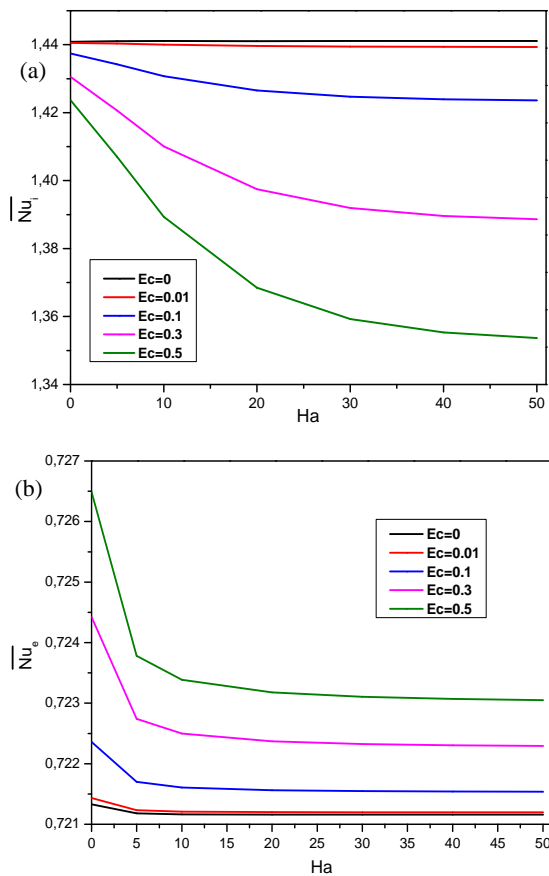
Fig. 8 shows the effect of Hartmann number on the local Nusselt number on the inner and outer surfaces, for an Eckert number Ec = 0.5. It is found that for high values of Hartmann number, the local Nusselt number on the inner and outer surfaces decreases.

Fig. 10. Effect of Eckert number on average Nusselt number on (a) inner and (b) outer surfaces of the cylinder against the Hartmann number, η = 0.5, Pr = 0.02, t^*=120.

It is observed from fig. 10 that the effect of increasing Hartmann number is the decrease the average Nusselt number on both surfaces of the cylinder. So a considerably increasing Hartmann number, which leads to a reduction of the centrifugal force, results in a progressive decrease in the Nusselet number.

From this figure, it can also be noticed that the average Nusselt number on the outer cylinder is lower than on the inner cylinder, because the velocity and temperature gradient are higher for the cold inner cylinder than for the outer cylinder. Also the results show the effects of viscous dissipation terms on the rate of heat transfer, the average Nusselt number increases with an increase in the Eckert number on the outer cylinder, but it decreases on the inner cylinder. In fact, as the Eckert number is large the heat generated in the annulus increases due to viscous dissipation, and thus the temperature of the fluid increases. This causes a decrease in the temperature gradient close to the inner cylinder, and an increase in the gradient in the vicinity of the outer cylinder.

In this part, some results are presented in different non-dimensional time values for the distribution of velocity and temperature in the annulus Figs (11, 12, 13 and 14).
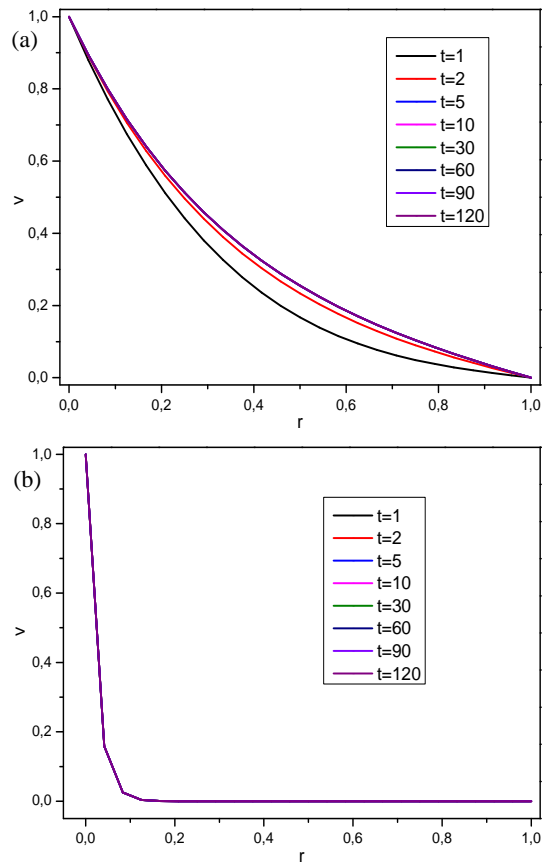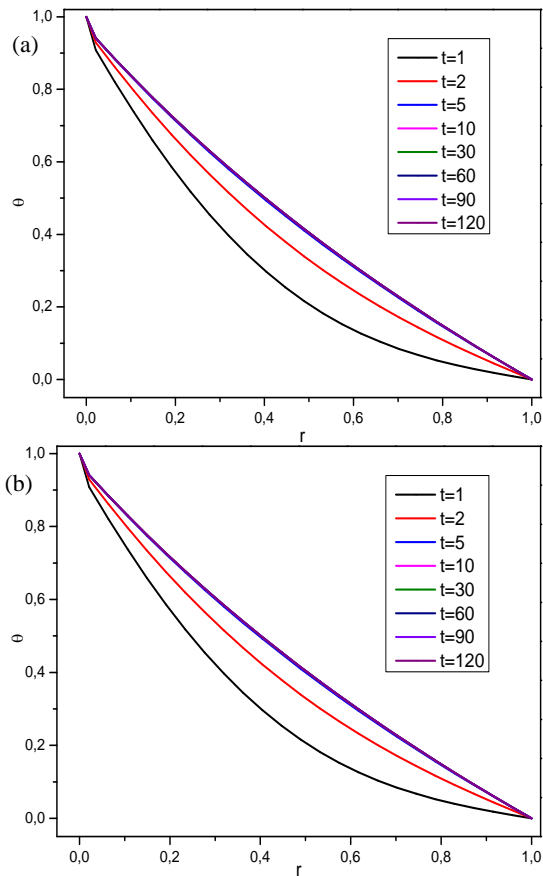

Fig. 11. Velocity distribution at different times at z/d=7 for (a) Ha = 2 and (b) Ha = 50 for Ta=20.
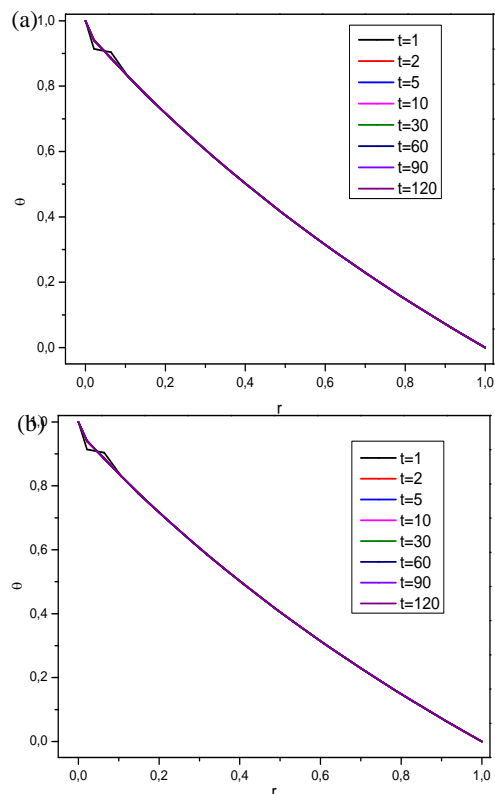
Fig.12. Temperature distribution at different times at z/d=7 for (a) Ha = 0.0 and (b) Ha = 50 for Ta=20, Ec=0,Pr=0,02

Fig. 13. Temperature distribution at different times at z/d=7 for (a) Ha = 2 and (b) Ha = 50 for Ta=20, Ec=0, Pr=1.

Fig. 14. Temperature distribution at different times at z/d=7 for (a) Ha = 2 and (b) Ha = 50 for Ta=20, Ec=0, Pr=7.
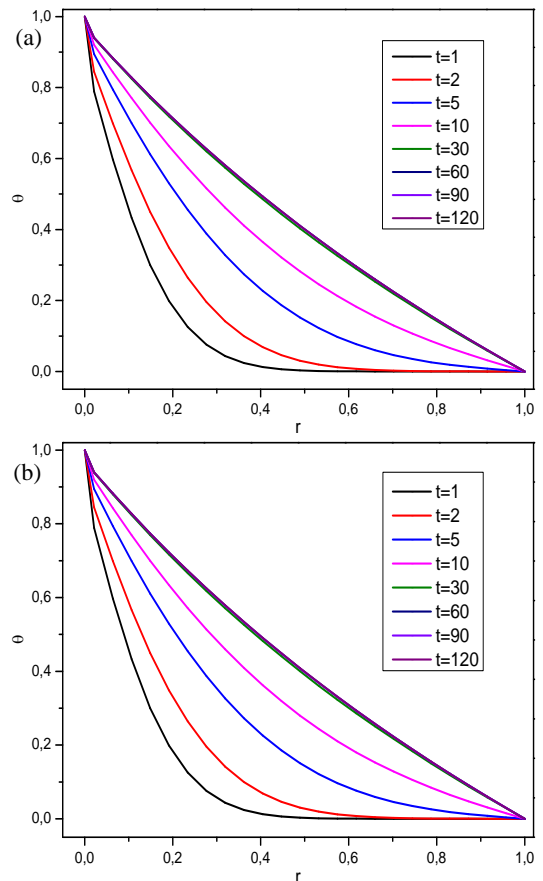
There is not much difference in velocity at $t= 10$ compared to $t = 120$, but comparing temperature distribution at $t=10$ with values greater than 10, it indicates that much more time is still needed to reach steady-state.

From fig. 12, we can notice that for a small value of Prandtl number (Pr=0,02), The effect of the time variation is found to be not significant on the temperature, it reaches faster a steady-state to the point that we can't notice the difference between the steady and unsteady states flows.

As we know, for larger fluid Prandtl number, the momentum flow transfer is faster than heat transfer. This can be seen clearly in Fig. 14 (for a fluid with Pr = 7) and the distribution of the azimuthal component of velocity reaching a steady-state quicker than the temperature at the mid-length.

CONCLUSION

In this study, the forced convection flow of an electrically conducting fluid between two horizontal concentric cylinders in the presence of an axial magnetic field and a temperature gradient considering the effects of viscous heat dissipation in the fluid has been investigated numerically and analytically. The velocity distribution in the annulus is obtained analytically in terms of the modified Bessel functions

whose argument contains Hartmann number and radial coordinate. To obtain the temperature, the expansions of the modified Bessel functions, with three terms which coincides better with the numerical results, are used in the energy equation.

It is found that the velocity decreases in the annulus with increase of Hartmann number. However an increase in Hartmann number does not affect the temperature. The effects of magnetic field strength and Eckert number on local and average Nusselt number have been examined. The results show that an increase in Hartmann number reduces the Nusselt number on both surfaces of the cylinders. Also it was noticed that as the Eckert number increases average Nusselt number increases on the outer cylinder, but opposite trend is observed on the inner cylinder. In addition, some results of the unsteady state have been discussed in this work.

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