A New Approach of Implementation of MMSE Demodulator for Massive MIMO Systems

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***Abstract*—Multiple-Input Multiple Output (MIMO) is the key technology in most advanced wireless communication standarts like 3GPP Long Term Evolution (LTE) and IEEE 802.11n/ac/ad (Wi-Fi). The cost of MIMO improvements is increased computational complexity of the signal processing at both ends and, as a consequence, growing complexity of the hardware. We propose a new approach of implementation of traditional MMSE demodulator reducing computational complexity for massive MIMO systems.**

I. Introduction

Nowadays MIMO systems is a key technology in radio communication standards such as LTE and Wi-Fi [1],[2],[11]. The use of MIMO technology in communication systems makes it possible to significantly increase the capacity [1], but involves the necessity of solving many problems of signal processing.

Development of the MIMO technology is associated with appearance of large-size MIMO systems, which have still more advantages of the multi-array systems. However, increased number of antennas is also increasing the number of arithmetical operations necessary for the signal demodulation [1]. This paper suggests applying of the Strassen algorithm and the 3M method for decreasing of the computational complexity in obtaining of the estimate, which is optimal in terms of the mean square error criterion, of the vector of transmitted symbols at the receiver end [3],[4].

II. System Model

Shown in Fig. 1 is an abstract model of the MIMO system, based on which different configurations of the system can be built.

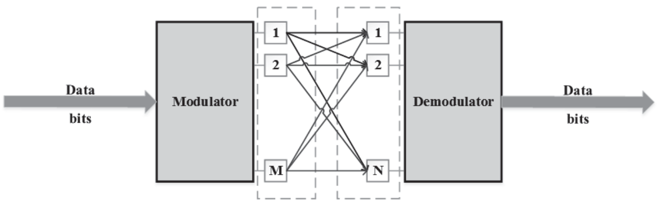

Fig. 1. MIMO structure

Here M is the number of antennas at the transmitter part, N is the number of antennas at the receiver end. At the receiver end the signals from each of the transmitting antennas are emitted simultaneously and within one and the same frequency band. In the channel properly these signals are subject to the influence of the Rayleigh fading and the additive Gaussian white noise. Therefore, an additive aggregate of M transmitted signals arrives at each of N receiving arrays.

Mathematical model of the MIMO system has the following representation:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \boldsymbol{\eta}, \tag{1}$$

where $\mathbf{y}$ is the vector of received complex counts with the dimensionality N, $\mathbf{H}$ is the matrix of the telecommunication channel complex transmission coefficients with the dimensionality $N \times M$, $\mathbf{x}$ is the vector of transmitted informational symbols with the dimensionality M, $\boldsymbol{\eta}$ is the complex random Gaussian noise vector in the telecommunication channel with the dimensionality N.

In general mathematical terms, the process of demodulation at the receiver end is reduced to solving of the given system of equations (1).

III. MIMO Demodulation

The problem of the received signal demodulation in the multi-array systems at the receiver end at the known matrix of the channel is one of the traditional aspects in the sphere of wireless telecommunication. The problem is in recovering of the transmitted signal at the receiver end from the received vector of counts at the known channel matrix and at known statistical characteristics of the noise.

There exist a certain number of known demodulation techniques [1]. Each of them has its own advantages and disadvantages as compared to the other. Depending upon specific conditions and criteria at developing of the telecommunication system it is necessary to make a choice in favor of one of the algorithms.

These algorithms allow computing the estimate of the vector of transmitted symbols at the receiver end. Let us consider these techniques.

Three main techniques used for computing the estimate of transmitted symbols are the following:

1) the ZF (Zero Forcing) method (sometimes it is called as a decorrelator);

2) the algorithm, which is optimal upon the MMSE (Minimum Mean Square Error) criterion;

3) the ML (Maximum Likelihood) method.

The linear ZF method is the simplest of all the above methods. Using this method, the estimate is computed under the equation:

$$\hat{\mathbf{x}}_{ZF} = \left(\mathbf{H}^H \mathbf{H}\right)^{-1} \mathbf{H}^H \mathbf{y}, \qquad (2)$$

where ^{H} is operation of Hermitian transpose.

According to (2) in order to compute the estimate, it is considered the matrix of the channel only, but availability of the noise is disregarded that results in decreasing of the resistance to noise.

Using of the MMSE algorithm provides for better estimate computation results because availability of the noise is taken into consideration. The estimate obtained as the result of the algorithm operation is represented mathematically as follows:

$$\hat{\mathbf{x}}_{MMSE} = \left(\mathbf{H}^H \mathbf{H} + 2\sigma^2 \mathbf{1}\right)^{-1} \mathbf{H}^H \mathbf{y}, \qquad (3)$$

where $2\sigma^2$ is the aggregate dispersion of the real and imaginary components of the Gaussian noise vector, $\mathbf{1}$ is the identity matrix with the dimensionality $N \times N$.

The ML method possesses the best characteristics of resistance to noise [1]. The estimate obtained with the help of this method minimizes the square of the discrepancy norm:

$$\hat{\mathbf{x}}_{ML} = \arg \min_{\mathbf{x} \in \mathbf{X}^M} \left\| \mathbf{y} - \mathbf{H}\mathbf{x} \right\|^2, \qquad (4)$$

where $\mathbf{X}^M$ is the discrete set of values of the M-dimensional vector of $\mathbf{x}$ complex informational symbols. The set $\mathbf{X}$ is determined by the type of the modulation used in the system [1]. To find the minimum, according to (5), it is necessary to perform enumeration upon all the possible combinations of the vector of complex informational symbols $\mathbf{x}$.

The estimation obtained based on the ZF method, and the estimate $\hat{\mathbf{x}}_{MMSE}$ which is optimal in terms of the minimum mean square error criterion, are the simplest in terms of computations among the above-mentioned ones, and, as the consequence thereof, they require less time for obtaining of the estimate, however, having lower resistance to noise as compared with the ML method. Implementation of the ML method requires execution of a great number of operations. The number of operations depends upon two parameters of the system – the number of transmitting arrays and the type of modulation used. For example, while using the modulation QAM-16 (Quadrature Amplitude Modulation) in the telecommunication system or QAM-64 (at the demodulation) it is required to compute the discrepancy 16^N or 64^N times correspondingly. Even in the case with a small number of arrays at the transmitter end, for example 8, the discrepancy would have to be computed $16^8 = 4294967296$ times for QAM-16, and 281474976710656 times for QAM-64. It is worthy of note that the above computations have to be performed during the period of time, which is not exceeding the duration of one informational symbol. Therefore, the MMSE demodulation algorithm is the basic one due to low computational expenses as compared with the ML method; it also possesses a higher resistance to noise as compared with the ZF method.

IV. Massive MIMO

Development of wireless communication systems resulted in origination of large-size MIMO systems, or the so-called massive MIMO [5]. Their main difference from the existing multi-array systems is in availability of the antenna arrays, the number of arrays in which exceeds by several times the number of arrays in traditional systems [5]. Reference to the traditional ones may include those multi-array systems, the configuration of which is described in the most recent versions of the standards for the wireless data transmission systems that are IEEE 802.11ac, IEEE 802.11ad and 4G LTE-A (Long Term Evolution Advanced): 8 antennas at the receiver end and 8 antennas at the transmitter. One antenna array of a large-size MIMO system may contain one or several hundred antennas [5].

Massive MIMO systems have a number of advantages as compared with the traditional multi-array systems, of which the following can be designated as the basic advantages [5]:

- enhancement of the network capacity and reliability of the network operation;
- possibility of simultaneous servicing several dozens of mobile terminals that is important under the conditions of a limited frequency resource.

In this condition, an apparent drawback as compared with the traditional multi-array systems is in the fact that increasing of the number of arrays at the receiver and transmitter ends results in increasing of the channel matrix dimensionality, and, consequently, in increasing of the number of mathematical operations necessary for obtaining of the estimate at demodulation [6]. Therefore, with the increased number of arrays the computational complexity while obtaining the estimates $\hat{\mathbf{x}}_{ZF}$ and $\hat{\mathbf{x}}_{MMSE}$ is increasing, however, the advantage of a low complexity as compared with obtaining of the estimate $\hat{\mathbf{x}}_{ML}$ is preserved.

V. Computational complexity

The mathematical algorithms can be classified in accordance with complexity of their execution [7]. Considering that the algorithm is composed of a sequence of strictly determined operations (commands) [7], the algorithms can be classified depending upon the number of operations required for obtaining of the result. The parameter, which is called the *computational complexity*, is often used for classification of the algorithms. The term is used for determining of the number of elementary arithmetical operations, which must be performed for obtaining the solution to a certain problem.

Let us consider in detail the computational complexity of obtaining the estimate of the vector of informational symbols using the MMSE demodulation algorithm (3). We shall regard the telecommunication systems, in which the number of arrays at the receiver end and the number of arrays at the transmitter end are equal; that is $N = M$.

We address to the equation (3) in order to obtain the estimate of MMSE. We shall separately consider all the operations used for obtaining the estimate upon the MMSE algorithm:

1) *Multiplication of the matrix with the dimensionality $N \times N$ by the matrix with the dimensionality $N \times N$;*
2) *Multiplication of the matrix with the dimensionality $N \times N$ by the vector-column with the dimensionality N* (this operation has to be performed twice);
3) *Summation of two matrices with the dimensionality $N \times N$*;
4) *Inversion of the matrix with the dimensionality $N \times N$.*

The channel matrix $\mathbf{H}$ and the vector of received counts $\mathbf{y}$ contain complex numbers, therefore, the number of operations must be considered for the complex numbers.

In accordance with (3) we can say that matrix $\mathbf{H}^H\mathbf{H}$ is Hermitian matrix [8]. Let us introduce the following notations: $\mathbf{H} = \begin{bmatrix} \mathbf{h}_{00} & \mathbf{h}_{01} \\ \mathbf{h}_{10} & \mathbf{h}_{01} \end{bmatrix}$, $\mathbf{H}^H = \begin{bmatrix} \mathbf{h}_{00}^H & \mathbf{h}_{10}^H \\ \mathbf{h}_{01}^H & \mathbf{h}_{11}^H \end{bmatrix}$. Consequently, the matrix product $\mathbf{H}^H\mathbf{H}$ is equal to:

$$\mathbf{H}^H\mathbf{H} = \begin{bmatrix} \mathbf{h}_{00}^H\mathbf{h}_{00} + \mathbf{h}_{10}^H\mathbf{h}_{10} & \mathbf{h}_{00}^H\mathbf{h}_{01} + \mathbf{h}_{10}^H\mathbf{h}_{11} \\ \mathbf{h}_{01}^H\mathbf{h}_{00} + \mathbf{h}_{11}^H\mathbf{h}_{10} & \mathbf{h}_{01}^H\mathbf{h}_{01} + \mathbf{h}_{11}^H\mathbf{h}_{11} \end{bmatrix}. \quad (5)$$

Because matrix $\mathbf{H}^H$ is Hermitian transpose of matrix $\mathbf{H}$, it is true $\left(\mathbf{h}_{01}^H\mathbf{h}_{00} + \mathbf{h}_{11}^H\mathbf{h}_{10}\right) = \left(\mathbf{h}_{00}^H\mathbf{h}_{01} + \mathbf{h}_{10}^H\mathbf{h}_{11}\right)^H$.

As a result, it is no need to compute full matrix $\mathbf{H}^H\mathbf{H}$ – we only need to compute elements of main diagonal and the elements below the main diagonal. The elements above the main diagonal is equal to the complex conjugate of the elements below the main diagonal [8].

The computational complexity of multiplication of the matrix with the dimensionality $N \times N$ by the self-adjoint matrix with the dimensionality $N \times N$ $\mathbf{H}^H\mathbf{H}$ can be estimate as $\left(2N^2 + 2N^2\left(N-1\right)\right)_{MULT}$ operations of multiplications and $\left(2N\left(2N-1\right) + N\left(N-1\right)\left(2N-1\right)\right)_{ADD}$ operations of summation [9]:

$$\begin{aligned} &Z_{TRMATCONJ}\left(N\right) = \left(2N^2 + 2N^2\left(N-1\right)\right)_{MULT} + \ldots \\ &\left(2N\left(2N-1\right) + N\left(N-1\right)\left(2N-1\right)\right)_{ADD} = \\ &= \left(2N^3\right)_{MULT} + \left(2N^3 - N^2 - N\right)_{ADD} \end{aligned} . \quad (6)$$

Inversion of the Hermitian matrix $\mathbf{H}^H\mathbf{H} + 2\sigma^2\mathbf{1}$ is suggested calculating with help of Frobenius method [10]. Let us introduce the following notation: $\mathbf{T} = \left(\mathbf{H}^H\mathbf{H} + 2\sigma^2\mathbf{1}\right)$. Matrix $\mathbf{T}$ with the dimensionality $N \times N$ can be divided into 4 matrices with the equal dimensionality $\frac{N}{2} \times \frac{N}{2}$ as following:

$$\mathbf{T} = \begin{bmatrix} \mathbf{t}_{00} & \mathbf{t}_{01} \\ \mathbf{t}_{10} & \mathbf{t}_{11} \end{bmatrix}. \quad (7)$$

As $\mathbf{T}$ is the Hermitian matrix it is possible to say that $\mathbf{t}_{00} = \mathbf{t}_{00}^H$, $\mathbf{t}_{01} = \mathbf{t}_{10}^H$, $\mathbf{t}_{01}^H = \mathbf{t}_{10}$ and $\mathbf{t}_{11} = \mathbf{t}_{11}^H$. Matrix $\mathbf{T}$ will be equal to:

$$\mathbf{T} = \begin{bmatrix} \mathbf{t}_{00} & \mathbf{t}_{01} \\ \mathbf{t}_{01}^H & \mathbf{t}_{11} \end{bmatrix}. \quad (8)$$

According to Frobenius method, inversion of matrix $\mathbf{T}$ is:

$$\mathbf{T}^{-1} = \begin{bmatrix} \mathbf{t}_{00}^{-1} + \mathbf{t}_{00}^{-1}\mathbf{t}_{01}\mathbf{G}^{-1}\mathbf{t}_{10}\mathbf{t}_{00}^{-1} & -\mathbf{t}_{00}^{-1}\mathbf{t}_{01}\mathbf{G}^{-1} \\ \mathbf{G}^{-1}\mathbf{t}_{10}\mathbf{t}_{00}^{-1} & \mathbf{G}^{-1} \end{bmatrix}, \quad (9)$$

where $\mathbf{G} = \mathbf{t}_{11} - \mathbf{t}_{10}\mathbf{t}_{00}^{-1}\mathbf{t}_{01}$ [10]. Therefore we can say that $\mathbf{t}_{00}^{-1}\mathbf{t}_{01} = \mathbf{t}_{01}^H\mathbf{t}_{00}^{-1}$. We use the following designation $\mathbf{W} = \mathbf{t}_{01}^H\mathbf{t}_{00}^{-1}$. Then we ca rewrite equation (9) as following:

$$\mathbf{T}^{-1} = \begin{bmatrix} \mathbf{t}_{00}{}^{-1} + \mathbf{W}^H\mathbf{G}^{-1}\mathbf{W} & -\left(\mathbf{G}^{-1}\mathbf{W}\right)^H \\ \mathbf{G}^{-1}\mathbf{W} & \mathbf{G}^{-1} \end{bmatrix}, \quad (10)$$

where $\mathbf{G} = \mathbf{t}_{11} - \mathbf{t}_{10}\mathbf{W}^H$.

Therefore, for calculating inversion of the Hermitian matrix $\mathbf{T}$ we need to perform following operations:

- 2 inversion of matrix with the dimensionality $\frac{N}{2} \times \frac{N}{2}$: $\mathbf{t}_{00}^{-1}$, $\mathbf{G}^{-1}$;
- 4 matrix products with the dimensionality $\frac{N}{2} \times \frac{N}{2}$: $\mathbf{t}_{01}^H\mathbf{t}_{11}^{-1}$, $\mathbf{G}^{-1}\mathbf{W}$, $\mathbf{W}^H\left(\mathbf{G}^{-1}\mathbf{W}\right)$, $\mathbf{t}_{10}\mathbf{W}^H$;
- 2 summation of matrix with the dimensionality $\frac{N}{2} \times \frac{N}{2}$: $\mathbf{t}_{00}^{-1} + \mathbf{W}^H\mathbf{G}^{-1}\mathbf{W}$, $\mathbf{t}_{11} - \mathbf{t}_{10}\mathbf{W}^H$.

For operations of matrices inversion in (10) Frobenius method needs to be applied recursively. Complexity of inversion of the Hermitian matrix can be estimate as $\left(2N^3 - 2N^2\right)_{MULT}$ operations of multiplications, $\left(2N^3 - 4N^2 + 2N\right)_{ADD}$ operations of summation and $\left(N\right)_{DIV}$ operations of divisions [9].

We suppose, that operations of multiplications and operations of divisions have the same complexity. Therefore, total complexity of inversion of the Hermitian matrix is equal to [9]:

$$Z_{INVCONJ}\left(N\right)=\left(2N^3-2N^2+N\right)_{MULT}+ \\ +\left(2N^3-4N^2+2N\right)_{ADD} . \tag{11}$$

Let us consider the number of elementary arithmetical operations required for calculating matrix-vector product $\mathbf{T}^{-1}\mathbf{v}$, where $\mathbf{v}=\mathbf{H}^H\mathbf{y}$. This operation can be represented as following:

$$\begin{bmatrix}\mathbf{v}_1\\ \mathbf{v}_2\end{bmatrix}=\begin{bmatrix}\mathbf{t}_{00} & \mathbf{t}_{01}\\ \mathbf{t}_{10} & \mathbf{t}_{11}\end{bmatrix}\begin{bmatrix}\mathbf{q}_1\\ \mathbf{q}_2\end{bmatrix}, \tag{12}$$

wherein

$$\begin{aligned}\mathbf{v}_1&=\mathbf{t}_{00}\mathbf{q}_1+\mathbf{t}_{01}\mathbf{q}_2\\ \mathbf{v}_2&=\mathbf{t}_{10}\mathbf{q}_1+\mathbf{t}_{11}\mathbf{q}_2\end{aligned} . \tag{13}$$

Because $\mathbf{T}$ is Hermitian matrix, the overall complexity of this operation is the following [9]:

$$\begin{aligned}&Z_{TRMATVECCONJ}\left(N\right)=\left(2N+4N\left(N-1\right)\right)_{MULT}+\\ &+\left(2\left(2N\left(N-1\right)\right)\right)_{ADD}=\\ &=\left(4N^2-2N\right)_{MULT}+\left(4N^2-4N\right)_{ADD}\end{aligned} . \tag{14}$$

Complexity operation of multiplication of the matrix with the dimensionality $N\times N$ by the vector-column with the dimensionality N can be estimate as:

$$Z_{TRMATVEC}\left(N\right)=8N^2-2N \tag{15}$$

The last operation of MMSE algorithm is summation of two matrices with the dimensionality $N\times N$. Because $\mathbf{1}$ is the identity matrix, complexity of this operation include multiplications of elements of main diagonal of matrix $\mathbf{1}$ by number $2\sigma^2$ [9]:

$$Z_{SUMMAT}\left(N\right)=N . \tag{16}$$

Therefore, total number of operations necessary for obtaining of the MMSE estimate (3) we can calculate using equations (9), (11), (14), (15), (16):

$$\begin{aligned}&Z_{MMSE}\left(N\right)=Z_{TRMATCONJ}\left(N\right)+Z_{INVCONJ}\left(N\right)+\\ &+Z_{TRMATVEC}\left(N\right)+Z_{TRMATVECCONJ}\left(N\right)+\\ &+Z_{SUMMAT}\left(N\right)=\\ &=2N^3+2N^3-N^2-N+\\ &+2N^3-2N^2+N+2N^3-4N^2+\\ &+2N+4N^2-2N+4N^2-4N+8N^2-\\ &-2N+N=8N^3+9N^2-3N\end{aligned} . \tag{17}$$

Number of arithmetical operation required for obtaining estimate upon the MMSE algorithm (3) is provided in Table I.

TABLE I. COMPUTATIONAL COMPLEXITY OF THE MMSE DEMODULATOR FOR DIFFERENT MIMO CONFIGURATIONS

MIMO configuration, $N\times N$	**Number of required operations for obtaining of the MMSE estimate,** $Z_{MMSE}\left(N\right)$
2×2	92
4×4	640
8×8	4 640
16×16	35 008
32×32	271 232
64×64	2 133 760
128×128	16 924 160

VI. NEW REALIZATION OF MMSE DEMODULATOR WITH REDUCED COMPUTATIONAL COMPLEXITY

We consider application of the Strassen algorithm and the 3M method to the algorithm of obtaining of the MMSE estimate (3). To decrease the computational complexity in obtaining the MMSE estimate at the demodulator end for the massive MIMO systems without altering the characteristics of the algorithm, which is optimal in terms of the minimum mean square error criterion, it is suggested to jointly apply two methods, the Strassen algorithm and the 3M method.

Both of the above methods are suggested for application to the operation of obtaining the product of square matrices consisting of complex transmission coefficients of the telecommunication channel, dimensionality of which is equal to the number of arrays at the receiver and the transmitter ends. In addition to the said operation, it is suggested to decrease using the 3M method the computational complexity of the operation of multiplication of the matrix by the vector. Also and matrix inversion are suggested to use for decrease computational complexity of inversion of the Hermitian matrix.

To perform computation of matrix product it is suggested a simultaneous use of the Strassen algorithm and the 3M method, as in Algorithm 1.

Algorithm 1 New algorithm for obtaining matrix product

Input: matrix $\mathbf{H}$

Output: matrix product $\mathbf{H}^H\mathbf{H}$

1. Compute $\mathbf{H}^H$
2. Assign $\mathbf{H}=\begin{bmatrix}\mathbf{h}_{00} & \mathbf{h}_{01}\\ \mathbf{h}_{10} & \mathbf{h}_{01}\end{bmatrix}$, $\mathbf{H}^H=\begin{bmatrix}\mathbf{h}_{00}^H & \mathbf{h}_{10}^H\\ \mathbf{h}_{01}^H & \mathbf{h}_{11}^H\end{bmatrix}$
3. For $j=0,1,\ldots\left(\dfrac{N}{2}-1\right)$ compute $\mathbf{h}_{00}^H\mathbf{h}_{00}$, $\mathbf{h}_{10}^H\mathbf{h}_{10}$, $\mathbf{h}_{01}^H\mathbf{h}_{01}$, $\mathbf{h}_{11}^H\mathbf{h}_{11}$ using 3M method as following:

$$\mathbf{F}_1=\mathbf{h}_{00}^H\mathbf{h}_{00}=\mathbf{h}_{00}^H\left[\mathbf{h}_{00}^{[j+1]}\right]=\left[\mathbf{f}_n^{[j+1]}\right]_1$$

$$\mathbf{F}_2=\mathbf{h}_{10}^H\mathbf{h}_{10}=\mathbf{h}_{10}^H\left[\mathbf{h}_{10}^{[j+1]}\right]=\left[\mathbf{f}_n^{[j+1]}\right]_2$$

$$\mathbf{F}_3 = \mathbf{h}_{01}^H \mathbf{h}_{01} = \mathbf{h}_{01}^H \left[\mathbf{h}_{01}^{[j+1]} \right] = \left[\mathbf{f}_n^{[j+1]} \right]_3$$

$$\mathbf{F}_4 = \mathbf{h}_{11}^H \mathbf{h}_{11} = \mathbf{h}_{11}^H \left[\mathbf{h}_{11}^{[j+1]} \right] = \left[\mathbf{f}_n^{[j+1]} \right]_4$$

where $n = \left[\frac{N}{2} - \left(\frac{N}{2} - j \right); \frac{N}{2} \right]$

4. Assign $\mathbf{h}_{00}^H$, $\mathbf{h}_{01}$, $\mathbf{h}_{01}^H$, $\mathbf{h}_{11}$ as following

$$\mathbf{h}_{00}^H = \left(\mathbf{h}_{00}^H\right)_R + i\left(\mathbf{h}_{00}^H\right)_I$$

$$\mathbf{h}_{01} = \left(\mathbf{h}_{01}\right)_R + i\left(\mathbf{h}_{01}\right)_I$$

$$\mathbf{h}_{01}^H = \left(\mathbf{h}_{01}^H\right)_R + i\left(\mathbf{h}_{01}^H\right)_I$$

$$\mathbf{h}_{11} = \left(\mathbf{h}_{11}\right)_R + i\left(\mathbf{h}_{11}\right)_I$$

where $(\ )_R$ – values of real part complex of matrix, $(\ )_I$ – values of imaginary part complex of matrix

5. Compute $\mathbf{h}_{00}^H \mathbf{h}_{01}$ and $\mathbf{h}_{10}^H \mathbf{h}_{11}$ using 3M method as following:

$$\mathbf{h}_{00}^H \mathbf{h}_{01} = \left(\left(\mathbf{h}_{00}^H\right)_R \left(\mathbf{h}_{01}\right)_R - \left(\mathbf{h}_{00}^H\right)_I \left(\mathbf{h}_{01}\right)_I \right) + \\ +i \begin{bmatrix} \left(\left(\mathbf{h}_{00}^H\right)_R + \left(\mathbf{h}_{00}^H\right)_I \right) \left(\left(\mathbf{h}_{01}\right)_R + \left(\mathbf{h}_{01}\right)_I \right) - \\ \left(\mathbf{h}_{00}^H\right)_R \left(\mathbf{h}_{01}\right)_R - \left(\mathbf{h}_{00}^H\right)_I \left(\mathbf{h}_{01}\right)_I \end{bmatrix}$$

$$\mathbf{h}_{10}^H \mathbf{h}_{11} = \left(\left(\mathbf{h}_{10}^H\right)_R \left(\mathbf{h}_{11}\right)_R - \left(\mathbf{h}_{10}^H\right)_I \left(\mathbf{h}_{11}\right)_I \right) + \\ +i \begin{bmatrix} \left(\left(\mathbf{h}_{10}^H\right)_R + \left(\mathbf{h}_{10}^H\right)_I \right) \left(\left(\mathbf{h}_{11}\right)_R + \left(\mathbf{h}_{11}\right)_I \right) - \\ \left(\mathbf{h}_{10}^H\right)_R \left(\mathbf{h}_{11}\right)_R - \left(\mathbf{h}_{10}^H\right)_I \left(\mathbf{h}_{11}\right)_I \end{bmatrix}$$

where $\left(\mathbf{h}_{00}^H\right)_R \left(\mathbf{h}_{01}\right)_R$, $\left(\mathbf{h}_{00}^H\right)_I \left(\mathbf{h}_{01}\right)_I$, $\left(\left(\mathbf{h}_{00}^H\right)_R + \left(\mathbf{h}_{00}^H\right)_I \right) \left(\left(\mathbf{h}_{01}\right)_R + \left(\mathbf{h}_{01}\right)_I \right)$, $\left(\mathbf{h}_{10}^H\right)_R \left(\mathbf{h}_{11}\right)_R$, $\left(\mathbf{h}_{10}^H\right)_R \left(\mathbf{h}_{11}\right)_R$, $\left(\left(\mathbf{h}_{10}^H\right)_R + \left(\mathbf{h}_{10}^H\right)_I \right) \left(\left(\mathbf{h}_{11}\right)_R + \left(\mathbf{h}_{11}\right)_I \right)$ for $\frac{N}{2} \geq 32$ compute using the Strassen algorithm; in case if $\frac{N}{2} < 32$ compute traditionally

6. Assign $\mathbf{H}^H \mathbf{H} = \begin{bmatrix} \mathbf{h}_{00}^H \mathbf{h}_{00} + \mathbf{h}_{10}^H \mathbf{h}_{10} & \mathbf{h}_{00}^H \mathbf{h}_{01} + \mathbf{h}_{10}^H \mathbf{h}_{11} \\ \left(\mathbf{h}_{00}^H \mathbf{h}_{01} + \mathbf{h}_{10}^H \mathbf{h}_{11} \right)^H & \mathbf{h}_{01}^H \mathbf{h}_{01} + \mathbf{h}_{11}^H \mathbf{h}_{11} \end{bmatrix}$

7. End

To decrease complexity of matrix inversion it is suggested to use Algorithm 2.

Algorithm 2 New algorithm for obtaining matrix inversion

Input: matrix $\mathbf{T}$

Output: matrix product $\mathbf{T}^{-1}$

1. Assign $\mathbf{T} = \begin{bmatrix} \mathbf{t}_{00} & \mathbf{t}_{01} \\ \mathbf{t}_{01}^H & \mathbf{t}_{11} \end{bmatrix}$
2. Compute $\mathbf{t}_{01}^H$ and using Frobenius method $\mathbf{t}_{00}^{-1}$ recursively
3. Assign $\mathbf{W} = \mathbf{t}_{01}^H \mathbf{t}_{00}^{-1}$, $\mathbf{G} = \left(\mathbf{t}_{11} - \mathbf{t}_{10} \mathbf{W}^H \right)$
4. Compute $\mathbf{W} = \mathbf{t}_{01}^H \mathbf{t}_{00}^{-1}$ using 3M method:

$$\mathbf{W} = \mathbf{t}_{01}^H \mathbf{t}_{11}^{-1} = \left(\left(\mathbf{t}_{01}^H\right)_R \left(\mathbf{t}_{11}^{-1}\right)_R - \left(\mathbf{t}_{01}^H\right)_I \left(\mathbf{t}_{11}^{-1}\right)_I \right) + \\ +i \begin{bmatrix} \left(\left(\mathbf{t}_{01}^H\right)_R + \left(\mathbf{t}_{01}^H\right)_I \right) \left(\left(\mathbf{t}_{11}^{-1}\right)_R + \left(\mathbf{t}_{11}^{-1}\right)_I \right) - \\ \left(\mathbf{t}_{01}^H\right)_R \left(\mathbf{t}_{11}^{-1}\right)_R - \left(\mathbf{t}_{01}^H\right)_I \left(\mathbf{t}_{11}^{-1}\right)_I \end{bmatrix}$$

where $\left(\mathbf{t}_{01}^H\right)_R \left(\mathbf{t}_{11}^{-1}\right)_R$, $\left(\mathbf{t}_{01}^H\right)_I \left(\mathbf{t}_{11}^{-1}\right)_I$, $\left(\left(\mathbf{t}_{01}^H\right)_R + \left(\mathbf{t}_{01}^H\right)_I \right) \left(\left(\mathbf{t}_{11}^{-1}\right)_R + \left(\mathbf{t}_{11}^{-1}\right)_I \right)$ for $\frac{N}{2} \geq 32$ compute using the Strassen algorithm; in case if $\frac{N}{2} < 32$ compute traditionally

5. Compute $\mathbf{W}^H$
6. Assign $\mathbf{t}_{10} = \left(\mathbf{t}_{10}\right)_R + i\left(\mathbf{t}_{10}\right)_I$, $\mathbf{W} = \mathbf{W}_R + i\mathbf{W}_I$, $\mathbf{W}^H = \left(\mathbf{W}^H\right)_R + i\left(\mathbf{W}^H\right)_I$
7. Compute $\mathbf{t}_{10} \mathbf{W}^H$ using 3M method as following:

$$\mathbf{t}_{10} \mathbf{W}^H = \left(\left(\mathbf{t}_{10}\right)_R \left(\mathbf{W}^H\right)_R - \left(\mathbf{t}_{10}\right)_I \left(\mathbf{W}^H\right)_I \right) + \\ +i \begin{bmatrix} \left(\left(\mathbf{t}_{10}\right)_R + \left(\mathbf{t}_{10}\right)_I \right) \left(\left(\mathbf{W}^H\right)_R + \left(\mathbf{W}^H\right)_I \right) - \\ \left(\mathbf{t}_{10}\right)_R \left(\mathbf{W}^H\right)_R - \left(\mathbf{t}_{10}\right)_I \left(\mathbf{W}^H\right)_I \end{bmatrix}$$

where $\left(\mathbf{t}_{10}\right)_R \left(\mathbf{W}^H\right)_R$, $\left(\mathbf{t}_{10}\right)_I \left(\mathbf{W}^H\right)_I$, $\left(\left(\mathbf{t}_{10}\right)_R + \left(\mathbf{t}_{10}\right)_I \right) \left(\left(\mathbf{W}^H\right)_R + \left(\mathbf{W}^H\right)_I \right)$ $\left(\left(\mathbf{t}_{01}^H\right)_R + \left(\mathbf{t}_{01}^H\right)_I \right) \left(\left(\mathbf{t}_{11}^{-1}\right)_R + \left(\mathbf{t}_{11}^{-1}\right)_I \right)$ for $\frac{N}{2} \geq 32$ compute using the Strassen algorithm; in case if $\frac{N}{2} < 32$ compute traditionally

8. Compute $\mathbf{G} = \left(\mathbf{t}_{11} - \mathbf{t}_{10} \mathbf{W}^H \right)$
9. Compute $\mathbf{G}^{-1}$ using Frobenius method recursively
10. Assign $\mathbf{G}^{-1} = \left(\mathbf{G}^{-1}\right)_R + i\left(\mathbf{G}^{-1}\right)_I$
11. Compute $\mathbf{G}^{-1}\mathbf{W}$, $\mathbf{W}^H \left(\mathbf{G}^{-1}\mathbf{W} \right)$ using 3M method as following:

$$\mathbf{U}=\mathbf{U}_R+i\mathbf{U}_I=\mathbf{G}^{-1}\mathbf{W}=\left(\left(\mathbf{G}^{-1}\right)_R\mathbf{W}_R-\left(\mathbf{G}^{-1}\right)_I\mathbf{W}_I\right)+$$
$$+i\begin{bmatrix}\left(\left(\mathbf{G}^{-1}\right)_R+\left(\mathbf{G}^{-1}\right)_I\right)\left(\mathbf{W}_R+\mathbf{W}_I\right)-\\ \left(\mathbf{G}^{-1}\right)_R\mathbf{W}_R-\left(\mathbf{G}^{-1}\right)_I\mathbf{W}_I\end{bmatrix}$$
$$\mathbf{W}^H\left(\mathbf{G}^{-1}\mathbf{W}\right)=\mathbf{W}^H\mathbf{U}=\left(\left(\mathbf{W}^H\right)_R\mathbf{U}_R-\left(\mathbf{W}^H\right)_I\mathbf{U}_I\right)+$$
$$+i\begin{bmatrix}\left(\left(\mathbf{W}^H\right)_R+\left(\mathbf{W}^H\right)_I\right)\left(\mathbf{U}_R+\mathbf{U}_I\right)-\\ \left(\mathbf{W}^H\right)_R\mathbf{U}_R-\left(\mathbf{W}^H\right)_I\mathbf{U}_I\end{bmatrix}$$

where $\left(\mathbf{G}^{-1}\right)_R\mathbf{W}_R$, $\left(\mathbf{G}^{-1}\right)_I\mathbf{W}_I$, $\left(\left(\mathbf{G}^{-1}\right)_R+\left(\mathbf{G}^{-1}\right)_I\right)\left(\mathbf{W}_R+\mathbf{W}_I\right)$, $\left(\mathbf{W}^H\right)_R\mathbf{U}_R$, $\left(\mathbf{W}^H\right)_I\mathbf{U}_I$, $\left(\left(\mathbf{W}^H\right)_R+\left(\mathbf{W}^H\right)_I\right)\left(\mathbf{U}_R+\mathbf{U}_I\right)$ for $\frac{N}{2}\geq 32$ compute using the Strassen algorithm; in case if $\frac{N}{2}<32$ compute traditionally

12. Compute $\mathbf{t}_{00}^{-1}+\mathbf{W}^H\mathbf{G}^{-1}\mathbf{W}$
13. Assign $\mathbf{T}^{-1}=\begin{bmatrix}\mathbf{t}_{00}^{-1}+\mathbf{W}^H\mathbf{G}^{-1}\mathbf{W} & -\left(\mathbf{G}^{-1}\mathbf{W}\right)^H\\ \mathbf{G}^{-1}\mathbf{W} & \mathbf{G}^{-1}\end{bmatrix}$
14. End

Last operation in equation (3) complexity of which is suggested to reduce is obtaining of matrix-vector product. New algorithm for this operation is listed below as Algorithm 3.

Algorithm 3 New algorithm for obtaining matrix-vector product

Input: matrix $\mathbf{H}^H$, vector $\mathbf{y}$

Output: matrix-vector product $\mathbf{H}^H\mathbf{y}$

1. Assign $\mathbf{H}^H=\left(\mathbf{H}^H\right)_R+i\left(\mathbf{H}^H\right)_I$
2. Assign $\mathbf{y}=\left(\mathbf{y}\right)_R+i\left(\mathbf{y}\right)_I$
3. Compute $\mathbf{H}^H\mathbf{y}$ using 3M method as following:
$$\mathbf{H}^H\mathbf{y}=\left[\left(\mathbf{H}^H\right)_R\left(\mathbf{y}\right)_R-\left(\mathbf{H}^H\right)_I\left(\mathbf{y}\right)_I\right]+$$
$$+i\begin{bmatrix}\left(\left(\mathbf{H}^H\right)_R+\left(\mathbf{H}^H\right)_I\right)\left(\left(\mathbf{y}\right)_R+\left(\mathbf{y}\right)_I\right)-\\ \left(\mathbf{H}^H\right)_R\left(\mathbf{y}\right)_R-\left(\mathbf{H}^H\right)_I\left(\mathbf{y}\right)_I\end{bmatrix}$$
4. End

New realization of MMSE demodulator consist in using Algorithm 1, Algorithm 2 and Algorithm 3 to equation (3) for obtaining the estimate $\hat{\mathbf{x}}_{MMSE}$.

Therefore, the number of arithmetical operations being required for obtaining the estimate $\hat{\mathbf{x}}_{MMSE}$, optimal in terms of the mean square error criterion, for different antennas configuration is equal to the number provided in Table II.

TABLE II. COMPUTATIONAL COMPLEXITY OF DEMODULATION IN MASSIVE MIMO SYSTEM

MIMO configuration	Number of required operations (multiplication and summation)		Ratio between the number of operations while using the MMSE algorithm and new realization of MMSE algorithm
	Traditional MMSE	New realization	
2×2	92	96	1,04
4×4	640	706	1,10
8×8	4 640	5 068	1,09
16×16	35 008	26 808	0,77
32×32	271 232	192 912	0,64
64×64	2 133 760	1 204 320	0,56
128×128	16 924 160	8 659 264	0,51

VII. CONCLUSION

Using of different methods of fast multiplication of the matrices allows decreasing the number of operations for execution of the algorithm without losing in the resistance to noise. The paper considers two methods for decreasing the computational complexity of the algorithms along with joint application of the above methods.

As it was mentioned earlier for massive MIMO systems it is typical to have a large dimensionality of the matrix of complex transmission coefficients of the telecommunication channel, in particular, $N>32$. In addition, the Strassen algorithm possesses the highest efficiency at recursive application to the matrices of large dimensionality ($N>40$). And considering that the matrices contain complex elements, it is reasonable to apply the 3M method. Thus, it is substantiated the efficiency of application of those methods in order to simplify obtaining of the MMSE estimate.

It is shown in the process of investigation that joint application of the Strassen algorithm and the 3M method is significantly decreasing the number of elementary operations required for obtaining of the estimate, which is optimal in terms of the minimum mean square error criterion. Obtaining of that estimate is necessary for the MMSE demodulation algorithm providing for better results in computation of the estimate as compared with the ZF algorithm, it also possesses a lower computational complexity as compared to the ML algorithm.

Therefore, application of the Strassen algorithm and the 3M method to the procedure of obtaining of the MMSE estimate allows decreasing a total number of the required elementary operations without losing in the resistance to noise and without altering the characteristics of the MMSE algorithm by 2 times for the matrices of complex transmission coefficients of the telecommunication channel with the large dimensionality (128×128), which can be found in the promise massive MIMO systems.

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