



## MHD Unsteady Memory Convective Flow through Porous Medium with Variable Suction

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### ABSTRACT

A free convective unsteady visco-elastic flow through porous medium of variable permeability bounded by an infinite vertical porous plate with variable suction, constant heat flux under the influence of transverse uniform magnetic field has been investigated in the present study. The permeability of porous medium fluctuates with time about the constant mean. Approximate solutions for mean velocity, transient velocity, mean temperature and transient temperature of non-Newtonian flow and skin friction are obtained. The effects of various parameters such as  $P_r$  (Prandtl number),  $G_r$  (Grashof number),  $M$  (Hartmann number),  $\omega$  (frequency parameter) and  $k_0$  (mean permeability parameter) on the above are depicted, skin friction, amplitude and phase are shown graphically and discussed. Expressions for fluctuating parts of velocity ' $M_r$ ' and ' $M_i$ ' are found and plotted graphically, effects of different parameters on them are discussed.

**Keywords:** Free convection, Walter's liquid 'B', Variable permeability, Porous medium and Variable suction.

### 1. INTRODUCTION

In recent years, the problem of free convection flow through a porous medium of variable permeability has attracted the attention of a number of scholars because of its possible application in design of steam displacement process in an oil recovery and various geothermal systems. In fact a porous material containing the fluid is non-homogeneous and there can be numerous inhomogeneities present in the porous medium. Therefore, the permeability of porous medium may not necessarily be constant. This is studied extensively by Chandrasekhara *et al.* (1984), Vedha Nayagam *et al.* (1987) Singh *et al.* (1989). Sreekanth *et al.* (1996) studied the hydro magnetic free convective flow through a porous medium with variable permeability. Singh and Kumar (2002) have discussed three dimensional free convective flow and heat transfer through porous medium with periodic permeability. Later, Vershney and Singh (2005) extended the above problem for mass transfer. Sharma *et al.* (2006) studied MHD fluctuating free convective flow with radiation embedded in porous medium having variable permeability and heat source / sink. Mahanti and Gaur (2009) have studied the effects of varying viscosity and thermal conductivity on steady free convective flow and heat transfer along an isothermal vertical plate in the presence of heat sink. Anjali and Ganga (2010) have studied the dissipation effects on MHD non linear flow and heat transfer past a porous surface with prescribed

heat flux. Humera *et al.* (2010) have studied the hydromagnetic free convective Reclin-Ericksen flow through a porous medium with variable permeability. Suneetha *et al.* (2011) studied the radiation and mass transfer effects on MHD free convective dissipative fluid in the presence of heat source / sink. Later Vasu *et al.* (2011) studied the radiation and mass transfer effects on transient free convection flow of a dissipative fluid past semi-infinite vertical plate with uniform heat and mass flux. Prasad *et al.* (2011) have studied the finite difference analysis of radioactive free convection flow past an impulsively started vertical plate with variable heat and mass flux. Seth *et al.* (2011) studied the effects of rotation and magnetic field on unsteady coquette flow in a porous channel. Singh and Kumar (2011) have studied the fluctuating heat and mass transfer on unsteady MHD free convection flow of radiating and reacting fluid past a vertical porous plate in slip-flow regime. Das *et al.* (2011) have studied the mass transfer effects on unsteady hydro magnetic convective flow past a vertical porous plate in a porous medium with heat source. Reddy and Reddy (2011) studied the mass transfer and heat generation effects on MHD free convective flow past an inclined vertical surface in a porous medium.

The aim of the authors is to extend the problem of Maharshi and Tak (2000) to memory fluid i.e. Walter's liquid model B' (1960, 1962) with variable suction. The mixture of polymethyl methacrylate and pyridine at 25<sup>0</sup>

C containing 30.5g of polymer per liter behaves very nearly as the above mentioned liquid.

## 2. FORMULATION OF THE PROBLEM

We consider the flow of convective memory fluid through a porous medium bounded by an infinite vertical porous plate with constant heat flux under the influence of uniform transverse magnetic field. The x - axis is taken along the plate in the upward direction and y - axis normal to it. All the fluid properties are assumed to be constant, except that influence of the density variations with temperature is considered only in the body force term. The magnetic field of small intensity  $H_0$  is induced in the y - direction. Since the fluid is slightly conducting, the magnetic Reynolds number is far lesser than unity hence the induced magnetic field is neglected in comparison with the applied magnetic field following Sparrow and Cess (1961). The viscous dissipation and Darcy's dissipation terms are neglected for small velocities following Rudraiah *et al.* (1979). The flow in the medium is entirely due to buoyancy force. So under these conditions the flow with variable suction is governed by the following equations:

$$v = -v_0(1 + \varepsilon e^{i\omega t}) \tag{1}$$

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = g\beta_1(T - T_\infty) + v \frac{\partial^2 u}{\partial y^2} - \frac{vu}{k(t)} \tag{2}$$

$$\beta \left( \frac{\partial^3 u}{\partial t \partial y^2} + v \frac{\partial^3 u}{\partial y^3} \right) - \left( \frac{\sigma \mu e^2 H_0^2}{\rho} \right) u \tag{3}$$

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T}{\partial y^2} \tag{4}$$

The boundary conditions are:

$$y = 0 : u = 0, \frac{\partial T}{\partial y} = \frac{-q}{\kappa} \tag{5}$$

$$y \rightarrow \infty : u = 0, T = T_\infty$$

where u and v are velocity components along x and y - directions,  $\omega$  is the frequency parameter  $\beta_1$  is the coefficient of volume expansion,  $\beta$  is kinematic visco-elasticity,  $\sigma$  is electrical conductivity,  $\mu_e$  is magnetic permeability,  $H_0$  is magnetic intensity,  $k$  is thermal conductivity,  $C_p$  specific heat at constant pressure, q is heat flux and the permeability of porous medium which is assumed to be of the form following Singh *et al.* (1989).

$$k(t) = k_0(1 + \varepsilon e^{i\omega t}) \tag{6}$$

where  $k_0$  is the mean permeability of the medium,  $\omega$  is the frequency of fluctuation, t the time and ( $\varepsilon \ll 1$ ) is a constant quantity, other symbols have usual meanings.

## 3. METHOD OF SOLUTION

Introducing the following non-dimensional quantities:

$$y' = \frac{yv_0}{g}, t' = \frac{tv_0^2}{4g}, \omega' = \frac{4g\omega}{v_0^2}, u' = \frac{u}{v_0} \tag{7}$$

$$G_r = \frac{g\beta_1 q g^2}{kv_0^4}, P_r = \frac{\mu C_p}{\kappa}, R_m = \frac{\beta v_0^2}{g^2} \tag{8}$$

$$k_0' = \frac{k_0 v_0^2}{g^2}, M = \frac{\sigma \mu e^2 H_0^2 g}{v_0^2 \rho}, \theta = \frac{(T - T_\infty) \kappa v_0}{q g}$$

the Eq. (2) and Eq. (3) in view of Eq.(5) and Eq. (6) become after suppressing primes,

$$\left( \frac{1}{4} \right) \frac{\partial u}{\partial t} - \left( 1 + \varepsilon e^{i\omega t} \right) \frac{\partial u}{\partial y} = G_r \theta + \frac{\partial^2 u}{\partial y^2} \tag{9}$$

$$-R_m \left[ \left( \frac{1}{4} \right) \frac{\partial^3 u}{\partial t \partial y^2} - \left( 1 + \varepsilon e^{i\omega t} \right) \frac{\partial^3 u}{\partial y^3} \right] - \frac{u}{k_0} \left( 1 + \varepsilon e^{i\omega t} \right) - Mu \tag{10}$$

$$\left( \frac{1}{4} \right) \frac{\partial \theta}{\partial t} - \left( 1 + \varepsilon e^{i\omega t} \right) \frac{\partial \theta}{\partial y} = \left( \frac{1}{P_r} \right) \frac{\partial^2 \theta}{\partial y^2} \tag{11}$$

The corresponding boundary conditions are:

$$y = 0 : u = 0, \frac{\partial \theta}{\partial y} = -1 \tag{12}$$

$$y \rightarrow \infty : u = 0, \theta = 0$$

The partial differential equations, Eq. (7) and Eq. (8) are reduced to ordinary differential equations by assuming the following series expressions for velocity and temperature fields.

$$u(y,t) = u_0(y) + \varepsilon e^{i\omega t} u_1(y) + \dots \tag{13}$$

$$\theta(y,t) = \theta_0(y) + \varepsilon e^{i\omega t} \theta_1(y) + \dots \tag{14}$$

substituting Eq. (10) and Eq. (11) in Eq. (7) and Eq. (8) and equating the coefficients of like powers of  $\varepsilon$  to zero and neglecting  $O(\varepsilon^2)$ , the following set of ordinary differential equations are obtained :

$$R_m u_0'''' + u_0'' + u_0' - \left( \frac{1}{k_0} + M \right) u_0 = -G_r \theta_0 \tag{15}$$

$$R_m u_1'''' + \left[ \left( 1 - \frac{iR_m \omega}{4} \right) u_1'' + u_1' - \left( \frac{i\omega}{4} + \frac{1}{k_0} + M \right) u_1 \right]' = -G_r \theta_1 - \frac{u_0}{k_0} - u_0' - R_m u_0'''' \tag{16}$$

$$\frac{\theta_0''}{P_r} + \theta_0' = 0 \tag{14}$$

$$\frac{\theta_1''}{P_r} + \theta_1' = \frac{i\omega\theta_1}{4} = -\theta_0' \tag{15}$$

The corresponding boundary conditions are:

$$y = 0: u_0 = 0, u_1 = 0, \theta_0' = -1, \theta_1' = 0 \tag{16}$$

$$y \rightarrow \infty: u_0 = 0, u_1 = 0, \theta_0 = 0, \theta_1 = 0$$

Equation (12) and Eq. (13) are third order differential equations when  $R_m \neq 0$  and we have two boundary conditions, so we follow Beard and Walter's (1964),

$$u_0 = u_{01} + R_m u_{02} + OR_m^2 \tag{17}$$

$$u_1 = u_{11} + R_m u_{12} + OR_m^2 \tag{18}$$

substituting these Eq. (17) and Eq. (18) into Eq. (12), equating different powers of  $R_m$  and neglecting  $O(R_m^2)$ , we get

$$u_{01}'' + u_{01}' - \left(\frac{1}{k_0} + M\right)u_{01} = -G_r\theta_0 \tag{19}$$

$$u_{11}'' + u_{11}' - \left(\frac{i\omega}{4} + \frac{1}{k_0} + M\right)u_{11} \tag{20}$$

$$= \frac{-u_{01}}{k_0} - u_{01}' - G_r\theta_1$$

$$u_{01}''' + u_{02}'' + u_{02}' - \left(\frac{1}{k_0} + M\right)u_{02} = 0 \tag{21}$$

$$u_{11}'' - \frac{i\omega}{4}u_{11}'' + u_{12}'' + u_{12}' - \left(\frac{i\omega}{4} + \frac{1}{k_0} + M\right)u_{12} \tag{22}$$

$$= \frac{-u_{02}}{k_0} - u_{02}' - u_{01}''$$

The boundary conditions are:

$$y = 0: u_{01} = u_{02} = u_{11} = u_{12} = 0 \tag{23}$$

$$y \rightarrow \infty: u_{01} = u_{02} = u_{11} = u_{12} = 0$$

The velocity and temperature fields are given by

$$u = u_0 + \varepsilon u_1 = u_{01} + R_m u_{02} + \varepsilon(u_{11} + R_m u_{12}) \tag{24}$$

$$\theta = \theta_0 + \varepsilon\theta_1 \tag{25}$$

Taking real part of solution for the velocity field and temperature field they can be expressed in terms of fluctuating part as

$$u(y, t) = u_0(y) + \varepsilon(M_r \cos \omega t - M_i \sin \omega t) \tag{26}$$

$$\theta = \theta_0(y) + \varepsilon(N_r \cos \omega t - N_i \sin \omega t) \tag{27}$$

Where

$$M_r + iM_i = u_1(y) \tag{28}$$

$$N_r + iN_i = \theta_1(y) \tag{29}$$

The expressions of transient velocity and transient temperature for  $\omega t = \pi / 2$  are given by

$$u\left(y, \frac{\pi}{2\omega}\right) = u_0(y) - \varepsilon M_i \tag{30}$$

$$\theta\left(y, \frac{\pi}{2\omega}\right) = \theta_0(y) - \varepsilon N_i \tag{31}$$

The skin friction at the plate in terms of amplitude and phase is:

$$C_f = \frac{T_\omega}{\rho v_0^2} = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{G_r}{P_1}(P_r - r_1)(1 + R_m P_3) + R_m r_3 + \varepsilon|\lambda|\cos(\omega t + \alpha) \tag{32}$$

Where

$$C_f = \frac{T_\omega}{\rho v_0^2} = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{G_r}{P_1}(P_r - r_1)(1 + R_m P_3) + R_m r_3 + \varepsilon|\lambda|\cos(\omega t + \alpha) \tag{33}$$

Where

$$\lambda_r + i\lambda_i = \lambda \text{ and } \tan \alpha = \frac{\lambda_r}{\lambda_i}$$

#### 4. DISCUSSION AND CONCLUSIONS

The (thermal Grashoff number)  $G_r$  signifies the relative effect of the thermal buoyancy to the viscous hydrodynamic force, so the rise in mean velocity is due to enhancement of thermal buoyancy force. In the neighborhood of plate steep rise is noted then gradually diminishing. From Eq. (6), we see that mean permeability parameter is in direct proportion to mean permeability enhancement of mean velocity is attributed to higher values of mean permeability parameter. (Prandtle number)  $P_r$  defined by the relative ratio of kinematics viscosity to thermal diffusivity viscous forces tend to slow down the velocity so reduction of mean velocity is due to increase in Prandtle number value. Application of transverse magnetic field gives rise to Lorentz force, which is resistive typed force analogous to drag force, higher values of magnetic field suppresses mean velocity. Transient velocity profiles are influenced by mean velocity and fluctuating part  $M_i$  as it contains both in its expressions. Transitive velocities are negative buoyancy force, Lorentz force and viscous force are likely to suppress the transient velocity parameter profiles whereas higher

values of mean permeability parameter  $k_0$  and frequency parameter  $\omega$  are influencing the same refer Fig. 1. and Fig. 2.

The dominant viscous forces bring down degree of hotness so the effect of Prandtl number is to subside the temperature distribution in thermal boundary layer refer Fig. 3 and Fig. 4.

Fluctuating parts  $M_r$  and  $M_i$  are negative buoyancy force and viscous forces tends to reduce the fluctuating parts  $M_r$  and  $M_i$  considerably whereas mean

permeability parameter and frequency parameter have same effect on  $M_r$  and  $M_i$  refer Fig. 5. and Fig. 6.

As expected amplitude is positive buoyancy force. Lorentz force, kinematic viscous force and mean permeability enhancing amplitude the less the frequency the more is amplitude is refer Fig.7. The phase is positive and the effects of  $P_r$ ,  $G_r$  and  $M$  are to influence the phase that means there is a phase lead refer Fig. 8.

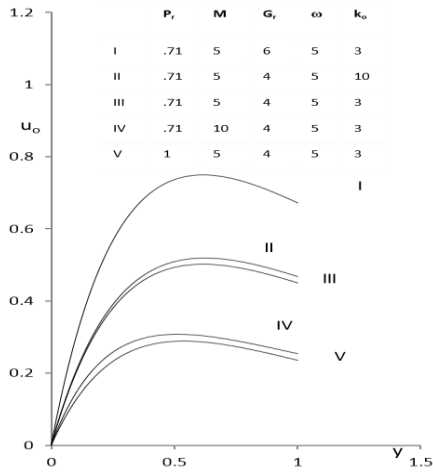


Fig. 1. Effect of  $k_0$ ,  $M$ ,  $G_r$ ,  $\omega$ ,  $P_r$  on mean velocity  $u_0$  versus  $y$  for  $R=0.05$

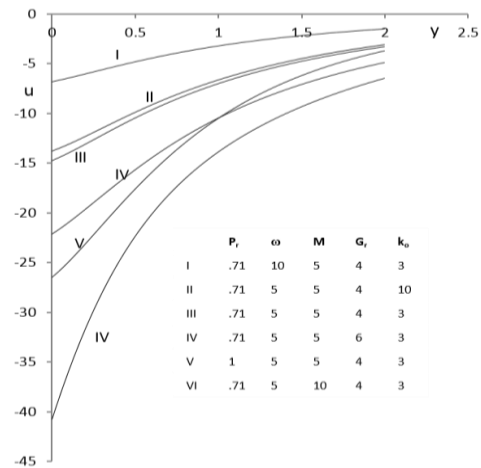


Fig. 2. Effect of  $k_0$ ,  $M$ ,  $G_r$ ,  $\omega$ ,  $P_r$  on transient velocity  $u$  versus  $v$  for  $R=0.05$  and  $\omega t=\pi/2$

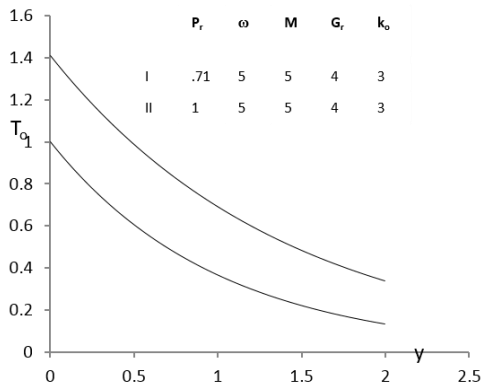


Fig. 3. Effect of  $k_0$ ,  $M$ ,  $G_r$ ,  $\omega$ ,  $P_r$  on mean temperature  $T_0$  versus  $y$  for  $R=0.05$  and  $\omega t=\pi/2$

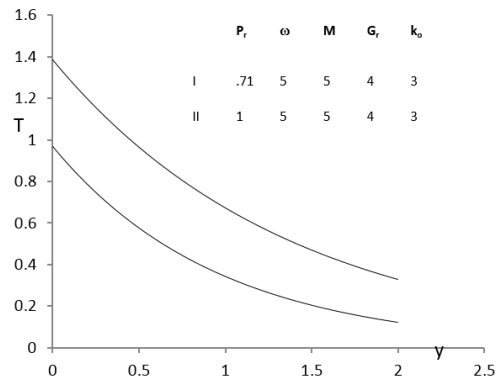


Fig. 4. Effect of  $k_0$ ,  $M$ ,  $G_r$ ,  $\omega$ ,  $P_r$  on mean temperature  $T$  versus  $y$  for  $R=0.05$

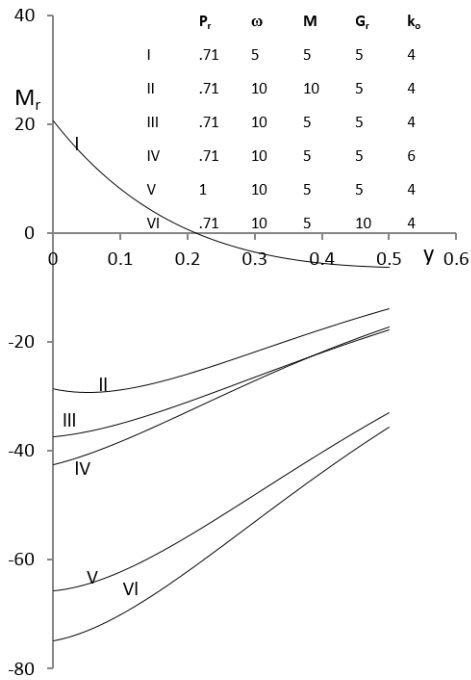


Fig. 5. Fluctuating parts of velocity profile  $M_r$

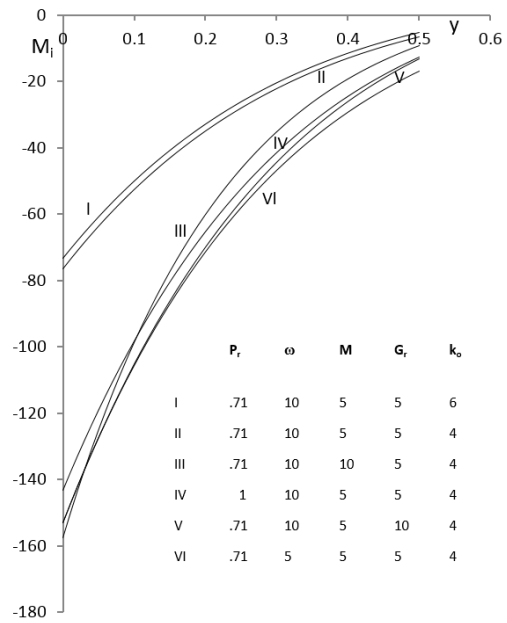


Fig. 6. Fluctuating parts of velocity profile  $M_i$

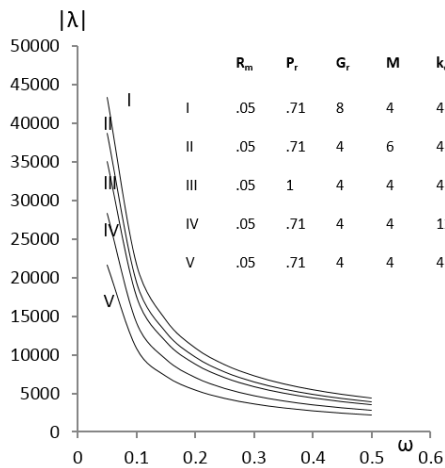


Fig. 7. Amplitude versus Frequency parameter  $\omega$

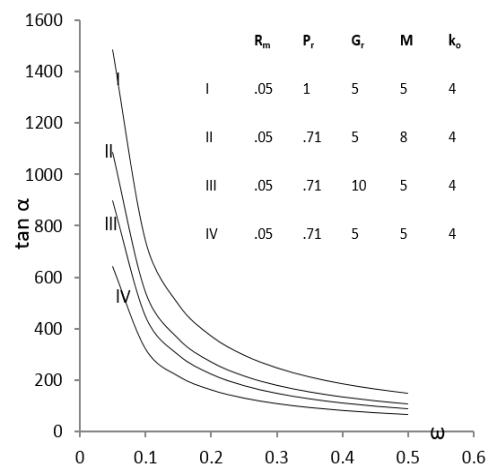


Fig. 8. Phase versus Frequency parameter  $\omega$

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