

# A NEW ALGORITHM OF ITERATIVE MIMO DETECTION AND Decoding Using Linear Detector and Enhanced Turbo Procedure in Iterative Loop

Mikhail Bakulin\*, Vitaly Kreyndelin\*, Andrey Rog<sup>†</sup>, Dmitry Petrov<sup>‡</sup>, Sergei Melnik<sup>§</sup>

\*Moscow Technical University of Communications and Informatics (MTUSI),  
Moscow, Russia, {m.g.bakulin, vitkrend}@gmail.com

<sup>†</sup>GlobalInformService, Moscow, Russia, andhorn@mail.ru

<sup>‡</sup> Peoples' Friendship University of Russia (RUDN University), Moscow, Russia

<sup>§</sup>Central Scientific Research Institute of Communication, Moscow, Russia, svmelnik@mail.ru

**Abstract**—In the paper we develop and evaluate a novel low complexity algorithm of iterative detection and decoding in multiple input multiple output (MIMO) system. It is based on a new enhanced Turbo procedure. Although the algorithm utilizes well-known components such as linear minimum mean square error (MMSE) detector and channel decoder with soft bits feedback, the new original procedure of getting extrinsic data essentially allows improving the receiver performance and reducing its complexity. Moreover, it is shown that proposed Turbo approach works even without channel decoder in the iteration loop. Thus, we obtain pure iterative MMSE detector with improved performance. Utilization of combined scheme with MMSE detector and channel decoder feedback demonstrates really outstanding performance. It is confirmed with simulations that the performance of proposed architecture exceeds traditional ML MIMO detector schemes that are designed with channel decoder but without iterative loop.

## I. INTRODUCTION

The concept of Multiple input multiple output (MIMO) communication has been introduced about two decades ago [1], [2]. Nowadays, it is an integral part of many communication standards. Transmitting and receiving data through multiple antennas brings significant increase in channel capacity, system throughput and enhances the efficiency of restricted frequency resource utilization. However, these obvious benefits are accompanied by additional complications. The main drawback is complexity increase, which is first of all, concerns the receiver part. An ideal maximum likelihood (ML) MIMO detector has exponential complexity with regard to the number of transmitting antennas, which makes it hardly implementable especially in case of 4x4 or 8x8 MIMO configurations. It has been already noticed, for instance, in LTE-A [3]. ML MIMO Detector and its reduced complexity modifications, like Spherical Decoder [4] or K-Best algorithm [5], [6] present one class of detectors, which provides close to optimal performance with the expense of high calculation complexity. On the other hand, there is another class of Linear MIMO Detectors: Zero Forcing, MMSE [7], OSIC [2], [8], [9]. Their complexity grows polynomially with the number of antennas. Hence, it is at least few times less than for ML designs. Of

course, complexity reduction is accompanied with performance degradation, which can achieve up to 5 dB and more in the case of large-scale MIMO systems.

Iterative MIMO Detection and Decoding is another class of MIMO detection algorithms, where MIMO detection and channel decoding are combined in common iteration loop [10]. Fig. 1 illustrates a general scheme realizing iterative detection and decoding. Such scheme is also named "Turbo" due to its similarity to Turbo code decoders. Different modules of MIMO detection/decoding scheme are enriched with additional (extrinsic) information about the signal to be detected. Such approach demonstrates really outstanding performance in the case of ML or Spherical MIMO Detector when it is combined together with Channel Decoder [10], [11]. However, this scheme looks even more bulky than pure ML and thus its implementation (especially for mobile devices) is problematic. There are iterative schemes which combine linear MMSE detection and Channel decoding [12]-[15]. They demonstrate improved performance at reasonable complexity and can be considered as a good trade-off for MIMO Receiver implementation. However, there is still big gap between MMSE-based iterative detection and ML-based one. In this study we present a new modified Turbo scheme, which utilizes MMSE detection kernel and original mechanism for obtaining extrinsic data in iterative way. The new scheme demonstrates superior performance compared with conventional iterative schemes and its complexity is also reduced.

The rest of the paper is organized as follows. In Section II, we describe few conventional schemes of iterative detection and decoding using ML and MMSE kernels and discuss the basic principles of Turbo approach. In Section III, we introduce the new MIMO iterative detection scheme (we named it Turbo MMSE Detection), which works without feedback from Channel Decode. Nevertheless, it still demonstrates enhanced performance. In Section IV, we expand our new Turbo approach to joint detection and decoding scheme with channel decoder feedback and demonstrate its excellent performance. It exceeds all previously known iteration algorithms with linear detector kernels. The final section is intended for conclusions.

## II. ITERATIVE DETECTION AND DECODING IN MIMO RECEIVER

Iterative decoding is a well-known technique, which is first of all utilized in decoding of concatenated code constructions, such as Turbo codes [16], [17]. The main idea of iterative decoding is to define bit probabilities of original bit sequence transmitted through disturbing channel by set of serially concatenated decoders, where the output of last decoder is directed to 1st decoder thus enclosing iteration loop. Each decoder estimates bit probabilities based on its unique bit dependency chain (or code construction) together with extrinsic information (probability) about estimated bits, which is obtained from other decoders and which is considered as prior probability. For particular decoder extrinsic information can not contain bit probabilities obtained in this decoder in previous iteration cycles. Similar decoding procedure can be applied to signal equalization process where the signal is equalized and decoded in consequent modules within joint iteration loop [18]. This allows to improve equalization accuracy, by utilizing extrinsic information from Channel Decoder feedback. Same approach can be applied to joint MIMO detection and decoding.

The typical MIMO system is described by matrix equation:

$$Y = HX + \eta, \quad (1)$$

where  $X = (x_1, \dots, x_M)^T$  is the transmitted signal vector,  $Y = (y_1, \dots, y_N)^T$  is received signal vector,  $\eta = (\eta_1, \dots, \eta_N)^T$  is additive noise vector,  $H$  is channel matrix of size  $N \times M$ , all variables are complex. Equation (1) describes signal transmission through flat MIMO channel, which is typical case for most MIMO OFDM systems utilized in most broadband wireless standards.

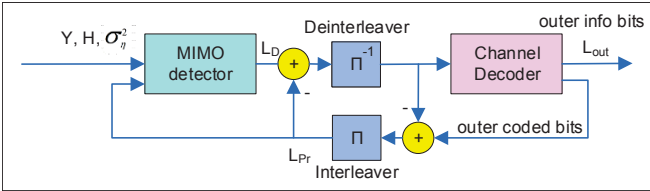


Fig. 1. Iterative detection and decoding scheme with ML kernel

### A. Iterative detection and decoding with ML or spherical detector kernel

The typical scheme of joint iterative detection and decoding is presented in Fig. 1. It definitely illustrates the process with ML detector kernel [10], [19]. Bit error probability in ML detector with Decoder feedback is defined as:

Bit error probability in ML detector with Decoder feedback is defined as:

$$L(b_{m,k}) = \ln \left( \frac{\sum_{X:f(b_{m,k}=1)} \prod_{n=1}^N e^{-\beta_{n,0}} \prod_{p=1}^M \prod_{\substack{t=1, \\ t \neq k, p=m}}^K \Pr(b_{p,t})}{\sum_{X:f(b_{m,k}=0)} \prod_{n=1}^N e^{-\beta_{n,1}} \prod_{p=1}^M \prod_{\substack{t=1, \\ t \neq k, p=m}}^K \Pr(b_{p,t})} \right) + \ln \left( \frac{\Pr(b_{m,k}=1)}{\Pr(b_{m,k}=0)} \right), \quad (2)$$

where  $L(b_{m,k})$  is logarithm likelihood ratio (LLR),  $m = 1, \dots, M$ ,  $k = 1, 2, \dots, K$ ,  $K$  - the number of bits in QAM constellation  $s : f(b_1, \dots, b_K)$ . Summing is performed through all possible combinations of symbols, containing bit  $b_{m,k}$  equals 0 or 1,  $\beta_{n,j=0/1}$  is Euclidean distance for each receiving antenna, defined by equation:

$$\beta_{n,j=0/1} = \frac{1}{2\sigma_\eta^2} \|y_n - h_n X_j\|^2. \quad (3)$$

In (3),  $y_n$  is a component of received vector,  $h_n$  - channel matrix row,  $X_j$  - transmitted vector, containing QAM symbol in layer  $m$  having bit  $b_{m,k} = 0/1$ ,  $\sigma_\eta^2$  - real (quadrature) noise variance, it is assumed that noise power is the same for all receiving antennas. Note, that second summation in (2) defines the prior probability (logarithm ratio of prior probabilities). This value is obtained from the output of Channel Decoder as well as prior probabilities  $\Pr(b_{p,t})$  for other bits composing  $X_j$ .

Prior data should contain only extrinsic information, therefore LLR data from the input of Channel Decoder are subtracted from LLRs on its output as it is show in the Fig. 1. For the same reason the second summation in (2) is eliminated from LLRs, before they are directed to Channel Decoder. The number of symbols engaged in the calculation of  $L(b_{m,k})$  in (2) is exponentially growing with regard to the number of transmitting antennas  $2^{M \cdot K}$ . This makes such approach to be hardly implementable.

Spherical Decoder reduces the number of symbols used in LLR calculation. Only the branches with high probability ratio are utilized in ultimate LLR calculation [10], [19]. This leads to significant complexity reduction, but still such approach has the complexity much larger than for the methods utilizing linear MIMO Detectors. The complexity increase ratio varies from few times up-to hundred times depending on MIMO configuration and reliable radius of decoding sphere, which in turn depends on the channel. That is why many investigations were devoted to finding the proper iterative scheme using linear MIMO Detector kernel, and particularly MMSE Detector as it provides the most accurate estimation of transmitted signal vector compared to other linear detectors.

### B. Conventional scheme of iterative detection and decoding with MMSE kernel

Before the actual algorithm description, it is necessary to note that further on we are considering real-valued signal transmission model. For this case, the equation (1) describing MIMO system still is correct, but complex vectors and matrices are transformed into real by means of unfolding procedure. It means that complex vector  $Y$  of length  $N$  is transformed into real-valued vector of length  $2N$ . It has real components of initial vector in its 1st half and image components in the 2nd half:

$$Y = (\Re(y_1), \dots, \Re(y_N), \Im(y_1), \dots, \Im(y_N))^T.$$

The same transformation is done for vectors  $\eta$  and  $X$ , which length becomes  $2M$ . Complex-valued matrix  $H$  of size  $N \times M$  becomes  $2N \times 2M$  matrix:

$$\begin{bmatrix} \Re(H) & -\Im(H) \\ \Im(H) & \Re(H) \end{bmatrix}$$

Basically, Iterative detection and decoding scheme using MMSE Detector looks similar to the scheme from Fig. 1. The main difference is in obtaining prior data from extrinsic LLRs at Channel Decoder output. MMSE Detector estimates transmitted signal vector  $X$ . Hence, it was proposed to form its prior estimation based on extrinsic bit probabilities (or LLRs) at Decoder output [12]-[15]. Therefore, it is assumed that components of signal vector are independent random variables and each component  $x_m$  has Gaussian distribution, characterized by its mean value  $\bar{x}_m$  and variance  $\sigma_{x_m}^2$ . Prior mean value  $\bar{x}_m$  and variance  $\sigma_{x_m}^2$  are formed using non-linear transformations:

$$\begin{aligned}\bar{x}_m &= E(x_m) = \sum_{s_q \in s} s_q \Pr(s_q) = \sum_{s_q \in s} s_q \prod_{t=1}^K \Pr(b_{q,t}), \\ E(x_m^2) &= \sum_{s_q \in s} s_q^2 \Pr(s_q) = \sum_{s_q \in s} s_q^2 \prod_{t=1}^K \Pr(b_{q,t}), \\ \sigma_{x_m}^2 &= \text{cov}(x_m) = E(x_m^2) - (E(x_m))^2.\end{aligned}\quad (4)$$

In (4),  $x_m$  represent quadrature (real or imaginary) component of QAM symbol,  $s_q$  is real or imaginary part of constellation point,  $\Pr(s_q)$  is probability of symbol  $s_q$ , which is defined as a product of bit probabilities  $\Pr(b_{q,t})$ . Each symbol  $s_q$  identifies few bits with indices  $t = 1, \dots, K$ . The bits are assumed to be independent. Thus, symbol probability is defined by their product.

Utilization of prior data in MMSE detector is performed in two steps. In the first step, soft interference cancellation is done, where for each component  $x_m$  the new observation vector with subtracted interference from other components is defined by

$$\tilde{Y} = Y - H\bar{X}_{-m}, \quad (5)$$

where  $\bar{X}_{-m} = (\bar{x}_1, \dots, \bar{x}_{m-1}, 0, \bar{x}_{m+1}, \dots, \bar{x}_{2M})^T$  contains prior estimations for all components except  $m$ . Setting  $\bar{x}_m = 0$  is done for obtaining extrinsic information about symbol on layer  $m$  after MMSE detection. MMSE estimation of  $\hat{x}_m$  with prior estimation of  $\bar{X}_{-m}$  is defined by equation:

$$\begin{aligned}\hat{x}_m &= G_m Y - G_m H \bar{X}_{-m} + E(x_m) = \\ &= G_m (Y - H \bar{X}_{-m}),\end{aligned}\quad (6)$$

where, as it was defined above  $E(x_m) = \bar{x}_m = 0$ ,  $G_m$  is MMSE filter defined by equation:

$$\begin{aligned}G_m &= \\ &= (\text{cov}(X, X))_{\text{Line}_m} H^T (H \text{cov}(X, X) H^T + \sigma_\eta^2 I)^{-1}\end{aligned}\quad (7)$$

In (7),  $\sigma_\eta^2$  is noise variance,  $(\text{cov}(X, X))_{\text{Line}_m}$  is line  $m$  of signal covariance matrix, defined by equation:

$$\begin{aligned}\text{cov}(X, X) &= \\ &= \text{diag}(\sigma_{x_1}^2, \dots, \sigma_{x_{m-1}}^2, \sigma_{s_m}^2, \sigma_{x_{m+1}}^2, \dots, \sigma_{x_{2M}}^2),\end{aligned}\quad (8)$$

where  $\sigma_{x_j}^2$  is prior variance defined in (4),  $\sigma_{s_m}^2$  - signal variance for layer  $m$  defined by signal power. For normalized power (having identity power on each complex layer)  $\sigma_{s_m}^2 = 1/2$ . Equation (7) can be transformed to:

$$G_m = \sigma_{s_m}^2 (H^T)_{\text{Line}_m} (H \text{cov}(X, X) H^T + \sigma_\eta^2 I)^{-1} \quad (9)$$

because  $\text{cov}(X, X)$  is a diagonal matrix. It is seen from (9), that MMSE filter is calculated for each real component of

transmitted symbol vector, which means it is necessary to invert  $2N \times 2N$  matrix  $2M$  times in each iteration. Estimation  $\hat{x}_m$  gives biased solution, which can be described by equation:

$$\hat{x}_m = \mu_m x_m + \nu_m, \quad (10)$$

where  $\mu_m$  is gain of MMSE filter and  $\nu_m$  is estimation noise, caused by thermal noise  $\eta$  and by interference from other layers. Again it is assumed that solution (10) produces noise characterized by Gaussian distribution with zero mean and variance  $\sigma_{\nu_m}^2$ . Taking into account that vector  $Y$  is defined by (1),  $\hat{x}_m$  is defined by (6), and  $G_m$  is defined by (9) it is easy to obtain  $\mu_m$  and  $\sigma_{\nu_m}^2$ :

$$\begin{aligned}\mu_m &= \\ &= \sigma_{s_m}^2 (H_{\text{col}_m})^T (H \text{cov}(X, X) H^T + \sigma_\eta^2 I)^{-1} H_{\text{col}_m}, \\ \sigma_{\nu_m}^2 &= \sigma_{s_m}^2 (1 - \mu_m),\end{aligned}\quad (11)$$

where  $H_{\text{col}_m}$  is column of  $H$  with index  $m$ . From (11) we can define probability density function for a symbol  $s_q$  transmitted on layer  $k$ :

$$p(\hat{x}_m | s_q) = \frac{1}{\sqrt{2\pi\sigma_{\nu_m}^2}} \exp\left(-\frac{(\hat{x}_m - \mu_m s_q)^2}{2\sigma_{\nu_m}^2}\right). \quad (12)$$

Using (12) it is possible to define LLR values for bits composing symbol  $x_m$  on the second step of MMSE detection. One can observe that no prior information about estimated component  $\hat{x}_m$  was used on the first step. However, this information is available after Channel Decoder and it can be utilized in de-mapping procedure. Similar to the approach defined by equation (2), we note that prior probability of symbol is defined by the product of its bit probabilities. Hence, LLR can be defined by equation:

$$\begin{aligned}L(b_{m,k}) &= \\ &= \ln \left( \frac{\sum_{s_0 \in s(b_m, k=1)} e^{-\frac{(\hat{x}_m - \mu_m s_0)^2}{2\sigma_{\nu_m}^2}} \prod_{t=1, t \neq k}^K \Pr(b_p, t)}{\sum_{s_1 \in s(b_m, k=0)} e^{-\frac{(\hat{x}_m - \mu_m s_1)^2}{2\sigma_{\nu_m}^2}} \prod_{t=1, t \neq k}^K \Pr(b_p, t)} \right) + \\ &\quad + \ln \left( \frac{\Pr(b_{m,k} = 1)}{\Pr(b_{m,k} = 0)} \right)\end{aligned}\quad (13)$$

In (13),  $\Pr(b_{p,t})$  is prior bit probability for bits composing symbol  $s$ , excluding sought bit. Summation is performed along symbols containing estimated bit equal to 1 in the numerator and to 0 in the denominator. Same as for ML kernel, the last summation is removed from  $L(b_{m,k})$  for getting extrinsic information to be directed to the Channel Decoder. Utilization of prior bit probabilities becomes possible because of bit interleaving. Thus, the bits composing one QAM symbol are located far away from each other in encoded block. They can be considered as independent after decoding procedure.

There are some modifications of iterative detecting and decoding with MMSE kernel which utilize more sophisticated algorithms. Particularly in [13], it was proposed to make hard interference cancellation for the most reliable symbols and soft interference cancellation for the others. It was shown that such approach can bring additional gain, but the actual gain was not high, while the complexity increased. In [13], it was

proposed to use two groups of symbols, which are subtracted in (5) for getting MMSE solution for a particular layer. First group contains symbols from the layers which already obtained MMSE estimation in current iteration cycle. Mean value and variance are defined from them based on posterior bit probabilities obtained after de-mapping. Second group contains symbols from layers not processed yet. Their mean value and variance are defined from prior bit probabilities obtained on previous iteration. According to [13], this approach brings additional gain, but at the same time it requires sorting of layers and additional re-modulation. This leads to essential complexity increase.

Summarizing this description, we can conclude, that described iterative method with MMSE detector kernel (hereafter, we will call this approach as conventional) brings additional gain compared with non-iterative linear detection. However, its performance shows serious degradation compared with iterative detection based on ML kernel. The examples of algorithm performance will be presented in following sections.

### III. NEW ITERATIVE MIMO DETECTION SCHEME WITH MMSE KERNEL

In this section we will present a new iteration scheme with MMSE kernel, which does not utilize Channel Decoder in the iteration loop. Before starting with algorithm description, we will introduce a new procedure for getting extrinsic information. Let's assume that we estimate a parameter  $x$ , which has Gaussian distribution and for which we have prior information about its mean value  $\bar{x}_{pr}$  and variance  $\sigma_{pr}^2$ . Suppose, that additionally we got a new measurements and defined a new estimation of mean value  $\bar{x}_{ms}$  and variance  $\sigma_{ms}^2$ . Estimated signal is defined by equation:

$$\bar{x}_{ms} = x + \eta, \quad (14)$$

where  $\eta$  is estimation error, which is supposed to have Gaussian probability density function (PDF).

Based on these data we can get posterior PDF for  $x$ :

$$P_{pos}(x) = C_1 \exp\left(-\frac{(x - \bar{x}_{pr})^2}{2\sigma_{pr}^2} - \frac{(x - \bar{x}_{ms})^2}{2\sigma_{ms}^2}\right), \quad (15)$$

where  $C_1$  is normalization constant. Transforming the expression inside the exponent we get:

$$P_{pos}(x) = C_1 \exp\left(-\frac{x^2 - 2x\frac{\bar{x}_{pr}\sigma_{ms}^2 + \bar{x}_{ms}\sigma_{pr}^2}{\sigma_{ms}^2 + \sigma_{pr}^2}}{2\frac{\sigma_{pr}^2\sigma_{ms}^2}{\sigma_{ms}^2 + \sigma_{pr}^2}}\right) \times \exp\left(-\frac{\bar{x}_{pr}^2\sigma_{ms}^2 + \bar{x}_{ms}^2\sigma_{pr}^2}{2\sigma_{ms}^2\sigma_{pr}^2}\right). \quad (16)$$

It is easy to see that (16) is still Gaussian PDF, having variance and mean value defined by (17):

$$\sigma_{pos}^2 = \frac{\sigma_{pr}^2\sigma_{ms}^2}{\sigma_{pr}^2 + \sigma_{ms}^2}, \quad (17)$$

$$\bar{x}_{pos} = \bar{x}_{pr}\frac{\sigma_{ms}^2}{\sigma_{pr}^2 + \sigma_{ms}^2} + \bar{x}_{ms}\frac{\sigma_{pr}^2}{\sigma_{pr}^2 + \sigma_{ms}^2},$$

$$P_{pos}(x) = C_1 \exp\left(-\frac{(x - \bar{x}_{pos})^2}{2\sigma_{pos}^2}\right) \exp\left(-\frac{\bar{x}_{pr}^2\sigma_{ms}^2 + \bar{x}_{ms}^2\sigma_{pr}^2}{2\sigma_{pr}^2\sigma_{ms}^2}\right) \times \exp\left(\frac{(\bar{x}_{pr}\sigma_{ms}^2 + \bar{x}_{ms}\sigma_{pr}^2)^2}{2\sigma_{pr}^2\sigma_{ms}^2(\sigma_{pr}^2 + \sigma_{ms}^2)}\right) = C_3 \exp\left(-\frac{(x - \bar{x}_{pos})^2}{2\sigma_{pos}^2}\right). \quad (18)$$

Correspondingly, in the case when we are solving backward task of getting extrinsic information from the posterior PDF, we obtain:

$$\sigma_{ms}^2 = \frac{\sigma_{pr}^2\sigma_{pos}^2}{\sigma_{pr}^2 - \sigma_{pos}^2}, \quad (19)$$

$$\bar{x}_{ms} = -\bar{x}_{pr}\frac{\sigma_{pos}^2}{\sigma_{pr}^2 - \sigma_{pos}^2} + \bar{x}_{pos}\frac{\sigma_{pr}^2}{\sigma_{pr}^2 - \sigma_{pos}^2}.$$

Knowing (19), one can proceed with the iterative detection. The block diagram of corresponding scheme is shown in Fig. 2.

MMSE detector estimates mean value  $\hat{X}_{MMSE}$  and variance  $\sigma_{MMSE}^2$  based on prior data obtained on the previous iteration, where prior data are also mean value  $\bar{X}_{pr}$  and variance  $\sigma_{pr}^2$ . MMSE detector produces new estimation in accordance with equations:

$$\begin{aligned} \hat{X}_{MMSE} &= \bar{X}_{pr} + G_{MMSE\_Pr}(Y - H\bar{X}_{pr}), \\ G_{MMSE\_Pr} &= V_{pr}H'(HV_{pr}H' + \sigma_{\eta}^2I)^{-1}, \\ V_{pr} &= \text{diag}(\sigma_{pr}^2), \\ \sigma_{MMSE}^2 &= \text{diag}(V_{MMSE}) = \\ &= \text{diag}(V_{pr} - G_{MMSE\_Pr}HV_{pr}). \end{aligned} \quad (20)$$

Note, that in (20)  $\sigma_{MMSE}^2$  and  $\sigma_{pr}^2$  are vectors,  $V_{pr}$  is a diagonal matrix containing variances  $\sigma_{pr}^2$  for each layer of estimated vector  $X$ , correspondingly  $\sigma_{MMSE}^2$  is a vector of variances obtained from diagonal of the matrix  $V_{MMSE}$ . Considering MMSE estimation as posterior PDF, which is assumed to be Gaussian, and recalling conclusions of (19), we can extract extrinsic data (external data or new observation), which will be used on the next processing step. Thus, following (19) we can define for each component  $m$  of vectors  $\bar{X}_{ex}$ ,  $\sigma_{ex}^2$ :

$$\begin{aligned} \bar{x}_{ex,m} &= -\frac{\sigma_{MMSE,m}^2}{\sigma_{pr,m}^2 - \sigma_{MMSE,m}^2}\bar{x}_{pr,m} + \\ &+ \frac{\sigma_{pr,m}^2}{\sigma_{pr,m}^2 - \sigma_{MMSE,m}^2}\hat{x}_{MMSE,m}, \\ \sigma_{ex,m}^2 &= \frac{\sigma_{pr,m}^2\sigma_{MMSE,m}^2}{\sigma_{pr,m}^2 - \sigma_{MMSE,m}^2}. \end{aligned} \quad (21)$$

In the derivation of (21) it was assumed, that the components of vectors are independent. Therefore, each component can be corrected separately. The last statement is not exactly true, but we can consider it as correct with a reasonable degree of accuracy.



Soft QAM Demapper produces LLR estimations based on obtained  $\hat{X}_{ex}$ ,  $\sigma_{ex}^2$ :

$$L(b_{m,k}) = \frac{\sum_{s_0 \in s(b_{m,k}=1)} e^{-\frac{(\bar{x}_{ex,m} - \mu_m s_0)^2}{2\sigma_{ex,m}^2}} \prod_{t=1, t \neq k}^K \Pr(b_p, t)}{\sum_{s_1 \in s(b_{m,k}=1)} e^{-\frac{(\bar{x}_{ex,m} - \mu_m s_1)^2}{2\sigma_{ex,m}^2}} \prod_{t=1, t \neq k}^K \Pr(b_p, t)} + \ln \left( \frac{\Pr(b_{m,k} = 1)}{\Pr(b_{m,k} = 0)} \right). \quad (22)$$

These data are directed to the output when the iteration loop ends, or to the next processing module fore re-modulation (or re-mapping). Re-modulator produces non-linear synthesizing of symbol vector mean  $\bar{X}_{ps}$  and its variance  $\sigma_{ps}^2$ :

$$\begin{aligned} \bar{x}_{ps,m} &= \sum_S s(b_{m,1}, \dots, b_{m,K}) \prod_{t=1}^K \Pr(b_{m,t}), \\ \sigma_{ps,m}^2 &= \sum_S |s(b_{m,1}, \dots, b_{m,K})|^2 \prod_{t=1}^K \Pr(b_{m,t}) - |\bar{x}_{ps,m}|^2, \end{aligned} \quad (23)$$

where summation in (23) is done among all constellation points (all bit combinations) of layer  $m$ . Considering  $\bar{X}_{ps}$  and  $\sigma_{ps}^2$  as posterior PDF with external data  $X_{ex1}$ ,  $\sigma_{ex1}^2$  we can extract extrinsic (prior) data applying (19):

$$\begin{aligned} \bar{x}_{pr,m} &= -\frac{\sigma_{ps,m}^2}{\sigma_{ex,m}^2 - \sigma_{ps,m}^2} \bar{x}_{ex,m} + \\ &+ \frac{\sigma_{ex,m}^2}{\sigma_{ex,m}^2 - \sigma_{ps,m}^2} \bar{x}_{ps,m}, \\ \sigma_{pr,m}^2 &= \frac{\sigma_{ps}^2 \sigma_{ex,m}^2}{\sigma_{ex,m}^2 - \sigma_{ps,m}^2}. \end{aligned} \quad (24)$$

Obtained data are used in MMSE detection in next the iteration cycle. In the first iteration, it is assumed that  $\sigma_{pr,m}^2 = \sigma_{ps,m}^2$ ,  $\bar{x}_{pr,m} = 0$ , so MMSE filter works same as in conventional MMSE detector.

To avoid negative variances in (24), it is necessary to check the condition  $\sigma_{ps,m}^2 < \sigma_{ex,m}^2$ . If it is not true, that means that the iteration for that particular layer does not bring accuracy improvement. Hence, this result should be discharged, i.e. in the next iteration prior data for the layer  $m$  are the same as in the iteration before. Proposed method that we called "Turbo MMSE" requires just two iterations. Further iterations do not bring noticeable performance improvement.

Fig. 3 and Fig. 4 demonstrate the performance of proposed algorithm. Simulations were performed for LTE-A system for MIMO configurations 8x8 and 4x4, and different modulations and code rates. For comparison we provide also results of two reference algorithms: MMSE and ML, which work in conventional non-iterative scheme. It can be seen that proposed iterative procedure demonstrates significant gain regarding pure MMSE but still there is the gap to ML. However, because, as it was mentioned before, ML complexity is much higher than MMSE, proposed iterative algorithm looks to be attractive

low complexity alternative, providing reasonable performance-complexity trade-off.

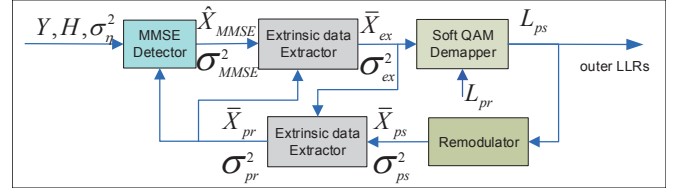


Fig. 2. Iterative Detection using MMSE kernel (no Decoder in feedback loop)

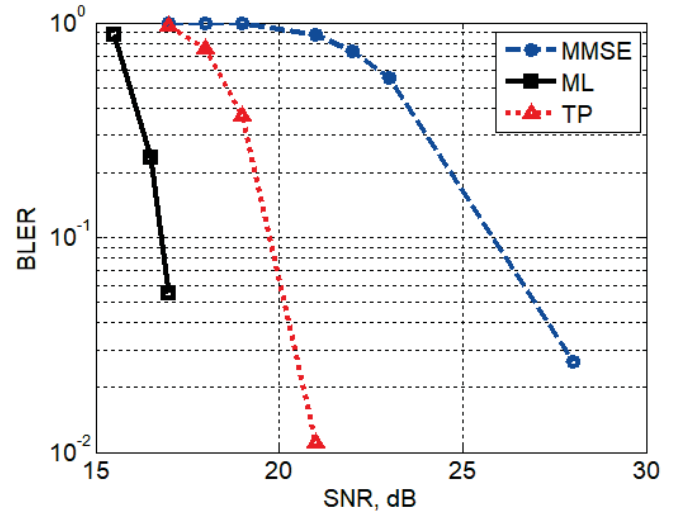


Fig. 3. Block Error Rate for 3 algorithms: MMSE, ML and Turbo MMSE (marked as TP). LTE-A system parameters: 8x8 SU-MIMO. Modulation (MCS15): QAM 16, code rate: 3/4; Channel: EPA-5, 1000 random realizations, perfect channel estimation

#### IV. NEW ITERATIVE MIMO DETECTION AND DECODING SCHEME WITH MMSE KERNEL

Iterative detecting demonstrates much better performance in case Channel Decoder is included in joint detection and decoding iteration loop. Fig. 5 presents modification of Turbo-MMSE scheme with included Channel Decoder in joint iteration detecting and decoding loop.

It can be seen that Decoder input is connected to the output of soft QAM Demapper in a way similar to the conventional scheme with ML kernel from Fig. 1. Whereas, its 1st output is interleaved and re-connected to Demapper through feedback line. The input LLRs are subtracted from LLRs output from Decoder for getting extrinsic data  $L_{EXTR}$  been used in next iteration cycle. 2nd output from Decoder is interleaved and connected to non-linear Remapper, producing estimations of  $\bar{X}_{ps}$  and  $\sigma_{ps}^2$ , defined by (23). One can observe, that proposed scheme contains two feedback lines: 1st local feedback line connects Decoder output and Demapper, while 2nd global feedback line closes main loop, where the data are passed through Remapper, then through Module, extracting extrinsic data, and finally are directed to MMSE detector. Note, that data processing procedure in global feedback line is the same

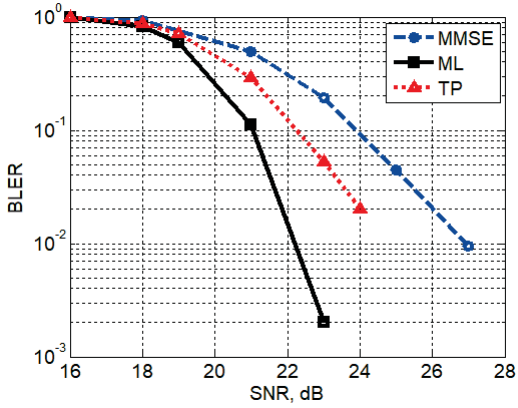


Fig. 4. Block Error Rate for 3 algorithms: MMSE, ML and Turbo-MMSE (marked as TP). LTE system parameters: 4x4 SU-MIMO. Modulation (MCS17): QAM 64, code rate: 1/2, Channel: EPA-5, 1000 random realizations, perfect channel estimation

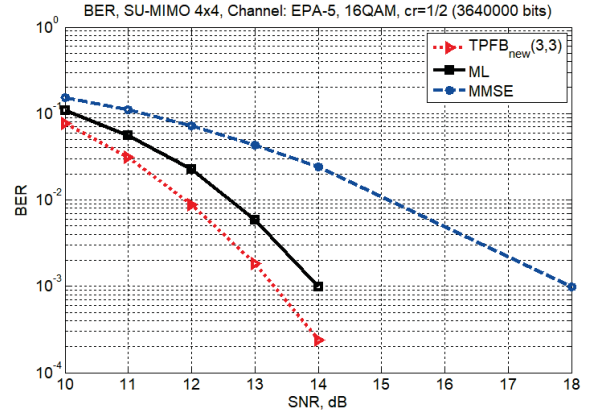


Fig. 6. Bit Error Rate for 3 algorithms: MMSE, ML and Iterative MIMO Detection and Decoding (marked as TPFBnew). LTE system parameters: 4x4 MIMO. Modulation: QAM 16, code rate: 1/2, Channel: EPA-5

as in Turbo MMSE, while data processing procedure for local feedback line looks close (but not exactly the same) to the procedure used in conventional iterative scheme with MMSE Kernel.

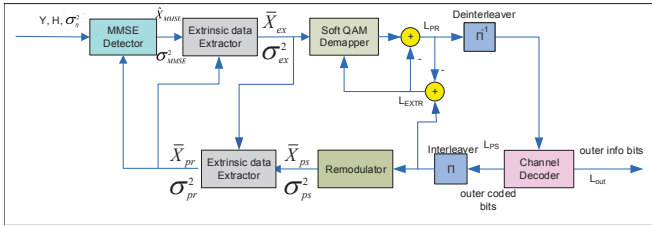


Fig. 5. New Iterative Detection and Decoding scheme using MMSE kernel

For the sake of clarity we recall that LLRs after de-mapping are defined by equation (13), where 2nd summand is removed from the output before directing to Decoder. Bit probabilities  $\Pr(b_{p,t})$  are obtained from extrinsic data obtained in Decoder. For that input to Decoder LLRs are subtracted from its output and interleaved LLRs. Also, same as in Turbo-MMSE case, if after current iteration for some particular layer there is no accuracy improvement, i.e.  $\sigma_{ps,m}^2 \geq \sigma_{ex,m}^2$ , the new prior data is discarded, i.e. prior data for next iteration are same as iteration before.

The proposed scheme with two feedback lines demonstrates excellent performance, which was proven by multiple simulations performed for different communication standards. Fig. 6 shows performance of proposed method for MIMO 4x4 for LTE. For comparison, two more algorithms are presented: ML and MMSE used without iterative feedback. In all cases the total number of Turbo Decoder loops is 6. In new iterative detection scheme with MMSE kernel, two detection loops were fulfilled, while inside one (external) detection loop Turbo-Decoder fulfilled 3 internal (decoding) iterations, so the total number of Decoder iterations was the same. The simulation was done without automatic repeat request (ARQ) mode.

Fig. 7 shows the advantage of proposed method for new broadcasting standard DVB-NGH. For comparison, we also

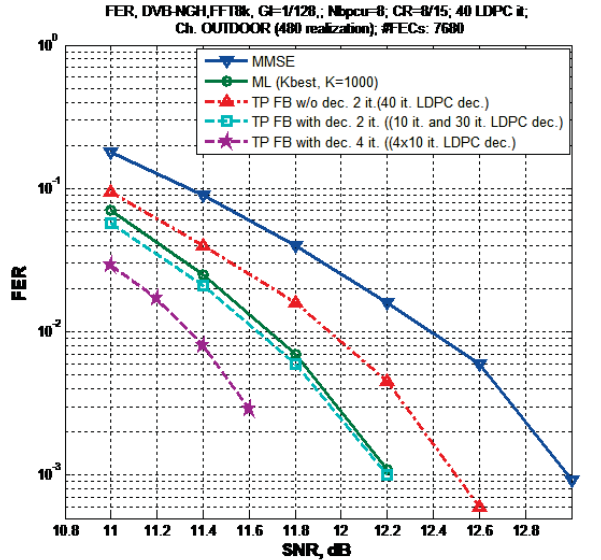


Fig. 7. Block Error Rate for 4 algorithms: MMSE, ML (without iterative decoding), conventional Iterative MIMO Detection and Decoding with MMSE kernel and feedback without decoder (marked as TP FB w/o dec.), new Iterative MIMO Detection and Decoding with MMSE kernel and feedback with decoder (marked TP FB with dec.). The figures in parentheses mean number of Decoder internal loops in each external Detector loop. Simulation of DVB-NGH system: code rate 8/15, MIMO 2x2, modulation 16QAM x 16QAM, Outdoor Channel Model

show the performance of few reference algorithms: ML and MMSE, which works without iterative feedback, and also conventional iterative detection and decoding with MMSE kernel, described in Section II. New iterative detection and decoding scheme with MMSE Kernel was tested for two modes. In mode 1 there was 2 (external) detecting cycles, while LDPC Decoder made 11 internal loops in first iteration and 29 internal loops in second iteration. In second mode there was 4 detection loops, while Decoder made 10 internal loops in each external iteration. Note, that total number of Decoder iterations was 40 (upper limit) for all tested algorithms.

One may observe that even with two iterations proposed

method demonstrates performance similar to ML. Its performance exceeds the performance of conventional iterative detecting and decoding with MMSE kernel, while its complexity is lower. Indeed, as it was mentioned in Section II.B, each external iteration requires  $2M$  real  $2NN$  matrix inversions, while the new algorithm requires just one. This significant complexity reduction accompanied with performance improvement makes proposed new scheme to be very attractive especially in applications, which do not require ARQ.

## V. CONCLUSION

In the paper we presented new Iterative Detection and Decoding approach based on utilization of MMSE MIMO detection. We showed that such algorithm can be applied to pure MIMO detection as well as to joint MIMO Detection and Decoding utilizing Channel Decoder in detection/decoding loop. We confirmed that proposed method has better performance than already existing iterative detection and decoding algorithms with MMSE kernel, whereas its complexity is much lower.

Proposed scheme is obviously attractive for broadcasting MIMO receivers [20],[21], which does not use ARQ. ARQ assumes additional re-transmissions, where the bits belonging to the same encoded block are transmitted once again (it may be the same bits or bits, which were punctured in previous transmission). In the case of iterative Detection and Decoding, ARQ re-transmission leads to complexity increase, because it is necessary to utilize additional sub-carriers in joint detection and decoding. Of course, developed approach is also suitable for MIMO systems with ARQ. We just mention potential implications because one should be careful in choosing the most efficient detecting/decoding scheme. For instance, proposed iterative detection and decoding has really low complexity compared to ML or Spherical Decoder. We also have all reasons to expect that it can be beneficial even in the case of increased design complexity due to re-transmission. Till now, we have not studied profoundly this approach for systems with ARQ, but we believe that it should be done in future in order to find the best trade-off between dozens of available MIMO detecting/decoding schemes.

Returning to broadcasting MIMO receivers, we would like to mention that iterative detection and decoding scheme has been already recommended for DVB-NGH in work [21]. There, such implementation resulted in superior performance. Therefore, our newly proposed low complexity algorithm is an attractive candidate for DVB-NGH systems. Its advantages become even more obvious in the cases when the number of antennas in MIMO system increases.

## ACKNOWLEDGMENT

The publication has been prepared with the support of the "RUDN University Program 5-100" and funded by RFBR according to the research projects No. 12-34-56789 and No. 12-34-56789.

## REFERENCES

- [1] P.W. Wolniansky, G.J. Foschini, G.D. Golden, and R.A. Valenzuela, "V-BLAST: An architecture for realizing very high data rates over the rich-scattering wireless channel," in *1998 International Symposium on Signals, Systems, and Electronics (ISSSE'98)*, Pisa, Italy, September 30, 1998.
- [2] G.D. Golden, G.J. Foschini, R.A. Valenzuela, and P.W. Wolniansky, "Detection algorithm and initial laboratory results using V-BLAST space-time communication architecture", *Electronic Letters*, vol.35, no.1, pp.14-15, Jan.7, 1999.
- [3] 3GPP Technical Specification, 36.306, V11.2.0, Dec., 2012.
- [4] B. Hassibi, H. Vikalo, "On the Sphere-Decoding Algorithm I. Expected Complexity", *IEEE Transactions On Signal Processing*, vol.53, no.8, Aug., 2005.
- [5] K. Higuchi, H. Kawai, N. Maeda, M. Sawahashi, "Adaptive selection of survival symbol replica candidates based on Maximum reliability in QRM-MLD for OFCDM MIMO multiplexing", in *Proc. IEEE GLOBECOMM 2004*, Nov. 29-Dec.3, 2004, pp.2480-2486.
- [6] M. Bakulin, V. Kreyndelin, A. Rog, D. Petrov, S. Melnik, "MMSE Based K-best Algorithm for Efficient MIMO Detection", in *9th International Congress on Ultra Modern Telecommunications and Control Systems and Workshops (ICUMT)*, Munich, Germany, 4-6 November, 2017.
- [7] D. Wubben, R. Bonke, V. Kuhn, K.-D. Kammeyer, "MMSE Extension of V-BLAST based on Sorted QR Decomposition", in *Vehicular Technology Conference (VTC 2003-Fall) IEEE 58th*, 2003, vol.1, pp.508-512.
- [8] M. Bakulin, V. Kreyndelin, A. Rog, D. Petrov, S. Melnik, "Low-Complexity Iterative MIMO Detection Based on Turbo-MMSE Algorithm", in *17-th International Conference. Internet of Things, Smart Spaces and Next Generation Networks and Systems*, St. Petersburg, Russia, Aug. 28-30, 2017, pp.550-560.
- [9] H. Lee, I. Lee, "New Approach for coded layered space-time OFDM systems", in *ICC 2005 IEEE International Conference on Communications*, vol.1, 16-20 May, 2005, pp. 608-612.
- [10] B. M. Hochwald, S. T. Brink, "Achieving Near-Capacity on a Multiple-Antenna Channel", *IEEE Transaction on Communications*, vol.51, no.3, March 2003.
- [11] H. Vikalo, B. Hassibi, and T. Kailath, "Iterative Decoding for MIMO Channels Via Modified Sphere Decoding", *IEEE Transaction on Wireless Communications*, vol.3, on.6, Nov. 2004.
- [12] M. Witzke, S. Baro, F. Schreckenbach, and J. Hagenauer, "Iterative detection of MIMO signals with linear detectors," in *36th Asilomar Conference On Signals, Systems And Computers ACSSC 2002*, Pacific Grove, CA, USA, Nov. 3-6, 2002.
- [13] A. Boronka, T. Rankl and J. Speidel, "Iterative Nonlinear Detection of MIMO Signals using an MMSEOSIC Detector and Adaptive Cancellation", in *5th International ITG Conference on Source and Channel Coding*, Erlangen, Jan. 14-16, 2004.
- [14] A. Gueguen, *Method and Device for Efficient Decoding of Symbols Transmitted in a MIMO Telecommunication System*, US patent, 7.292.658 B2.
- [15] J.W. Choi, A.C. Singer, J. Lee, N.I. Cho "Improved Linear Soft-input Soft-output Detection Via Soft Feedback Successive Interference Cancellation", *IEEE Transaction on Communications*, vol.58, no.3, March 2010.
- [16] B. Vucetic, J. Yuan, *Turbo Codes: Principles and Applications*, Kluwer Academic Publishers, 2000.
- [17] A. Giulietti, B. Bougard, L.V. der Perre, *Turbo Codes : Desirable and Designable*, Kluwer Academic Publishers, 2004.
- [18] R. Koetter, A. Singer, and M. Tuchler, "Turbo equalization", *Signal Processing Magazine, IEEE*, vol.21, no.1, pp.67-80, Jan. 2004.
- [19] H. Vikalo, B. Hassibi, T. Kailath, "Iterative Decoding for MIMO Channels Via Modified Sphere Decoding", *IEEE Transaction on Wireless Communications*, vol.3, no.6, Nov. 2004.
- [20] D. Gozálviz, J. López-Sánchez, D. Gómez-Barquero, N. Cardona, "Combined Time And Space Diversity: Mobile Reception in DVB-T and DVB-T2 Systems", *IEEE Vehicular Technology Magazine*, Dec. 2012.
- [21] Digital Video Broadcasting (DVB), Next Generation broadcasting system to Handheld, physical layer specification (DVB-NGH), DVB Document A160, Nov. 2012.