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An Explicit Model for Concentration Distribution using Biquadratic-Log-Wake Law in an Open Channel Flow

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ABSTRACT

The log-wake law with biquadratic boundary correction for the vertical velocity distribution which was changed from cubic boundary correction by Guo for the pipe data is applied to turbulent flow in open-channels. The biquadratic-log-wake law is tested with experimental data from Coleman, Lyn, Wang and Qian and Kironoto and Graf. It shows that the biquadratic-log-wake law matches well with flume data. A new mathematical model for vertical concentration distribution using the biquadratic-log-wake law is proposed and tested with the existing laboratory data. This study reflect the fact that sediment suspension has significant effects on both von Karman constant and Coles' wake strength.

Keywords: Open-channel, Velocity distribution, Boundary correction, Sediment suspension, von Karman constant, Coles' wake strength.

NOMENCLATURE

A	relative density	R_i	Richardson number
B	additive constant in log law	S	channel slope
C	instantaneous volumetric sediment concentration	y	vertical distance from bed
C_{avg}	local time average volumetric sediment concentration	γ	proportionality constant
$C_{\xi a}$	volumetric sediment concentration at distance $\xi = \xi_a$	δ	maximum velocity distance from bed
C_0	bed concentration	ε_s	sediment diffusion coefficient
C_1	sediment concentration at $\xi = 1$	ε_m	momentum diffusion coefficient
C_m	mean concentration	κ	von Karman constant of mixture
d	sediment particle diameter	κ_0	von Karman constant for clear water
d_*	dimensionless sediment particle diameter	μ_f	dynamic viscosity of water
g	gravitational force	ν	kinematic viscosity of sediment water mixture
h	flow depth	ξ	$(= y/\delta)$ normalized distance
u_{max}	maximum velocity at $y = \delta$	ξ_a	normalized reference level
u	time average velocity at a distance y	Π	Coles' wake parameter
u_*	shear velocity	ρ	density of water sediment mixture
		ρ_f	density of clear water
		ω_0	sediment particle settling velocity

1. INTRODUCTION

Studies of velocity and concentration profile in a steady uniform sediment-laden open-channel flow are important subjects in sediment transport. They have fundamental importance in river mechanics. Numerous investigations related to sediment-laden turbulent flow in rectangular open-channels have been undertaken to examine the vertical distribution of velocity and concentration. In hydraulic open-channel flow over a smooth bed surface without sediment, the vertical distribution of mean velocity profile is usually described by log law throughout the depth. The log law in terms of inner variables is usually expressed as

$$\frac{u}{u_*} = \frac{1}{\kappa}\ln\frac{yu_*}{\nu} + B \tag{1}$$

where u is time-averaged velocity in main flow direction, u_* is the shear velocity, κ is the von Karman constant, y is vertical distance from channel bed, ν is the kinematic viscosity and B is the additive constant in the log law. Later on, Vanoni (1946), Einstein and Chien (1955), Elata and Ippen (1961), Wang and Qian (1989) and many researchers showed that log law remains valid in sediment-laden flows. They concluded that the von Karman constant decreases with sediment suspension. Barenblatt (1996) applied the log law in sediment-laden flows and pointed out that the application of the log law in sediment-laden flow is limited to the overlap zone i.e. the log law may not be valid in the wake layer and near the water surface. Several researchers Coles (1956) and Nezu and Rodi (1986) suggested that the deviated velocity profile from the standard log-law cannot be fitted only by adjusting the von Karman constant κ and additive constant B in wake layer.

Coles (1956) first introduced the wake function $W(\xi)$, and combined the logarithmic law with the wake function and produced the log-wake law

$$\frac{u}{u_*} = \left(\frac{1}{\kappa}\ln\frac{yu_*}{\nu}+B\right)+W(\xi) \tag{2}$$

in which the terms in the parentheses are the logarithmic law. $W(\xi)$ is the wake function which determines the deviation of the logarithmic law away from the wall, $\xi(= y/\delta)$ is the relative distance from the wall where δ is the distance of maximum velocity from the bed. Hinze (1975) gives an expression for the wake function which satisfies Coles' data i.e.

$$W(\xi) = \frac{2\Pi}{\kappa}\sin^2\frac{\pi\xi}{2} \tag{3}$$

in which Π is the Coles' wake strength. Finally, using Hinze's Eq. (3) the log-wake law i.e. Eq. (2) can be written as

$$\frac{u}{u_*} = \left(\frac{1}{\kappa}\ln\frac{yu_*}{\nu}+B\right)+\frac{2\Pi}{\kappa}\sin^2\frac{\pi\xi}{2} \tag{4}$$

Coleman (1981) applied the log-wake law to the open-channels. He analysed the effect of sediment suspension on the von Karman constant κ and the wake strength Π and concluded that the von Karman constant κ remains the same as clear water; on the other hand Coles' wake strength Π increases with the Richardson Number. According to Guo and Julien (2001) the Richardson number is defined as

$$R_i = \frac{g\delta}{u_*^2}\frac{\rho-\rho_f}{\rho_f}\frac{C_0-C_1}{1+[(\rho-\rho_f)/\rho_f]C_m} \tag{5}$$

where g is the gravitational force, δ is the maximum velocity distance from bed, ρ is the density of water sediment mixture, ρ_f is the density of clear water, C_0 is the bed concentration, C_1 is the sediment concentration at $\xi = 1$, C_m is the mean concentration. It reflects the density gradient intensity. The Richardson number becomes strong with the high density gradient. Coleman (1981) further pointed out that by incorrect extension of the log law to the wake layer, one obtained the wrong result that κ decreases with the sediment suspension; though von Karman constant may decrease with sediment suspension in the log-wake model (Lyn 1986). For open-channel flows, many researchers gave different values of Π. Coleman (1981) obtained an average value 0.19, Nezu and Rodi (1986) suggested a range from 0 to 0.2 and Kirkgoez (1989) got value 0.1. Also for smooth bed Π = - 0.077 was reported by Cardoso *et al.* (1989). So the value of Π is not universal for clear water and sediment-laden flow and the value may be positive or negative.

Although log law and log-wake law can predict the velocity profile in clear water and sediment-laden flow, both of them cannot satisfy the boundary condition at the boundary surface (Guo and Julien 2001). Guo and Julien (2001) proposed a clear water velocity profile satisfying the derivative boundary condition as

$$\frac{u_{\max}-u}{u_*} = -\frac{1}{\kappa}\ln\xi + \Omega\cos^2\frac{\pi\xi}{2} - \frac{1-\xi}{\kappa} \tag{6}$$

where $u_{\max}$ is the maximum velocity at distance $y = \delta$ or $\xi = 1$, Ω is the wake strength. He further pointed out that modified log-wake law i.e., Eq. (6) is also valid in sediment-laden flow and showed how the model parameters κ and Ω vary with sediment-suspension.

Guo and Julien (2003) developed a modified log-wake law for turbulent flow in smooth pipes. Later on, Guo *et al.* (2005) applied the modified log-wake law in zero-pressure-gradient (ZPG) turbulent boundary layer with proper modification. The modified log-wake law (MLWL) for ZPGBL reads as

$$\frac{u_{\max}-u}{u_*} = -\frac{1}{\kappa}\ln\xi + \frac{2\Pi}{\kappa}\cos^2\frac{\pi\xi}{2} - \frac{1-\xi^3}{3\kappa} \tag{7}$$

where Π is the Coles' wake strength. Here the last term denotes the cubic boundary correction. Guo and Julien (2008) applied this Eq. (7) to clear water flows in open-channels.

Guo (2006) studied the Zagarola and Smith (1997) data and argued that if the cubic boundary correction term is changed to biquadratic term the data fits more accurately with experimental data. This paper calls the log-wake law with the biquadratic boundary correction as biquadratic-log-wake law. Extension of the biquadratic-log-wake law to the sediment-laden open channels is a part of this paper.

Rouse (1937) derived the formula for vertical concentration distribution using the classical log law. Suspension-sediment concentration distribution phenomenon can be described by using many theories, such as: diffusion theory, energy theory, mixture theory, similarity theory and stochastic theory. The only difference, if any, is in the expression for the sediment diffusion coefficient ε_s (Ni and Wang 1991). For all of the aforementioned theories, one can find that they agree with some experimental data, but not all. There are limitations for the applicability of each of the theories. There is no general expression of sediment exchange coefficient which satisfies all experimental data set. The logarithmic law is not appropriate for narrow sediment-laden open-channels. Therefore, considering the biquadratic-log-wake law a new mathematical model for vertical concentration distribution is developed.

The main objectives of this paper are: (1) to show that the biquadratic-log-wake law is valid in both clear water and sediment-laden open channel flow; (2) to study the effect of sediment suspension on model parameters; (3) to test the developed analytical model for the vertical distribution of concentration with the existing laboratory data.

2. Velocity Defect Form of Biquadratic Log -wake Law (BLWL)

The log-wake law with the biquadratic boundary correction according to Guo (2006) reads as

$$\frac{u}{u_*}=\left(\frac{1}{\kappa}\ln\frac{yu_*}{\nu}+B\right)+\frac{2\Pi}{\kappa}\sin^2\frac{\pi\xi}{2}-\frac{\xi^4}{4\kappa} \tag{8}$$

The terms in the parentheses are the logarithm law of the wall; the sine square term is the law of the wake (Coles 1956) and the biquadratic function satisfies the maximum velocity condition. To eliminate the additive constant B one can assume the maximum velocity u = $u_{\max}$ at ξ = 1 to the log wake law with new boundary correction. From Eq. (8) one obtains

$$\frac{u_{\max}}{u_*}=\left(\frac{1}{\kappa}\ln\frac{\delta u_*}{\nu}+B\right)+\frac{2\Pi}{\kappa}-\frac{1}{4\kappa} \tag{9}$$

Subtracting Eq. (8) from Eq. (9) gives the velocity defect form of the log wake law with new boundary correction

$$\frac{u_{\max}-u}{u_*}=-\frac{1}{\kappa}\ln\xi+\frac{2\Pi}{\kappa}\cos^2\frac{\pi\xi}{2}-\frac{1-\xi^4}{4\kappa} \tag{10}$$

3. Vertical Concentration Distribution Model Using BLWL

In steady, uniform and fully developed open channel flow, carrying suspended load, the volume fraction of sediment concentration C is described by the advection-diffusion equation. Taking the x-axis along the bed in the main direction of flow and the y-axis vertically upwards, the steady-state equation can be expressed as follows (Graf 1971)

$$\omega_0 C(y)+\varepsilon_s(y)\frac{dC(y)}{dy}=0 \tag{11}$$

where ω_0 is the settling velocity of sediment particle, $C(y)$ is the sediment concentration at y and ε_s is the sediment diffusion coefficient. This equation is in fact a sediment mass conservation equation, where the mass flux $\varepsilon_s(dC/dy)$ in y-direction is balanced by sediment settling flux $-\omega_0 C(y)$. Equation (11) shows that different mathematical models of the distribution of sediment concentration may be derived by using different models of sediment diffusion coefficient.

According to the Reynolds analogy, the sediment diffusion coefficient ε_s is assumed to be proportional to the momentum diffusion coefficient ε_m as

$$\varepsilon_s=\gamma\varepsilon_m \tag{12}$$

where γ is the proportionality constant. In this equation, γ describes the difference between diffusivity of momentum (diffusivity of a fluid particle) and diffusivity of sediment particles. The momentum diffusion coefficient ε_m in fluid sediment mixture is given by Einstein and Chien (1955) as

$$\varepsilon_m=\frac{\tau_t}{\rho_f(1+AC)\dfrac{du}{dy}} \tag{13}$$

where ρ_f is the density of water and $A(=\rho_s/\rho_f-1)$ is a constant. This equation reduces to Boussinesq's formula $\tau_t=\rho_f\,\varepsilon_m(du/dy)$ for clear water flow. For two-dimensional open channel flow, turbulent shear stress τ_t can be expressed as

$$\frac{\tau_t}{\tau_0}=1-\frac{y}{h} \tag{14}$$

Combining Eq. (12), Eq. (13) and Eq. (14) one can express the sediment diffusion coefficient in the following form as

$$\varepsilon_s=\frac{\gamma u_*^2(1-\xi)}{(1+AC)\dfrac{du}{dy}} \tag{15}$$

where $u_*(=\sqrt{(\tau_0/\rho_f)})$ is the shear velocity, $\xi(=y/h)$ is the dimensionless distance from channel bed with respect to the flow depth h and C is the volumetric concentration at distance y. Substitution of Eq. (15) into the Eq. (11) gives

$$\frac{dC}{C(1+AC)}=-\frac{\omega_0}{\gamma u_*^2}\frac{\frac{du}{dy}}{1-\xi}dy \tag{16}$$

For the flow in narrow channel, maximum velocity occurs below the free surface, which is called velocity dip position. One can use the velocity dip position δ to normalize the distance y from the bed. By substituting ξ = y/δ into Eq. (16) and rewriting, this equation becomes

$$\frac{dC}{C(1+AC)} = -\frac{\omega_0}{\gamma u_*^2} \frac{\frac{du}{d\xi}}{1-\xi} d\xi \tag{17}$$

The velocity gradient can be expressed from Eq. (10) using the polynomial approximation of the sine square function $\sin^2(\pi\xi/2) \approx 3\xi^2 - 2\xi^3$ (White 1991) as

$$\frac{du}{dy} = \frac{u_*}{\kappa\xi} + \frac{12\Pi u_*}{\kappa}(\xi - \xi^2) - \frac{\xi^3 u_*}{\kappa} \tag{18}$$

Substituting Eq. (18) into Eq. (17) and integrating between ξ_a and ξ one can get the equation of sediment concentration distribution as

$$\ln\left(\frac{C(1+AC_{\xi_a})}{C_{\xi_a}(1+AC)}\right) = -\frac{\omega_0}{\kappa\gamma u_*}\left[\ln\frac{\xi}{\xi_a} + \left(\frac{\xi^3}{3} - \frac{\xi_a^3}{3}\right) + (12\Pi+1)\left(\frac{\xi^2}{2} - \frac{\xi_a^2}{2}\right) + (\xi - \xi_a)\right] \tag{19}$$

where $C_{\xi a}$ is the reference concentration of sediment at the reference level ξ_a. Suspended sediment concentration distribution equation can be expressed explicitly and more precisely from Eq. (19) as

$$\frac{C}{C_{\xi_a}} = \frac{1}{1 + (1 + AC_{\xi_a})\left[\exp\left\{\frac{\omega_0}{\kappa\gamma u_*}F(\xi)\right\} - 1\right]} \tag{20}$$

where $F(\xi)$ is a function of ξ which is given by

$$F(\xi) = \left[\ln\frac{\xi}{\xi_a} + \left(\frac{\xi^3}{3} - \frac{\xi_a^3}{3}\right) + (12\Pi+1)\left(\frac{\xi^2}{2} - \frac{\xi_a^2}{2}\right) + (\xi - \xi_a)\right] \tag{21}$$

The concentration distribution throughout the water depth can be calculated analytically from this equation if the reference level ξ_a and reference concentration $C_{\xi a}$ are known. Rouse equation can be obtained from Eq. (20) if one considers that velocity distribution follows the log-law and $A \approx 0$.

4. Comparison Of The BLWL For Clear Water And Sediment- Laden Flow

The experimental data obtained by Coleman (1986), Lyn (1986), Wang and Qian (1989) and Kironoto and Graf (1994) (clear water only) are used to test the validity of biquadratic-log-wake law in clear water and sediment-laden flow. In all data sets biquadratic-log-wake law matches well with existing flume data. The aspect ratio (ratio of channel width to flow depth) in all the data set are less than 5. So the maximum velocity occurs below the water surface in case of clear water and sediment-laden open channels. Figures (1)-(4) show the result in clear water. Figure 1(a) is in rectangular coordinates and Fig. 1(b) is in semi-log plot and similarly the others. Figures (5)-(7) show the result in sediment-water mixture.

To estimate the model parameters κ and Π, a least square method is used (Guo 1998). From the obtained values of κ and Π by the least square method one can see that the computed result matches well with the measured data of researchers.

Coleman's (1981, 1986) data are useful to test any mathematical model of velocity and concentration distribution in sediment-laden flows. To test the model Eq. (10) in clear water and sediment-laden open channels, Coleman's (1986) data are used. The bottom and side walls were assumed to be smooth. The flow conditions i.e. $h \approx 1.69$ mm, $b = 356$ mm, $S = 0.002$, $u_* = 0.041$ m/s were same for all runs. Figure (1) shows a comparison between Coleman's (1986) data and computed velocity from biquadratic-log-wake law for clear water runs 1, 21 and 32. The comparison of computed and observed velocity profiles for runs 3, 7, 11, 16, 24 and 31 are shown in Fig. (5). From both figures one can conclude that BLWL is valid in clear water and sediment-laden open channels.

Similarly Lyn's (1986) data are used to test the validity of the biquadratic-log-wake law in smooth open-channels. The values of the flow parameters are given in Table 2. To test the biquadratic-log-wake law in clear water, four clear water data C-1, C-2, C-3 and C-4 are used. For all these runs the value of the von Karman constant κ is kept to be 0.41. From Table 2 one can see that the value of Coles' wake strength can be negative. The results for the clear water are plotted in Fig. (2) and for the sediment-water mixture are plotted in Fig. (6). From both the figures it is clear that the biquadratic-log-wake law holds good in both clear water and sediment-water mixture flows.

Wang and Qian's (1989) clear water (CW) and clear water + plastic particle experiments (SF, SM and SC) are used to test the validity of the biquadratic-log-wake law in smooth narrow open-channels. The boundaries are smooth for all runs. Other flow conditions i.e., total flow depth $h = 8$, 9, 10 cm, channel width $b = 30$ cm, and bed slope $S = 0.01$ are kept same for all runs. The shear velocity u_* is assumed to be ≈ 0.07 m/s for fine plastic particles and 0.0916 m/s for medium and coarse plastic particles. Calculated values of the flow parameters for all runs (except SF5) are shown in Table 3. Computed and observed values of velocity profile for clear water runs CW1, CW2, CW3 and CW4 are presented in Fig. (3). Figure (7) shows the result in sediment-laden flows. From the figures one can see that the biquadratic-log-wake law describes the velocity profile well for both clear water and sediment-laden open-channel flows.

In a similar way, four data sets of Kironoto and Graf (1994) are used to test the biquadratic-log-wake law in clear water flows. The values of the parameters are given in Table 4. Comparison of four runs UGA3, UGA5, UGB3 and UGB5 with the computed values are presented in Fig. (4) which shows that biquadratic-log-wake law is applicable in clear water flows.

4.1 Effect of density gradient on model parameters

To test the effect of density gradient (R_i) on the main model parameters κ and Π for velocity distribution, Coleman's (1981) data are used. Coleman's (1981) experiment can be treated as density sediment-laden flow with dilute concentration where maximum volumetric sediment concentration is 2.3%. The calculated values of κ, Π and R_i are given in Table 1. Figure (8) shows that variation of κ and Π with Richardson number R_i. From the figure one can see that both κ and Π initially decreases with the increase of Richardson number and then gradually increases. So the density gradient has significant effect on both von Karman constant and Coles wake strength. A cubic relation between κ and R_i by fitting the data (except run 22) as

$$\frac{\kappa}{\kappa_0} = -6.1\times10^{-7} R_i^3 + 1.9\times10^{-4} R_i^2 - 0.15 R_i + 1 \tag{22}$$

where κ_0 = 0.41 is the von Karman constant for clear water. Similarly a quadratic relation between Π and R_i by fitting data as in Fig. 8(b) gives

$$\Pi = 6\times10^{-5} R_i^2 - 7\times10^{-3} R_i + 0.4 \tag{23}$$

One can predict von Karman coefficient κ and Coles' wake parameter Π from the above proposed formulae for density sediment-laden flows.

4.2 Effect of average concentration on model parameters

Wang and Qian's (1989) clear water (CW) and clear water + plastic particle experiments (SF, SM and SC) are used to test the effect of average concentration on von Karman constant and Coles' wake strength. The specific gravity of plastic particles is 1.05. So mixture of clear water and plastic particle flows can be assumed as quasi-neutral sediment-laden flows (Guo 1998). Therefore the effect of density gradient can be neglected. Figure 9(a) shows how the von Karman constant κ varies with average concentration C_{avg}. A linear relation between κ and C_{avg} by fitting the data gives as

$$\frac{\kappa}{\kappa_0} = 1 - 1.098 C_{avg} \tag{24}$$

where κ_0 = 0.41 is the value of the von Karman constant for clear water which is compatible with the previous result of other researchers. The von Karman constant κ decreases with the average sediment concentration C_{avg}.

Similarly the variation of Coles' wake strength Π with the variation of average concentration C_{avg} is plotted in Fig. 9(b). It shows that Coles' wake strength Π increases with the average value of the sediment concentration C_{avg}. From Wang and Qian's (1989) data one can obtain a linear relation between Π and C_{avg} by fitting the data as

$$\Pi = 0.081 + 0.66 C_{avg} \tag{25}$$

One can use above equations to predict von Karman coefficient κ and Coles' wake parameter Π for neutral or quasi-neutral sediment-laden flows where the effect of density gradient is negligible.

4.3 Combined effect of density gradient and average concentration on model parameters

A composite expression for the effects of average concentration C_{avg} and Richardson number R_i on the von Karman constant may be expressed as

$$\frac{\kappa}{\kappa_0} = -6.1\times10^{-7} R_i^3 + 1.9\times10^{-4} R_i^2 - 0.15 R_i + 1 - 1.098 C_{avg} \tag{26}$$

A similar expression can be found in Guo's (1998) paper. On the other hand, a single composite expression for the Coles' wake parameter for the effects of average concentration and Richardson number is not possible. The Coles' wake parameter reflects the effect of side wall on velocity distribution which also depends on the aspect ratio of open channels. For wide open channels, the effect of Coles' wake parameter may be neglected i.e. $\Pi \approx 0$ and for narrow open channels (where aspect ratio is less than or equal to five) value of Π linearly decreases to zero when aspect ratio increases to five (Guo 1998).

5. Comparison Of The Concentration Model With Experimental Data

Data of Coleman (1986), Lyn (1986) and Wang and Qian (1989) are used to validate the present model. The calculated values of sediment concentration are obtained from Eq. (20). The main parameters for the concentration distribution model are the sediment particle settling velocity ω_0 and the proportionality constant γ. In this paper, the sediment concentration is normalized by the value $C_{\xi a}$ at $\xi = \xi_a$. Here the reference level ξ_a is taken as the lowest normalized distance from the bed from the available data. Then $C_{\xi a}$ is the volumetric concentration at $\xi = \xi_a$ from the available data. Values of another two parameters namely κ and Π are same as they are in the velocity model.

Settling velocity of sediment particles is calculated from the formula proposed by Zhiyao *et al.* (2008) as

$$\omega_0 = \frac{\nu}{d} d_*^3 \left[38.1 + 0.93 d_*^{12/7}\right]^{-7/8} \tag{27}$$

where d_* is the dimensionless sediment particle diameter, d is the sediment particle diameter and ν is the kinematic viscosity for the sediment-water mixture which is given as

$$\nu = \frac{\mu_f\,(1+2.5C_{avg}+6.25C_{avg}^2+15.62C_{avg}^3)}{\rho_f\,(1+AC_{avg})} \qquad (28)$$

where μ_f is the dynamic viscosity of water, C_{avg} is the local time average volumetric sediment concentration and ρ_f is the density of clear water. For all the data sets γ is taken as fitting parameter.

Coleman's (1986) experiments were performed in smooth rectangular open-channel. The concentration was measured at 12 points over the entire flow depth. Three types of sands with diameter d = 0.105, 0.210 and 0.420 mm were used in the experiment. Computed value of the sediment settling velocity ω_0 for all runs are given in Table 1. From the table one can observe that sediment particle settling velocity decreases with the increase of sediment concentration. The calculated concentration distribution with the observed data of Coleman (1986) for runs 3, 7, 11, 13, 16, 20, 24 and 31 are plotted in Fig. (10). From the figure one can see that the present model matches well with the observed data. From the figure it is also clear that the value of γ decreases with the increase of sediment concentration.

Lyn (1986) performed experiments in sediment-laden, uniform open-channel turbulent flow over flat beds. For equilibrium bed conditions Lyn used three different grain sizes: for bed 1565EQ, d = 0.15 mm; for 1965EQ, d = 0.105 mm and for 2565EQ, d = 0.24 mm. In this data set also the reference level is chosen at the lowest available height of observed concentration data set for experiment 1565EQ, 1965EQ and 2565EQ. The calculated value of settling velocity of sediment particle is given in Table 2. A comparison between the proposed model and the flume data of Lyn (1986) is presented in Fig. (11). From the figure it is clear that the model matches well with the data.

Similarly, Wang and Qian's (1989) data are used to test the concentration model. In this paper, three types of runs are used: SF (clear water + fine plastic particle), SM (clear water + medium plastic particle) and SC (clear water + coarse plastic particle). The diameters of the plastic particles are d = 0.268 mm (fine), 0.96 mm (medium) and 1.42 mm (coarse). The settling velocity of plastic particles is presented in Table 3. Calculated results of sediment distribution with the Wang and Qian's (1989) data are plotted in Fig. (12) for runs SF3, SF4, SM2, SM3, SC3 and SC4. From the figure it is clear that for all fine, medium and coarse plastic particles the computed value of concentration matches well with the experimental data.

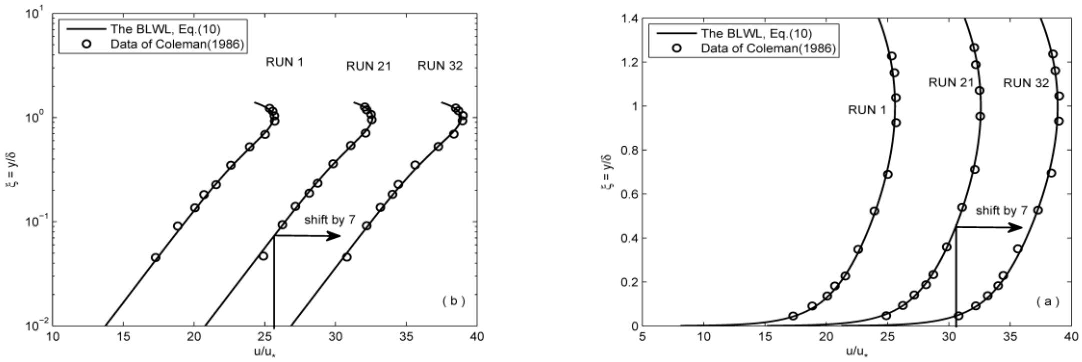

Fig. 1. Comparison of the BLWL with Coleman's (1986) clear water data (Run 1, 21 and 32)

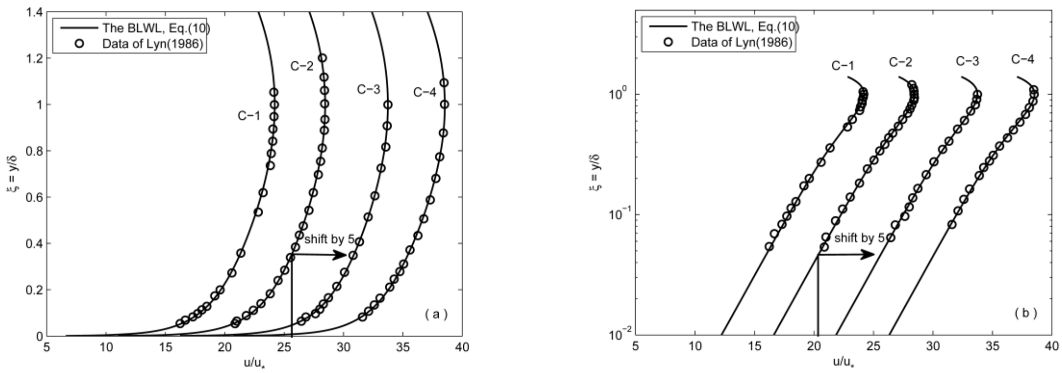

Fig. 2. Comparison of the BLWL with Lyn's (1986) clear water data (Run C1, C2, C3 and C4)

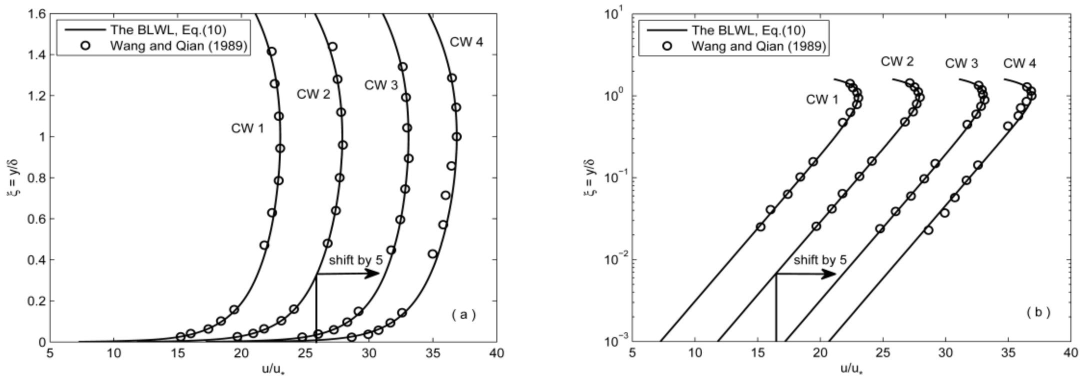

Fig. 3. Comparison of the BLWL with Wang and Qian's (1989) clear water data (Run CW1, CW2, CW3 and CW4)

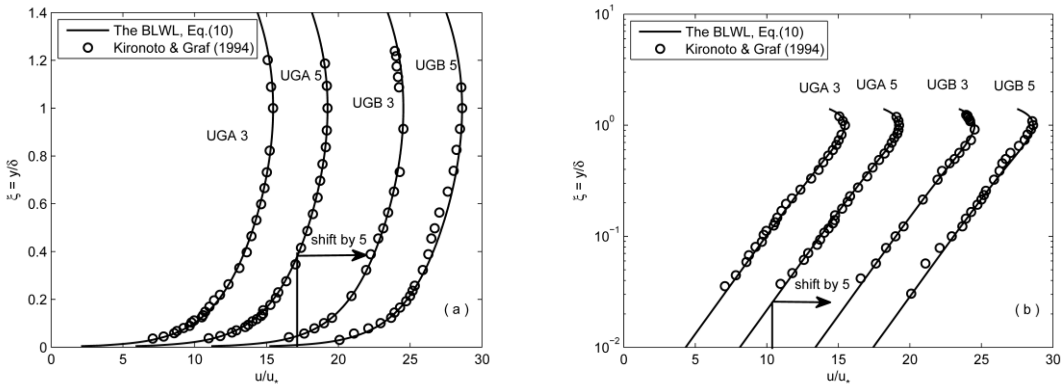

Fig. 4. Comparison of the BLWL with Kironoto and Graf's (1994) clear water data (Run UGA3, UGA5, UGB5 and UGB5)

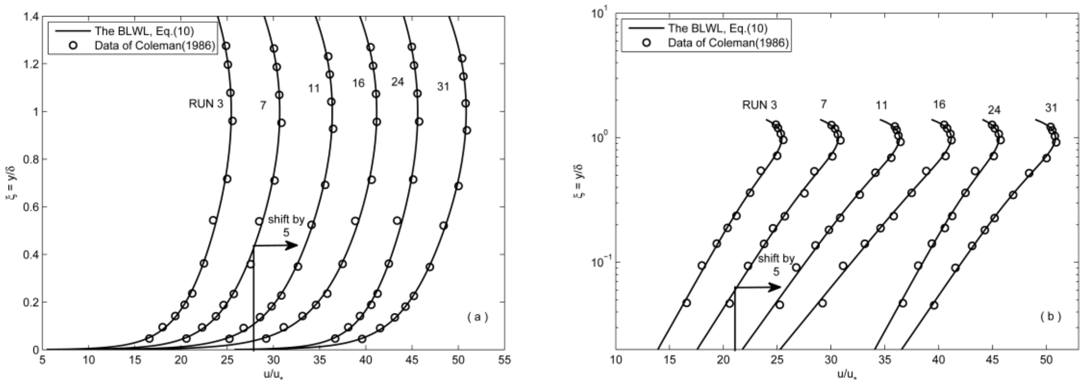

Fig. 5. Comparison of the BLWL with Coleman's (1986) sediment water mixture data (Run 3, 7, 11, 16, 24 and 31)

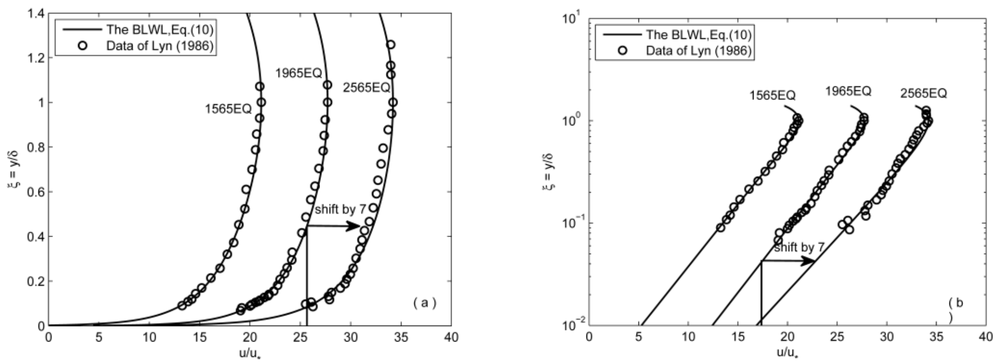

Fig. 6. Comparison of the BLWL with Lyn's (1986) sediment water mixture data (Run 1565EQ, 1965EQ and 2565EQ)

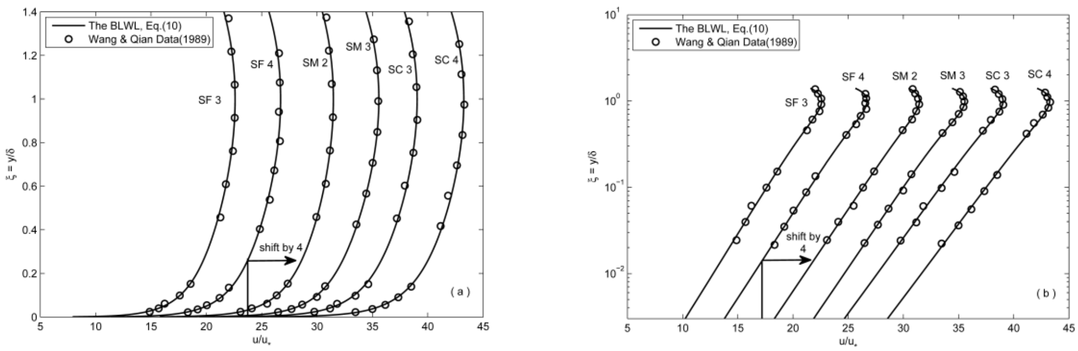

Fig. 7. Comparison of the BLWL with Wang and Qian's (1989) sediment water mixture data (Run SF1, SF2, SM2, SM3, SC3 and SC

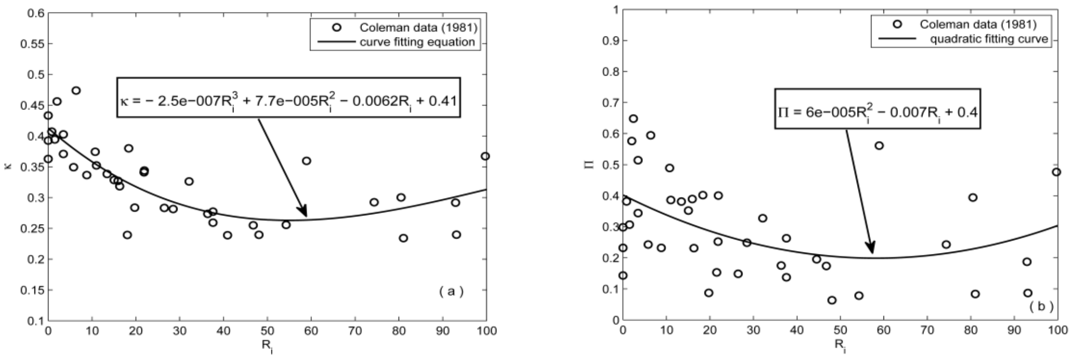

Fig. 8. Effect of density gradient on (a) von Karman constant and (b) Coles' wake parameter

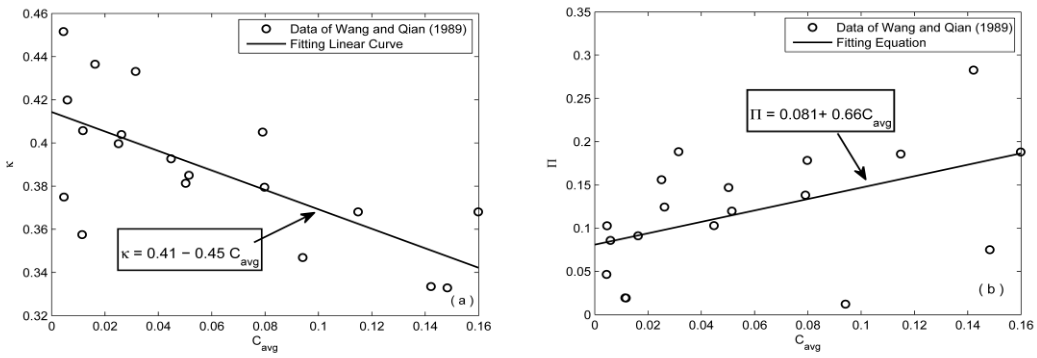

Fig. 9. Effect of average concentration on (a) von Karman constant and (b) Coles' wake parameter

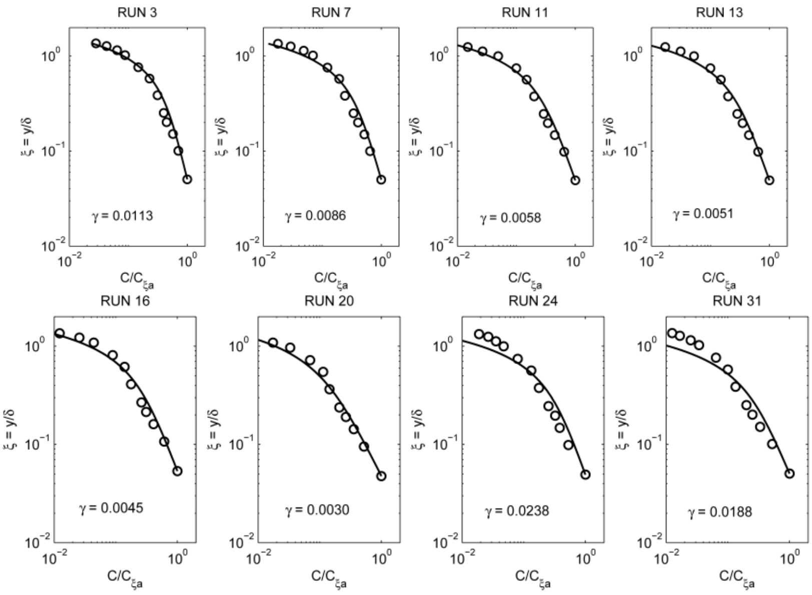

Fig. 10. Calculated and measured sediment concentration profiles for runs 3, 7, 11, 13, 16, 20, 24 and 31 of Coleman (1986)

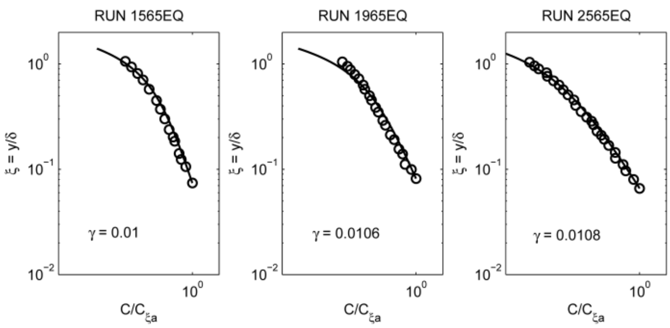

Fig. 11. Calculated and measured sediment concentration profiles for runs 1565EQ, 1965EQ and 2565EQ of Lyn (1986)

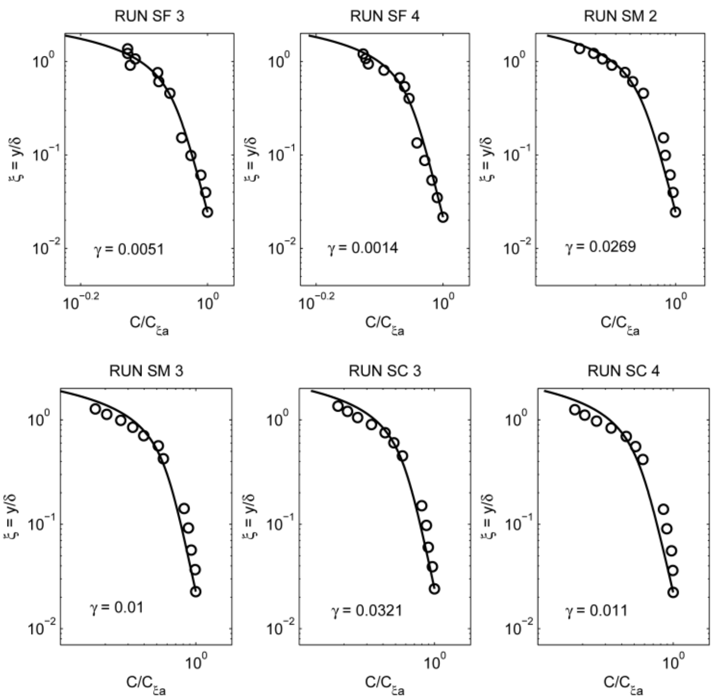

Fig. 12. Calculated and measured sediment concentration profiles for runs SF3, SF4, SM2, SM3, SC3 and SC4 of Wang and Qian (1989)

Table 1 Values of parameters of Coleman's (1981, 1986) experimental data

RUN	h (mm)	u_{max} ($m=s$)	δ (mm)	C_0 ($\times 10^{-3}$)	C_1 ($\times 10^{-4}$)	C_m ($\times 10^{-3}$)	R_i	κ	Π	ω_0 (m/s)
1	172	1.05	132	0.000	0.000	0.000	0.0000	0.4100	0.2558	0.0066
2	171	1.05	120	1.390	0.920	0.305	1.4991	0.3943	0.3067	0.0065
3	172	1.05	119	3.200	1.580	0.580	3.4824	0.3707	0.3440	0.0063
4	171	1.05	128	4.900	1.880	0.870	5.7993	0.3493	0.2424	0.0060
5	171	1.04	122	7.800	2.800	1.120	8.8178	0.3366	0.2318	0.0057
6	170	1.05	119	10.000	3.720	1.450	11.0060	0.3521	0.3860	0.0054
7	171	1.06	120	13.500	4.400	1.680	15.0490	0.3283	0.3520	0.0050
8	173	1.04	130	13.500	4.100	1.860	16.3357	0.3184	0.2311	0.0046
9	172	1.04	136	15.500	3.750	2.500	19.7257	0.2837	0.0868	0.0043
10	171	1.06	134	21.000	3.800	2.790	26.4840	0.2831	0.1485	0.0038
11	169	1.08	122	25.000	6.000	3.080	28.5189	0.2813	0.2483	0.0036
12	173	1.06	134	29.000	6.400	3.400	36.3887	0.2735	0.1749	0.0034
13	171	1.07	122	33.000	8.200	3.580	37.5814	0.2771	0.2629	0.0031
14	171	1.06	122	33.000	7.800	4.020	37.6010	0.2591	0.1372	0.0028
15	171	1.07	119	42.000	8.800	4.150	46.7974	0.2548	0.1736	0.0027
16	171	1.08	122	39.000	8.000	4.400	40.9002	0.2525	0.1950	0.0025
17	171	1.07	131	39.000	5.950	4.700	48.0717	0.2395	0.0634	0.0025
18	172	1.05	131	44.000	6.250	4.820	54.2820	0.2557	0.0783	0.0024
19	170	1.07	131	75.000	5.900	4.800	93.1240	0.2398	0.0866	0.0023
20	170	1.07	126	68.000	6.600	5.030	81.0288	0.2340	0.0835	0.0021
21	169	1.05	128	0.000	0.000	0.000	0.0000	0.4100	0.2458	0.0216
22	170	1.03	122	2.130	0.650	0.245	2.4249	0.5511	0.6480	0.0212
23	170	1.05	119	5.700	1.170	0.560	6.3914	0.4737	0.5940	0.0207
24	169	1.06	122	15.800	1.590	0.815	18.3496	0.3799	0.4017	0.0201
25	167	1.06	124	17.500	2.980	1.210	18.0625	0.3017	0.1529	0.0193
26	171	1.04	130	17.800	3.000	1.430	21.8546	0.3409	0.2518	0.0186
27	168	1.07	122	19.000	2.850	1.890	21.9171	0.3439	0.4002	0.0177
28	170	1.06	122	27.800	3.320	2.000	32.1619	0.3261	0.3273	0.0168
29	168	1.08	130	57.000	2.980	1.790	74.3522	0.2924	0.2424	0.0160
30	168	1.09	137	71.000	2.730	2.490	92.9203	0.2915	0.1869	0.0151
31	172	1.07	119	71.000	4.400	2.680	80.4960	0.3002	0.3940	0.0146
32	173	1.02	131	0.000	0.000	0.000	0.0000	0.4100	0.2858	0.0549
33	174	1.04	128	0.695	0.175	0.065	0.8349	0.4070	0.3817	0.0547
34	172	1.05	131	1.630	0.241	0.103	2.0254	0.4562	0.5761	0.0546
35	172	1.06	122	3.000	0.415	0.177	3.4745	0.4026	0.5142	0.0545
36	171	1.09	122	9.200	0.620	0.267	10.7301	0.3744	0.4891	0.0541
37	167	1.08	119	11.800	0.800	0.365	13.4214	0.3383	0.3810	0.0540
38	167	1.12	114	16.100	1.060	0.455	15.9497	0.3270	0.3891	0.0537
39	171	1.12	114	62.000	1.120	0.510	58.9378	0.3594	0.5611	0.0535
40	171	1.11	125	100.000	1.150	0.545	99.7121	0.3671	0.4760	0.0535

Table 2 Values of parameters of Lyn's (1986) experimental data

RUN	h (cm)	u_{max} (m/s)	$u*$ (cm/s)	δ (cm)	κ	Π	ω_0 (m/s)
C-1	6.54	0.7530	3.11	5.66	0.4100	0.2780	0.0000
C-2	6.53	0.8750	3.73	5.22	0.4100	0.2480	0.0000
C-3	5.75	0.8570	3.61	5.46	0.4100	0.2580	0.0000
C-4	5.69	1.0190	4.33	4.88	0.4100	0.3250	0.0000
1565EQ	6.45	0.7562	3.58	5.66	0.2808	0.0464	0.0123
1965EQ	6.51	0.7767	3.75	5.74	0.2878	0.0216	0.0184
2565EQ	6.54	0.8585	4.25	4.85	0.2427	-0.0740	0.0262

Table 3 Values of parameters of Wang and Qian's (1989) experimental data

RUN	h (cm)	u_{max} (m/s)	δ (cm)	C_{avg} (%)	κ	Π	ω_0 (m/s)
CW1	10	2.111	6.362	0.0000	0.4100	-0.0900	0.00000
CW2	10	2.098	6.254	0.0000	0.4100	-0.0300	0.00000
CW3	10	2.115	6.712	0.0000	0.4100	-0.0700	0.00000
CW4	8	2.003	7.000	0.0000	0.4100	-0.0100	0.00000
SF1	10	2.120	6.660	0.4650	0.3750	0.1027	0.01430
SF2	10	2.090	6.600	1.1483	0.3575	0.0195	0.00380
SF3	10	2.070	6.570	2.5100	0.3997	0.1560	0.00080
SF4	10	2.080	7.440	5.0233	0.3813	0.1468	0.00020
SF6	9	2.160	9.010	14.2125	0.3335	0.2827	0.00001
SM1	10	2.110	7.060	0.4483	0.4516	0.0464	0.09060
SM2	10	2.150	6.550	1.6275	0.4365	0.0911	0.02260
SM3	10	2.160	7.070	3.1450	0.4331	0.1883	0.00670
SM4	10	2.190	7.190	5.1433	0.3850	0.1196	0.00250
SM5	10	2.200	8.330	7.9050	0.4050	0.1381	0.00110
SM6	10	2.210	9.400	9.4117	0.3469	0.0121	0.00070
SM7	10	2.230	8.680	15.9842	0.3680	0.1881	0.00030
SC1	10	2.120	6.430	0.5917	0.4199	0.0858	0.11810
SC2	10	2.100	6.790	1.1725	0.4057	0.0192	0.06920
SC3	10	2.110	6.640	2.6167	0.4039	0.1244	0.02000
SC4	10	2.130	7.190	4.4783	0.3926	0.1028	0.00720
SC5	10	2.150	7.350	7.9758	0.3795	0.1783	0.00230
SC6	10	2.170	7.540	11.4883	0.3680	0.1856	0.00110
SC7	10	2.160	7.730	14.8292	0.3328	0.0750	0.00070

Table 4 Values of parameters of Kironoto and Graf's (1994) experimental data

RUN	h (cm)	u_{max} (m/s)	u_* (cm/s)	δ (cm)	κ	Π
UGA3	28.50	0.572	3.70	22.40	0.41	0.105
UGA5	28.50	0.570	4.00	21.40	0.41	0.106
UGB3	29.00	0.465	3.20	22.91	0.41	0.104
UGB5	29.00	0.463	3.40	22.91	0.41	0.115

6. Conclusion

The following conclusions can be drawn from the above discussion:

1. The biquadratic-log-wake law (BLWL) originally developed by Guo (2006) can be applied to describe the velocity profile in clear water as well as sediment-laden open-channel flows. It can also describe the dip phenomenon.

2. The concentration distribution model developed in this work using the biquadratic-log-wake law can describe the vertical concentration distribution throughout the depth. For higher concentration it deviates from the observed data. This deviation occurs as we start with the Rouse diffusion equation which is applicable only for low concentrated flows.

3. The Richardson number R_i and average concentration C_{avg} have significant effect on von Karman constant κ and Coles' wake strength Π. Both κ and Π initially decrease with the increase of Richardson number then gradually increase. On the other hand, κ decreases with the increase of C_{avg} and Π increases with the increase of C_{avg}.

4. The proportionality constant γ between sediment diffusion coefficient ε_s and momentum diffusion coefficient ε_m gradually decreases with the increase of sediment suspension.

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