A Multiclass Retrial System With Coupled Orbits And Service Interruptions: Verification of Stability Conditions

Ioannis Dimitriou
University of Patras, Greece
idimit@math.upatras.gr

Evsey Morozov, Taisia Morozova
Institute of Applied Mathematical Research, Karelian Research Centre of RAS
Petrozavodsk State University
Petrozavodsk, Russia
emorozov@karelia.ru, tiamorozova@mail.ru

***Abstract*—In this work, we investigate the stability conditions of a multiclass retrial system with coupled orbit queues and service interruptions. We consider a single server system accepting N classes of customers according to independent Poisson inputs and with class-dependent, arbitrarily distributed service times. An arriving customer who finds the server unavailable upon arrival, joins the corresponding orbit queue according to its class. We assume that the first (oldest) blocked customer in an orbit queue attempts to connect with the server after an exponentially distributed service time, which depends both on its class, and on the current state (busy or idle) of the other orbit queues. During service times, interruptions occur according to class-dependent Poisson process, following by class-dependent arbitrarily distributed setup times. We consider both preemptive-repeat identical, and preemptive-resume interruptions. Potential applications of such a system can be found in the modelling of relay-assisted cooperative wireless networks. We focus on the *non-symmetrical orbits* and perform simulation experiments for the system with three classes of customers to verify stability conditions for both types of the server interruptions.**

I. Introduction

This research focuses on the numerical investigation of the necessary stability conditions of a single-server multiclass retrial system with coupled orbit queues and *service interruptions*. Thus we generalize the model considered in previous works [16], [18], [19] where a reliable server has been assumed. This system is characterized by the fact that the retransmission rate of an orbit queue depends on the state of the *other orbit queues*, which in turn provides a remarkable flexibility to the orbits to adapt their characteristics with ultimate goal the optimal system's performance.

In general, sufficient and necessary stability conditions are different, and in our previous our works we extensively studied the "gap" between these conditions. Indeed, as it has been shown by simulation in [16], [19] the sufficient condition is redundant, and necessary stability conditions solely allow to delimit stability region with a high accuracy. As we show in this note, this effect holds for the system with server interruptions as well.

A. Related work

Queues with repeated attempts have been extensively studied so far. For more details on the theoretical development, the interested reader is referred to the seminal books in [10], [2], and the survey papers [1], [13] (not exhaustive list).

Clearly, the stationary analysis, and more importantly the investigation of the stability conditions in a multiclass system with repeated attempts, is much more challenging than that of the single-class variant. For sake of clarity we mention the works in [3], [4], in which necessary and sufficient conditions were investigated by using the *regenerative approach* [20] for multiclass retrial systems under typical, i.e., *non*-queue aware constant retrial policy. Simulation experiments were also performed to validate the theoretical findings. Quite recently, stability analysis of a multiclass retrial system under classical retrial policy was also investigated in [17].

Recently, stability conditions for a novel class-dependent, queue-aware constant retrial policy, i.e., coupled orbit queues were investigated in [16], [18], using the regenerative approach, for the model with arbitrary number of orbit queues. For more details on the use of regenerative approach on the stability conditions of stochastic systems see e.g., [4], [3]. The stability conditions for a two class retrial system with coupled orbit queues have been obtained in [7], [8], [9] by using the well-known Foster-Lyapunov drift criteria [11].

B. Our contribution

This work is devoted to the investigation of the stability conditions of a retrial system with an arbitrary number of orbit queues under a class-dependent queue-aware constant retrial policy and service interruptions. This research is a continuation and extension of our previous works [16], [18], [19]. Using the regenerative approach, we extend the works [16], [18], [19] with an important goal to investigate the impact of *service interruptions* on the stability conditions. For such a model we consider *preemptive repeat different* and *preemptive resume* service policy. We show that the regenerative approach is an adequate method to handle it.

Potential application of our system model with coupled orbits can be found in the modelling of relay-assisted cooperative wireless systems [21] with interacting nodes. In such systems the service rate of the nodes are severely affected by the wireless interference, as a result of the number of active (non-empty) nodes. This feature is adequately modelled in our system by the concept of coupled orbit queues. Moreover, in modern cognitive radio networks [15] wireless nodes are able to dynamically adjust retransmission rates to improve spectrum utilization [5], [7].

Furthermore, as it has been mentioned in [18], this model is suitable to describe dynamics of cellular networks, in which the transmission rate in a particular cell decreases as the number of users in the neighboring cells increase [5]. A similar effect is observed in the processor sharing models [6], [14].

This paper is organized as follows. In Section II we describe the basic model with server interruptions and, in particular, give a modified definition of the *configuration of the orbits.* In Section III, we describe in detail the server interruptions mechanism and show how to calculate the mean generalized service time for both types of interruptions, using Laplace-Stieltjes Transform. In Section V, we provide theoretical results describing the stationary regime of the system. Finally, simulation experiments are presented in Section 5 which demonstrate that the necessary stability condition is close to be the stability criterion of the system with server interruptions.

II. The system model

We consider a single server multiclass retrial system accepting N classes of customers according to Poisson independent inputs. The service station can handle at most one customer, and class-i customer arrives according to Poisson process with rate λ_i, $i = 1, \dots, N$. Denote $\lambda = \sum_i \lambda_i$, the total input rate and let $p_i = \lambda_i/\lambda$, be the probability that an arbitrary arrival belongs to class $i = 1, \dots, N$. Then, interarrival times of the input are exponential with rate λ. We also assume service times of class-i customers, $\{S_n^{(i)}, n \geq 1\}$, to be independent identically distributed (iid) with with cumulative distribution function (cdf) B_i, probability density function (pdf) b_i, Laplace-Stieltjes Transform (LST) β_i^* and service rate γ_i, $i = 1, \dots, N$.

This model is characterized by the following features: the server is subject to class-dependent interruptions, following by also class-dependent setup periods. In particular, we assume that during the service time of a type-i customer, interruptions occur according to a Poisson process with rate ν_i, $i = 1, \dots N$.

Upon service interruption, the server becomes unavailable for an arbitrarily distributed time $R^{(i)}$ (called *setup time*) with pdf r_i, LST r_i^* and the mean $\mathsf{E}R^{(i)} = \bar{r}_i$, $i = 1, \dots, N$. During setup time customers continue to arrive at the system and join the corresponding orbits. The interrupted customer *remains in the service area* to return for service after the setup period.

We consider both *preemptive repeat different*, and *preemptive resume* service discipline. In the former case, the service time of the preempted customer begins again from scratch, but each time another interruption is cleared, i.e., upon setup time completion, a new independent (potential) service time begins. Thus, a customer departs from the system when, for the first time, such a service time elapses without interruption. In the latter case, upon the setup completion, the customer resumes his service from the interruption point. In both cases we call a *generalized service time* the total time a customer spends in the service area [12].

As in [16], [18], [19], we also adopt a novel class-dependent queue-aware constant retrial policy which is described as follows. A customer, meeting server busy upon arrival, joins a class-dependent FIFO (First In First Out) orbit queue. The customer at the head of orbit i repeats his attempt until he finds server idle to occupy it. The time between successive attempts from each orbit i is exponentially distributed with a rate which depends on the current *configuration of other orbits: busy or empty*. Thus, each orbit acts as a FIFO queueing system with a state-dependent "service" rate, and this dependence is a key property of the model.

To be more precise, for each i, we define the set $\mathcal{G}(i)$ of N-dimensional vectors

$$J_n^{(i)} = \{j_{n,1}^{(i)}, \dots, j_{n,i-1}^{(i)}, 1, j_{n,i+1}^{(i)}, \dots, j_{n,N}^{(i)}\}$$

with binary components $j_{n,k}^{(i)} \in \{0, 1\}$, if $k \neq i$, while the ith component always equals 1, $j_{n,i}^{(i)} = 1$. Each vector $J_n^{(i)}$ is called *configuration* and has the following interpretation: if the kth orbit is non-idle (busy), we put $j_{n,k}^{(i)} = 1$, otherwise, $j_{n,k}^{(i)} = 0$. We assume that $\mathcal{G}(i)$ is the ordered set of possible configurations, then index n denotes the nth element of this set. We stress that each configuration from the set $\mathcal{G}(i)$ relates to the case when orbit i is *non-idle*. For a given configuration $J_n^{(i)}$, we denote $\mu_n^{(i)}$ the retransmission rate of orbit i. Finally, we denote M_i the set of rates for all configurations belonging to the set $\mathcal{G}(i)$, $i = 1, ..., N$.

We recall that this construction has been proposed in [16], and it generalizes the setting studied in previous works [18], [7], [8], [9], where it is assumed that orbit i has rate μ_i if at least one (other) orbit is busy, otherwise, the rate is μ_i^*, $i = 1, \dots, N$. In this case each the set $\mathcal{G}(i)$ contains only two configurations, $J_1^{(i)}$, $J_2^{(i)}$, such that $\max_{k \neq i} j_{1,k}^{(i)} = 0$, and $\max_{k \neq i} j_{2,k}^{(i)} = 1$. Moreover, in this case $M_i = \{\mu_i^*, \mu_i\}$.

Note that in general case the capacity of the M_i equals 2^{N-1}, and thus the number of possible configurations of the system equals $N2^{N-1}$. Also we note that, in general, not all different configurations have different retrial rates, and it means that the input rate, from a fixed orbit, is *insensitive* to switching between these configurations. It reflects such situation in practice when the effect of interference caused by the change of the state (idle/non-idle) of some orbits turns out to be negligible for the analysis of the system.

III. Calculation of the mean generalized service time

Clearly, due to the presence of interruptions, the "service" time of a customer will definitely be extended. To cope with this issue, we introduce the concept of the *generalized* service time, as the time elapsed from the instant a customer starts his service until the instant the server is ready to start the service of another customer. Denote $C_n^{(i)}$ the generalized service time of the nth arriving customer, who belongs to class i, and let $C^{(i)}$ be the generic generalized class-i service time. Then, $\{C_n^{(i)}, n \geq 1\}$ are the iid random variables, with rate

$$\gamma_i := \frac{1}{\mathsf{E}C^{(i)}} < \infty, \, i = 1, \dots, N.$$

Similarly as in [16], denote $A_i(t)$ the number of class-i customers arriving in the interval $[0, t]$. Then, the work which class-i customers bring in the system in the interval $[0, t]$ equals,

$$V_i(t) := \sum_{n=1}^{A_i(t)} C_n^{(i)}, \tag{1}$$

and the summary amount of the work that arrive in the system during $[0, t]$ equals,

$$V(t) := \sum_{i=1}^{N} V_i(t) = \sum_{i=1}^{N} \sum_{n=1}^{A_i(t)} C_n^{(i)}, \, t \geq 0. \tag{2}$$

Before proceeding further, we provide some expressions for the generalized service times that are necessary in the following.

A. Preemptive repeat different service discipline

We first consider the *preemptive repeat different* setting. Denote the density $c^{(i)}(t)$ of the generalized service time of a class-i customer, that is,

$$\mathsf{P}\Big(C^{(i)} \in (t, \, t + dt)\Big) =: c^{(i)}(t)dt.$$

Then, denoting the *tail distribution* $\bar{B}_i = 1 - B_i$, we obtain the following expression for this density:

$$c^{(i)}(t) = e^{-\nu_i t} b_i(t) + \nu_i e^{-\nu_i t} \bar{B}_i(t) * r_i(t) * c^{(i)}(t), \tag{3}$$

where $*$ means convolution.

The 1st term in the right-hand side of (3) corresponds to the case when no interruption occurs during the service time of a class-i customer, and thus, the generalized service times of a type i customer equals his service time. The 2nd term in the right-hand side of (3) describes the case where an interruption occurs at some instant $u < t$ during the service time of a class-i customer. Then, the server is immediately stop working, and becomes unavailable for a setup period of type i. The interrupted class-i customer remains in the service area, awaiting the server to return after the setup. Then, the interrupted customer starts his service from scratch, and thus, a new generalized service time begins. This situation is continued until the instant a service time is completed without interruption.

Note that in this case the *clear* service time (i.e., the time that a customer spends receiving service) of a customer is extended as many times an interruption occurs. In particular, each time an interruption occurs, it cancels the service time a customer already received until this instant, and after a setup period, the customer has to repeat from scratch his service. Denote the LST of the generalized service time

$$c_i^*(s) := \int_0^\infty e^{-st} c^{(i)}(t) dt, \; s > 0.$$

Then, using (3) and the property of the LST of the convolution, we obtain, after some algebra,

$$c_i^*(s) = \frac{\beta_i^*(\nu_i + s)}{1 - \frac{\nu_i(1 - \beta_i^*(\nu_i + s))}{\nu_i + s} r_i^*(s)}. \tag{4}$$

Using (4), we derive both the mean generalized service time

$$\mathsf{E}C^{(i)} = -\frac{d}{ds} c_i^*(s)|_{s=0} = \frac{1 - \beta_i^*(\nu_i)}{\beta_i^*(\nu_i)} \left(\frac{1}{\nu_i} + \bar{r}_i \right), \tag{5}$$

and the traffic intensity of class-i customers:

$$\rho_i \;\; = \;\; \lambda_i \mathsf{E}C^{(i)}, \;\; i = 1, \dots, N. \tag{6}$$

B. Preemptive-resume service discipline

In this case, the clear service time that a customer receives, is equal to his original service time. Contrary to the case discussed previously, the original service time of a customer is extended to a time period which is equal to the summary setup period (i.e., the sum of the setup periods that occur during a service time). Then, we obtain the following expression for the density of the generalized service time, for each $i = 1, \dots, N$:

$$c^{(i)}(t) = e^{-\nu_i t} b_i(t) + b_i(t) \sum_{n=1}^{\infty} e^{-\nu_i t} \frac{(\nu_i t)^n}{n!} * r_i^{(n)}(t), \tag{7}$$

where $r_i^{(n)}(t)$ denotes the nth convolution of the density $r_i(t)$ with itself. The 2nd term in the right-hand side of (7) corresponds to the case when, during the *clear (original)* service time of a class-i customer, exactly n interruptions occur (with the probability $e^{-\nu_i t} \frac{(\nu_i t)^n}{n!}$ within time interval of the length t), and then we take into account these n setup times by means of the convolution $r_i^{(n)}(t)$.

By applying similar arguments as above, we can easily obtain the following expression for the LST of the generalized service time density for the preemptive-resume service discipline:

$$c_i^*(s) = \beta_i^*(\nu_i(1 - r_i^*(s)) + s). \tag{8}$$

Now, by a standard way, we obtain the mean generalized service time:

$$\begin{aligned} \mathsf{E}C^{(i)} &= -\frac{d}{ds} c_i^*(s)|_{s=0} = \mathsf{E}S^{(i)}(1 + \nu_i \bar{r}_i), \qquad (9) \\ \rho_i &= \lambda_i \mathsf{E}C^{(i)}, \, i = 1, \dots, N. \end{aligned}$$

Remark 1. One can give an intuitive explanation of the expression (9) rewritten in the following form:

$$\mathsf{E}C^{(i)} = \mathsf{E}S^{(i)} + \nu_i \mathsf{E}S^{(i)}\bar{r}_i.$$

The 1st term is evident, while the 2nd term shows that the average number of the interruptions during original service time equals $\nu_i \mathsf{E}S^{(i)}$ (by the Wald's identity), and this quantity then is multiplied by the average duration of the setup time $\bar{r}_i$. Thus the term $\nu_i \mathsf{E}S^{(i)}\bar{r}_i$ equals an extra time caused by the interruptions. Moreover, we note that expression (9) is consistent with the case when the server is reliable, that is when the interruption rate $\nu_i = 0$ for each i, because in this case (9) becomes $\mathsf{E}C^{(i)} = \mathsf{E}S^{(i)}$.

IV. Performance analysis

As in [16], we apply the regenerative approach to obtain the stationary performance measures or the corresponding bounds. A remarkable feature, demonstrating the power and flexibility of the regenerative approach, is that, in this new setting, we can easily establish the performance results presented below by the same arguments that have been applied in works [18], [16] for the systems with reliable server. By this reason we omit the corresponding details of the proof.

Let $\mathbf{N}(t) = (N_1(t), \dots, N_N(t))$ be the N-dimensional vector describing the number of customers in the corresponding orbit queues at instant t. Denote $W_i(t)$ the remaining work in orbit i, at instant t^-, and consider the process $X(t) := N(t) + Q(t)$, $t \geq 0$, where $Q(t) \in \{0, 1\}$ denotes the number of customers in the server, and $N(t) = \sum_i N_i(t)$ is the summary size of all orbit queues, at instant t^-. The process $X =: \{X(t), t \geq 0\}$ is regenerative, and its regenerations are defined as the arrival instants when customers see an *idle system*. As in [16], denote generic regeneration period T and recall that the system is stable if the process X is positive recurrent that is the mean regeneration period if finite, $\mathsf{E}T < \infty$.

Remark 2. It is worth mentioning that the instances $\{T_n\}$ are also regenerations of the multidimensional queue-size process $\{\mathbf{N}(t)\}$. However, instead of this vector process, we study much more simple regenerative process X, and this dimension reduction is a key advantage of the regenerative approach.

Recall that due to service interruptions the server is occupied by a customer for a generalized service period. Denote $\hat{B}_i(t)$ the summary time, in the interval $[0, t]$, when the server is occupied by class-i customers, and let $W_i(t)$ be the remaining workload of class-i customers in the system at instant t^-. As in [16], we can deduce from the balance equation $V_i(t) = W_i(t) + B_i(t)$ for the summary work $V_i(t)$ generated by class-i customers in the interval $[0, t]$, that the stationary busy probability $\mathsf{P}_b^{(i)}$ that the server is occupied by class-i customers is

$$\lim_{t\to\infty} \frac{\hat{B}_i(t)}{t} = \mathsf{P}_b^{(i)} = \rho_i, \ i = 1, \dots, N. \tag{10}$$

Analogously, we obtain the *stationary busy probability* of the server as the limit w.p.1,

$$\lim_{t\to\infty} \frac{V(t)}{t} = \sum_{i=1}^{N} \rho_i = \mathsf{P}_b =: \rho. \tag{11}$$

We emphasize that the stationary busy probability of the server P_b in this setting includes the time when the server is blocked because of the interruptions.

Now we introduce the maximal and the minimal possible retrial rate from orbit i:

$$\hat{\mu}_i = \max_{J_n^{(i)} \in \mathcal{G}(i)} \mu_n^{(i)}, \ \mu_i^0 = \min_{J_n^{(i)} \in \mathcal{G}(i)} \mu_n^{(i)}, \ i = 1, \dots, N.$$

Then, analysis developed in [16] can be extended to our model to establish the following statement.

Theorem 1. *The necessary stability condition of the system under consideration is*

$$\sum_i \rho_i < \min_{1\leq i\leq N} \Big[\frac{\hat{\mu}_i}{\lambda_i + \hat{\mu}_i}\Big]. \tag{12}$$

In the next section we also verify the following *sufficient stability condition* obtained in [18]:

$$\sum_{i=1}^{N} \rho_i + \max_{1\leq i\leq N} \frac{\lambda}{\mu_i^0 + \lambda} < 1. \tag{13}$$

These conditions have been proved for a less general model with *two-state configurations* [18], but they can be readily extended to the current model [19].

We also note that, exactly as in [16], one can establish the following bounds of the stationary probability $\mathsf{P}_0^{(i)}$ that the *server is idle and orbit i is busy*:

$$\frac{\lambda_i}{\hat{\mu}_i}\rho \leq \mathsf{P}_0^{(i)} \leq \frac{\lambda_i}{\mu_i^0}\rho, \ i = 1, \dots, N. \tag{14}$$

In the next section, we verify by simulation stability conditions (12) for a set of parameters for the system with 3 classes of customers.

V. Simulation results

In this section we verify stability conditions contained in Theorem 1 for a particular case of a 3-class system.

We simulate this type of system using *discrete-event modeling*. In more detail, we consider the system only at such time instants (called *key instants*), when one of the following events occurs: *arrival* to the system, *departure* from the system, *retrial or interruption*. We denote Z_j the instant when the jth key instant occurs. There exist another events, say, service beginning instant or the setup time-over, but they match with key instants. For instance, a service beginning instant may happen only if a new customer meets free server, or a customer makes a successful retrial. These instants are defined recursively as follows:

$$Z_{j+1} = Z_j + \min\left\{t_A^{(j)}, t_D^{(j)}, t_R^{(j)}, t_I^{(j)}\right\}, \ j \geq 1, \qquad (15)$$

where $t_A^{(j)}$, $t_D^{(j)}$, $t_R^{(j)}$ and $t_I^{(j)}$ are the time intervals since instant Z_j until the next arrival, departure, retrial or interruption, respectively. (Z_1 is the first arrival instant.)

Recall that there are two possible types of the server interruptions in this system. In the first scenario, called *preemptive repeat different service* (REPEAT), the interrupted class-i customer starts the service over again, with the new independent service time sampled from the given service time distribution B_i. In the second scenario, *preemptive-resume service* (RESUME), the class-i interrupted customer continues being served after each setup instant until he accumulates his predefined service time.

In all experiments, we study the dynamics of orbit sizes $N_i(t)$ in an *exponential model* vs. number of key instances (x-axis), and verify the stability conditions for both types of the interruptions. Moreover, in each experiment we consider 600 arrivals, and in summary sample 300 such independent experiments. Then we average these observations (calculate the sample mean estimate) to obtain smooth output plots.

The following retrial rates are used in *all experiments*:

$$\begin{aligned} M_1 &= \left\{\mu_{00}^1 = 20, \, \mu_{10}^1 = 30, \, \mu_{01}^1 = 20, \, \mu_{11}^1 = 25\right\}, \\ M_2 &= \left\{\mu_{00}^2 = 20, \, \mu_{10}^2 = 30, \, \mu_{01}^2 = 20, \, \mu_{11}^2 = 25\right\}, \\ M_3 &= \left\{\mu_{00}^3 = 20, \, \mu_{10}^3 = 30, \, \mu_{01}^3 = 20, \, \mu_{11}^3 = 25 \right. . \end{aligned} \qquad (16)$$

Note that these parameters reflect a *partial symmetry* of the system: the identical response of each orbit on the *identical configurations*. However, in general it is *not a symmetric system*, because *not all* corresponding parameters are identical [19].

Denote $1/\bar{r}_i = \alpha_i$ the rate of the setup time when a class-i customer is being served. Fig. 3 shows dynamics of the orbits for the REPEAT interruptions model and the following input parameters:

$$\begin{aligned} &\lambda_1 = 2, \ \ \lambda_2 = 5, \ \lambda_3 = 3, \\ &\gamma_1 = 10, \, \gamma_2 = 20, \, \gamma_3 = 13, \\ &\nu_1 = 5, \quad \nu_2 = 5, \quad \nu_3 = 5, \\ &\alpha_1 = 15, \, \alpha_2 = 30, \, \alpha_3 = 20. \end{aligned}$$

To calculate the traffic intensity ρ, we need to find LST β_i^* in formula (5). Note that for the exponential service time with parameter γ_i, we obtain

$$\beta_i^*(\nu_i) = \frac{\gamma_i}{\gamma_i + \nu_i}.$$

As a result, these parameters give ρ = 0.85, see (11). On the other hand, it is easy to calculate, that the r.h.s. of conditions (12) and (13), equals 0.9 and 0.8, respectively, so only condition (12) hold while condition (13) is violated. As we see, despite breaking the sufficient condition, all orbits are stable as shown at Fig. 3.

For the next experiment with the REPEAT interruptions model, we take the following set of system parameters:

$$\begin{aligned} &\lambda_1 = 2, \, \lambda_2 \ = 5, \, \lambda_3 = 3, \\ &\gamma_1 = 5, \, \gamma_2 = 10, \, \gamma_3 = 15, \\ &\nu_1 = 3, \ \ \nu_2 = 3, \ \ \nu_3 = 3, \\ &\alpha_1 = 15, \alpha_2 = 20, \alpha_3 = 15. \end{aligned}$$

This choice gives ρ = 1.4, which evidently violates both necessary and sufficient stability conditions. Note that this result is mainly caused by the choice of the setup rates, which are taken smaller than that in the 1st experiment, making setup time, and hence generalized service time, longer. As a result, Fig. 4 demonstrates that all orbits indeed become unstable and grow approximately linearly. Moreover, this linear grow is specific for each orbit, and it is mainly because of different values of the traffic intensities $\rho_i = \lambda_i/\gamma_i$.

The following simulation results describe the dynamics of orbits with RESUME interruptions. In this case we use the following system parameters:

$$\begin{aligned} &\lambda_1 = 3, \, \lambda_2 \ = 3, \, \lambda_3 = 3, \\ &\gamma_1 = 10, \, \gamma_2 = 20, \, \gamma_3 = 13, \\ &\nu_1 = 5, \nu_2 = 5, \nu_3 = 5, \\ &\alpha_1 = 15, \alpha_2 = 30, \alpha_3 = 20. \end{aligned}$$

In this case we obtain ρ = 0.9, while the r.h.s of (12) equals 0.83. Thus, condition (13) is violated while condition (12) is satisfied, and Fig. 1 indicates that dynamics of the orbits is indeed similar to a stable dynamics observed on Fig. 3.

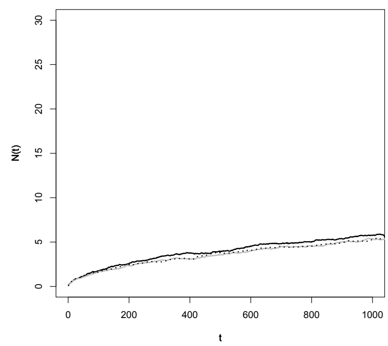

Fig. 1. RESUME mode: condition (13) is violated, condition (12) holds, all orbits are stable

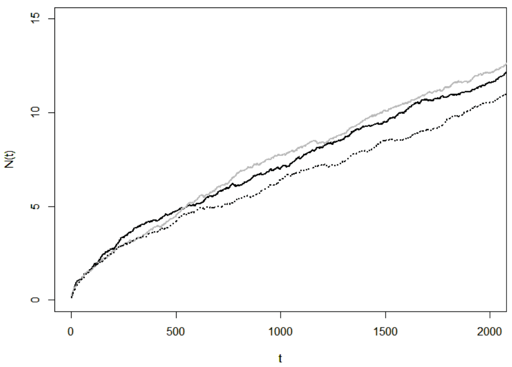

Fig. 2. RESUME mode: both conditions are violated, all orbits are unstable

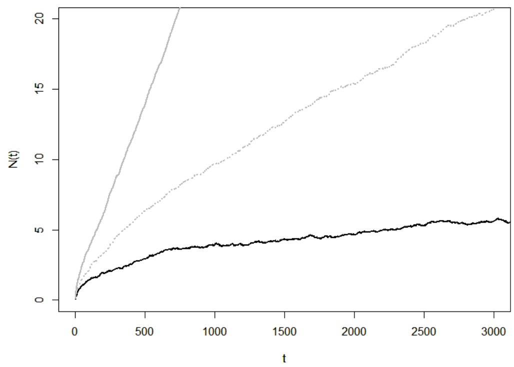

Fig. 4. REPEAT mode: both conditions are violated, all orbits are unstable

Finally, for the RESUME interruptions system with the parameters

$$\lambda_1 = 3,\ \lambda_2 = 3,\ \lambda_3 = 3,$$
$$\gamma_1 = 5,\ \gamma_2 = 10,\ \gamma_3 = 15,$$
$$\nu_1 = 5,\ \nu_2 = 5,\ \nu_3 = 5,$$
$$\alpha_1 = 15, \alpha_2 = 20, \alpha_3 = 15,$$

we obtain ρ = 1.8. Thus, both stability conditions are violated, and Fig. 2 reflects this property. It is also important to note that, despite the orbits are non-symmetric, their dynamic is quite similar. We suppose that it is because the traffic intensities are equal, $\rho_i = 0.6$ (while other parameters remain different).

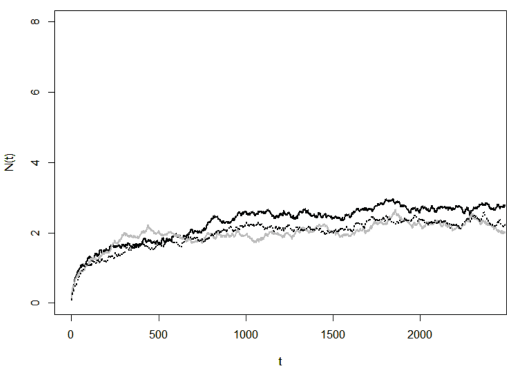

Fig. 3. REPEAT mode: condition (13) is violated, condition (12) holds, all orbits are stable

It is worth mentioning that the results demonstrated by Fig. 3 and Fig. 1 seem to be unexpected, because in both cases the sufficient stability condition is violated, while all orbits remain stable. This result has been first detected in the work [16] and it supports our conjecture that condition (13) is indeed redundant and the necessary stability condition (12) is close to be the stability criterion (or even is stability criterion) of the system.

VI. Conclusion

In this work, we introduce a retrial model with coupled orbit queues and two type of interruptions: preemptive-repeat different service, and preemptive-resume service. In this system, a new customer meeting server unavailable joins the corresponding infinite capacity orbit. The retrial rate from orbit i depends on the current configuration of other orbits: busy or idle, which gives rise to a novel class-dependent, queue-aware constant retrial policy. Service interruptions occurs according to a class-dependent Poisson process following a class-dependent setup periods. For both types of the models, we formulate and verify by simulation the stability conditions. These conditions have been proved earlier in our previous works [18], [16] for the system with reliable server. But they are readily extended, again by the regenerative approach, to the system with interruptions. This research verifies by simulation that the necessary stability conditions indeed are stability criterion for the model with coupled orbits and unreliable server when setup times have class-dependent general distributions. Moreover, this work again shows that regenerative approach is a powerful method to analysis complicated models of the modern communication systems.

For a future research it would be important to simulate non-exponential models, however, in the model with REPEAT service interruptions, it can be a hard problem to calculate the *LST of a non-exponential* service time distribution present in (5). Another goal of a future research is to verify stability condition related to each orbit separately, that is, instead of (12), verify condition $\rho < \frac{\hat{\mu}_i}{\lambda_i + \hat{\mu}_i}$ for each orbit i. In addition, we are planning to extend the observed model by making it possible to switch between the interruption modes depending on the customer's class. It is very motivated setting, for instance, for Windows and Unix-like operations system because both of these modes are supported for multitasking, and the interruption type depends on which task has been sent to the server.

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REFERENCES

[1] Artalejo, J.R.: Accessible bibliography on retrial queues: Progress in 2000-2009. Mathematical and Computer Modelling **51**(9-10), 1071-1081 (2010)

[2] Artalejo, J.R., Gomez-Corral, A.: Retrial Queueing Systems: A Computational Approach. Springer (2008)

[3] Avrachenkov, K., Morozov, E., Nekrasova, R., Steyaert, B.: Stability analysis and simulation of N-class retrial system with constant retrial rates and Poisson inputs. Asia Pac. J. Oper. Res. **31**(2), 1440002 (18 pages) (2014)

[4] Avrachenkov, K., Morozov, E., Steyaert, B.: Sufficient stability conditions for multi-class constant retrial rate systems. Queueing Syst. **82**, 149-171 (2016)

[5] Bonald, T., Borst, S., Hegde, N., Proutiere, A.: Wireless data performance in multi-cell scenarios, in: Proc. ACM Sigmetrics/Performance '04, pp. 378-388, ACM, New York (2004)

[6] Bonald, T., Massoulié, L., Proutiére, A., Virtamo, J.: A queueing analysis of max-min fairness, proportional fairness and balanced fairness. Queueing Syst. **53**(1-2), 65-84 (2006)

[20] Morozov, E., Delgado, R.: Stability analysis of regenerative queues. Automation and Remote control **70**, 1977-1991 (2009)

[7] Dimitriou, I.: A two class retrial system with coupled orbit queues. Prob. Engin. Infor. Sc. **31**(2), 139-179 (2017)

[8] Dimitriou, I.: A queueing system for modeling cooperative wireless networks with coupled relay nodes and synchronized packet arrivals. Perform. Eval. **114**: 16-31 (2017)

[9] Dimitriou, I.: Modeling and analysis of a relay-assisted cooperative cognitive network. In: N. Thomas, M. Farshaw (eds.), ASMTA 2017, LNCS, Springer, Newcastle, UK (2017)

[10] Falin G.I., Templeton, J.G.D.: Retrial Queues. Chapman & Hall, London (1997)

[11] Fayolle G., Malyshev V.A., Menshikov M.V.: Topics in the constructive theory of countable Markov chains. Cambridge University Press (1995)

[12] Gaver, D.P.: A Waiting Line with Interrupted Service, Including Priorities. Journal of the Royal Statistical Society. Series B (Methodological), **24** (1), 73-90 (1962).

[13] Kim, J., Kim, B.: A survey of retrial queueing systems. Annals of Operations Research **247**(1), 3-36 (2016)

[14] Liu, X., Chong, E., Shroff, N.: A framework for opportunistic scheduling in wireless networks. Comp. Netw. **41**, 451-474 (2003)

[15] Mitola, J., Maguire, G.: Cognitive radio: making software radios more personal. IEEE Pers. Commun. **6**(4), 13-18 (1999)

[16] E Morozov, T.Morozova. Analysis of a generalized system with coupled orbits. Submitted for FRUCT23, 2018.

[17] Morozov, E., Phung-Duc, T.: Stability analysis of a multiclass retrial system with classical retrial policy. Perform. Eval. **112**: 15-26 (2017)

[18] Morozov, E., Dimitriou, I. Stability Analysis of a Multiclass Retrial System with Coupled Orbit Queues, Proceedings of 14th European Workshop, EPEW 2017, Berlin, Germany, September 7-8, 2017, $DOI 10.1007/978-3-319-66583-2_6$

[19] Evsey Morozov, Taisia Morozova, and Ioannis Dimitriou. Simulation of multiclass retrial system with coupled orbits. Proceedings of SMARTY'18: First International Conference Stochastic Modeling and Applied Research of Tech- nology Petrozavodsk, Russia, September 21-25, 2018,pp. 6-16.

[21] Sadek, A., Liu, K., Ephremides, A.: Cognitive multiple access via cooperation: Protocol design and performance analysis. IEEE Trans. Infor. Th. **53**(10), 3677-3696 (2007)