



Two-Fluid Mixed Magnetoconvection Flow in a Vertical Enclosure

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ABSTRACT

The problem of steady, laminar flow and heat transfer of an electrically conducting fluid through vertical channel in the presence of uniform transverse magnetic field is formulated using a two-fluid continuum model. Combined free and forced convection inside the channel is considered. The effects of viscous and ohmic dissipations are included in the energy equation. Both walls are kept either at the same or different temperatures such as isoflux-isothermal and isothermal-isoflux conditions. Governing equations in cartesian co-ordinates are solved analytically using regular perturbation technique to develop the expression for velocity and temperature. Velocity, temperature and Nusselt number are presented graphically. Effects of pertinent parameters, such as Hartmann number, electric field load parameter, viscosity ratio, width ratio and conductivity ratio are determined.

Keywords: Two-fluid, Magnetoconvection, Vertical enclosure, Isoflux-isothermal.

NOMENCLATURE

<p>A constant defined in Eq. (8)</p> <p>b thermal expansion coefficient ratio $\left(\frac{\beta_2}{\beta_1}\right)$</p> <p>$B_0$ magnetic field</p> <p>Br Brinkman number $\left(\frac{\mu_1 U_0^{(1)^2}}{K_1 \Delta T}\right)$</p> <p>$C_p$ specific heat at constant pressure</p> <p>E_0 applied electric field</p> <p>E electric field load parameter $\left(\frac{E_0}{B_0 U_0^{(1)}}\right)$</p> <p>$g$ acceleration due to gravity</p> <p>Gr Grashof number $\left(\frac{g \beta_1 h_1^3 \Delta T}{\nu_1^2}\right)$</p> <p>$GR$ mixed convection parameter $\left(\frac{Gr}{Re}\right)$ defined in Eq. (23)</p> <p>h width ratio $\left(\frac{h_2}{h_1}\right)$</p> <p>$\bar{u}_i$ average velocity</p> <p>T_1, T_2 temperature of the boundaries</p> <p>X, Y space co-ordinates</p>	<p>h_1 width of the region-I</p> <p>h_2 width of the region-II</p> <p>K ratio of the thermal conductivities $\left(\frac{K_1}{K_2}\right)$</p> <p>$K_1$ thermal conductivity of the fluid in region-I</p> <p>K_2 thermal conductivity of the fluid in region-II</p> <p>M Hartmann number $\left(h_1 B_0 \sqrt{\frac{\sigma_e}{\mu_1}}\right)$</p> <p>$m$ ratio of the viscosities $\left(\frac{\mu_1}{\mu_2}\right)$</p> <p>$n$ ratio of the densities $\left(\frac{\rho_2}{\rho_1}\right)$</p> <p>$p$ dimensional pressure</p> <p>Re Reynolds number $\left(\frac{U_0^{(1)} h_1}{\nu_1}\right)$</p> <p>$T$ temperature</p> <p>$U_0^{(i)}$ reference velocity $\left(-\frac{dP}{dX} \frac{h_i^2}{48 \mu_i}\right)$</p> <p>$u$ velocity</p> <p>Greek symbols</p> <p>β coefficient of thermal expansion</p> <p>ν kinematic viscosity</p>
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α	thermal diffusivity
σ_e	electrical conductivity
μ	viscosity
ρ	density of the fluid

\mathcal{E}	dimensionless parameter defined in Eq.(42)
ΔT	difference in temperature ($T_2 - T_1$)
θ_i	nondimensional temperature $\frac{(T_i - T_0)}{\Delta T}$

1. INTRODUCTION

Mixed convection is defined as heat transfer situations where both natural convection and forced convection heat transfer mechanisms interact. In a vertical passage, the internal main flow can be either upward or downward. The upward forced flow is termed ‘assisted’ flow because the natural convection created by buoyancy is in the same direction as the bulk flow. In contrast, the downward flow is called ‘opposed’ flow based on its direction opposite to the natural convection. In the past twenty years, mixed convection in a vertical heated channel has received considerable attention due to its extensive practical applications, including turbine rotor blade internal cooling systems, cooling of nuclear reactors and electronic components. Tao (1960) analyzed laminar fully developed mixed convection flow in a vertical parallel-plate channel with uniform wall temperatures. Aung and Worku (1986a, 1986b) discussed the theory of combined free and forced convection in a vertical channel with flow reversal conditions for both developing and fully developed flows. Aung and Worku (1986b) assumed that the walls of the channel were having asymmetric temperatures. The case of developing mixed convection flow in ducts with asymmetric wall heat fluxes was analyzed by the same authors (Aung and Worku, 1987). A comprehensive review of the literature dealing with mixed convection in internal flow was reported by Aung (1987). Cheng *et al.* (1990), Hamadah and Wirtz (1991) and Ingham *et al.* (1988) also reported on, flow reversal situation in mixed convection in a vertical channel for different wall heating conditions.

The problem concerning the flow of immiscible fluids has a definite role in chemical engineering and in medicine. In view of this, Bird *et al.* (1960) obtained an exact solution for the laminar flow of two immiscible fluids between two parallel plates. Mitra (1982) analyzed the unsteady flow of two electrically conducting fluids between two rigid parallel plates.

Since the last three decades research in nuclear power safety has motivated the study of the buoyancy-driven convection in fluids with volumetric heat generation. Recently, the problem of convective heat transfer in a layered system attracted attention due to possible core melt stratification in a postulated severe accident scenario in a light water reactor (Sehgal, 2001). Besides the reactor safety research, thermal convection in a double-layer system is of interest in chemical engineering and geophysics. Much research effort has been directed at the problem of natural convection in a uniform pool with internal heat generation (Dhir, 1997). However, only a few studies have focused on the subject of thermal convection in volumetrically heated stratified layers. Fieg (1976) investigated the natural convection characteristics of two stratified immiscible liquid layers with internally heated lower layer. The

temperature was maintained equal at the top and bottom boundaries. Heptane and water were used as lighter and heavier liquids, respectively. The important conclusion was that the two layers behaved as if separated by a rigid highly conductive wall. Schramm and Reineke (1978) studied experimentally and numerically the natural convection in a rectangular channel filled with two immiscible fluids of different physical properties. Kulacki and Nguen (1981) studied hydrodynamic instability and thermal convection in a horizontal layer of two immiscible fluids with internal heat generation in the lower layer.

There has been some theoretical and experimental work on the stratified laminar flow of two immiscible liquids in a horizontal pipe (see, Charles and Lilleleht, 1965, Bentwich, 1964, and Pacham and Shail, 1971). The interest in this configuration stems from the possibility of reducing the power required to pump oil in a pipeline by the suitable addition of water. Shail (1973) investigated theoretically the possibility of using a two-phase system to obtain increased flow rates in an electromagnetic pump. Hartmann flow of a conducting fluid and non-conducting fluid layer contained in a channel has been studied by Shail (1973). His results predicted that an increase of the order 30% can be achieved in the flow rate for suitable ratios heights and viscosities of the fluids. Malashetty and Leela (1991, 1992) reported closed-form solutions for the two-phase flow and heat transfer situation in a horizontal channel for which both phases are electrically conducting. Malashetty and Umavathi (1997) studied two-phase MHD flow and heat transfer in an inclined channel in the presence of buoyancy effects for the situation where only one of the phases is electrically conducting. Malashetty *et al.* (2000, 2001) analyzed the problem of fully developed two fluid magnetohydrodynamic flows with and without applied electric field in an inclined channel. Recently, Malashetty *et al.* (2006) studied the magneto convection in a vertical channel.

Much work is not found in the literature in the presence of electromagnetic field. Keeping in view numerous applications of two-immiscible fluids, it is the objective of the paper to investigate theoretically, mixed convection of electrically conducting and electrically non-conducting immiscible fluids in a vertical channel.

2. MATHEMATICAL FORMULATION

The geometry under consideration illustrated in Fig. 1 consists of two infinite parallel plates maintained at different or equal constant temperatures extending in the X and Z directions. The region $-\frac{h_1}{2} \leq Y \leq 0$ is occupied by a viscous, electrically conducting fluid of density ρ_1 , viscosity μ_1 , thermal conductivity K_1 , thermal expansion coefficient β_1 and electrical conductivity σ_e .

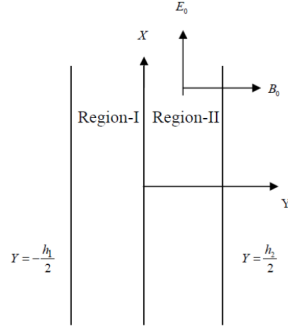


Fig. 1. Physical configuration.

The region $0 \leq Y \leq \frac{h_2}{2}$ is occupied by another viscous, non-conducting fluid of density ρ_2 , viscosity μ_2 , thermal conductivity K_2 , and thermal expansion coefficient β_2 . A uniform magnetic field B_0 is applied normal to the plates and the uniform electric field E_0 is applied across the channel. The fluids are assumed to have constant properties except the density in the buoyancy term in momentum equation. A fluid rises in the channel driven by buoyancy forces. The transport properties of both the fluids are assumed to be constant. The fluids in both the regions are considered to be incompressible and the flow is steady, laminar and fully developed. It is assumed that the only non-zero component of the velocity \vec{q} is the X -component U_i ($i=1,2$). Thus, as a consequence of the mass balance equation, one obtains

$$\frac{\partial U_i}{\partial X} = 0 \quad (1)$$

so that U_i depends only on Y .

The stream wise and the transverse momentum balance equations yields (Arpaci and Larsen, 1984).

Region-I

$$g\beta_1(T_1 - T_0) - \frac{1}{\rho_1} \frac{\partial P}{\partial X} + \nu_1 \frac{d^2 U_1}{dY^2} - \frac{\sigma_e}{\rho_1} (E_0 + B_0 U_1) B_0 = 0 \quad (2)$$

The Y -momentum balance equation can be expressed as

$$\frac{\partial P}{\partial Y} = 0 \quad (3)$$

Region-II

$$g\beta_2(T_2 - T_0) - \frac{1}{\rho_2} \frac{\partial P}{\partial X} + \nu_2 \frac{d^2 U_2}{dY^2} = 0 \quad (4)$$

$$\frac{\partial P}{\partial Y} = 0 \quad (5)$$

where $P = p + \rho_0 g x$ (for $P_1 = P_2 = P$) is the difference between the pressure and the hydrostatic pressure. On account of Eqs. (3) and (5), P depends only on X so that Eqs. (2) and (4) can be rewritten as

Region-I

$$g\beta_1(T_1 - T_0) - \frac{1}{\rho_1} \frac{dP}{dX} + \nu_1 \frac{d^2 U_1}{dY^2} - \frac{\sigma_e}{\rho_1} (E_0 + B_0 U_1) B_0 = 0 \quad (6)$$

Region-II

$$g\beta_2(T_2 - T_0) - \frac{1}{\rho_2} \frac{dP}{dX} + \nu_2 \frac{d^2 U_2}{dY^2} = 0 \quad (7)$$

Let us assume that the walls of the channel are isothermal. In particular, the temperature of the boundary at $Y = -\frac{h_1}{2}$ is T_1 , while the temperature at

$Y = \frac{h_2}{2}$ is T_2 , with $T_2 \geq T_1$. These boundary conditions

are compatible with Eqs. (6) and (7) if and only if $\frac{dP}{dX}$ is independent of X . Therefore, there exists a constant A such that

$$\frac{dP}{dX} = A \quad (8)$$

On account of Eq. (8) and by evaluating the derivatives of Eqs. (6) and (7) with respect to X , one obtains

$$\frac{dT_1}{dX} = 0 \quad (9)$$

$$\frac{dT_2}{dX} = 0 \quad (10)$$

so that the temperature also depends only on Y . By taking into account the effect of viscous dissipation, the energy balance equation can be written as

Region-I

$$\alpha_1 \frac{d^2 T_1}{dY^2} + \frac{\nu_1}{C_p} \left(\frac{dU_1}{dY} \right)^2 + \frac{\sigma_e}{\rho_1 C_p} (E_0 + B_0 U_1)^2 = 0 \quad (11)$$

Region-II

$$\alpha_2 \frac{d^2 T_2}{dY^2} + \frac{\nu_2}{C_p} \left(\frac{dU_2}{dY} \right)^2 = 0 \quad (12)$$

Equations (6), (11), (7) and (12) allow one to obtain differential equation for U_i namely

Region-I

$$\frac{d^4 U_1}{dY^4} - \frac{\sigma_e B_0^2}{\mu_1} \frac{d^2 U_1}{dY^2} - \frac{\rho_1 g \beta_1}{K_1} \left(\frac{dU_1}{dY} \right)^2 + \frac{g \beta_1 \sigma_e}{K_1 \nu_1} (E_0 + B_0 U_1)^2 \quad (13)$$

Region-II

$$\frac{d^4 U_2}{dY^4} = \frac{\rho_2 g \beta_2}{K_2} \left(\frac{dU_2}{dY} \right)^2 \quad (14)$$

The boundary conditions on U_i are both no-slip conditions

$$U_1 = 0 \quad \text{at } Y = -\frac{h_1}{2} \quad (15)$$

$$U_2 = 0 \quad \text{at } Y = \frac{h_2}{2} \quad (16)$$

and those induced by the boundary conditions on T and by Eqs. (6) and (7) are

$$U_1(0)=U_2(0) \quad (17)$$

$$\frac{d^2U_1}{dY^2} = \frac{A}{\mu_1} + \frac{\sigma_e E_0 B_0}{\mu_1} + \frac{g\beta_1 \Delta T R_r}{2\nu_1} \quad \text{at } Y = -\frac{h_1}{2} \quad (18)$$

$$\frac{d^2U_2}{dY^2} = \frac{A}{\mu_2} - \frac{g\beta_2 \Delta T R_r}{2\nu_2} \quad \text{at } Y = \frac{h_2}{2} \quad (19)$$

$$\mu_1 \frac{dU_1}{dY} = \mu_2 \frac{dU_2}{dY} \quad \text{at } Y = 0 \quad (20)$$

$$\frac{d^2U_1}{dY^2} - \frac{\sigma_e}{\mu_1} (E_0 + B_0 U_1) B_0 = \frac{\mu_2 \rho_1 \beta_1}{\mu_1 \rho_2 \beta_2} \frac{d^2U_2}{dY^2} + \frac{A}{\mu_1} \left(1 - \frac{\rho_1 \beta_1}{\rho_2 \beta_2} \right) \quad \text{at } Y = 0 \quad (21)$$

$$\frac{d^3U_1}{dY^3} - \frac{\sigma_e B_0^2}{\mu_1} \frac{dU_1}{dY} = \frac{\mu_2 \rho_1 K_2 \beta_1}{\mu_1 \rho_2 K_1 \beta_2} \frac{d^3U_2}{dY^3} \quad \text{at } Y = 0 \quad (22)$$

The Eqs. (13) to (22) can be written in the dimensionless form by employing the dimensionless quantities

$$u_1 = \frac{U_1}{U_0^{(1)}}; u_2 = \frac{U_2}{U_0^{(2)}}; \theta_1 = \frac{T_1 - T_0}{\Delta T}; \theta_2 = \frac{T_2 - T_0}{\Delta T}; y_1 = \frac{Y_1}{h_1}; y_2 = \frac{Y_2}{h_2};$$

$$Gr = \frac{g\beta_1 \Delta T h_1^3}{\nu_1^2}; Re = \frac{U_0^{(1)} h_1}{\nu_1}; Br = \frac{\mu U_0^{(1)2}}{K_1 \Delta T}; GR = \frac{Gr}{Re}; R_r = \frac{T_2 - T_1}{\Delta T}$$

$$M^2 = \frac{\sigma_e B_0^2 h_1^2}{\mu_1}; E = \frac{E_0}{B_0 U_0^{(1)}}; \quad (23)$$

The reference velocity $U_0^{(i)}$ and the reference temperature T_0 are given by

$$U_0^{(1)} = -\frac{Ah_1^2}{48\mu_1}; U_0^{(2)} = -\frac{Ah_2^2}{48\mu_2}; T_0 = \frac{T_1 + T_2}{2} \quad (24)$$

Moreover, the temperature difference ΔT is given $\Delta T = T_2 - T_1$ if $T_1 < T_2$ (25)

As a consequence, the dimensionless parameter R_r can only take the values 0 or 1. That is R_r is 1 for asymmetric heating i.e. $T_1 < T_2$, while R_r is 0 for symmetric heating i.e. $T_1 = T_2$, respectively. The dimensionless mean velocity \bar{u}_i and the dimensionless bulk temperature θ_{bi} are given by

$$\bar{u}_1 = \int_{-1/4}^0 u_1 dy \quad (26)$$

$$\bar{u}_2 = \int_0^{1/4} u_2 dy \quad (27)$$

$$\theta_{b1} = \frac{1}{u_1} \int_{-1/4}^0 u_1 \theta_1 dy \quad (28)$$

$$\theta_{b2} = \frac{1}{u_2} \int_0^{1/4} u_2 \theta_2 dy \quad (29)$$

Equation (8) implies that A can be either positive or negative. If $A > 0$, then $U_0^{(i)}$, Re and GR are negative, i.e. the flow is downward. On the contrary, if $A < 0$, the flow is upward, so that $U_0^{(i)}$, Re and GR are positive.

Using Eqs. (23) and (24), the Eqs. (13) to (22) become

Region-I

$$\frac{d^4u_1}{dy^4} - M^2 \frac{d^2u_1}{dy^2} = GR Br \left(\left(\frac{du_1}{dy} \right)^2 + M^2 E^2 + M^2 u_1^2 + 2M^2 E u_1 \right) \quad (30)$$

Region-II

$$\frac{d^4u_2}{dy^4} = mnKbh^4 GR Br \left(\frac{du_2}{dy} \right)^2 \quad (31)$$

The boundary and interface conditions become

$$u_1 = 0; \quad \frac{d^2u_1}{dy^2} = -48 + M^2 E + \frac{GR R_r}{2}$$

$$\text{at } y = -\frac{1}{4}$$

$$u_2 = 0; \quad \frac{d^2u_2}{dy^2} = -48 - \frac{nb GR R_r}{2}$$

$$\text{at } y = \frac{1}{4}$$

$$u_1 = mh^2 u_2; \quad \frac{du_1}{dy} = h \frac{du_2}{dy} \quad \text{at } y = 0$$

$$\frac{d^2u_1}{dy^2} - M^2 u_1 - M^2 E = \frac{1}{nb} \left(\frac{d^2u_2}{dy^2} + 48(1 - nb) \right)$$

at $y = 0$

$$\frac{d^3u_1}{dy^3} - M^2 \frac{du_1}{dy} = \frac{1}{nbKh} \frac{d^3u_2}{dy^3} \quad \text{at } y = 0 \quad (32)$$

3. SOLUTION

3.1 Case-I

The solutions of Eqs. (30) and (31) using Eq. (32) in the absence of viscous dissipation, so that $Br = 0$, the velocities become

Region-I

$$u_1 = B_1 + B_2 y + B_3 \text{Cosh}(My) + B_4 \text{Sinh}(My) \quad (33)$$

Region-II

$$u_2 = B_5 + B_6 y + B_7 y^2 + B_8 y^3 \quad (34)$$

Using the Eq. (23) in Eqs. (6) and (7), we obtain the energy balance equation as

Region-I

$$\theta_1 = -\frac{1}{GR} \left(48 + \frac{d^2 u_1}{dy^2} - M^2 (E + u_1) \right) \quad (35)$$

Region-II

$$\theta_2 = -\frac{1}{nbGR} \left(48 + \frac{d^2 u_2}{dy^2} \right) \quad (36)$$

Using the velocities obtained in Eqs. (33) and (34), the energy balance Eqs. (35) and (36) can be evaluated and are not presented.

3.2 Case- II

The solutions of Eqs. (30) and (31) can be obtained when buoyancy forces are negligible and viscous dissipation is dominating, i.e., $GR = 0$, so that a purely forced convection occurs. For this condition solutions of Eqs. (30) and (31), using boundary and interface conditions given by Eq. (32) for $GR = 0$, the velocities are given by

Region-I

$$u_1 = D_1 + D_2 y + D_3 \text{Cosh}(My) + D_4 \text{Sinh}(My) \quad (37)$$

Region-II

$$u_2 = D_5 + D_6 y + D_7 y^2 + D_8 y^3 \quad (38)$$

Using Eq. (23) in Eqs. (11) and (12), the energy balance equations become

Region-I

$$\frac{d^2 \theta_1}{dy^2} = -Br \left(\left(\frac{du_1}{dy} \right)^2 + M^2 E^2 + M^2 u_1^2 + 2M^2 E u_1 \right) \quad (39)$$

Region-II

$$\frac{d^2 \theta_2}{dy^2} = -Br Km h^4 \left(\frac{du_2}{dy} \right)^2 \quad (40)$$

The boundary and interface conditions for temperature are

$$\begin{aligned} \theta_1 \left(-\frac{1}{4} \right) &= -\frac{R_r}{2} \\ \theta_2 \left(\frac{1}{4} \right) &= \frac{R_r}{2} \\ \theta_1 &= \theta_2 ; \frac{d\theta_1}{dy} = \frac{1}{Kh} \frac{d\theta_2}{dy} \quad \text{at } y = 0 \end{aligned} \quad (41)$$

Using Eq. (37), (38) and (41), the energy balance Eqs. (39) and (40) can be evaluated and not presented.

3.3 Perturbation Solution

We define the dimensionless parameter

$$\varepsilon = \frac{Gr}{Re} Br = GR Br \quad (42)$$

Equation (42) shows that ε does not depend on the reference temperature difference ΔT . The fact that the

product $GR Br (= \varepsilon)$ is very small and hence can be exploited to use the regular perturbation method. To this end the solutions are assumed in the form:

$$u_i(y) = u_{i0}(y) + \varepsilon u_{i1}(y) + \varepsilon^2 u_{i2}(y) + \dots = \sum_{n=0}^{\infty} \varepsilon^n u_{in}(y) \quad (43)$$

Using Eq. (43) in Eqs. (30) and (31) and equating the coefficients of like powers ε to zero, we obtain the zeroth and first-order equations as follows

Isothermal-isothermal ($T_1 - T_2$) wall conditions

Region-I

Zeroth-order equations:

$$\frac{d^4 u_{10}}{dy^4} - M^2 \frac{d^2 u_{10}}{dy^2} = 0 \quad (44)$$

First-order equations:

$$\frac{d^4 u_{11}}{dy^4} - M^2 \frac{d^2 u_{11}}{dy^2} = \left(\frac{du_{10}}{dy} \right)^2 + M^2 E^2 + M^2 u_{10}^2 + 2M^2 E u_{10} \quad (45)$$

Region-II

Zeroth-order equations:

$$\frac{d^4 u_{20}}{dy^4} = 0 \quad (46)$$

First-order equations:

$$\frac{d^4 u_{21}}{dy^4} = mnKbh^4 \left(\frac{du_{20}}{dy} \right)^2 \quad (47)$$

The corresponding boundary and interface conditions reduces to Zeroth-order equations:

$$\begin{aligned} u_{10} &= 0 ; \frac{d^2 u_{10}}{dy^2} = -48 + M^2 E + \frac{GR R_r}{2} \quad \text{at } y = -\frac{1}{4} \\ u_{20} &= 0 ; \frac{d^2 u_{20}}{dy^2} = -48 - \frac{nbGR R_r}{2} \\ \text{at } y &= \frac{1}{4} \end{aligned}$$

$$u_{10} = mh^2 u_{20} ; \frac{du_{10}}{dy} = h \frac{du_{20}}{dy} \quad \text{at } y = 0$$

$$\frac{d^2 u_{10}}{dy^2} - M^2 u_{10} - M^2 E = \frac{1}{nb} \left(\frac{d^2 u_{20}}{dy^2} + 48(1-nb) \right) \quad \text{at } y = 0$$

$$\frac{d^3 u_{10}}{dy^3} - M^2 \frac{du_{10}}{dy} = \frac{1}{nbKh} \frac{d^3 u_{20}}{dy^3} \quad \text{at } y = 0 \quad (48)$$

First-order equations:

$$u_{11} = 0 ; \frac{d^2 u_{11}}{dy^2} = 0 \quad \text{at } y = -\frac{1}{4}$$

$$u_{21} = 0 ; \frac{d^2 u_{21}}{dy^2} = 0 \quad \text{at } y = \frac{1}{4}$$

$$u_{11} = mh^2 u_{21} ; \frac{du_{11}}{dy} = h \frac{du_{21}}{dy} \quad \text{at } y = 0$$

$$\frac{d^2 u_{11}}{dy^2} - M^2 u_{11} = \frac{1}{nb} \frac{d^2 u_{21}}{dy^2};$$

$$\frac{d^3 u_{11}}{dy^3} - M^2 \frac{du_{11}}{dy} = \frac{1}{nbKh} \frac{d^3 u_{21}}{dy^3} \quad \text{at } y = 0 \quad (49)$$

Solutions of zeroth-order Eqs. (44) and (46) using boundary and interface conditions (48) are

$$u_{10} = C_1 + C_2 y + C_3 \text{Cosh}(My) + C_4 \text{Sinh}(My) \quad (50)$$

$$u_{20} = C_5 + C_6 y + C_7 y^2 + C_8 y^3 \quad (51)$$

Solutions of first-order Eqs. (45) and (47) using boundary and interface conditions (49) are

$$\begin{aligned} u_{11} = & C_9 + C_{10} y + C_{11} \text{Cosh}(My) + C_{12} \text{Sinh}(My) + \\ & g_{10} \text{Cosh}(2My) + g_{11} \text{Sinh}(2My) + g_{12} y^2 \text{Cosh}(My) \\ & + g_{13} y^2 \text{Sinh}(My) + g_{14} y \text{Cosh}(My) \\ & + g_{15} y \text{Sinh}(My) + g_{16} y^4 + g_{17} y^3 + g_{18} y^2 \end{aligned} \quad (52)$$

$$u_{21} = g_{19} y^8 + g_{20} y^7 + g_{21} y^6 + g_{22} y^5 + g_{23} y^4 + \frac{C_{13}}{6} y^3 + \frac{C_{14}}{2} y^2 + C_{15} y + C_{16} \quad (53)$$

Using Eq. (23) in Eqs. (6) and (7), we obtain the energy balance equations as

Region-I

$$\theta_1 = -\frac{1}{GR} \left(48 + \frac{d^2 u_1}{dy^2} - M^2 (E + u_1) \right) \quad (54)$$

Region-II

$$\theta_2 = -\frac{1}{nbGR} \left(48 + \frac{d^2 u_2}{dy^2} \right) \quad (55)$$

Using velocities obtained in Eqs. (50) to (53) the energy balance Eqs. (54) and (55) can be evaluated and are not presented.

Isoflux-isothermal ($q_1 - T_2$) wall conditions

For this case, the thermal boundary conditions for the channel walls can be written in the dimensional form as

$$\begin{aligned} q_1 = -K_1 \frac{dT_1}{dY} & \quad \text{at } Y = -\frac{h_1}{2} \\ T = T_2 & \quad \text{at } Y = \frac{h_2}{2} \end{aligned} \quad (56)$$

The dimensionless form of the above equations can be obtained by using the Eq. (23) along with $\Delta T = \frac{q_1 h_1}{K_1}$ to

give

$$\begin{aligned} \frac{d\theta_1}{dy} = -1 & \quad \text{at } y = -\frac{1}{4} \\ \theta_2 = \frac{R_{qt}}{2} & \quad \text{at } y = \frac{1}{4} \end{aligned} \quad (57)$$

where $R_{qt} = (T_2 - T_0)/\Delta T$ is the thermal ratio parameter for isoflux-isothermal walls. Other than the no-slip conditions at the channel walls, two more boundary

conditions in terms of U_1 are required to solve Eq. (13). These are induced by the conditions given by Eq. (57) and the other obtained from Eq. (6) as follows.

Differentiating Eq. (6) with respect to Y with $\frac{dP}{dX} = A$ gives

$$\frac{d^3 U_1}{dY^3} - \frac{\sigma_e B_0^2}{\mu_1} \frac{dU_1}{dY} + \frac{g\beta_1}{v_1} \frac{dT_1}{dY} = 0 \quad (58)$$

Equation (58) is non-dimensionalised by using the Eq. (23) to give

$$\frac{d^3 u_1}{dy^3} - M^2 \frac{du_1}{dy} + GR \frac{d\theta_1}{dy} = 0 \quad (59)$$

Evaluating the Eq. (59) at the left wall ($y = -1/4$)

yields

$$\frac{d^3 u_1}{dy^3} - M^2 \frac{du_1}{dy} = GR \quad \text{at } y = -\frac{1}{4} \quad (60)$$

The other boundary condition at the right wall can be shown to be the same as that given for the isothermal-isothermal wall with R_T is replaced by R_{qt} such that

$$\frac{d^2 u_2}{dy^2} = -48 - \frac{nbGR R_{qt}}{2} \quad \text{at } y = \frac{1}{4} \quad (61)$$

The integrating constants appeared in the Eqs. (50) to (53) are evaluated using boundary conditions (48), (49), (60) and (61) and are not presented

Isothermal-isoflux ($T_1 - q_2$) wall conditions

For this case, the thermal boundary conditions for the channel walls can be written in the dimensional form as

$$\begin{aligned} q_2 = -K_2 \frac{dT_2}{dY} & \quad \text{at } Y = \frac{h_2}{2} \\ T = T_1 & \quad \text{at } Y = -\frac{h_1}{2} \end{aligned} \quad (62)$$

The dimensionless form of above equations can be obtained by using the Eq. (23) along with

$$\Delta T = \frac{q_2 h_2}{K_2}$$

to give

$$\begin{aligned} \frac{d\theta_2}{dy} = -1 & \quad \text{at } y = \frac{1}{4} \\ \theta_1 = \frac{R_{tq}}{2} & \quad \text{at } y = -\frac{1}{4} \end{aligned} \quad (63)$$

where $R_{tq} = (T_1 - T_0)/\Delta T$ is the thermal ratio parameter for isothermal-isoflux walls. Other than the no-slip conditions at the channel walls, two more boundary conditions in terms of U_2 are required to solve Eq. (14). These are induced by the conditions given in Eq. (63) and the other obtained from Eq. (7) as follows.

Differentiating Eq. (7) with respect to Y with $\frac{dP}{dX} = A$ gives

$$\frac{d^3 U_2}{dY^3} + \frac{g\beta_2}{v_2} \frac{dT_2}{dY} = 0 \quad (64)$$

Equation (64) is non-dimensionalised by using the Eq. (23) to give

Table 1 Temperature values for different values of GR and ε .

y	GR=-500,ε=-0.1			GR=500,ε=0.1		
	E=-1	E=0	E=1	E=-1	E=0	E=1
-0.25	-0.5	-0.5	-0.5	-0.5	-0.5	-0.5
-0.175	-0.34976	-0.34976	-0.34975	-0.34964	-0.34964	-0.34962
-0.1	-0.19955	-0.19955	-0.19952	-0.19954	-0.19953	-0.19949
-0.025	-0.04945	-0.04944	-0.04941	-0.04947	-0.04946	-0.04942
0	5.56E-04	5.61E-04	5.89E-04	5.44E-04	5.50E-04	5.78E-04
0.025	0.05054	0.05055	0.05057	0.05054	0.05054	0.05057
0.1	0.20047	0.20047	0.20048	0.20044	0.20044	0.20045
0.175	0.35037	0.35037	0.35037	0.35023	0.35023	0.35024
0.25	0.5	0.5	0.5	0.5	0.5	0.5

$$\frac{d^3 u_2}{dy^3} + GRnb \frac{d\theta_2}{dy} = 0 \tag{65}$$

Evaluating the Eq. (65) at the right wall ($y=1/4$) yields

$$\frac{d^3 u_2}{dy^3} = GRnb \quad \text{at } y = \frac{1}{4} \tag{66}$$

The other boundary condition at the right wall can be shown to be the same as that given for the isothermal-isothermal wall with R_T is replaced by R_{Tq} such that

$$\frac{d^2 u_1}{dy^2} = -48 + \frac{GR R_{Tq}}{2} \quad \text{at } y = -\frac{1}{4} \tag{67}$$

The integrating constants appeared in the Eqs. (50) to (53) are evaluated using boundary conditions (48), (49), (66) and (67) and are not presented.

3.4 Nusselt Number

The heat transfer parameter on the wall expressed in terms of the Nusselt number, in non-dimensional form become

$$Nu_- = (1+h) \frac{d\theta_1}{dy} \quad \text{at } y = -\frac{1}{4}$$

$$Nu_+ = \left(1 + \frac{1}{h}\right) \frac{d\theta_2}{dy} \quad \text{at } y = \frac{1}{4}$$

The expressions for Nu_- and Nu_+ can be directly obtained and are not presented.

4. RESULTS AND DISCUSSION

In this section the fluid flow and heat transfer results for an electrically conducting fluid flow in a vertical enclosure are discussed in the presence of an applied magnetic field B_0 normal to gravity and applied electric field E_0 parallel to gravity considering both viscous and ohmic dissipations. The electric loading parameter $E=0$ corresponds to short circuit configuration and $E \neq 0$ corresponds to open circuit, E may be positive

or negative depending either on the polarity of E_0 or on the sign of $T_2 - T_1$. If the polarity of E_0 is fixed as positive, then E may be positive or negative depending on $T_2 > T_1$ or $T_2 < T_1$ respectively. The effect of electromagnetic force when $E=0$ is found to accelerate the flow and hence acts as a MHD generator. Further, the direction of the flow when $E > 0$ is opposite to that when $E < 0$ and hence the present configuration can be used for effectively flow reversal situation required in many practical problems.

The basic equations governing the flow are solved using regular perturbation method. The product of $GRBr$, where GR is the ratio of Grashof number to Reynolds number and Br is the Brinkmann number is used as the perturbation parameter.

The flow field for asymmetric heating are obtained and depicted in Figs. 2 to 10 and also shown in Tables 1 to 3 for the values of the parameters fixed as $m=1, b=1, h=1, K=1, n=1, M=2, GR=500$ and $\varepsilon=0.1$ expect the varying one. Equations (33) and (34) are the velocity field in both the regions in the absence of Brinkmann number and solutions are depicted graphically for $E=2, M=2$, in Fig. 2. There is a flow reversal near the cold wall at $y=-1/4$ for $GR=400$ and there is symmetric profile for $GR=0$.

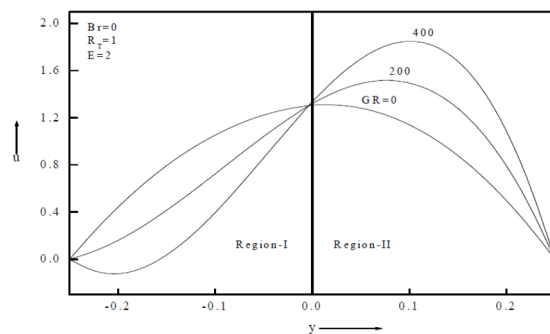


Fig. 2. Velocity profiles for different values of GR

Table 2 Temperature profiles for different values of Hartmann number M.

y	M=2			M=6		
	E=-1	E=0	E=1	E=-1	E=0	E=1
-0.25	-0.5	-0.5	-0.5	-0.5	-0.5	-0.5
-0.175	-0.34976	-0.34976	-0.34975	-0.34974	-0.34977	-0.3497
-0.1	-0.19955	-0.19955	-0.19952	-0.19954	-0.19955	-0.19944
-0.025	-0.04945	-0.04944	-0.04941	-0.04945	-0.04943	-0.04928
0	5.56E-04	5.61E-04	5.89E-04	5.48E-04	5.77E-04	7.25E-04
0.025	0.05054	0.05055	0.05057	0.05054	0.05056	0.0507
0.1	0.20047	0.20047	0.20048	0.20047	0.20047	0.20055
0.175	0.35037	0.35037	0.35037	0.35037	0.35035	0.35038
0.25	0.5	0.5	0.5	0.5	0.5	0.5

Table 3 Temperature profiles for different values of viscosity ratio m.

y	m=0.1			m=4		
	E=-1	E=0	E=1	E=-1	E=0	E=1
-0.25	-0.5	-0.5	-0.5	-0.5	-0.5	-0.5
-0.175	-0.34992	-0.34993	-0.34993	-0.34931	-0.34931	-0.3493
-0.1	-0.19987	-0.19989	-0.19987	-0.19868	-0.19868	-0.19865
-0.025	-0.04986	-0.04987	-0.04986	-0.04826	-0.04824	-0.04821
0	1.35E-04	1.25E-04	1.39E-04	0.00183	0.00185	0.00188
0.025	0.05013	0.05012	0.05013	0.05186	0.05187	0.05191
0.1	0.2001	0.2001	0.2001	0.20157	0.20158	0.2016
0.175	0.35008	0.35007	0.35007	0.35112	0.35112	0.35113
0.25	0.5	0.5	0.5	0.5	0.5	0.5

Equations (44) and (45) are the solutions for temperature field in the absence of Grashof number which is depicted graphically in Fig. 3 for different values of Brinkmann number. The temperature field increases with increasing values of Brinkmann number. Fig. 2 ($Br = 0$) and 3 ($GR = 0$) are the similar graphs obtained by Umavathi (2006) and Baraletta (1998) for one fluid model considering permeable fluid and viscous fluid respectively.

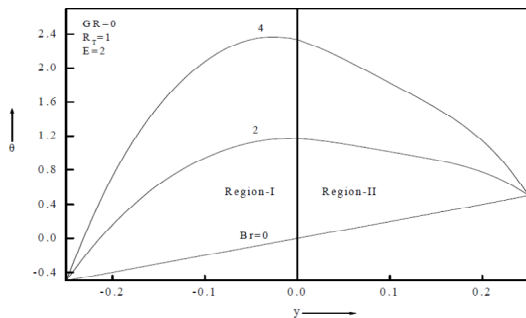


Fig. 3. Temperature profiles for different values of Br.

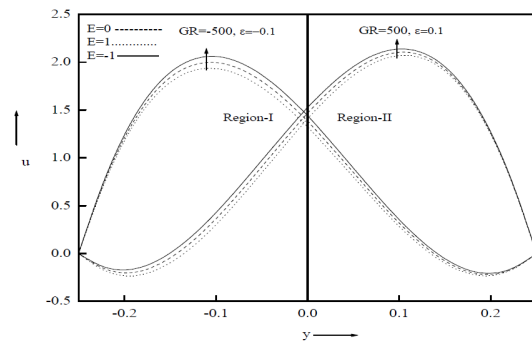


Fig. 4. Velocity profiles for different values of GR and ϵ .

The effect of GR and ϵ on the velocity is shown in Fig. 4 for short and open circuits. For positive values of GR and ϵ the flow reversal is near the cold wall and for negative values of GR and ϵ the flow reversal is near the hot wall for open and short circuits. The effect of GR and ϵ on temperature for $E = 0, \pm 1$ is shown in Table 1.

Table 4 Nusselt values for different values of Hartmann number M .

M	E=-1		E=0		E=1	
	Nu ₋	Nu ₊	Nu ₋	Nu ₊	Nu ₋	Nu ₊
2	14.42359639	3.979748009	14.42348712	3.980019086	14.42395032	3.980101942
4	4.658277671	3.979524633	4.657859532	3.980437548	4.659464179	3.980711141
6	4.136257668	3.979341515	4.135371929	3.980957939	4.13834317	3.981427017

The variation of temperature for positive and negative GR and ε , the values are same up to three decimal places and effect is almost invariant for both open and short circuits.

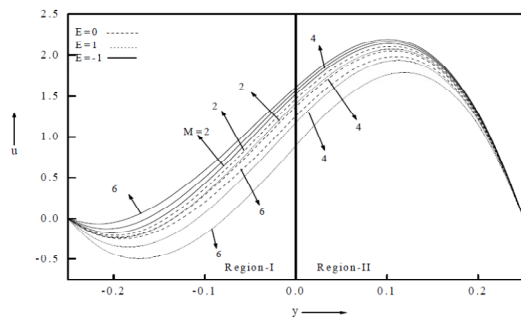


Fig. 5. Velocity profiles for different values of Hartmann number M .

The effect of Hartmann number on velocity is shown in [Fig. 5](#). It is observed that the Hartmann number is to decrease the velocity in both the regions. This graph also shows the effect of electric field load parameter E on the flow for both open and short circuits. Effect of Hartmann number is to suppress the velocity. The effect of Hartmann number on temperature varies from fifth decimal place as shown in [Table 2](#) for both open and short circuits.

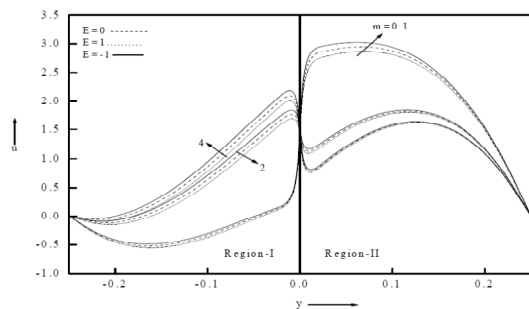


Fig. 6. Velocity profiles for different values of viscosity ratio m .

[Figure 6](#) shows the effect of viscosity ratio m on the velocity. It is found that larger the value of viscosity of conducting fluid in region-I compared to the viscosity of another conducting fluid in region-II, the larger the flow field in region-I and smaller the flow field in region-II for both open and short circuits. For $m < 1$, flow reversal is observed near the cold wall. The effect of viscosity ratio m on the temperature is very sensitive and it varies from fourth decimal place as shown in [Table 3](#) for $E = 0, \pm 1$.

The effect of width ratio h on velocity and temperature are shown in [Figs. 7](#) and [8](#) respectively. As the width ratio h increases velocity decreases significantly in region-II when compared to region-I. It is found that smaller the width of region-II compared to region-I, smaller the temperature.

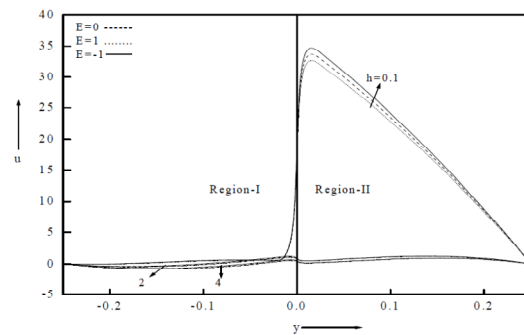


Fig. 7. Velocity profiles for different values of width ratio h .

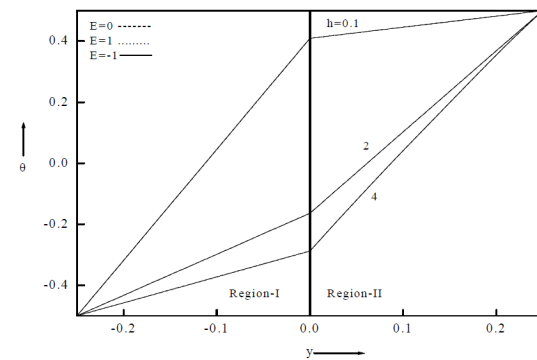


Fig. 8. Temperature profiles for different values of width ratio h .

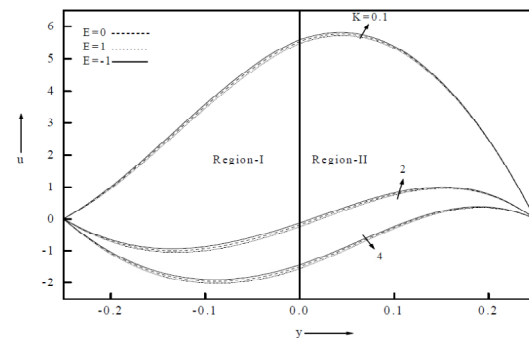


Fig. 9. Velocity profiles for different values of thermal conductivity ratio K .

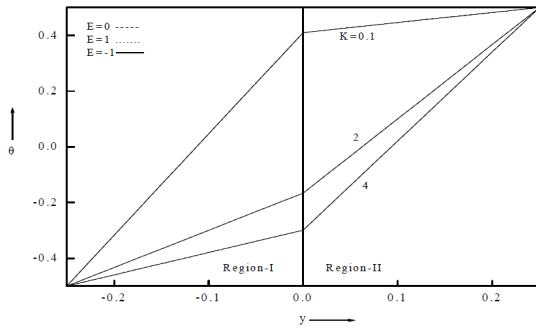


Fig. 10. Temperature profiles for different values of thermal conductivity ratio K .

It is interesting to note that the velocity and temperature are invariant for $E = 0, \pm 1$ for varying width ratio h .

The effect of thermal conductivity ratio K on the flow is shown in Figs. 9 and 10. It is seen that increasing the value of K decreases both velocity and temperature in both the regions. There is a flow reversal near the cold wall. There is no effect of electric field load parameter E on the flow for varying K .

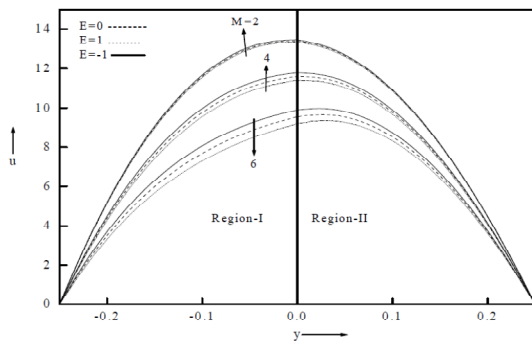


Fig. 11. Velocity profiles for different values of Hartmann number H in isoflux-isothermal wall conditions.

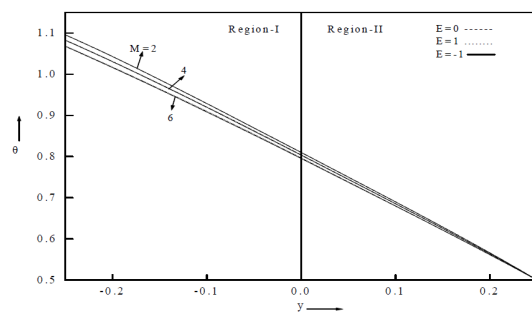


Fig. 12. Temperature profiles for different values of Hartmann number M in isoflux-isothermal wall conditions.

Figures 11 to 14 shows the effect of Hartmann number M on the flow for isoflux-isothermal and isothermal-isoflux wall conditions for $E = 0, \pm 1$. The effect of Hartmann number M is to suppress the velocity and temperature in both the regions as seen in Figs.11 and 12 respectively. The velocity is suppressed for both open and short circuits where as the effect of electric load parameter E on temperature is invariant.

For isothermal-isoflux wall conditions, the effect of Hartmann number is to promote the velocity and temperature in both the regions but the direction of flow is reverse when compared to isoflux-isothermal wall conditions. In this case also the effect of electric field parameter E is significant on velocity and invariant on temperature.

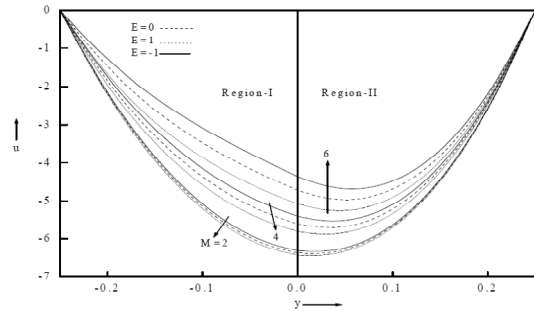


Fig. 13. Velocity profiles for different values of Hartmann number M in isothermal-isoflux wall conditions.

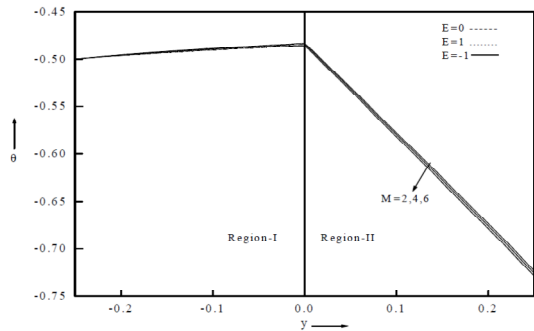


Fig. 14. Temperature profiles for different values of Hartmann number M in isothermal-isoflux wall conditions.

The variations of Nusselt number for different values of Hartmann number for both open and short circuits is shown in Table 4. For $E = -1$, as the Hartmann number M increases, rate of heat transfer decreases near the cold wall and near the hot wall but magnitude is very large near the cold wall. Similar result is observed for short circuit ($E = 0$) and for open circuit ($E = 1$) on Nusselt number for varying Hartmann number.

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