K-MESON NUCLEON SCATTERING

by

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PREFACE

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The thesis is based on four papers by the author. Except where stated in the text, the work described is original and has not been submitted in this or any other University for a degree.

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ABSTRACT

In Chapter I, the various attempts by different authors, to determine the parity of K-mesons and its coupling constants, are surveyed. The details of the investigation carried out by the author and his conclusions are given. Two types of dispersion relations were used: (1) Matthew-Salam type which is not sensitive to low energy data but weighs the high energy region; (2) Igi type which is sensitive to low energy K^{+} p data but considerably convergent in the high energy region. With Igi type dispersion relation, two possible cases were investigated: (i) where the $K^{+}p$ total cross-section was constant in the low energy region; (ii) where it varied in a linear way within the experimental error.

In Chapter II, a dispersion technique suggested by Feldman, Matthews and Salam, is used to derive the static equations for pion-hyperon scattering and it is shown that the inclusion of the $\bar{K}N$ channel does not change the conclusions on the $J = 3/2$, $I = 1$ p-wave, π-Y resonance, given by Amati, Stanghellini and Vitale and by Capps.

In Chapter III, assuming that the isotropy of KN scattering in $I = 1$ is due to the cancellation of p-wave contributions coming from the hyperon cuts and the two-pion cut, the KN scattering in $I = 1$ state and in $I = 0$, S, $P_{1/2}$, $P_{3/2}$ states is investigated. Qualitative agreement with the present experimental situation is obtained. Using crossing symmetry, the two-pion contribution in $\bar{K}N$ scattering is also considered.

In Chapter IV, the recently discovered Y* ($m_{Y^*} \sim 1385$ MeV, $\frac{\Gamma}{2} \sim 15$ MeV) is assumed to be a $\bar{K}N$ I = 1 bound system, and an effective range theory for $\bar{K}N$ scattering in I = 1 state is formulated. With a large effective range for I = 1 state and a zero effective range for I = 0 state, parameters which fit the present low energy K^-p scattering data and give constructive Coulomb nuclear interference are found. These parameters indicate that the real part of the scattering length in I = 0 state (i.e. a_0) is small. This smallness of a_0 can be explained by postulating the existence of a $\bar{K}N$ I = 0 bound state lying below the $\Sigma\pi$ threshold. Also a $\Sigma\pi$ resonance (I = 0) just below the $\bar{K}N$ threshold ($\delta_{\Sigma\pi} = 90^\circ$) is expected.

I. THE PARITY OF K-MESONS AND DISPERSION RELATIONS

I.1 Introduction

It was pointed out by a number of authors[1,2,3] that the forward scattering dispersion relations for the K-meson nucleon scattering may offer a powerful means to determine the parity of K-meson relative to the hyperons and the nucleon, and also the strength of the K-meson interactions. The first numerical attempt in this direction, using the experimental data, was made by Matthews and Salam[1]. They found that, for K^+p potential repulsive, an attractive K^-p potential implies pseudescalar K-mesons (Λ and Σ parities assumed positive) and a repulsive K^-p potential implies scalar K-mesons. In both cases, the coupling constants obtained by them were of the order of unity. The main sources of error in their evaluation came from (i) lack of experimental information on total cross-sections of K^+p and K^-p scattering; (ii) the contribution from the unphysical continuum, and (iii) the behaviour of σ^-ab near threshold. Conclusion similar to that of Matthews-Salam was reached by Igi[4], who used a subtracted form of dispersion relation. He found that if K^-p interaction is repulsive, then K-coupling is scalar and is of the order of unity. If, on the other hand, K^-p interaction is attractive, the coupling could be either scalar or pseudoscalar, depending on the energy dependence of the K^+p scattering cross-section at low energies. In this case, for scalar K-meson, the coupling constant is ~ 1 while for p.S K-meson the coupling constant is ~ 4.

The dispersion relation used by Matthews and Salam is not very sensitive to low energy data, but slowly convergent in the high energy region, while that used by Igi is very sensitive to low energy K^+p data, but much more convergent in the high energy region. This consideration, together with the fact that by early 1959, experimental results both in the low energy region[5,6] and in the high energy region[7] have increased considerably, prompted the author[8] to re-examine the question of the K-meson parity determination using the new data and the dispersion relation of Matthews-Salam and of Igi. The details of his calculation and results will be presented in the following sections.

Several other authors have also attempted to determine the parity of K-mesons and the K-meson coupling constants from forward scattering dispersion relations using different forms. Galzenati and Vitale[9] used two dispersion relations in subtracted form, considering them as independent relations, for the study of the dependence of the real part of K^-p scattering amplitude at zero energy on the values of the coupling constants. Their results, when compared with the existing experimental information, strongly indicate that the K-meson is pseudoscalar with respect to both the Λ and the Σ hyperon, and that the sign of $D_-(\omega)$ at low energy is positive, giving therefore constructive Coulomb interference in the K^-p elastic scattering. The magnitude of the coupling constant obtained by them is of the order of 5. Amati[10] has derived an effective range formula from the forward scattering dispersion relation for K^+p scattering, which does not contain the experimentally uncertain zero energy K^- proton scattering amplitude. He finds that the low energy dependence of σ^+ indicates equal Λ and Σ parities with opposite K parity (K.p.s.). Selleri[11] has used the effective range formulation of Amati without the restrictive assumption of constant σ^+ and adopting for the scattering amplitude in the unphysical region the solutions given by Dalitz and Tuan[12]. He finds that the experimental evidence of weak energy dependence of σ^+ indicates a p.s.K-meson (p.s. at least with respect to one of the hyperons). Karplus, Kerth and Kycia[13] have also used the Dalitz-Tuan K^-p scattering parameters and the then existing data to study the K-meson hyperon coupling terms occurring in the dispersion reactions. They concluded that the experimental data was not sufficiently accurate for any definite conclusion on K-meson parity. Sugano and Komatsuzawa[14], using Igi's form of dispersion relation for the charge exchange scattering $K^+n \rightarrow K^0p$ and for the ordinary elastic scattering $K^+p \rightarrow K^+p$, inferred that $P_{K\Lambda}$ is odd and $g_{\Lambda K}^2 \approx 5$ while $P_{K\Sigma}$ is undetermined and $g_{\Sigma K}^2 \approx 0$.

The major difficulties which arise in the application of K-meson dispersion relations are (i) contribution from the high energy region,

(ii) sensitivity on the low energy dependence of K^+p cross-section or on the experimentally uncertain sign of K^-p scattering length, and (iii) the contribution from the unphysical region. Using perturbation theory Tuan[15] has estimated the contribution of the unphysical region. On the basis of their scattering length solution for K^-p scattering, Dalitz and Tuan[12] have pointed out that the contribution from the unphysical region in the dispersion relation used by Gobel and by Matthews and Salam may be considerable, so that the conclusion[(1)] that the parity of K-meson can be deduced from the sign of K^-p potential is no longer clearly established. Dalitz and Tuan also indicated that the possibility of a resonance in the unphysical region may mean a very large contribution from this region than so far considered. The discovery of a $\Lambda\pi$[16] resonance at an energy 50 MeV below K^-N threshold seems to indicate such a Dalitz-Tuan resonance[17]. Nogami[18] very recently has used the dispersion theoretic analysis, assuming that the situation implied by Dalitz-Tuan (a-) solution to be correct. He finds that no definite conclusion can be reached, though it is very likely that $P_{K\Lambda}$, the $K\Lambda$ parity relative to N, is odd and $P_{K\Sigma}$ is even and hence $P_{\Sigma\Lambda}$ is odd.

From this survey, the only conclusion we can draw is that our present status of knowledge does not provide us with an unambiguous answer to the question of K-meson parity on the basis of dispersion relations.

I.2 Dispersion Relation For K-meson-Nucleon Scattering

Investigation of the analytic structure of K-meson nucleon dispersion theory, from the viewpoint of rigorous proof[19,20] has shown that even for the case of forward scattering, a completely adequate proof is not possible, unless certain physically unrealistic inequalities are satisfied for the masses of the particles involved. However, as a tool for the analysis of experimental data, dispersion relations have served useful purposes in pion-nucleon scattering[21], before rigorous proof was given[22]. With this spirit, formal derivation of K-N relativistic dispersion relations, following conventional method[23-25], has been carried out by Sakurai[26], by Amati and Vitale[27] and by Igi[3].

Writing the forward amplitude for $K^{\pm}$-p scattering as $T^{\pm}(w)$, we divide it into dispersive part $D_{\pm}(w)$ and absorptive part $A_{\pm}(w)$,

$$T_{\pm}(w) = D_{\pm}(w) + i\,\epsilon(w)\,A_{\pm}(w) \tag{I.1}$$

where w is the K-meson lab. energy and $\epsilon(w)$ in the sign function,

$$\epsilon(w) = 1 \text{ for } w > 0$$
$$= -1 \text{ for } w < 0$$

Putting $T^{(1)}(w) \equiv \frac{1}{2}\left[T-(w) + T+(w)\right] \equiv D^{(1)}(w) + i\epsilon(w)A^{(1)}(w)$, (I.2)

$T^{(2)}(w) \equiv \frac{1}{2}\left[T-(w) - T-(w)\right] \equiv D^{(2)}(w) + i\epsilon(w)A^{(2)}(w)$, (I.3)

$D^{(i)}(w)$ have simple behaviour when $w \to -w$; $D^{(1)}$ and $A^{(2)}$ are even functions whereas $D^{(2)}(w)$ and $A^{(1)}(w)$ are odd. Hence, the following dispersion relations can be written:

$$\tfrac{1}{2}\left[D_-(w) + D_+(w)\right] = \frac{1}{\pi}\int_0^\infty dw' \frac{2w'\,\tfrac{1}{2}\left[A_-(w') + A_+(w')\right]}{w'^2 - w^2}, \tag{I.4}$$

$$\tfrac{1}{2}\left[D_-(w) - D_+(w)\right] = \frac{2w}{\pi}\int_0^\infty dw' \frac{\tfrac{1}{2}\left[A_-(w') - A_+(w')\right]}{w'^2 - w^2} \tag{I.5}$$

When the convergence of the integrals (4) and (5) are not guaranteed, we may increase the power in the denominators in the following way:

$$\tfrac{1}{2}\left[D_-(w) + D_+(w)\right] - \tfrac{1}{2}\left[D_-(w_K) + D_+(w_K)\right] = \frac{2(w^2-w_K^2)}{\pi}\int_0^\infty dw' \frac{w'\,\tfrac{1}{2}\left[A_-(w') + A_+(w')\right]}{(w'^2-w_K^2)(w'^2-w_K^2)} \tag{I.6}$$

$$\tfrac{1}{2}\left[D_-(w) - D_+(w)\right] - \left(\frac{w}{w_K}\right)\tfrac{1}{2}\left[D_-(w_K) - D_+(w_K)\right] = \frac{2w(w^2-w_K^2)}{\pi}\int_0^\infty dw' \frac{\tfrac{1}{2}\left[A_-(w') - A_+(w')\right]}{(w'^2-w_K^2)(w'^2-w^2)} \tag{I.7}$$

where w_K is an arbitrary energy larger than m_K.

The contribution to the amplitudes $A_\pm(w)$ from the energy region $0 < w < m_K$ consists of two parts: one discrete part coming from the Λ, Σ 'bound states' and another continuous part coming from the 'unphysical region' $w_{\Lambda\pi} \leq w \leq m_K$ where

$$w_{\Lambda\pi} = \frac{(\Lambda + \pi)^2 - N^2 - K^2}{2N} \; . \tag{I.8}$$

(The particle symbols have been used to denote their masses). The unphysical region occurs because of the possibility of $\Lambda + \pi$ and $\Sigma + \pi$ intermediate states with thresholds below $\bar{K}N$ threshold. The position of the discrete states are given by

$$w_y = \frac{y^2 - N^2 - K^2}{2N} \qquad (y = \Lambda, \Sigma) \qquad (I.9)$$

For the region of integration extending from m_K to ∞, we have the optical theorem

$$\sigma_{\pm}(w) = \frac{4\pi}{k} A_{\pm}(w) \qquad (I.10)$$

where k is the lab. momentum of the K-meson and $\sigma_{\pm}$ is the total cross-section for $K^{\pm}$-p scattering

Substitution of (I.10) in (I.4) and (I.5) leads to

$$D_+(w) = \frac{1}{4\pi^2}\int_{m_K}^{\infty} dw' k' \left[\frac{\sigma_+(w')}{w'-w} + \frac{\sigma_-(w')}{w'+w}\right] + \frac{1}{\pi}\int_0^{w_{\Lambda\pi}} dw' \frac{A_-(w')}{w'+w} + \int_{w_{\Lambda\pi}}^{m_K} dw' \frac{A_-(w')}{w'+w} \qquad (I.11)$$

$$D_-(w) = \frac{1}{4\pi^2}\int_{m_K}^{\infty} dw' k' \left[\frac{\sigma_-(w')}{w'-w} + \frac{\sigma_+(w')}{w'+w}\right] + \frac{1}{\pi}\int_0^{w_{\Lambda\pi}} dw' \frac{A_-(w')}{w'-w} + \int_{w_{\Lambda\pi}}^{m_K} dw' \frac{A_-(w')}{w'-w} \qquad (I.12)$$

The first integrals are already expressed by experimentally measurable quantities. The second integrals represent the bound state contributions and can be expressed in terms of renormalized coupling constants. For the third integrals, one has to make some sort of approximation[1,4,15].

When the convergence is not good, equation (I.6) and (I.7) can be applied. We may then obtain a dispersion relation of the type

$$D_+(w) - \frac{1}{2}\left(1+\frac{w}{w_K}\right)D_+(w_K) - \frac{1}{2}\left(1-\frac{w}{w_K}\right)D_-(w_K)$$

$$= \frac{(w^2-w_K^2)}{4\pi^2}\int_{m_K}^{\infty} dw' \frac{k'}{(w'^2-w_K^2)}\left[\frac{\sigma_+(w')}{w'-w} + \frac{\sigma_+(w')}{w'+w}\right] + \frac{(w^2-w_K^2)}{\pi}\int_0^{w_{\Lambda\pi}} \frac{dw' A_-(w')}{(w'^2-w_K^2)(w'+w)} + \frac{(w^2-w_K^2)}{\pi}\int_{w_{\Lambda\pi}}^{m_K} dw' \frac{A_-(w')}{(w'^2-w_K^2)(w'+w)} \qquad (I.13)$$

The bound state contributions can be written down using the conventional second order perturbation theory and interpreting the coupling constants as the renormalized coupling constants[28].

If p_1 and p_2 are the initial and final 4-momenta of the nucleon and q_1 and q_2 those of the k^-meson, then the Born amplitude is given by

$$T(K^-+p \to K^-+p) = \bar{U}(p_2)\left[\frac{i\slashed{Q} - N \mp \Sigma}{S-\Sigma^2} \cdot \frac{g^2_{\Sigma K}}{4\pi} + \frac{i\slashed{Q} - N \mp \Lambda}{S-\Lambda^2} \cdot \frac{g^2_{\Lambda K}}{4\pi}\right] U(p_1)$$

$$s = -(p_1 + q_1)^2$$

$$Q = \tfrac{1}{2}(q_1 + q_2)$$

$$= -\left[\frac{\frac{S-N^2-K^2}{2N} + N \pm \Sigma}{S-\Sigma^2} \cdot \frac{g^2_{\Sigma K}}{4\pi} + \frac{\frac{S-N^2-K^2}{2N} + N \pm \Lambda}{S-\Lambda^2} \times \frac{g^2_{\Lambda K}}{4\pi}\right] \tag{I.14}$$

where the plus (or minus) sign is to be taken if the interation is scalar (or pseudoscalar) i.e. if $p_{ky} = +1$ (or -1).

The imaginary part of the Born amplitude is obtained by giving s a small positive imaginary part and taking the limit when this goes to zero. We then find that the discrete contribution to A_- from the Σ, Λ bound states to be

$$A_- = \left[\left(\frac{\Sigma^2-N^2-K^2}{2N} + N \pm \Sigma\right)\frac{g^2_{\Sigma K}}{4\pi}\right]\pi\,\delta(\Sigma^2 - s) + \left[\left(\frac{\Lambda^2-N^2-K^2}{2N} + N \pm \Lambda\right)\frac{g^2_{\Lambda K}}{4\pi}\right]\pi\,\delta(\Lambda^2 - s) \tag{I.15a}$$

The K-meson energy in the laboratory system is related with s by

$$w = \frac{s - N^2 - K^2}{2N} \quad ;$$

changing the variable from s to w, we get,

$$A_-(w) = \frac{1}{2N}\left[w_\Sigma + N \pm \Sigma\right]\left(\frac{g^2_{\Sigma K}}{4\pi}\right)\pi\,\delta(w_\Sigma - w) + \frac{1}{2N}\left[w_\Lambda + N \pm \Lambda\right]\left(\frac{g^2_{\Lambda K}}{4\pi}\right)\pi\,\delta(w_\Lambda - w) \tag{I.15b}$$

where w_y has been given before (equation 9).

Using equation (I.15b), we find that the bound state terms to the dispersion relations - (I.11), (I.12) and (I.13) are respectively

$$\frac{1}{2N}\cdot\frac{w_\Sigma + N \pm \Sigma}{w_\Sigma + w}\left(\frac{g^2_{\Sigma K}}{4\pi}\right) + \frac{1}{2N}\cdot\frac{w_\Lambda + N \pm \Lambda}{w_\Lambda + w}\left(\frac{g^2_{\Lambda K}}{4\pi}\right) , \tag{I.16a}$$

$$\frac{1}{2N}\cdot\frac{w_\Sigma + N \pm \Sigma}{w_\Sigma - w}\left(\frac{g^2_{\Sigma K}}{4\pi}\right) + \frac{1}{2N}\cdot\frac{w_\Lambda + N \pm \Lambda}{w_\Lambda - w}\left(\frac{g^2_{\Lambda K}}{4\pi}\right) \tag{I.16b}$$

and

$$-\frac{k^2}{w_\Sigma + w}\cdot\frac{1}{K^2}\cdot\frac{w_\Sigma + N \pm \Sigma}{2N}\cdot\left(\frac{g^2_{\Sigma K}}{4\pi}\right) - \frac{k^2}{w_\Lambda + w}\cdot\frac{1}{K^2}\cdot\frac{w_\Lambda + N \pm \Lambda}{2N}\left(\frac{g^2_{\Lambda K}}{4\pi}\right) \tag{I.16c}$$

where the +ve sign corresponds to $P_{ky} = +1$ i.e. NKy interaction scalar and the -ve sign to $P_{ky} = -1$ i.e. NKy interaction pseudoscalar (P_{ky} denotes the relative parity of K-meson and hyperon, the parity of nucleon being sonsidered positive).

Igi[4] has put the bound state term (I.16c) in the following approximate form

$$\frac{k^2}{w} \cdot 2F \equiv \frac{k^2}{w}\left[\frac{1}{2N} \cdot \frac{1}{W_\Sigma + N \mp \Sigma}\left(\frac{g^2_{\Sigma K}}{4\pi}\right) + \frac{1}{2N} \cdot \frac{1}{W_\Lambda + N \mp \Lambda}\left(\frac{g^2_{\Lambda K}}{4\pi}\right)\right] \tag{I.16d}$$

neglecting terms of the order $\frac{w_y}{w}$ (here, the -ve sign corresponds to to scalar and the +ve sign to pseudoscalar interaction).

Matthews and Salam[1] used the dispersion relation obtained by subtracting (11) from (12) while Igi[4] used the relation (13) at K-N threshold.

I.3 Analysis Of Experimental Results

Before considering any experimental numbers, we first introduce some parameters which will be used in our analysis.

We define an amplitude T which is related with the S-matrix in the following way

$$S_{fi} = \delta_{fi} + i\,(2\pi)^4\,\delta(p + q - p^1 - q^1)\left(\frac{m_N^2}{4\,p_o p'_o q_o q'_o}\right)^{1/2} \frac{4\pi W}{m_N}\, T_{fi}$$

where W is the total energy in the c.m. system. The diffefential cross-section in c.m. is related with the amplitude T by

$$\frac{d\,\sigma_{fi}}{d\Omega} = \frac{k_f}{k_i}\left|T_{fi}\right|^2$$ where k_i, k_f are the initial and final relative momentum.

For energy near threshold T_{fi} is the s-wave scattering amplitude. In the case of elastic scattering,

$$T = \frac{1}{k\cot\delta - ik}$$ where k is the relative momentum in c.m. and δ is the phaseshift.

For K^+p scattering, there is no inelastic process, so that the phase shift is real. We now define the s-wave scattering length for K^+p elastic scattering by

$k\cot\delta = \pm\frac{1}{a}$ where the positive or the negative sign is to be taken according as the potential is attractive or repulsive[1]. In the former case, the phase shift is positive while in the latter it is negative.

The amplitude M^+ of MS is related with T^+ by

$$M^+ = \frac{4\pi W}{N} T^+ = \frac{4\pi W}{N}\,\frac{1}{\pm\frac{1}{a} - ip_c} \quad (p_c = \text{c.m. momentum}) \qquad \text{(I.17a)}$$

so that near the threshold

$$\mathrm{Re}M^{+} = \pm \frac{4\pi W}{N} a \quad . \tag{I.17b}$$

For $K^{-}p$ scattering, we have competing inelastic processes. The s-wave elastic cross-section in c.m. system is related with the forward scattering amplitude by

$$\frac{\sigma_{el}^{-}}{4\pi} = |T^{-}|^{2} = \left(\frac{N}{4\pi W}\right)^{2} \left[(\mathrm{Re}\,M^{-})^{2} + (\mathrm{Im}\,M^{-})^{2}\right] \tag{I.18}$$

Using the optical theorem, we have,

$$\mathrm{Im}\,M^{-} = \frac{W}{N} p_c \sigma_T^{-} \quad .$$

We now get

$$(\mathrm{Re}\,M^{-})^{2} = \left(\frac{4\pi W}{N}\right)^{2} \frac{\sigma_{el}}{4\pi} - \left(\frac{W}{N}\right)^{2} p_c^{2} (\sigma_T^{-})^{2} \tag{I.19}$$

It is worth noting that the amplitude used by Igi is related to that of MS by

$D^{\pm} + iA^{\pm} = \frac{M^{\pm}}{4\pi}$. Equations (17) and (19) gives for w = 1 (we use the unit $c = \hbar = K = 1$, the unit of length being $\frac{c\hbar}{K} = 0.4 \times 10^{-13}$ cm)

$$D^{+}(1) = \frac{N+1}{N} (\pm a), \quad D^{-}(1) = \frac{N+1}{N} (\pm b) \tag{I.20}$$

where the parameter 'b' is defined by

$$\pm b = \pm \sqrt{\frac{\sigma_{el}^{-}}{4\pi} - \frac{p_c^{2}(\sigma_T^{-})^{2}}{16\pi^{2}}}$$

The dispersion relation used by Matthews and Salam is obtained by subtracting (11) from (12) at threshold. This can now be written as

$$(\pm b) - (\pm a) = \frac{1}{6\pi^2}\left[\int_K^\infty k' \left[\sigma_{sc}^-(w') - \sigma_{sc}^+(w')\right] \left(\frac{1}{w'-K} - \frac{1}{w'+K}\right) dw' + \int_{K_3}^\infty |k'|\, \sigma_{ab}(w') \left(\frac{1}{w'-K} - \frac{1}{w'+K}\right) dw' + [BS]\right] \quad (I.21)$$

Here σ_{sc}^- is the elastic plus charge exchange scattering and (BS) denotes the 'bound state' terms.

In calculating the integrals in MS dispersion relation, we have taken σ_{sc}^+ to be constant throughout at a value 15 mb; σ^- $(= \sigma_{sc}^- + \sigma_{ab}^-)$ has also been taken by us to be constant at a value 40 mb from 120 MeV K-meson energy to 2 Gev K-meson K.E.[7]. In the region 0 - 120 MeV K.E. the elastic scattering contribution has been evaluated graphically using the combined emulsion and bubble chamber data[5]. The charge exchange scattering has been taken by us to be one-fifth of the elastic scattering. The contribution due to absorption in the 0- 120 MeV K.E. region as well as in the unphysical region $(w_{\Lambda\pi} - m_k)$ has been evaluated assuming a constant value 7 of $|k|\,\sigma_{ab}^-$.

The MS dispersion relation can now be written in the following form :

$$(\pm b) + a = \frac{1}{6\pi^2}\left[\int_1^{1+t} k' \sigma_{sc}^-(w') \left(\frac{1}{w'-1} - \frac{1}{w'+1}\right) dw' + \int_{w_{\Lambda\pi}}^{1+t} |k'|\, \sigma_{ab}^- \left(\frac{1}{w'-1} - \frac{1}{w'+1}\right) dw' + \sigma^- \int_{1+t}^{5} k' \left(\frac{1}{w'-1} - \frac{1}{w'+1}\right) dw' - \sigma^+ \int_1^5 k' \left(\frac{1}{w'+1} - \frac{1}{w'+1}\right) dw' + [BS]\right] \quad (I.22)$$

where $t = \frac{120}{m_k}$ and we have taken K^+p potential as repulsive. We take

$$\frac{\sigma_{el}^-}{4\pi} = 4.5 \text{ mb}^5.$$

Inserting the calculated values term by term we get

$$(\pm 1.68) + 0.86 = 0.72 - 0.14 + 1.37 - 0.73 + [BS] \qquad (I.23)$$

If we now take the positive sign corresponding to K^-p potential attractive, we get

$$[BS] = 1.32 \qquad \text{(K-meson pseudoscalar)} \qquad (I.24)$$

and taking the negative sign, corresponding to K^-p potential repulsive,

$$[BS] = -2.04 \qquad \text{(K-meson scalar)} \qquad (I.25)$$

Igi's dispersion relation at threshold have been used by us and for this purpose, w is put equal to 1 + h and then the limit when h is negligible compared to 1 is taken. The first ferm in Igi's formula becomes

$$\tfrac{1}{4}\left[D_-(1) - D_+(1)\right] + \tfrac{1}{2}\left(\frac{\partial D_+}{\partial W_1}\right)_{W_1 = 0} \qquad (I.26)$$

where W_1 = K-meson K.E. in lab.

From equation (19) $\left(D_+ = \frac{\text{Re}M+}{4\pi}\right)$, for the case of K^+p scattering,

$$\left(\frac{\partial D_+}{\partial W_1}\right)_{W_1 = 0} = \frac{M_p}{M_{p+1}} a^3 - \frac{a}{M_{p+1}} \qquad (I.27)$$

Assuming that σ_{sc}^+ is a constant, we now write Igi's dispersion relation at threshold in the following form:

$$(\pm b) + a + \left(\frac{\partial D_+}{\partial W_l}\right)_{W_l=0} \cdot \frac{2M_p}{M_p+1} = \frac{M_p}{M_p+1} \cdot \frac{1}{\pi^2}\left[\int_{1+t}^{\infty} \frac{\sigma^+ dw'}{k'(w'-w)} + \int_{1}^{1+t} \frac{\sigma_{sc}(w')dw'}{k'(w'+w)} + \int_{w_{K\pi}}^{1+t} \frac{|k'|\sigma_{ab}dw'}{k'^2(w'+w)} + \int_{1+t}^{\infty} \frac{\sigma^- dw'}{k'(w'+w)}\right] + \frac{4M_p}{M+1} \cdot 2F \qquad (I.28)$$

Inserting values term by term , we have

$$(\pm 1.68) + 0.86 + 0.16 = \frac{0.66}{\pi^2}\left[-9.38 + 10.27 - 2.10 + 16.77\right] + 4\,\frac{M_p}{M_p+1}\,.2F \qquad (I.29)$$

If we take the positive sign, we get

$$4\,\frac{M_p}{M_p+1}\,.2F = 1.67 \qquad (I.30)$$

which corresponds to pseudoscalar K-meson.

On the other hand, if we take the negative sign, we get

$$4\,\frac{M_p}{M_p+1}\,.2F = -1.68 \qquad (I.31)$$

which corresponds to scalar K-meson.

If now, instead of taking σ^+ constant, a linear variation of it with energy of the form $\sigma^+(w') = m\,(w' - w) + \sigma(w)$ is considered in the energy range 1 to 1 + t and then a constant value $\sigma^+(w') = \sigma^+(1 + t)$ up to infinity, we shall have

$$\left(\frac{\partial D+}{\partial W_1}\right)_{W_1 = 0} = \frac{M_p}{M_p+1}\, a^3 - \frac{1 + M_p}{M_p} \cdot \frac{m}{8\,\pi\, a} - \frac{a}{M_p+1}$$

and $$\int_1^{\infty} \frac{\sigma^+(w')dw'}{k'(w'-w)} = m \log \frac{1 + [t/(2+t)]^{1/2}}{1 - [t/(2+t)]^{1/2}} + mt^{1/2}(2+t)^{1/2} - \sigma + (1+t) \qquad (I.32)$$

Kaplon has reported a K^+p cross-section equal to (13.5 ± 2.8) mb in the energy range (20 - 100) MeV. The maximum value of 'm' permitted within this experimental error is obtained by taking $\sigma^+ = 13.5 - 2.8$ at 20 MeV K.E. and $\sigma^+ = 13.5 + 2.8$ at 100 Mev K.E. This value of m comes out to be 21.61. Using this value we shall now get, instead of equation (1.29), the following equation, term by term:

$$(\pm 1.68) + 0.68 - 2.57 = \frac{0.66}{\pi^2}\left[19.66 + 10.27 - 2.10 \quad + 16.77\right] + 4\,\frac{M_p}{M_p + 1}\,2F \,. \qquad (I.33)$$

If we take the positive sign, corresponding to K^-p potential attractive, then

$$4\,\frac{M_p}{M_p + 1}\,.2F = -3.17 \qquad (I.34a)$$

If the negative sign is taken, corresponding to K^-p potential repulsive, then

$$4\,\frac{M_p}{M_p + 1}\,.2F = -6.52 \qquad (I.34b)$$

Therefore, we find that K-meson comes out scalar irrespective of the sign of the K^-p potential.

I.4 Conclusion

Our conclusion is that the Matthews-Salam dispersion relation gives a pseudoscalar or scalar K-meson according to K^-p potential attractive or repulsive. However, the experimental data of Burrowes et al do not indicate that the contribution in the dispersion relation due to the energy region beyond $5m_k$, where the integrals have been cut off, will be negligible. Igi's dispersion relation also gives a pseudoscalar or scalar K-meson, if σ^+ is taken to be constant in the low energy region and the K^-p potential is considered attractive or repulsive respectively. However, within the limit of the present data on K^+p scattering, we can have K-meson scalar, irrespective of the sign of K^-p potential, if we take a linear variation of σ^+ with energy. We have calculated the coupling constants from equations (24), (25) and (30), (31). Taking $g_{N\Lambda k} = g_{N\Sigma k}$, our result is

$$\frac{g^2}{4\pi} = \left.\begin{array}{l} 7.26 \text{ p.s.} \\ 0.78 \text{ s.} \end{array}\right\} \text{Matthews-Salam}$$

$$\frac{g^2}{4\pi} = \left.\begin{array}{l} 4.60 \text{ p.s.} \\ 0.32 \text{ s.} \end{array}\right\} \text{Igi'(constant } \sigma^+)$$

The above conclusions were reached by assuming $\Lambda - \Sigma$ relative parity even and by using the high energy data of Burrowes et al[7] and the low energy data reported at the CERN conference[5]. The assumption of same parity for Λ and Σ is, at present, a very questionable one. The high energy data have considerably increased now and cross-sections up to 8 BeV/c are available. Besides, for the low energy K^-p scattering, we shall put forward an effective range theory in Chapter IV. We, therefore, felt that it would be worthwhile to re-examine the question of parity determination. We have used our theory to evaluate the dispersion integrals in the unphysical region and in the low energy region and the new data for the high energy region. The details are given in appendix 1.

II. PION-HYPERON SCATTERING

II.1 Introduction

Feldman, Matthews and Salam[29] suggested that we may write down a dispersion relation for the quantity $B_{1\pm}\,(T_{1\pm})^{-1}$, where $T_{1\pm}$ is the amplitude for $J = 1 \pm \frac{1}{2}$ scattering and $B_{1\pm}$ is the corresponding Born amplitude. Now, unitarity gives $\mathrm{Im}\,(T_{1\pm})^{-1} = -k$ on the right hand physical cut ($S_J = 1 + 2ik^{1/2}T_J k^{1/2}$). So if the approximation of neglecting the left hand cut can be made, then, we have the dispersion relation

$$B_{1\pm}(w)\ \mathrm{Re}\,(T_{1\pm}(w))^{-1} = 1 - \frac{w - w_B}{\pi}\, P\int \frac{B_{1\pm}(w')\, k'dw'}{(w' - w)(w' - w_B)} \qquad \text{(II.1)}$$

where w_B is the pole of the Born term and a subtraction has been made at this point. The reason for normalizing at the Born pole is that for w near w_B the amplitude $T_{1\pm}(w)$ can be taken to be equal to the Born amplitude. The integral term in equation (II.1) indicates how the contribution from the right hand physical cut (or the unitarity cut) is taken into account.

Using equation (II.1) FMS reproduced very simply the Chew-Mandelstam[30] result on π-π scattering and the Chew-Low equation for the (3,3) amplitude in π- N scattering.

In deriving the static equations for π- Y scattering, we shall take into account the Σ-Λ mass difference and we shall find that $B_{1\pm}(w)$ breaks up into two or three terms, the poles of which are different. Equation (II.1) then has to be extended. To this end, we proceed in the following way.

Let us suppose, $B_{1\pm}(w) = B^1_{1\pm}(w) + B^2_{1\pm}(w)$ where $B^1_{1\pm}(w)$ has the pole at $w = w_1$ and $B^2_{1\pm}(w)$ at $w = w_2$. First of all, we notice that equation (II.1) essentially means we have written down a Cauchy integral for $\dfrac{BT^{-1} - 1}{w - w_B}$ with a contour which runs above and below the right hand cut. Of course, we have neglected the contributions from the left hand cuts. Let us now write down Cauchy integrals for the quantities

$\dfrac{B_1(w)T^{-1}(w) - C_1(w)}{w - w_1}$ and $\dfrac{B_2(w)T^{-1}(w) - C_2(w_2)}{w - w_2}$ where $C_1(w_1)$ and $C_2(w_2)$ are arbitrary normalization constants. Then we have,

$$B_1(w)T^{-1}(w) = C_1(w_1) + \frac{(w - w_1)}{\pi}\int \frac{B_1(w')\,\mathrm{Im}\,T^{-1}(w')\,dw'}{(w' - w)\,(w' - w_1)} \qquad \text{(II.2)}$$

$$B_2(w)T^{-1}(w) = C_2(w_2) + \frac{(w - w_2)}{\pi}\int \frac{B_2(w')\,\mathrm{Im}\,T^{-1}(w')\,dw'}{(w' - w)\,(w' - w_1)} \qquad \text{(II.3)}$$

We add equations (II.2) and (II.3). Taking $w_1 = w_2$ and comparing with (II.1) we find

$$C_1(w_1) + C_2(w_2) = 1 \qquad \text{(II.4)}$$

If the poles w_1 and w_2 are near each other we can take the above normalization to be valid and this, at onee, leads to

$$B_{1\pm} T^{-1}_{1\pm} = 1 + \frac{(w - w_1)}{\pi} \int \frac{B_{1\pm}(w') \mathrm{Im}\, T^{-1}_{1\pm}(w')\, dw'}{(w' - w)(w' - w_1)} +$$

$$\frac{(w - w_2)}{\pi} \int \frac{B_{1\pm}(w') \mathrm{Im}\, T^{-1}_{1\pm}(w')\, dw'}{(w' - w)(w' - w_2)} \qquad \text{(II.5)}$$

$$\text{or } B_{1\pm}(w)\, \mathrm{Re}\, T^{-1}_{1\pm}(w) = 1 - \frac{(w - w_1)P}{\pi} \int \frac{B_{1\pm}(w')\, k^1\, dw'}{(w' - w)(w' - w_1)}$$

$$- \frac{(w - w_2)P}{\pi} \int \frac{B_{1\pm}(w')\, k^1\, dw'}{(w' - w)(w' - w_2)} \qquad \text{(II.6)}$$

In the next section, we shall first derive the pion-hyperon static, equations exactly in the Chew-Low form, neglecting the $\Sigma - \Lambda$ mass difference and using equation (II.1). Then we shall derive the static equations using equation (II.6), where we shall take into account the $\Sigma - \Lambda$ mass difference.

Amati, Stanghellini and Vitale[32] have studied the low energy pion-hyperon scattering using a field theoretic model which takes into account the $\Sigma - \Lambda$ mass difference and the possible inequality of $\Sigma\pi$ and $\Lambda\pi$ coupling constants. This model consists of a fixed Y which can appear either as Σ or Λ, interacting with the π and the K-meson fields, treated in the one meson approximation. Using our equation (II.6), we shall see that we can reproduce their results very simply. This technique of ASV is similar to that developed by Bosco, Fubini and Stanghellini[33] which leads to the same results as the Chew and Low[34] formalism for the pion-nucleon scattering. Relativistic, fixed momentem transfer dispersion relations for pion-hyperon scattering have been derived by Capps and Nauenberg[35]. They have also written down the p-wave static equations in the Chew-Low form with somewhat different approximations. Resonances in the pion-hyperon system, in the Chew-Low approximation, have also been investigated by Nauenberg[36].

Experimentally, a pion-hyperon resonance has been discovered by Alston et al[16] in I = 1 state. Amati, Stanghellini and Vitale[37] put forward the attractive idea that this resonance is possibly the analog of (3,3) π-N resonance, expected on the basis of global symmetry. Using their static model, they calculated the position of this resonance for I = 1 and I = 2 states of the pion-hyperon system. ASV, in their calculation, disregard the $\bar{K}$ N channel which is coupled with the π Y channels. The reason for this is that threshold for the process $Y + \pi \rightarrow \bar{K} + N$ is of the same order as the one for two-pion production and the two-pion production is neglected in their model in the spirit of one meson approximation. However, from the study of low energy K^-p scattering, Dalitz and Tuan[38] have remarked that the $\bar{K}$ N interactions are quite strong. We, therefore, felt it worthwhile to investigate the effect of the strongly coupled $\bar{K}$ N channel on the position of this I = 1 resonance[39]. The details of our calculations are given in section II.4.

Following ASV, we have assumed the parity of Σ and Λ to be the same. There is not any clear experimental justification of this. On the other hand, Nambu and Sakurai[40] have cited arguments which indicate Σ and Λ have opposite parities. Also, if the above resonance is the analog of (3,3) πN resonance, it should have spin $\frac{3}{2}$. Though the experimental spin determination is not conclusive now, yet there is indication that the spin may be $\frac{1}{2}$.[41] This has prompted Duimio and Wolters[42] to study the consequences of the hypothesis of opposite parities of Σ and Λ, using the same approach of ASV. They obtain two resonances for the state vectors

$$|\pi\Lambda\rangle_p + |\pi\Sigma\rangle_s$$

and

$$|\Sigma\rangle + |\pi\Lambda\rangle_s + |\pi\Sigma\rangle_p$$

which correspond to I = 1, J = ½ andparity + and - respectively (Λ-parity is assumed to be +1). No $\pi\Lambda$ resonance is obtained in I = 1 and J = $\frac{3}{2}$ state. If the observed resonance is associated with the second state vector given above, then they predict another resonance occurring at an energy $w > 2.5$, corresponding to the first. The FMS technique adopted by us has also been used by Wali, Fulton and Feldman[43] to investigate the observed $\Lambda\pi$ resonance, assuming odd Σ-Λ parity but no $\Sigma-\Lambda$ difference. They find that there can exist a resonance in the I = 1 and J = ½ state andthey can fit all the known data to a reasonable set of renormalized coupling constants.

Also, they find it impossible to fit the resonance data, if $\bar{K}$ N channel is completely ignored. This is contrary to our case (Σ-Λ even parity), where $\bar{K}$ N channel seems to have very little effect.

II.2 (a) <u>Pion-Hyperon Static Equations</u>

We neglect the $\Sigma - \Lambda$ mass difference and consider orthogonal combinations of $\pi + \Sigma$ and $\pi + \Lambda$ states which diagonalize the Born amplitude. In our case if the Born amplitude is diagonalized, then $C'(\equiv BT^{-1})$ is diagonalized, so that $T = B/C'$ is also diagonalized.

These combinations are

$$\Psi_r = \frac{1}{\left(f_\Sigma^2 + 2f_\Lambda^2\right)^{1/2}}\left[\sqrt{2}\, f_\Lambda \Psi_{1\Lambda,1+} - f_\Sigma \Psi_{1\Sigma,1+}\right]$$

$$\Psi_s = \frac{1}{\left(f_\Sigma^2 + 2f_\Lambda^2\right)^{1/2}}\left[f_\Sigma \Psi_{1\Lambda,1+} + \sqrt{2}\, f_\Lambda \Psi_{1\Sigma,1+}\right]$$

The Born approximation for the scattering in these two states are given by

$$B_r = \frac{1}{2}\left(f_\Sigma^2 + f_\Lambda^2\right)\frac{4}{3}\,\frac{k^2}{w}$$

$$B_s = -f_\Lambda^2\,\frac{2}{3}\,\frac{k^2}{w} \qquad \text{(II.7)}$$

Using equation (1) we now get

$$\frac{\lambda_\alpha k^3 \cot \delta_\alpha}{w} = 1 - w r_\alpha \qquad \text{(II.8)}$$

where $r_\alpha = \dfrac{\lambda_\alpha}{\pi}\displaystyle\int^{w} \frac{k'^3\,dw'}{w'^2(w'-w)}$, $\lambda_\alpha = \frac{1}{2}\left(f_\Lambda^2 + f_\Sigma^2\right)\frac{4}{3}$ for $\alpha = r$
$\qquad = -f_\Lambda^2\,\frac{2}{3}$ for $\alpha = s$

We find that for $\alpha = r$, r_α is positive, so that we can expect a resonance at $w = \frac{1}{r_\alpha}$. This has already been pointed out by Capps[44].

For $\alpha = s$, r_α is -ve and there does not occur any resonance.

Putting $f_\Lambda = f_\Sigma = f$, we find that the scattering in the state

ψ_r corresponds to the (3,3) pion-nucleon scattering and the resonance is just the analog of the (3,3) pion-nucleon resonance[45,46]. However, it is worth noting that its appearance does not depend on the assumption of global symmetry[47]. The scattering in the state ψ_s corresponds to (1,3) pion-nucleon amplitude. We shall now obtain the (3,1) and (1,1) amplitude for π- Y scattering.

Assuming $f_\Sigma = f_\Lambda = f$, the (3,1) and (1,1) π- Y states are given by[45]

$$\psi_{r'} = \frac{1}{\sqrt{3}}\left[\sqrt{2}\,\psi_{1\Lambda,1-} - \psi_{1\Sigma,1-}\right]$$

$$\psi_{s'} = \frac{1}{\sqrt{3}}\left[\psi_{1\Lambda,1-} + \sqrt{2}\,\psi_{1\Sigma,1-}\right]$$

we again get,

$$\frac{\lambda_\alpha k^3 \cot\delta_\alpha}{W} = 1 - W r_\alpha \tag{II.9}$$

where

$$\alpha = r' , \quad \lambda_{r'} = -\frac{2}{3} f^2$$

$$\alpha = s' , \quad \lambda_{s'} = -\frac{8}{3} f^2$$

and r_α is given by the same expression as before. $\alpha = r'$ corresponds to (3,1) amplitude and $\alpha = s'$ to (1,1) amplitude.

II.2 (b) Pion-Hyperon Static Equations

We shall now take into account the $\Sigma - \Lambda$ mass difference and shall derive the static equations, using our dispersion theoretic technique[39]. We shall arrive at the same equations obtained by Amati, Stanghellini andVitale[32] using their field theoretical model.

Since ASV discuss their results in terms of the K-matrix , so we introduce it also in our discussion. The K-matrix in our case is

$$K = k^{1/2}\,(\mathrm{Re}T^{-1})^{-1}\,k^{1/2}$$
$$= k^{1/2}\,C^{-1}\,B\,k^{1/2} \qquad \text{(II.10)}$$

where $C \equiv B\,\mathrm{Re}T^{-1}$

Let us first consider

$$I = 0,\ J = \frac{3}{2}\ ,\ l = 1$$

In this case the Born approximation is given by

$$B_3^0 = \frac{2}{3}\,k^2\left[\frac{f_\Lambda^2}{w - 2\Delta} - \frac{2f_\Sigma^2}{w - \Delta}\right],\quad w = W - m_\Lambda\ ,\quad \Delta = m_\Sigma - m_\Lambda$$

The first term has the pole at $w = 2\Delta$ while the second term has at $w = \Delta$. Following our dispersion representation (6) we have

$$C_3^0 = 1 - \frac{(w - 2\Delta)}{\pi}\int\left(\frac{2}{3}\,f_\Lambda^2\right)\frac{k'^3 dw'}{(w' - 2\Delta)^2\,(w' - w)} - \frac{(w - \Delta)}{\pi}\times$$
$$\int\left(-\frac{4}{3}\,f_\Sigma^2\right)\frac{k'^3 dw'}{(w' - \Delta)^2\,(w' - w)} = 1-(w - 2\Delta)\ 2f_\Lambda^2\ I +$$
$$(w - \Delta)\ 4\ f_\Sigma^2\ I \qquad \text{(II.11)}$$

where $I = \frac{1}{3\pi}\int\frac{k'^3 dw'}{w'^2\,(w' - w)}$ and we have put $\Delta = o$ inside the integral following ASV. Comparing our result with that of ASV, we find that C_3^o is exactly D_3^o of ASV.

Now using equation (II.10), we get,

$$\tan \delta_3^0 = \frac{k^3}{D_3^o}\frac{2}{3}\left[\frac{f_\Lambda^2}{w - 2\Delta} - \frac{2f_\Sigma^2}{w - \Delta}\right] \qquad \left(\mathrm{Re}\,T^{-1} = k\cot\delta\right) \tag{II.12}$$

which is the same as that of ASV, if we neglect quantities of the order $4f^4 I\Delta$ in the numerator.

Next, we consider $I = 1$, $J = \frac{3}{2}$ p-wave:

Here we have two channels and the Born approximations are given by

$$B^3_{\Sigma\Sigma} = \frac{2}{3} k_\Sigma^2 \left[- \frac{f_\Lambda^2}{w - 2} + \frac{f_\Sigma^2}{w - \Delta}\right],$$

$$B^3_{\Sigma\Lambda} = - \frac{2\sqrt{2}}{3} k_\Sigma \; k_\Lambda \; \frac{f_\Sigma \; f_\Lambda}{w}, \tag{II.13}$$

$$B^3_{\Lambda\Lambda} = \frac{2}{3} k_\Lambda^2 \frac{f_\Lambda^2}{w + \Delta}.$$

Correspondingly, we have the following dispersion representations:

$$C^3_{\Sigma\Sigma} = 1 + (w - 2\Delta)\, 2f_\Lambda^2\, I - 2\,(w - \Delta)\, f_\Sigma^2\, I,$$

$$C^3_{\Sigma\Lambda} = C^3_{\Lambda\Sigma} = 2\sqrt{2}\, f_\Sigma \; f_\Lambda \; wI$$

$$C^3_{\Lambda\Lambda} = 1 - 2(w + \Delta)\, f_\Lambda^2 \; I \tag{II.14}$$

From (10) we find that the elements of K-matrix contain Det C in the denominator. From (14), we get

$$\mathrm{Det}C_3^1 = 1 - 2\, wf_\Sigma^2\, I - 2\,(3f_\Lambda^2 - f_\Sigma^2)\, I\Delta - 4f_\Lambda^2\,(f_\Lambda^2 + f_\Sigma^2)\, I^2 w^2$$
$$+ 4I^2 \Delta \; wf_\Lambda^4 + 4f_\Lambda^2 \; I^2 \Delta^2 \,(2f_\Lambda^2 - f_\Sigma^2)$$

Det C^1_3 is exactly equal to D^1_3 of ASV. We now have

$$\overset{3}{K}_{\Sigma\Sigma} = \frac{2}{3}\frac{k^3_\Sigma}{D^1_3}\left[\left(-\frac{f^2_\Lambda}{w-2\Delta} + \frac{f^2_\Sigma}{w-\Delta}\right)\left(1 - (w+\Delta)\ .2f^2_\Lambda\ I\right) + 4f^2_\Sigma\ f^2_\Lambda\ I\frac{k_\Lambda}{k_\Sigma}\right] . \qquad \text{(II.15)}$$

If we again neglect quantities of the order of $4f^4 I\Delta$ in our numerator as well as in that of ASV, we arrive at the same result. This happens for all the K-matrix elements, so that we shall just write down the Born matrix and the corresponding dispersion representation taken by us

$$I = 2, \quad J = \frac{3}{2}, \quad l = 1 ,$$

$$B^2_3 = \frac{2}{3} k^2_\Sigma\left[\frac{f^2_\Lambda}{w-2\Delta} + \frac{f^2_\Sigma}{w-\Delta}\right] ,$$

$$C^2_3 = 1 - 2(w-2\Delta)\, f^2_\Lambda\, I - 2(w-\Delta) f^2_\Sigma\, I .$$

$$I = 0, \quad J = \frac{1}{2}, \quad l = 1 ,$$

$$B^0_1 = -\frac{k^2_\Sigma}{3}\left[\frac{f^2_\Lambda}{w-2\Delta} + \frac{9f^2_\Lambda}{w} - \frac{2\ f^2_\Sigma}{w-\Delta}\right] ,$$

$$C^0_1 = 1 + (w-2\Delta) f^2_\Lambda\, I + 9wf^2_\Lambda I - 2(w-\Delta) f^2_\Sigma I .$$

$$I = 1, \quad J = \frac{1}{2}, \quad l = 1 ,$$

$$B^1_{\Sigma\Sigma} = \frac{k^2_\Sigma}{3}\left[\frac{f^2_\Lambda}{w-2\Delta} - \frac{7f^2_\Sigma}{w-\Delta}\right] ,$$

$$C^1_{\Sigma\Sigma} = 1 - (w-2\Delta) f^2_\Lambda I + 7(w-\Delta) f^2_\Sigma I ,$$

$$B^1_{\Lambda\Sigma} = \sqrt{\tfrac{2}{3}}\ k_\Sigma\ k_\Lambda \left[\frac{1}{w} - \frac{3}{w-\Delta}\right] f_\Sigma\ f_\Lambda\ ,$$

$$C^1_{\Lambda\Sigma} = \sqrt{2}\ f_\Sigma\ f_\Lambda\ I\left(-w + 3(w-\Delta)\right),$$

$$B^1_{\Lambda\Lambda} = \frac{k_\Lambda^2}{3}\ f_\Lambda^2 \left[\frac{1}{w+\Delta} + \frac{3}{w-\Delta}\right] ,$$

$$C^1_{\Lambda\Lambda} = 1 + (w+\Delta)\ f_\Lambda^2\ I + 3(w-\Delta) f_\Lambda^2\ I .$$

$$\text{Det } C^1_1 = 1 + \left(7f_\Sigma^2 + 3f_\Lambda^2\right) Iw - 7\Delta f_\Sigma^2\ I + I^2 w^2 \left(-4f_\Lambda^4 + 20f_\Lambda^2\ f_\Sigma^2\right) + I^2 \Delta\, w\left(10f_\Lambda^4 - 18f_\Sigma^2\ f_\Lambda^2\right) + I^2 \Delta^2 \left(-4f_\Lambda^4 - 4f_\Sigma^2\ f_\Lambda^2\right),$$ which is exactly equal to D^1_1 of ASV.

$I = 2,\ J = \frac{1}{2},\ l = 1$,

$$B^2_1 = -\frac{k_\Sigma^2}{3}\left[\frac{f_\Lambda^2}{w - 2\Delta} + \frac{f_\Sigma^2}{w-\Delta}\right] ,$$

$$C^2_1 = 1 + (w - 2\Delta)\ f_\Lambda^2\ I + (w-\Delta)\ f_\Sigma^2\ I .$$

Our C^2_3, C^o_3 and C^2_1 are respectively equal to D^2_3, D^o_3 and D^2_1 of ASV.

Summarizing, we may say that if we neglect quantities of the order of $4f^4 I\Delta \cong \frac{\Delta}{\Omega} f^2$ ($\Omega = 290$ MeV in the case of global symmetry, so that $\frac{\Delta}{\Omega} \approx \frac{1}{5}$) compared to f^2 in our numerators, we get the same equation as those of ASV. Since our denominators are exactly equal to those of ASV, so the position of any resonance predicted by them will also be given by our procedure at the same energy (the condition for resonance used by ASV is that the denominator of K-matrix elements should vanish; we have discussed this point in the appendix, using the 2 x 2 channel as a simple illustration.)

II.3. Pion-Baryon Interaction And The Pion-Hyperon Static Equations

It is well-known[46-48] that under the hypothesis of charge independence, Yukawa type interactions, equality of the ($\Sigma\Lambda\pi$) and ($\Sigma\Sigma\pi$) coupling constants and Σ , Λ even parity, the pion-baryon interactions take the form

$$H_\pi = ig_1 \bar{N}_1 \underset{\sim}{\tau} \gamma_5 N_1 \cdot \underset{\sim}{\pi} + ig_2 \left[\bar{N}_2 \underset{\sim}{\tau} \gamma_5 N_2 + \bar{N}_3 \underset{\sim}{\tau} \gamma_5 N_3 \right] \cdot \underset{\sim}{\pi} + ig_4 \bar{N}_4 \underset{\sim}{\tau} \gamma_5 N_4 \cdot \underset{\sim}{\pi} \qquad \text{(II.16)}$$

where

$$N_1 = \begin{pmatrix} p \\ n \end{pmatrix} , \quad N_2 = \begin{pmatrix} \Sigma_+ \\ Y \end{pmatrix} , \quad N_3 = \begin{pmatrix} Z \\ \Sigma_- \end{pmatrix} , \quad N_4 = \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix}$$

$$Y \equiv \frac{\Lambda - \Sigma^0}{\sqrt{2}} , \qquad Z \equiv \frac{\Lambda + \Sigma^0}{\sqrt{2}} .$$

The symmetry involved in (16) i.e. the isospinor description of all baryons (equivalently, the assumption $g_2 = g_3$) is referred to as "Restricted Symmetery"[49]. The hypothesis of "Global Symmetry" is to enlarge the above symmetry by assuming

$$g_1 = g_2 = g_4 \qquad . \qquad \text{(II.17)}$$

H_π given by equation (16) conserves the usual isotopic spin and the doubtlet spin which is ½ for members of the four doublets N_1, N_2, N_3 and N_4 and 1 for the pions. Thus the πN_1, πN_2, πN_3 and πN_4 systems can be I = ½ or I = $\frac{3}{2}$ states and the scattering of each of these systems in a given angular momentum states can be described by two scattering amplitudes[45,49] one for the I = ½ and one for I = $\frac{3}{2}$ state. This is the consequence of "Restricted Symmetry.

If we further assume global symmetry, i.e. equation (II.17), then the πN_2 and πN_3 scattering amplitudes in $I = \frac{1}{2}$ and $I = \frac{3}{2}$ states can be equated to the corresponding πN_1 scattering amplitudes.

In deriving the pion-hyperon static equations, we have put $\Delta = 0$ ($\Delta = m_\Sigma - m_\Lambda$) in integrals of the type

$$I = \frac{1}{3\pi} \int \frac{k'^3 \, dw'}{w'^2 (w' - w)}$$

. This approximation gives corrections of the order of $\frac{\Delta}{w \text{ cut-off}}$. Global symmetry and physical intuition suggests that the cut-off energies are of the order of baryon masses. So any corrections to I due to the mass[53] differences and to possible different cut-off energies will always be of the order of $\frac{\Delta}{M_Y}$. These corrections are smaller than the Δ-correction as we shall see below.

In the derivation of pion-hyperon static equations in section II.2, we have taken into account the $\Sigma - \Lambda$ mass difference and the inequality of coupling constants. In the restricted symmetry scheme, both corrections are due to renormalization effect of heavy mesons. The mass differences is known experimentally ($\Delta \simeq 80$ Mev); if the renormalization effect on the coupling constants is of the same order of magnitude, we can expect $\delta = f_\Lambda^2 - f_\Sigma^2$ to be rather small, perhaps such that $\frac{\Delta}{M_Y} \sim \frac{\delta}{f_Y^2}$.

If we examine the function Det C_3^1 and C_3^2, we find that they can vanish for $\frac{1}{2} f_\Sigma^2 \leqslant f_\Lambda^2 \leqslant 2f_\Sigma^2$ while C_3^0, C^0, Det C_1^1 and C_1^2 do not vanish. This means that under this restriction on coupling constants, we can expect resonance in $J = \frac{3}{2}$, $I = 1$ or 2 states only. The $I = 1$, $J = \frac{3}{2}$ resonance is obviously the analog of πN (3,3) resonance and as we have seen in section II.2a is expected from global symmetric considerations. It is important to note that for each value of I and J, the K-matrix for the different reactions have the same denominator (viz. Det C) for the same value of the total energy. This means that if $\pi - \Lambda$ is resonant, $\pi - \Sigma$ is also resonant just at the same total energy.

We now discuss the effect of the mass difference Δ and the inequality of coupling constants $\zeta \left(= \dfrac{f_\Lambda^2 - f_\Sigma^2}{f_\Lambda^2 + f_\Sigma^2} \right)$ on the positions of $J = \frac{3}{2}$, $I = 2$ and $I = 1$ resonances.

We have, $D_3^2 = C_3^2 = 1 - 4f^2 I w + 2f^2(3 + \zeta)\, I\Delta$,

where $f^2 = \frac{1}{2}\left(f_\Sigma^2 + f_\Lambda^2\right)$, so that $D_3^2 = 0$ gives the position of resonance for $J = \frac{3}{2}$, $I = 2$ to be[37]

$$w_r^{(2)} = \Omega + \frac{3}{2}\Delta + \frac{1}{2}\zeta\Delta \qquad \text{(II.18)}$$

where $4f^2 I \equiv \frac{1}{\Omega}$.

For $J = \frac{3}{2}$, $I = 1$,

$$D_3^1 = \mathrm{Det}\ C_3^2$$

$$= \left[1 + 2f^2(w - \Delta)I\right]\left[1 - 4f^2\left(w + \frac{\Delta}{2}\right)I\right] + 2f^2 \delta\, Iw - 8f^2 \delta\, I\Delta$$

$$- 8f^4 \delta\, I^2 w^2 + 8f^4 \delta\, I^2 w \Delta \quad . \tag{II.19}$$

(neglecting higher order terms)

For $\delta = 0$ we at once see that the position of resonance is given by

$1 - 4f^2(w + \frac{\Delta}{2})\ I = 0$ or

$$w_r^{(1)} = \Omega - \frac{\Delta}{2} \tag{II.20}$$

Let us now consider the case when $\delta \neq 0$ but small

$\left(\delta \lesssim 0.3 \text{ for } \tfrac{1}{2}f_\Sigma^2 \leqslant f_\Lambda^2 \leqslant 2f_\Sigma^2 \right)$.

We write

$$D_3^1 = \left[1 + 2f^2(w - \Delta + x)\ I\right]\left[1 - 4f^2\left(w + \frac{\Delta}{2} + y\right)\ I\right] \tag{II.21}$$

and compare (II.21) with (II.19).

This leads to $y = \frac{5}{6} \Delta\delta$ so that the position of the

resonance becomes[37]

$$w_r^{(1)} = \Omega - \frac{\Delta}{2} - \frac{5}{6}\Delta\delta \tag{II.22}$$

Comparing (II.18) and (II.22) we find a clear cut prediction of the

state, then there will be a resonance in $I=2, J=\frac{3}{2}$ state

theory. If there is a resonance in $I = 1$, $J = \frac{3}{2}$ at a higher energy,

given by $(\Delta + \frac{4}{3}\delta\Delta) = (80 + 105\,\delta)$ MeV, which is independent of

the actual position of any of these resonances.

It is worth pointing out that, if there is not any restricted symmetry, so that f_Λ and f_Σ are widely different, then we can have resonances in other states than those mentioned above[36,32]. This can be seen by inspecting C_3^0, C_1^0, Det C_1^1 and C_1^2. For example for $f_\Lambda^2 \gg f_\Sigma^2$, we can have resonances in I = 0, $J = \frac{3}{2}$ or I = 1, $J = \frac{1}{2}$ states whereas for $f_\Lambda^2 \ll f_\Sigma^2$ in I = 0, $J = \frac{1}{2}$ state. The limit $g_{\Sigma\Lambda\pi} \ll g_{\Sigma\Sigma\pi}$ is very hard to reconcile with the near equality of Λ- nucleon and nucleon-nucleon forces, indicated by hyper-nucleon interations and with the K^- capture experiments in deuterium which indicate that there exists a large $\Sigma - \Lambda$ exchange[36]. The possibility of $f_\Lambda \gg f_\Sigma$ however is compatible with these results. The only state which does not have a resonance for all choices of coupling constants is the I = 2, $J = \frac{1}{2}$ state.

II.4. Effect of $\bar{K}N$ Channel On The Position Of $I = 1$, $J = \frac{3}{2}$ π-Y Resonance

Recent experimental analysis[16] of pion-spectra in the reaction $K^- + p \longrightarrow \Lambda + \pi^+ + \pi^-$ seems to indicate a $\Lambda\pi$ resonant state in $I = 1$. As we have already mentioned, for $\frac{1}{2} f_\Sigma^2 \leqslant f_\Lambda^2 \leqslant 2f_\Sigma^2$, a π - Y p-wave resonance is possible only for $J = \frac{3}{2}$, $I = 1$ and 2. The position of the $I = 1$ resonance[37] is

$$w_r^{(1)} = \Omega - \frac{\Delta}{2} - \frac{5}{6}\delta\Delta \quad . \tag{II.22}$$

Since the $\Lambda\pi$ and $\Sigma\pi$ channels are strongly coupled with $\bar{K}N$ channel, so even though this resonance is well below the $\bar{K}$ N threshold ($\sim$ 50 MeV), it is interesting to see how far the position of this resonance is affected by the presence of this channel. Some authors have already considered the indication of a $Y\pi$ resonance in $K^- + p \longrightarrow Y + \pi$ production processes[50, 51].

To investigate the effect of $\bar{K}N$ channel on the position of $I = 1$ resonance, we have to calculate Det C taking into account the $\bar{K}N$ channel.

In this case the K-matrix is a 3 by 3 matrix.

The Born approximations are

$$B_{NN} = 0 \quad ,$$

$$B_{N\Sigma} = \frac{4}{3}\left(\frac{m_y}{m_N}\right)^{\frac{1}{2}} k_N \, k_\Sigma \, \frac{f_N g_\Sigma}{w - \Delta} \quad , \tag{II.23}$$

$$B_{N\Lambda} = \frac{2\sqrt{2}}{3} \left(\frac{m_y}{m_N}\right)^{1/2} k_N\, k_\Lambda\, \frac{f_N g_\Lambda}{w} \quad .$$

Correspondingly, for final $\bar{K}N$ channel,

$$C_{NN} = 1 \quad ,$$

$$C_{N\Sigma} = -(w-\Delta)\, 4g_\Sigma\, f_N I_1 \quad , \qquad \text{(II.24)}$$

$$C_{N\Lambda} = -w\,2\sqrt{2}\,g_\Lambda\, f_N I_1 \quad ,$$

where $I_1 = \frac{1}{3\pi}\int_{w_T} \frac{k'_N k' dw'}{w'^2(w'-w)}$, w_T = threshold of $\bar{K}N$ channel;

g_Σ and g_Λ are $K\Sigma N$ and $K\Lambda N$ coupling constants;

$$g_\Sigma^2 = \left(\frac{g_{\Sigma k}^2}{4\pi}\right)\cdot\frac{1}{4m_y^2}, \quad g_\Lambda^2 = \left(\frac{g_{\Lambda k}^2}{4\pi}\right)\cdot\frac{1}{4m_y^2}, \quad f_N^2 = \left(\frac{g_{\pi N}^2}{4\pi}\right)\cdot\frac{1}{4m_N^2},$$

$$f_\Sigma^2 = \left(\frac{g_{\Sigma\pi}^2}{4\pi}\right)\cdot\frac{1}{4m_y^2}, \quad f_\Lambda^2 = \left(\frac{g_{\Lambda\pi}^2}{4\pi}\right)\cdot\frac{1}{4m_y^2} \quad .$$

For initial $\bar{K}N$ channel,

$$C_{\Sigma N} = -(w-\Delta)\, 4g_\Sigma\, f_N I_2 \quad ,$$

$$C_{\Lambda N} = -w\,2\sqrt{2}\, g_\Lambda\, f_N I_2 \quad \text{where } I_2 = \frac{1}{3\pi}\int_{w_T} \frac{k'^2_N\, k' dw'}{w'^2(w'-w)} \quad .$$

We now have

$$\text{Det } C = \left[1 + 2f^2(w-\Delta)\, I\right]\left[1 - 4f^2\left(w+\frac{\Delta}{2}\right) I\right] + 2f^2\delta I w - 8f^2\delta I\Delta$$
$$-8f^4\delta I^2 w + 8f^4\delta I^2 w\Delta - (w-\Delta)^2\, 16 f_N^2 g_\Sigma^2 I_1 I_2 -$$
$$-8w^2 f_N^2\, g_\Lambda^2 I_1 I_2 \quad , \qquad \text{(II.25)}$$

neglecting other higher order terms. The last two terms in (25) take into account the effect of the $\bar{K}N$ channel. Writing as before

$$\mathrm{Det}\, C = \left[1 + 2f^2(w - \Delta + x')\, I\right]\left[1 - 4f^2(w + \frac{\Delta}{2} + y')\, I\right] \qquad \text{(II.26)}$$

and comparing (25) and (26), we get the following equation,

$$x'(w + \frac{\Delta}{2}) + y'(w - \Delta) = \delta w^2 - \delta w \Delta + (w - \Delta)^2\, 2C_\Sigma + w^2 C_\Lambda, \qquad \text{(II.27a)}$$

$$x' - 2y' = \delta w - 4\delta\Delta , \qquad \text{(II.27b)}$$

where $C_\Sigma = \left(\dfrac{g_\Sigma^2 \; f_N^2}{f^4}\right)$, $C_\Lambda = \left(\dfrac{g_\Lambda^2 \; f_N^2}{f^4}\right)$,

$$f^2 = \frac{1}{2}(f_\Lambda^2 + f_\Sigma^2), \qquad \delta = \left(\frac{f_\Lambda^2 - f_\Sigma^2}{f_\Lambda^2 + f_\Sigma^2}\right) .$$

From (6a) and (6b)

$$y' = \frac{5}{6}\Delta\delta + \frac{2(w - \Delta)^2 C_\Sigma + w^2 C_\Lambda}{3w} . \qquad \text{(II.28)}$$

Putting (II.28) in (II.26), we find that the position of the resonance is now given by

$$w^2(3 + 2C_\Sigma + C_\Lambda) + 3w\left[\frac{\Delta}{2} + \frac{5}{6}\Delta\delta - \Omega\right] - 4\Delta w C_\Sigma + 2\Delta^2 C_\Sigma = 0 . \qquad \text{(II.29)}$$

Neglecting the last term, we get

$$w_r^{(1)} = \frac{3\left[\Omega - \frac{\Delta}{2} - \frac{5}{6}\Delta\delta\right] + 4\Delta C_1}{3 + 2C_\Sigma + C_\Lambda} \qquad \text{(II.30)}$$

As we shall see below, C_Σ and C_Λ are small quantities, so that (II.30) can be rewritten in the following form

$$w_r^{(1)} = \Omega - \frac{\Delta}{2} - \frac{5}{6}\Delta\delta - \frac{2}{3} C_\Sigma \left(\Omega - \frac{5}{2}\Delta\right) - \frac{C_\Lambda}{3}\left(\Omega - \frac{\Delta}{2}\right) . \tag{II.31}$$

The last two terms represent the effect of $\bar{K}N$ channel.

To estimate C_Σ and C_Λ, we have evaluated the integrals I_1 and I_2 using a cut off at baryon mass and putting $w = o$ inside the integral[34]. Also, we have taken the effective ranges of pion-hyperon and pion-nucleon to be equal as a first approximation. This gives $4f^2 I = 1/\Omega \simeq$ effective range of π-N scattering

$$= \frac{4}{3\pi} f^2_N (1 + 2) \int_{\mu_\pi}^{m_N} \frac{k'^3 \, dw'}{w'^3} .$$

The factor 2 comes from the crossing term in the static equation for π-N effective range[52,53].

This gives

$$f^2 I = f^2_N \frac{(4.56 \times 3)}{3\pi} \times \mu_\pi . \qquad \left(\int_{\mu_\pi}^{m_N} \frac{k'^3 dw'}{w'^3} = 4.56 \mu_\pi \right)$$

Also, we get $I_1 \simeq \frac{1}{3\pi} \int_{\mu_\pi}^{m_y} \frac{k'^2 k'_N \, dw'}{w'^2 . \, w'} = \frac{3.44}{3\pi} \mu_\pi$,

$$I_2 \simeq \frac{1}{3\pi} \int_{\mu_\pi}^{m_y} \frac{k' k'^2_N \, dw'}{w'^3} = \frac{2.85}{3\pi} \mu_\pi ,$$

so that

$$\frac{I_1 I_2}{f^4 I^2} = \frac{3.44 \times 2.85}{f_N^4 \times 9 \times (4.56)^2} \simeq \frac{1}{f_N^4} \times \frac{1}{20} \; .$$

$$\therefore \; C_\Sigma = \left(\frac{g_\Sigma^2}{f_N^2}\right) \cdot \frac{1}{20} \; , \qquad C_\Lambda = \left(\frac{g_\Lambda^2}{f_N^2}\right) \cdot \frac{1}{20} \; .$$

Taking $\Omega = 290$ MeV, $\Delta = 80$ MeV,

$$C_\Sigma \, \frac{2}{3} \left(\Omega - \frac{5\Delta}{2} \right) + C_\Lambda \cdot \frac{1}{3} \left(\Omega - \frac{\Delta}{2} \right) = 3 \left(\frac{g_\Sigma^2}{f_N^2} \right) + 4 \left(\frac{g_\Lambda^2}{f_N^2} \right) \text{ MeV} \, .$$

Putting this in equation (II.31) we find that even if $\bar{K}N$ coupling constant is comparable to that of πN, the correction due to $\bar{K}N$ channel is small, being of the order $\lesssim 5$MeV .

II.5 Discussion

The Chew-Mandelstam technique[30] of finding the scattering amplitude is to write $T = \frac{N}{D}$ where N contains the left-hand cut and D contains the right-hand cut. This technique has been extended to the multi-channel case by Bjorken[54]. The FMS technique[29] is to write down a dispersion relation for BT^{-1} using the analyticity property of partial wave amplitudes. In the case where N is approximated by the Born term and we keep only the right-hand physical cut, both these techniques give the same result.

FMS wrote down the dispersion relation in $S = (W^2)$ plane, However, a complete dispersion relation for $B_{1\pm}\,(T_{1\pm})^{-1}$ or for that matter, for any $T_{1\pm}$, in the S plane should involve not only the first Riemann sheet ($\sqrt{s} = +W$) but also the second Riemann sheet ($\sqrt{s} = -W$). This is evident when we write a dispersion relation in the W-plane, where we have not only the physical right hand cut, but also a 'left hand physical cut'[55]. In the s-plane this 'left hand physical cut' goes to a right hand cut on the second Riemann sheet. Of course, this cut is related to the right hand cut on the first sheet by Macdowell's reflection principle[56]

$$f_{1+}(S_{II}) = -f_{(1+1)-}(S_I). \qquad \begin{array}{l}\sqrt{S_I} = +W \\ \sqrt{S_{II}} = -W\end{array}$$

In all the cases we have discussed, the pole of the Born term is near the right hand cut on the first sheet, so that Born is dominating and to neglect all other cuts (including the right hand cut in the second sheet) can be taken as a first approximation. However, for a partial wave for which the pole of the Born term does not lie near the right hand cut on the fist sheet, as in the s-wave $\bar{K}N$ processes, this approximation does not work.

III. K-MESON NUCLEON SCATTERING

III.1 Introduction

Experimentally, it is now known that the KN scattering in I = 1 state is pure s-wave up to K-meson lab energy 315 MeV[5,57-60]. This is indicated by the isotropy of the angular distribution in K^+p scattering. On the other hand, the I = 0 state scattering indicates p-wave interaction even at an energy 100 MeV[57-60]. Besides, analysis of emulsion and counter data tend to indicate that the I = 0 s-wave interaction is rather weak while the I = 1 s-wave interaction is strong and repulsive[20].

These features of KN low energy scattering have led the theoreticians to many speculations. In particular direct K-meson pion interactions have been introduced by Barshay[61] andby Yamaguchi[62]. In the model proposed by Barshay for K^+N scattering, an exchange of pion takes place between the kaon and nucleon through the Hamiltonian $\lambda K.K\pi\cdot\pi$. In this model, the s-wave in both the I=0 and I = 1 states are determined by the interference of the effects due to (a) the exchange of two pions and (b) the hyperon intermediate states arising from the direct interaction of K-mesons, hyperons and nucleons. Barshay finds that the two pion exchange gives a repulsive potential independent of the K-meson parity. Farther, assuming odd $\Sigma-\Lambda$ parity, he tries to explain why the s-wave scattering is weaker than that in I = 1 state. However, Barshay's model fails to give an explanation of why the I = 1 scattering is a pure s-wave even at high energy, which is one of the salient features of K^+N scattering. Sakurai[63], using his theory of strong interactions, attempts to explain the K+N scattering by assuming the exchange of two unstable vector bosons, one having the quantum numbers I = 1, J = 1, G parity even (resonating two pions) and the other having I = 0, J = 1, G parity odd (bound or resonating three pions) between the K-meson and the nucleon . Ceolin et al have shown that with a direct K-Y-N scalar coupling[64] the KN potential comes out attractive in disagreement with experiment, while a direct K-Y-N pseudoscalar coupling[65] doesnot reproduce the experimental behaviour. They concluded that to explain K^+N scattering direct K-Y-N coupling, even in the case of odd Σ-Λ parity, is inadequate. Also application of fixed source dispersion theory, which has been successful in explaining low energy π-N scattering[25], leads to predictions in contrast to experimental results. In particular, an I = 1, J = $\frac{3}{2}$ resonance is predicted[66,67]. Of course, these predictions can considerably change depending on the contribution of the crossing terms due to $\bar{K}N$ reactions[68,69].

Our purpose here is to assume an explanation of the isotropy of K-N scattering in I = 1 state, based on our present ideas of strong interactions and see how far we can then explain the other main features of KN scattering. Also, we shall investigate some consequences of our approach to the low energy $\bar{K}N$ scattering[70].

III.2. Kinematics, Crossing Relations and Analyticity

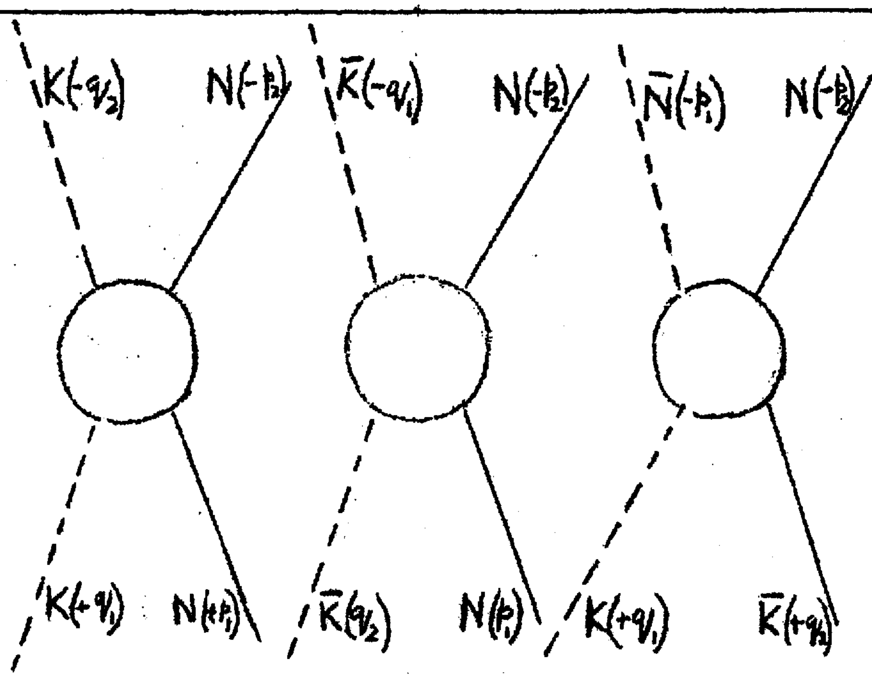
We denote the four-momenta of the incoming nucleon and K-meson by p_1 and q_1 and those of the outgoing particles by $-p_2$ and $-q_2$. The invariant Mandelstam variables are then

$$s = -(p_1 + q_1)^2 \qquad \text{(III.1a)}$$

$$\bar{s} = -(p_1 + q_2)^2 \qquad \text{(III.1b)}$$

$$t = -(p_1 + p_2)^2 \qquad \text{(III.1c)}$$

(we use $p \cdot q = \underline{p} \cdot \underline{q} - p_o q_o$)



(a) (b) (c)

Diagramatic representations of (a) channel I, (b) channel II, and (c) channel III.

The Lorintz invariants defined by equations (III.1 a, b, c) are the squares of the energies in the barycentric system of the three reactions:

I $\quad K + N \longrightarrow K + N$

II $\quad \bar{K} + N \longrightarrow \bar{K} + N$

III $\quad K + \bar{K} \longrightarrow N + \bar{N}$

When the four-momenta are on the mass shell, i.e. $p_1^2 = p_2^2 = -m_N^2$ and $q_1^2 = q_2^2 = -m_k^2$, where m_N and m_K are the masses of the nucleon and the kaon, the four-momentaconservation implies

$$s + t + \bar{s} = 2m_N^2 + 2\,m_k^2 = \Sigma \qquad \text{(III.2)}$$

In the barycentric system of channel I, the three variables s, $\bar{s}$ and t are related to the centre of mass energy W, the c.m. momentum k and the c.m. scattering angle θ by

$$s = W^2 = m_N^2 + m_k^2 + 2k^2 + 2\,(m_N^2 + k^2)^{1/2}(m_k^2 + k^2)^{1/2},$$

$$\bar{s} = -2k^2(1 + Z) + \left[(k^2 + m_N^2)^{1/2} - (k^2 + m_k^2)^{1/2}\right]^2 \qquad \text{(III.3)}$$

$$= \frac{1 - Z}{2}\,\frac{(m_N^2 - m_k^2)^2}{W^2} - \frac{1 + Z}{2}\,(W^2 - 2m_N^2 - 2m_k^2),$$

$t = -2k^2(1 - Z)$ where $Z = \cos\theta$.

Besides, the following two relations are very useful:

$$k^2 = \frac{\left[s - (m_N + m_k)^2\right]\left[s - (m_N - m_k)^2\right]}{4s} \qquad \text{(III.4)}$$

$$E = (k^2 + m_N^2)^{1/2} = \frac{W^2 + m_N^2 - m_k^2}{2W}$$

The S-matrix for channel I can be written as

$$S_I = \delta_{fi} - i(2\pi)^4 \, \delta(p_1 + p_2 + q_1 + q_2) \left(\frac{m_N^2}{4p_{10}\, p_{20}\, q_{10}\, q_{20}} \right)^{1/2}$$

$$\bar{U}(-p_2) \left[\xi_j^+ \, \eta_l^+ \, T_I \, \eta_k \, \xi_i \right] U(p_1) \qquad \text{(III.5)}$$

where the nucleon spinors are normalized according to

$$\bar{U}(p)\, U(p) = 1 \quad , \qquad \bar{U}(p) = U^+(p)\, \gamma_4 \, , \; \gamma_4 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

and ξ, η are the isotopic spinors of the nucleon and the kaon, and i, j, k. l are the isotopic spin indices of theincoming and the outgoing nucleons, and the incoming and the outgoing kaons respectively (i, j, k, l = 1 for p and $\bar{p}$, K^+ and K^-, and 2 for n, $\bar{n}$, K^0 and $\bar{K}^0$) The amplitude T_I can be decomposed into two Lorentz invariant functions:

$$T_I = -A(s, \bar{s}, t) + i \, \frac{(q_1 - q_2)}{2} \cdot \gamma \; B\,(s, \bar{s}, t) \qquad \text{(III.6}$$

The S-matrix in channel II can be written as

$$S_{II} = \delta_{fi} - i(2\pi)^4 \, \delta(p_1 + p_2 + q_1 + q_2) \left(\frac{m_N^2}{4p_{10}\, p_{20}\, q_{10}\, q_{20}} \right)^{1/2}$$

$$\bar{U}(-p_2) \left[\xi_j^+ \, \eta_i^+ \, T_{II} \, \eta_k \, \xi_i \right] U(p_1) \qquad \text{(III.7)}$$

Now, l is the isotopic spin index of the incoming antikaon and k is that of the outgoing antikaon. T_{II} can also be decomposed into two Lorentz invariant functions:

$$T_{II} = -\bar{A}\,(\bar{s}, s, t) + i\,\frac{(q_2 - q_1)}{2}\cdot\gamma\;\bar{B}\,(\bar{s}, s, t) \tag{III.8}$$

For channel III, the S-matrix takes the form

$$S_{III} = -i\,(2\pi)^4\,\delta(p_1 + p_2 + q_1 + q_2)\left(\frac{m_N^{\;2}}{4p_{10}\,p_{20}\,q_{10}\,q_{20}}\right)^{1/2}$$

$$\bar{U}(-p_2)\left[\xi_j^{\,+}\,\eta_l^{\,+}\,T_{III}\,\eta_k\,\xi_i\right]\mathcal{V}(-p_1) \tag{III.9}$$

where $\mathcal{V}(p)$ is normalized so that

$\bar{\mathcal{V}}(p)\,\mathcal{V}(p) = -1$ and $\bar{U}(p)\,\mathcal{V}(p) = 0$, and i, j, k, k, are the isotopic spin indices of the outgoing antinucleon , nucleon and incoming antikaon and kaon respectively. We decompose T_{III}, like T_I and T_{II} in the following way:

$$T_{III} = -C\,(t, s, \bar{s}) + i\,\frac{(q_1 - q_2)}{2}\cdot\gamma\;D(t, s, \bar{s}) \tag{III.10}$$

The principle of crossing relation, in the present context[71], states that the three amplitudes ($\xi_j^{\,+}\,\eta_l^{\,+}\;T_X\;\eta_i\;\xi_k$), where X = I, II, III, are the same analytic function matrix (in the γ-space and in the isotopic spin space) of two variables, say s and t, the distinction arising merely from assigning different ranges to the variablesof the amplitude. Theproof of this statement is given in our appendix 3.

Physically, it means something like the following. For definiteness, we consider the reaction (I) $K^+ + p \rightarrow K^+ + p$.

This reaction is expressed by the amplitude $(\xi_1^{+} \eta_1^{+} T_I \eta_1 \xi_1)$ which is a function of s and t. The principle of crossing relation states that the crossed process

(II) $K^- + p \rightarrow K^- + p$

is described by $(\xi_1^{+} \eta_1^{+} T_{II} \eta_1 \xi_1) = (\xi_1^{+} \eta_1^{+} T_I \eta_1 \xi_1)$, the same quantity which describes the original process I. Since the physical ranges of the variables in reactions I and II are non-overlapping, a procedureof continuation from one range of variables to another is called for to give the principle any physical significance. If, however, one knows the analytic properties of the function, then the procedure of continuation can be found. This is one of the places where the double dispersion representation plays the role of a dynamical postulate. In the conventional field theory, the principle of crossing relation is a consequence of the existence of field operators and the definition of S-matrix. In quantum electrodynamics, it has been known as the substitution law[17, 72]. The importance of this principle in elementary particle physics was first pointed out by Gell-Mann and Goldberger[73].

To investigate the consequences of the principle of crossing relation, we need the isotopic spin decomposition of the three scattering amplitudes. In the kaon-nucleon system, there are two isotopic spin states, I = 0 and I = 1. Since the total isotopic spin is concerved in strong interactions, we can write

$$T_I = T_I^{(0)}\ 1^{KN}(0) + T_I^{(1)}\ 1^{KN}(1) \tag{III.11}$$

where $1^{kN}(I)$ is the isotopic spin projection operator for the total isotopic spin I of the KN system such that

$$\xi_j^+ \eta_l^+ \ell^{KN}(I)\, \eta_k \xi_i = \sum_{I_z} \langle N(j), K(l) | I, I_z \rangle \langle I, I_z | N(i), K(k) \rangle$$

$$\ell^{KN}(I)\, \ell^{KN}(I') = \delta_{II'}\, \ell^{KN}(I) \tag{III.12}$$

Explicit representations of the isotopic spin operators have been worked out by Lee[71]. One obtains the result

$$\xi_j^+ \eta_l^+ \ell^{KN}(I)\, \eta_k \xi_i = \begin{cases} \frac{1}{2}\left(\delta_{ji}\delta_{lk} + \delta_{jk}\delta_{li}\right) & I = 1 \\ \frac{1}{2}\left(\delta_{ji}\delta_{lk} - \delta_{jk}\delta_{li}\right) & I = 0 \end{cases} \tag{III.13}$$

For antikaon-nucleon scattering, T_{II} can be decomposed into the isotopic spin eigenamplitudes:

$$T_{II} = T_{II}^{(0)}\ 1^{\bar{K}N}(0)\ )\ T_{II}^{(1)}\ 1^{\bar{K}N}(1) \tag{III.14}$$

where $1^{\bar{K}N}(I)$ is the isotopic spinprojection operator for the total isotopic spkn I state of the $\bar{K}N$ system, andone obtains

$$\xi_j^+ \eta_l^+ \ell^{\bar{K}N}(I)\, \eta_k \xi_i = \begin{cases} \frac{1}{2}\delta_{li}\delta_{kj} & I = 0 \\ \delta_{ji}\delta_{lk} - \frac{1}{2}\delta_{li}\delta_{kj} & I = 1 \end{cases} \tag{III.15}$$

In a similar way, we can write

$$T_{III} = T_{III}^{(0)}\, 1^{k\bar{k}:N\bar{N}}_{(0)} + T_{III}^{(1)}\, 1^{\bar{k}k:N\bar{N}}_{(1)} \tag{III.16}$$

and we obtain

$$\xi_j^{\dagger}\eta_l^{\dagger}\,\ell^{k\bar{k}:N\bar{N}}_{(I)}\,\eta_k\xi_i = \begin{cases} \frac{1}{2}\delta_{ji}\delta_{lk} & I = 0 \\ \delta_{li}\delta_{kj} - \frac{1}{2}\delta_{ji}\delta_{lk} & I = 1 \end{cases} \tag{III.17}$$

Using equations (III.11), (III.13), (III.14) and (III.15), and the crossing relation

$$\left(\xi_j^{+}\eta_l^{+}\, T_I\, \eta_k \xi_i \right) = \left(\xi_j^{+}\eta_l^{+}\, T_{II}\, \eta_k \xi_i \right) ,$$

we get

$$\begin{aligned} &\frac{1}{2}\left(\delta_{ji}\delta_{lk} - \delta_{jk}\delta_{li}\right)\left(-A^{(0)} + i\gamma.\,\frac{q_1-q_2}{2}\,B^{(0)}\right) \\ &\qquad + \frac{1}{2}\left(\delta_{ji}\delta_{lk} + \delta_{jk}\delta_{li}\right)\left(-A^{(1)} + i\gamma.\,\frac{q_1-q_2}{2}\,B^{(1)}\right) \\ &= \frac{1}{2}\delta_{jk}\delta_{li}\left(-\bar{A}^{(0)} + i\gamma.\,\frac{q_2-q_1}{2}\,\bar{B}^{(0)}\right) \\ &\qquad + \left(\delta_{ji}\delta_{lk} - \frac{1}{2}\delta_{li}\delta_{kj}\right)\left(-\bar{A}^{(1)} + i\gamma.\,\frac{q_2-q_1}{2}\,\bar{B}^{(1)}\right) . \end{aligned} \tag{III.18}$$

Identifying similar terms on the left and on the right, we obtain,

$$\begin{aligned} A^{(I)}(s, \bar{s}, t) &= \sum_{I'} \alpha_{II'}\, \bar{A}^{(I')}(\bar{s}, s, t) \\ B^{(I)}(s, \bar{s}, t) &= - \sum_{I'} \alpha_{II'}\, \bar{B}^{(I')}(\bar{s}, s, t) \end{aligned} \tag{III.19}$$

where the crossing matrix is given by

$$\alpha_{II'} = \begin{array}{c c} & \begin{array}{cc} I'=0 & I'=1 \end{array} \\ \begin{array}{c} I=0 \\ I=1 \end{array} & \begin{pmatrix} -1 & 3 \\ 1 & 1 \end{pmatrix} \end{array} . \tag{III.20}$$

From the crossing relations

$$\left(\xi_j^+ \eta_l^+ \, T_I \eta_k \xi_i\right) = \left(\xi_j^+ \eta_l^+ \, T_{III} \eta_k \xi_i\right) ,$$

$$\left(\xi_j^+ \eta_l^+ \, T_{II} \eta_k \xi_i\right) = \left(\xi_j^+ \eta_l^+ \, T_{\overline{III}} \eta_k \xi_i\right) ,$$

we obtain

$$\begin{aligned} A^{(I)}(s, \bar{s}, t) &= \sum_{I'} \beta_{II'} \, C^{(I')}(t, s, \bar{s}) , \\ B^{(I)}(s, \bar{s}, t) &= \sum_{I'} \beta_{II'} \, D^{(I')}(t, s, \bar{s}) , \\ \bar{A}^{(I)}(\bar{s}, s, t) &= \sum_{I'} \bar{\beta}_{II'} \, C^{(I')}(t, s, \bar{s}) , \\ \bar{B}^{(I)}(\bar{s}, s, t) &= -\sum_{I'} \bar{\beta}_{II'} \, D^{(I')}(t, s, \bar{s}) , \end{aligned} \tag{III.21}$$

where

$$\beta_{II'} = \frac{1}{2}\begin{pmatrix} 1 & -3 \\ 1 & 1 \end{pmatrix} , \quad \bar{\beta}_{II'} = \frac{1}{2}\begin{pmatrix} 1 & 3 \\ 1 & -1 \end{pmatrix} \tag{III.22}$$

In discussing the principle of crossing relation, we indicated the necessity for knowing the analytic properties of the scatting amplitudes in s and t. Actually, there is another reason to search for the analytic properties of the amplitudes in two variables. When one attempts to use "one-dimensional" dispersion relations, which exhibits the analytic properties in one variable s for fixed t in a dynamical calculation, such as the electromagnetic structure of the nucleon, one is constantly baffled because the dependence on t plays a crucial role. Mandelstam[74] first put forward a prescription for obtaining the analytic properties of amplitudes as functions of two complex variables, the energy and the momentum transfer. He begins with the assumption that an amplitude is an analytic function in the entire space of these two variables except for cuts along certain hyperplanes. He then determines the location of cuts from the requirement that the amplitude must satisfy one-dimensional dispersion relations for all the three reactions I, II, III. The application of Cauchy's theorem twice in the hyperspace of the complex s and t leads to a double dispersion representation. This representation, which satisfies the principle of crossing relation has been heralded as a possible basis of S-matrix theory of strong interactions[75]. However, in order to apply Mandelstam representation to handle general S-matrix elements, we have to deal with states involving more than two particles, which with our present theoretical tools has not been achieved yet. Obviously one may ask, how can we expect to deduce any meaningful consequences from an incomplete theory? The answer rests on two general features of Mandelstam representation[75]:

(a) The location of singularities is determined by the total "masses" of the actual physical systems; the higher the mass, the farther from the origin is the associated singularity. Now, among the strongly interacting particles there are none of zero mass; thus, the total "mass" of strongly interacting physical systems systematically tends to increase with the number of particles, and the singularities near the origin tend to be determined by one- and two-particle configuration. If there are aspects of the physical problem that are controlled mainly by "near-by" singularities, then one can make meaningful comparison of theory with experiment without a complete understanding of "faraway" singularities in which multi-particle configuration play a role.

(b) The "strength" of singularities is related to physical cross-sections and restricted by unitarity so that in a limited region of complex plane the behaviour of an S-matrix element tends to be controlled by the closest singularities. More precisely, an analytic function is determined through the Cauchy relations by a kind of Coulomb's law for potential due to point changes (poles) and line changes (branch cuts). The line-charge "density" is the discontinuity across the cut, which is proportional to physical cross-section and therefore limited in magnitude. There is assurance therefore that the "Coulomb's law", reciprocal dependence on distance, which favours nearby singularities, will not be overwhelmed by an increasing strength of singularity with distance. From a practical stand point this feature of the S-matrix approach is of tremendous importance to a theory of strong interactions permitting an orderly and systematic series of approximationS whose validity is subject to realistic appraisal without any assumptions as to the magnitudes of the coupling constants.

From the one-dimensional dispersion relations for reactions I-III, one finds that our amplitude $A^{I}(s, \bar{s}, t)$ (equation III.6) has poles at $\bar{s} = m_{\Lambda}^{2}, m_{\Sigma}^{2}$, corresponding to the single-hyperon intermediate states in reaction II, and the branch cuts along $s \geq (m_N + m_K)^2$ associated with the possible intermediate states in reaction I, along $\bar{s} \geq (m_{\Lambda} + \mu_{\pi})^2$ associated with theintermediate states in reaction II, and finally along $t \geq (2\mu_{\pi})^2$ for states in reaction III. The Mandelstam double disperion representation for $A^{(I)}(s, \bar{s}, t)$ can now be written in the symmetrical form

$$A^{(I)}(s, \bar{s}, t) = \sum_{Y=\Lambda,\Sigma} \frac{\alpha_Y^{(I)}}{m_Y^2 - \bar{s}} + \frac{1}{\pi^2}\int_{(m_N+m_K)^2}^{\infty} ds' \int_{(2\mu_\pi)^2}^{\infty} dt' \frac{A_{13}(s',t')}{(s'-s)(t'-t)}$$
$$+ \frac{1}{\pi^2}\int_{(m_\Lambda+\mu_\pi)^2}^{\infty} d\bar{s}' \int_{(2\mu_\pi)^2}^{\infty} dt' \frac{A_{23}(\bar{s}',t')}{(\bar{s}'-s)(t'-t)} + \frac{1}{\pi^2}\int_{(m_N+m_K)^2}^{\infty} ds' \int_{(m_\Lambda+\mu_\pi)^2}^{\infty} d\bar{s}' \frac{A_{12}^{(I)}(s',\bar{s}')}{(s'-s)(\bar{s}'-\bar{s})}$$
$$+ \frac{1}{\pi}\int_{(m_\Lambda+\mu_\pi)^2}^{\infty} d\bar{s}' \frac{A_2^{(I)}(\bar{s}')}{\bar{s}'-\bar{s}} + \int_{(2\mu_\pi)}^{\infty} \frac{1}{\pi} \cdot dt' \frac{A_3^{(I)}(t')}{t'-t} \qquad \text{(III.23)}$$

where the asymptotic behaviour of the amplitude is assumed to be given correctly by perturbation theory. Similar expression holds for $B^{(I)}$, with $\alpha_y^{(1)}$ replaced by $\beta_y^{(I)}$ and $A_{ij}^{(I)}$ by $B_{ij}^{(I)}$. The spectral functions $A_{ij}^{(I)}$, $B_{ij}^{(I)}$ are real and non-vanishing in the ranges which are bounded asymptotically by the limits of integrations.

The analytic properties of the partial wave amplitudes

$$\left\{A_1^{(I)}(S);\ B_1^{(I)}(S)\right\} = \int_{-1}^{+1} dZ\ P_1(Z) \left\{ \begin{array}{l} A^I\left[(S), t\ (s,\ Z)\right]; \\ B^1\left[s, t(s,\ Z)\right] \end{array} \right\} \qquad \text{(III.24)}$$

have been discussed by MacDowell[56], by Oehme[76] and by Frazer and Fulco[55]. All the singularities in $A_1^I(S)$ and $B_1^I(S)$ are associated with the possible intermediate states in the three channels and are referred to as "dynamical" singularities. The singularities of $A^{(I)}(S)$, $B^{(I)}(S)$ arise when the denominators in the double dispersion representation (i.e. equation III.23) vanish upon integration over Z from -1 to +1.

The vanishing of each denominator

$$\bar{s}\ (s,\ Z) - m_y^2 = 0 \quad , \quad y = \Lambda, \Sigma \qquad \text{(III.25)}$$

in the pole terms of the double dispersion representation, as Z varies from -1 to +1, gives rise to two branch cuts in s associated with the single hyperon intermediate states in channel II. These are

$$-\infty < s \leq 0 \quad ,$$

$$\frac{(m_N^2 - m_k^2)^2}{m_y^2} \leq s \leq 2(m_N^2 + m_k^2) - m_y^2 \quad . \qquad \text{(III.26)}$$

Vanishing of the denominators $\bar{s}' - \bar{s}\ (s,Z) = 0$, $\bar{s}' \geq (m_\Lambda + \mu_\pi)^2$ gives a branch cut along the real axis

$$-\infty < s \leq 2(m_N^2 + m_k^2) - (m_\Lambda + \mu_\pi)^2 \quad . \qquad \text{(III.27)}$$

Next, we consider $t' - t\ (s) = 0$ (III.28) for $t' \geq (2\mu_\pi)^2$, $-1 \leq Z \leq 1$. It is convenient to consider t as a function of k^2 and Z, rather than s and Z. One finds equation (III.28) is satisfied if $k^2 \leq -\mu_\pi^2$. The line segment $-m_k^2 \leq k^2 \leq -\mu_\pi^2$ gives rise to a branch cut

$$m_N^2 - m_k^2 \leq s \leq m_N^2 + m_k^2 - 2\mu_\pi^2 + 2\left(m_N^2 - \mu_\pi^2\right)^{1/2} \times \left(m_k^2 - \mu_\pi^2\right)^{1/2} \quad ; \qquad \text{(III.29)}$$

we use the relation $s = m_N^2 + m_k^2 + 2k^2 + 2\sqrt{(m_N^2 + k^2)(m_k^2 + k^2)}$.

For $-m_N^2 \leq k^2 \leq -m_k^2$, we get a circular branch cut with radius

$$|s| = m_N^2 - m_k^2 \qquad \text{(III.30)}$$

For $-\infty < k^2 \leq -m_N^2$, we get the branch cut $-\infty < s \leq -m_N^2 + m_K^2$. (III.31)

Finally, $s' - s = 0$, $s' \geq (m_N + m_k)^2$ gives the physical cut

$$\infty > s \geq (m_N + m_k)^2 . \qquad \text{(III.32)}$$

The partial wave amplitudes $A_l^{(I)}$, $B_l^{(I)}$ are not very convenient quantities to deal with, despite their comparative simple analytic properties. The main reasons are that they are not the eigenamplitudes of the total angular momentum and that the unitarity in the physical region is not readily expressible in terms of these amplitudes. The amplitude $f_{l\pm}$ in the state of parity $-(-1)^l$ and total angular momentum $j = l \pm \frac{1}{2}$ can be expressed in terms of the invariant amplitudes $A^{(I)}$ and $B^{(I)}$ 25:

$$f_{l\pm} = \frac{1}{16\pi W}\left\{(E+m)\left[A_l + (W-m_N)B_l\right] + (E-m)\left[-A_{l\pm 1} + (W+m)B_{l\pm 1}\right]\right\} \qquad \text{(III.32)}$$

The partial wave $f_{1\pm}$ is related to the phase shift by

$$f_{1\pm} = \frac{e^{i\delta_{1\pm}} \sin \delta_{1\pm}}{k} .$$

In addition to the dynamic singularities of A_1 and B_1, f_1 hasgot "kinematical" singularities arising from W and E; the combination

$$\frac{E \pm m}{W} = \frac{s \pm 2s^{\frac{1}{2}}m + m^2 - m_k^2}{2s}$$

brings in an additional pole at $s = 0$ and the branch cut of $s^{\frac{1}{2}}$ into $f_{l\pm}$. One can avoid the kinematical singularities by working with the amplitude $h_1(W) = \frac{1}{k^{2l}} \frac{W}{E + m} f_{l+}(W)$ of Frazer and Fulco[55].

In non-relativisitc treatment, where one is interested in an energy region near the threshold and only "nearby" singularities are taken into account, the amplitudes $h_1(W)$ and $f_1(W)$ give the same results. The unitarity condition is very simply expressed in terms of the reciprocral of the amplitude $f_{1\pm}$ viz.

$$\mathrm{Im} \left(\frac{1}{f_{1\pm}}\right) = - k \qquad \text{(III.33)}$$

III.3 Basic Assumption Regarding KN Scattering in I = 1 state

We begin with the consideration that for low energy KN scattering, the important left-hand contributions come from the hyperon cuts and the two-pion cut, which comes very close to the threshold. The hyperon cuts near the physical region of KN scattering are given by

$$\left(\frac{m_N^2 - m_k^2}{m_y}\right) \leq W \leq \sqrt{2\,(m_N^2 + m_k^2) - m_y^2} \qquad (y = \Lambda, \Sigma) \quad \text{(III.34)}$$

i.e. $4\,\mu_\pi \leq W \leq 6.9\,\mu_\pi$

The two-pion cut extends from the left up to

$$W = \sqrt{m_N^2 - \mu_\pi^2} + \sqrt{m_k^2 - \mu_\pi^2} \qquad \text{(III.35)}$$

$$= 9.97 \qquad (\gtrsim 35 \text{ MeV below } \bar{K}N \text{ threshold})$$

We approximate the two-pion cut by a single pole. This single pole approximation may be regarded as a sharp two-pion resonance, as considered by Frazer and Fulco[77] and also by Bowcock et al[78] and by Frautschi[79] to explain the nucleon electromagnetic structure and the small pion-nucleon phase shifts.

Let us denote the two-pion contribution to A and B in equation (III.6) by $A^{\pi\pi}(s, t)$ and $B^{\pi\pi}(s,t)$. Now, the single pole approximation of the two-pion cut leads to the following forms:

$$A^{\pi\pi}(s,\ t) = \left(s + \frac{t_r}{2} - \frac{\Sigma}{2}\right)\frac{a}{t - t_r} \tag{III.36a}$$

$$B^{\pi\pi}(s,\ t) = \frac{b}{t - t_r} \tag{III.36b}$$

where we regard 'a' and 'b' as unknown parameters; 't_r' is the position of the pole. The forms (III.36a) and (III.36b), including the kinematic factor in the fisrt, are suggested from the works of Frautschi and Walecka[80] in π- N scattering, of Lee[71] and of Ferrari, Frye, Pasterala[81] in KN, $\bar{K}$N scattering.

From equations (III.36), we find that the two-pion contribution to I = I, $p_{3/2}$ KN scattering is

$$\begin{aligned} f_{1+}^{\pi\pi(1)}(W) = \frac{1}{32\pi W^2}\Bigg\{ & \left[(W+m_N)^2 - m_K^2\right]\left[-a' + (W-m_N)\,b\right]\frac{1}{2k^2}\left[-2 + A\ln\frac{A+1}{A-1}\right] \\ & + \left[(W-m_N)^2 - m_K^2\right]\left[a' - (W+m_N)\,b\right] \times \\ & \frac{1}{2k^2}\left[-3A + \frac{3A^2-1}{2}\ln\frac{A+1}{A-1}\right]\Bigg\} \\ \simeq - \frac{1}{32\pi W^2} & \left[(W+m_N)^2 - m_K^2\right]\left[a' + (W-m_N)\,b\right]\frac{4k^2}{3(t_r+2k^2)^2} \end{aligned} \tag{III.3}$$

where $a' \equiv \left(s + \frac{t_r}{2} - \frac{\Sigma}{2}\right)a$,

$$A \equiv \left(\frac{t_r}{2k^2} + 1\right)$$

The two-pion contribution to I = 1, $p_{1/2}$ KN scattering is

$$f_{1-}^{\pi\pi(1)}(W) \simeq \frac{1}{32\pi W^2}\left\{-\left[(W+m_N)^2-m_K^2\right]\left[a'+(W-m)b\right]\frac{4}{3}\frac{k^2}{(t_r+2k^2)^2} + \frac{2W}{E+m_N}\left[a'-(W+m_N)b\right]\frac{2k^2}{(t_r+2k^2)}\right\} \qquad \text{(III.38)}$$

The contribution due to the hyperon cuts (i.e. the usual 'Born approximations') to I = 1, $p_{3/2}$ KN scattering is

$$f_{1+}^{B(1)}(W) \simeq \frac{1}{32\pi W^2}\left[(W+m_N)^2-m_K^2\right]\frac{\left[W+m_Y-2m_N\right]}{\alpha^2}\left(g^2_{\Lambda K}+g^2_{\Sigma K}\right)\frac{4}{3}k^2 \qquad \text{(III.39)}$$

and to I = 1, $p_{1/2}$ scattering is

$$f_{1-}^{B(1)}(W) \simeq \frac{1}{32\pi W^2}\left\{\left[2W(E+m_N)\left[g^2_{\Lambda K}+g^2_{\Sigma K}\right]\frac{\left[W+m_Y-2m_N\right]}{\alpha^2}\frac{4}{3}k^2 + \frac{2W}{E+m_N}\left[g^2_{\Lambda K}+g^2_{\Sigma K}\right]\frac{\left[m_Y-W-2m_N\right]}{\alpha}\cdot 2k^2\right.\right. \qquad \text{(III.40)}$$

where $\alpha \equiv W^2 + m_y^2 - 2(m_N^2 + m_k^2) - 2k^2$

and we have assumed even Λ-Σ parity, K pseudoscalar and have neglected Σ-Λ mass difference.

We now assume that the explanation of the isotropy of KN scattering in I = 1 state is that the two-pion amplitudes for $p_{1/2}$ and $p_{3/2}$ states cancel the corresponding Born amplitudes.

This assumption at once gives up the following two equations.

$$f_{1+}^{B(1)}(W) + f_{1+}^{\pi\pi(1)}(W) = 0 \qquad \text{(III.41a)}$$

$$f_{1-}^{B(1)}(W) + f_{1-}^{\pi\pi(1)}(W) = 0 \qquad \text{(III.41b)}$$

From these two equations we obtain,

$$a' + (W - m_N)\, b = (g_{\Lambda K}^2 + g_{\Sigma K}^2)\,(W + m_y - 2m_N)\left(\frac{t_r + 2k^2}{\alpha}\right)^2 \tag{III.42a}$$

$$a' - (W + m_N)\, b = (g_{\Lambda K}^2 + g_{\Sigma K}^2)\,(W + 2m_N - m_y)\,\frac{(t_r + 2k^2)}{\alpha} \tag{III.42b}$$

We can now find out our parameters 'a' and 'b'. In the static limit, they are given by

$$a = \frac{1}{\left(2m_N\, m_K + \frac{t_r}{2}\right)} \cdot \frac{1}{2\, W_0}\,(g_{\Lambda K}^2 + g_{\Sigma K}^2)\,\frac{t_r}{\alpha(W_0)} \times \Bigg[(W_0 + m_N)(W_0 + m_y - 2m_N)\,\frac{t_r}{\alpha(W_0)} + (W_0 - m_N)\,(W_0 + 2m_N = m_y) \Bigg] \tag{III.43a}$$

$$b = \frac{1}{2\, W_0}\,(g_{\Lambda K}^2 + g_{\Sigma K}^2)\,\frac{t_r}{\alpha(W_0)}\Bigg[(W_0 + m_y - 2m_N)\,\frac{t_r}{\alpha\,(W_0)} - (W_0 + 2m_N - m_y) \Bigg] \tag{III.43b}$$

where $W_0 = m_N + m_k$

III.4 KN Scattering In Other States

Since the two parameters in equations (III.36a, b) are now known, so we should be able to make definite predictions on s-wave scattering in I = 1 state and on s, $p_{1/2}$ and $p_{3/2}$ scattering in I = 0 state, taking only the two-pion contributions and the Born amplitudes.

The two-pion contribution to I = 1, s-wave KN scattering is now

$$f_{0+}^{\pi\pi(1)}(W) \simeq \frac{1}{32\pi W^2}\left[(W+m_N)^2-m_K^2\right].-\left[a'+(W-m_N)b\right]\frac{2}{t_\gamma+2k^2}$$

$$= -\frac{1}{32\pi W^2}\left[(W+m_N)^2-m_K^2\right]\left(g_{\Lambda K}^2+g_{\Sigma K}^2\right)\frac{2(W+m_Y-m_N)}{\alpha}\cdot\left(\frac{t_\gamma+2k^2}{\alpha}\right)$$

(using equation III.42a) (III.44)

The negative sign in front of (III.44) shows that the two-pion contribution is repulsive.

The Born amplitude for I = 1, s-wave KN scattering is

$$f_{0+}^{B(1)}(W) \simeq -\frac{1}{32\pi W^2}\left[(W+m_N)^2-m_K^2\right]\left(g_{\Lambda K}^2+g_{\Sigma K}^2\right)\frac{2(W+m_Y-m_N)}{\alpha}$$

(III.45)

which is also repulsive.

We therefore expect repulsive interactions for I = 1 s-wave KN scattering. We defer the question of quantitative agreement with experimental results, in this case, for the moment and switch over to I = 0.

Crossing symmetry shows that the two-pion I = 1, J = 1 resonant state gives $f_{0+}^{\pi\pi(0)}(W) = -3 f_{0+}^{\pi\pi(1)}(W)$. The Born term in this case (I = 0, $s_{1/2}$) is repulsive. Since $f_{0+}^{\pi\pi(1)}(W)$ is repulsive, so $f_{0+}^{\pi\pi(0)}(W)$ should be attractive and three times larger. This at once shows why we can expect rather weak s-wave I = 0 interaction. To put this quantitatively, we find

$$f_{0+}^{B(0)}(W) + f_{0+}^{\pi\pi(0)}(W) = \frac{1}{32\pi W^2}\left[(W+m_N)^2 - m_K^2\right]\frac{2(W+m_Y-2m_N)}{\alpha}\times$$
$$\left[-(3g_{\Sigma K}^2 - g_{\Lambda K}^2) + (g_{\Sigma K}^2 + g_{\Lambda K}^2)\,3\left(\frac{t_r+2k^2}{\alpha}\right)\right] \quad \text{(III.46)}$$

We can now derive an expression for the s-wave I = 0 scattering length, using the relation

$$k \cot \delta = \frac{1}{N} \quad \text{(III.47)}$$

where N is the amplitude due to the left-hand cuts; equation (III.47) holds so long as the rescattering term i.e. the contribution from the unitarity cut is not important. From (III.46) and (III.47), we get

$$a_{00} = 0.08\left[\left(\frac{3g_{\Sigma K}^2}{4\pi} - \frac{g_{\Lambda K}^2}{4\pi}\right) - \left(\frac{g_{\Sigma K}^2 + g_{\Lambda K}^2}{4\pi}\right)\frac{3\,t_r}{\alpha(W_0)}\right] \text{Fermi} \quad \text{(III.48)}$$

where $\tan \delta_{00} = -a_{00}\,k$.

For $t_r = 22\,\mu_\pi^2$ (Bowcock et al[78]), $\frac{t_r}{\alpha(W_0)} = 0.38$. From (III.48), we then find that a suitably chosen ratio of $\frac{g_{\Sigma K}^2}{g_{\Lambda K}^2}$ will give agreement with results known from experiments.

Next, we proceed to calculate the I = 0, $p_{3/2}$ and $p_{1/2}$ amplitudes, using equations (III.42a, b) and bearing in mind that the I = 0 two-pion contribution is -3 times that of I = 1 contribution. We obtain

$$f_{1+}^{(0)}(KN\to KN) = f_{1+}^{B(0)} + f_{1+}^{\pi\pi(0)}$$
$$= \frac{[(W+m_N)^2-m_K^2]}{32\pi W^2}\cdot\frac{2(W+m_Y-2m_N)}{\alpha}\times \left(3g_{\Sigma K}^2+g_{\Lambda K}^2\right)\frac{4k^2}{3\alpha} \qquad \text{(III.49)}$$

and
$$f_{1-}^{(0)}(KN\to KN) = f_{1-}^{B(0)} + f_{1-}^{\pi\pi(0)}$$
$$= f_{1+}^{(0)}(KN\to KN) - \frac{1}{32\pi W^2}\cdot\frac{2W}{E+m_N}\times 2(W+2m_N-m_Y)\cdot\frac{2k^2}{\alpha}\left(3g_{\Sigma K}^2+g_{\Lambda K}^2\right) \qquad \text{(III.50)}$$

The scattering lengths corresponding to these amplitudes are given by

$$a_{03} = -0.04\times\frac{1}{11.5}\left[3\,\frac{g_{\Sigma K}^2}{4\pi}+\frac{g_{\Lambda K}^2}{4\pi}\right]\text{Fermi}^3 \qquad \text{(III.51}$$

($\tan\delta_{03} = -a_{03}^{3}k^3$) and

$$a_{01} = 0.02\times\frac{1}{11.5}\left[3\,\frac{g_{\Sigma K}^2}{4\pi}+\frac{g_{\Lambda K}^2}{4\pi}\right]\text{Fermi}^3 \qquad \text{(III.52)}$$

($\tan\delta_{01} = -a_{01}^{3}k^3$)

Expression (III.51) and (III.52) indicate that the I = 0, $p_{3/2}$ is attractive and stronger than I = 0, $p_{1/2}$ which is repulsive. These results are in qualitative agreement with the analysis of emulsion results[82] and quantitative agreement can be obtained by choosing suitable values of $g_{\Sigma K}^2$ and $g_{\Lambda K}^2$. However, our present knowledge of $g_{\Sigma K}^2$ and $g_{\Lambda K}^2$ as well as the status of emulsion data do not justify anything much beyond qualitative agreement.

We now come back to our discussion of I = 1, s-wave scattering. In this case an effective range formula is usually applied to fit the experimental results[83]. In our language, this means that we have not only the left-hand contributions, but also the right-hand contributions coming from the unitarity cut (i.e. the rescattering term is important). The amplitude in this case will be given by[29,30],

$$k\cot\delta_{10} = \frac{1 - \dfrac{(W-W_1)}{\pi}\displaystyle\int \left[f_0^{\pi\pi(1)}(W') + f_0^{B(1)}(W')\right]\frac{k'dW'}{(W'-W)(W'-W_1)}}{f_{0+}^{\pi\pi(1)}(W) + f_{0+}^{B(1)}(W)} \qquad \text{(III.53)}$$

where W_1 is the subtraction point and at this point the physical amplitude is put equal to $f_0^{\pi\pi(1)} + f_0^{B(1)}$. By adjusting the point W_1 and taking a suitable cut off, we can now fit the experimental scattering length and effective range. For the purpose of illustration, taking $W_1 = W_o = m_N + m_K$, we find the scattering length

$$a_{10} = 0.08 \left(1 + \frac{t_r}{\alpha(W_o)}\right)\left(\frac{g_{\Sigma K}^2 + g_{\Lambda K}^2}{4\pi}\right) \quad \text{Fermi} \qquad \text{(III.54)}$$

and we get r = 0.5 f with a cut off $k_{max} = 3.6\,\mu_\pi$. Here $k\cot\delta_{10} = -\frac{1}{a_{10}} + \frac{1}{2} rk^2$ (I = 1, s-wave).

III.5 Two-Pion Contributions to $\bar{K}N$ Interaction

We now investigate the sign and magnitude of the two pion contribution in $\bar{K}N$ scattering on the basis of our knowledge of this in KN scattering. Use of crossing symmetry[71] shows that if we denote by $\bar{f}_l^{\pi\pi\binom{1}{0}}(\bar{W})$ the two pion contribution in $\bar{K} + N \rightarrow \bar{K} + N$ (energy of this process being $\bar{W}$), then

$$\bar{f}_l^{\pi\pi\binom{1}{0}}(\bar{W}) = f_l^{\pi\pi\binom{1}{0}}(\bar{W})$$

i.e. the two-pion pole gives the same contribution in $\bar{K}N$ and KN scattering. This means in I = 1, this contribution is repulsive and in I = 0 it is three times larger and attractive. Ferrari, Frye and Pusterala[84] have tried to determine the sign of the two-pion contribution following the procedure which has been applied in πN case. They approximate the two-pion cut by

$$\operatorname{Im} G(s) = R' \delta(s-a)$$
$$\left(G(s) = \frac{\sqrt{s}\,(E+m)}{k\cot\delta - ik} = \sqrt{s}\,(E+m)\,f(W) \right) \tag{III.55}$$

which gives the contribution

$$-\frac{1}{\pi}\,\frac{R'}{s-a} \tag{III.56}$$

to their amplitude G (S). The position of their pole is given by $\sqrt{a} = 9.6\,\mu_\pi$ which is very near the physical threshold $W_o = 10.2\,\mu_\pi$.

Let us now try to find out R^1 by equating (III.56) at threshold with that given by our $\bar{f}_0^{\pi\pi(1)}(\bar{W})$. We find

$$R' = -2.5\, f_0^{\pi\pi(1)}(\bar{W}_0)\, M_N^4 \qquad \text{(III.57)}$$

and $$f_0^{\pi\pi(1)}(\bar{W}_0) = -\left(\frac{g_{\Sigma K}^2 + g_{\Lambda K}^2}{4\pi}\right)\frac{t_r}{\alpha(W_0)} \times 0.08 \text{ Fermi} \qquad \text{(III.58)}$$

$$\frac{t_r}{\alpha(W_0)} = \begin{cases} 0.20 & \text{for } t_r = 12\mu_\pi^2 \\ 0.38 & \text{for } t_r = 22\mu_\pi^2 \end{cases}$$

We see that R^1 has the positive sign as found by them. Taking $t_r = 12\ \mu_\pi^2$, $\frac{g_{\Sigma K}^2 + g_{\Lambda K}^2}{4\pi} = 10$, we get $R^1 = 0.40\ M_N^4$ Fermi. Since the coupling constants and the value of t_r are not well established, we cannot say anything much about the magnitude of R^1. However, it seems our value of R^1 is possibly smaller by a factor of 2 to that of Ferrari et al[81,84].

Finally, a word about the interpretation of two-pion contribution in the physical $\bar{K}N$ scattering region is not out of place. Ferrari et al[81] have defined the interaction as the discontinuity across the left-hand cut and then from equation (III.55) considering the positive sign of R', they have interpreted the interaction as attractive in I = 1 state. However, if we take the usual field theoretic definition that an interaction is attractive (repulsive) if the phase shift due to it alone is positive (negative), then from (III.56) we find that their interpretation should be that the interaction is repulsive. This is essentially borne out by our equation (III.58) for $f_0^{\pi\pi(1)}(\bar{W})$. For I = 0, the interaction will be attractive.

and the net effect of two pion interaction in K^-p scattering is attractive. Another point worth making is that our amplitudes do not have the energy dependence $\propto \frac{1}{W-\sqrt{a}}$ asindicated by (III.56) which is rather strong because of the closeness of $\sqrt{a}$ to the physical threshold; so we should expect much less energy dependence of k cot δ than that of Ferrari et al[84].

III.6 Conclusion

Summarizing, we can say that by assuming the isotropy of KN scattering in I = 1 state as the result of the cancellation of the two-pion and the Born contributions in $p_{3/2}$ and $p_{1/2}$ states, we have reproduced all the qualitative features of low energy KN scattering as indicated by our present experimental knowledge; also we have some insight to the two-pion contribution to $\bar{K}N$ scattering. The solution of the experimental results, which agrees with all our theoretical considerations, is the D solution of Rochester Conference[60] and the B solution of Melkankoff et all[82], characterised by weak $s_{1/2}$, attractive $p_{3/2}$ and repulsive $p_{1/2}$ in I = 0 state and a repulsive pure s-wave interaction in I = 1 state.

IV LOW ENERGY K^--p

SCATTERING AND $\bar{K}N$ BOUND STATES

IV.1 Introduction

The major theoretical effort in the analysis of low energy K^--p scattering processes has been based on the phenomenological zero range theory[85,86,87], proposed by Jackson, Ravenhall and Wyld. In the zero range theory one assumes k cot δ to be essentially constant and equal to the reciprocal of the complex scattering length. The principal justification of the zero range theory is its simplicity; it needs minimum number of parameters. All present experimental data are consistent with the zero effective range parameters. Besides, no theory has been advanced which would give an estimate of the effective range (except, of course, the argument that on the basis of conventional Yukawa theory, one expects this to be of the order of $\hbar/m_K c$ corresponding to the exchange of a K-meson and $\hbar/m_K c \simeq 0.4$ Fermi, is small). Explicit parametrization of the $\bar{K}N$ scattering and reaction amplitudes has been done by Jackson and Wyld[86] and by Dalitz and Tuan[87] which under simple assumptions of energy dependence, leads to the zero range theory. These authors have also given formulae, taking into account the kinematic effect, arising from the energy difference of K^-p and $\bar{K}$-n thresholds and the Coulomb effect. A great deal of effort has also been devoted in studying the effect of $\bar{K}N$ interactions on global symmetry and restricted symmetry and the indications of this effect in low energy K^-p data[45,88-90]. The flattening of the K^-p elastic scattering cross-section, indicated by the preliminary experimental data, has also received considerable attention[84,86,91].

The recent discovery of a $\Lambda\pi$ resonance[16,92,93] (called Y^*) in $K^- + p \rightarrow \Lambda^0 + \pi^+ + \pi^-$ process at an energy 1382 ± 20 MeV, has renewed interest in the study of $\bar{K}N$ interactions. Two years ago, Dalitz and Tuan[17] suggested that with their (a-) and (b-) scattering length solutions, there should be resonances in $I = 1$ and $I = 0$ pion-hyperon scattering states. Such a resonance can be interpreted as a bound state of $\bar{K}N$ system which is metastable because of the pion-hyperon interaction[94,95]. At present, (a-) scattering length solution has been found which predicts the position and width of Y^*, as determined by experiment[94]. However, one crucial point here is that the Dalitz-Tuan (a-) solution requires destructive Coulomb-nuclear interference while there is no evidence for it. Though the present experimental data do not provide any unambiguous answer to this question, the results reported at the Kiev Conference[96] indicate constructive Coulomb-nuclear interference. The present emulsion data[97,98] seem to favour the constructive Coulomb interference. We, therefore, thought it would be worthwhile to see if we could have positive Coulomb interference as well as Y^* andinvestigate the consequences of such a theory.

From the very beginning, we assume the existence of Y^* as a bound state of $\bar{K}N$ $I = 1$ system (mass $\sim$1385 MeV) which is stable if we switch off the pion-hyperon interaction. In that case we have the well-known effective range expansion[75,99] for $k \cot \delta$ (δ real in this situation) where the scattering length is connected with the binding energy and the effective range by equation (IV.2). We investigate the case where we can have $k \cot \delta$ always positive and this leads to a very large effective range (equation IV.4). We then assume that when the pion-hyperon interaction is switched on, the scattering length, the effective range and the binding energy all become complex, the imaginary part of the binding energy being completely determined by the experimental half-width of Y^*. We assume the zero range theory to be valid for the $I = 0$ state. We find that we can explain the existing experimental data with a large effective range in $I = 1$ state which gives positive Coulomb interference. Our set of parameters indicate that the real part of the scattering length for $I = 0$ to be small (reminiscent of the (a+) solution of Dalitz-Tuan). We have investigated this point theoretically and find that by assuming the existence of $\bar{K}N$ $I = 0$ bound state, with mass below the $\Sigma\pi$ threshold, we can explain this. We also expect a narrow $\Sigma\pi$ resonance just below $\bar{K}N$ threshold in $I = 0$ state ($\delta_{\Sigma\pi} = 90^\circ$).

IV.2 Basic Formulation And Results

As we have said, we begin with the assumption that Y* is a stable bound state of $\bar{K}N$ system in the absence of pion-hyperon interaction. We then have the effective range expansion[99,75]

$$k \cot \delta = \frac{1}{a} + \frac{1}{2} rk^2 \quad , \qquad (\delta \text{ real}) \qquad \text{(IV.1)}$$

$$\frac{1}{a} = -\gamma + \frac{1}{2} r \gamma^2 \quad , \qquad \text{(IV.2)}$$

$$\gamma^2 = 2\mu B \quad , \qquad \text{(IV.3)}$$

where 'B' is the binding energy ($\simeq$50 MeV) and $\mu = \frac{m_N m_K}{m_N + m_K}$.

Examining (1) and (2), we find three interesting cases:[100]

(i) $\frac{1}{2}r \approx 0$; this gives $k \cot\delta = \frac{1}{a} = -\gamma$ and is the zero range theory when we have a bound state. The phase shift, in this case, begins at π at the threshold and gradually decreases with energy.

(ii) $\frac{1}{\gamma} > \frac{1}{2}r > 0$; in this case, $\frac{1}{a}$ is negative, so that somewhere in the physical region $k \cot \delta = 0$ or δ is $\frac{\pi}{2}$. The phase shift begins at π at the threshold and decreases so as to fall through $\frac{\pi}{2}$. In this case a situation giving a Breit-Wigner type of resonance is conceivable[100].

(iii) $\frac{1}{2}r > \frac{1}{\gamma}$; in this case $\frac{1}{a}$ is always positive. The phase shift begins at zero at the threshold and increases with energy. $k \cot \delta$ is always positive. We shall take this to be the right situation for $\bar{K}N$ elastic scattering in I = 1 state. Then, we at once get a lower limit of the effective range:

$$\frac{1}{2}r > 1.1 \quad \text{Fermi} \quad (\frac{1}{\gamma} = 1.1) \tag{IV.4}$$

which is obviously very large.

Let us now suppose that the pion-hyperon interaction is switched on so that instead of equations (1) and (2) we have

$$k \cot \delta = \frac{1}{A} + \frac{1}{2} RK^2 \tag{IV.5}$$

$$\frac{1}{A} = -\alpha + \frac{1}{2} R\alpha^2 \tag{IV.6}$$

where δ, A, R are all complex quantities and

$$\alpha^2 = 2\mu(B + i\eta) \tag{IV.7}$$

$$\eta = \frac{\Gamma}{2} = \text{half width of } Y^*$$

The Dalitz-Tuan[17,94] bound state theory for (a-) solution follows simply now, if we put R = 0; this gives

$$k \cot \delta = \frac{1}{A} = -\alpha$$

Taking A = a + ib we get,

$$\frac{a}{a^2 + b^2} = -\mathrm{Re}\,\alpha \ , \quad \frac{b}{a^2 + b^2} \doteq \mathrm{Im}\ \alpha$$

The position of the bound state is now given by

$$m_N + m_K - B = m_N + m_K - \frac{\mathrm{Re}(\alpha^2)}{2\mu}$$

$$= m_N + m_K - \frac{1}{2\mu}\left(\frac{a}{a^2 + b^2}\right)^2$$

$\mathrm{Re}(\alpha)^2 \simeq (\mathrm{Re}\,\alpha)^2$

if $(\mathrm{Im}\,\alpha)^2 \ll (\mathrm{Re}\,\alpha)^2$

$$\simeq m_N + m_K - \frac{1}{2\mu}\frac{1}{a^2}$$

$(|a| \gg b)$

while its width is given by

$$\eta = \frac{\Gamma}{2} = \frac{\mathrm{Im}(\alpha^2)}{2\mu} = \frac{1}{2\mu}\, 2\, \mathrm{Re}\,\alpha\ \mathrm{Im}\,\alpha = \frac{1}{\mu}\, \frac{-a}{a^2+b^2} \cdot \frac{b}{a^2+b^2}$$

$$\simeq \frac{1}{\mu} \quad \frac{b}{|a|^3}$$

Taking $a = -1.09$ and $b = 0.2o$, we at once get the observed position and half-width of Y*.

We next try to fit the present experimental data with a large effective range ($\mathrm{Re}\,\frac{1}{2} R > 1.1$). The formula for σ_{el}, σ_{ex} and $\sigma_{ab}^{1,0}$ have been derived by Jackson and Wyld[86] and by Dalitz and Tuan[87]. The only thing we have to do is to replace A_1 in these formulae by A_1 (K) where $\frac{1}{A_1(K)} = \frac{1}{A_1} + \frac{1}{2} Rk^2$ and $\frac{1}{A_1}$ is given by equation (6).

We find that we can fit the present low energy data with an effective range

$\frac{1}{2} R = 1.29 - i\ 0.277$ Fermi and

$A_o = 0 + i\ 1.1$ " (IV.8)

which gives at 175 MeV/c K^- momentum, with no Coulomb included, the following cross-sections

$$\sigma_{el} = 77.2\text{mb}$$
$$\sigma_{ex} = 14.0\text{mb}$$
$$\sigma(\Sigma^- + \Sigma^+) = 38.4\text{mb}$$

assuming $\epsilon = \left(\frac{\Lambda}{\Lambda + \Sigma}\right) = 0.5$. Also the ratio $\frac{\sigma_o}{\sigma_1}$ (k = 0) comes out to be 4.15 which is within the present experimental value $\epsilon = 5.5 \pm 1.4$. In Fig. 1 we have plotted σ_{el} when Coulomb effect is not included as well as when it is included (the mass difference of K^-p and $\bar{K}_0 n$ thresholds has, of course, always been taken into account). The cut off used by us for Coulomb scattering is the same as that of Jackson and Wyld[86]. In Fig. 2, we have plotted $|T|^2$ i.e. $\left(\frac{1}{4\pi} \frac{k\sigma_{ex}}{k'}\right)$ following a suggestion by Matthews[101]. It is worth mentioning that we have chosen our effective range such that at 175 MeV/c our scattering lengths $A_1(k)$ and A_0 coincide with the Dalitz (a+) solution. We have also verified that our $\sigma(\Sigma^- + \Sigma^+)$ agree with the Kiev data[96] (Fig. 3)

Finally, a few words about the $\frac{\Sigma^-}{\Sigma^+}$ ratio. This is given by

$$\frac{\Sigma^-}{\Sigma^+} = \frac{1 + \frac{2}{3} J^2 - \sqrt{6}\, J \cos \phi}{1 + \frac{2}{3} J^2 + \sqrt{6}\, J \cos \phi}$$

where $\frac{T^1_{\bar{K}N:\Sigma\pi}}{T^0_{\bar{K}N:\Sigma\pi}} = Je^{-i\phi}$, ϕ being the phase difference of Σ-production amplitudes in isotopic spin 0 and 1.

$$\phi = \phi(E_t) + \tan^{-1} \frac{ka_0}{1 + kb_0} - \tan^{-1} \frac{ka_1 - (a_1 s + b_1 r)k^2}{1 + kb_1 + (a_1 r - b_1 s)k^2} \qquad \text{(IV.9)}$$

where $\phi(E_t)$ is the phase difference at the k^-p threshold and $\frac{1}{2}R \equiv r + is$. Experimentally, $\phi(E_t) \sim \pm 60^\circ$ which is very large and has been a headache for theoreticians. From global symmetry conditions[102], one expects this to be small. In the next section we shall attempt to give an explanation of this.

Using equation (9), we find that the relative phase changes by -32°, for the effective range used by us, when we go from the threshold to $P_L = 175$ MeV/c. If now $\phi(E_t) = -60^{\circ}$, this would mean $\frac{\Sigma^-}{\Sigma^+} \approx 1$, since ϕ would be -90°. This is, of course, indicated by the present experimental data. In this context, it is possibly worth mentioning a remark by Dalitz and Tuan[87] that, if Coulomb interference is constructive, then the (a+) solution, together with $\phi(E_t) \sim -60^{\circ}$, giving an upward cusp in $\frac{\Sigma^-}{\Sigma^+}$ ratio at $\bar{K}^0 n$ threshold, seems to be the best candidate (the upward cusp is reminiscent of the large $\frac{\Sigma^-}{\Sigma^+}$ ratio obtained by the Berkeley group in the momentum range 50-100 MeV/c).

IV.3 $\bar{K}N$ I = 0 Bound State

As we have seen, our set of parameters which fit the experimental data (when no Coulomb is taken into account) corresponds to Re Ao = 0. This really indicates that Re Ao is small (the corresponding Dalitz (a+) solution with errors is Ao = 0.05 $\pm$ 0.2 + i (1.1 $\pm$ 0.25); A_1 = 1.45 $\pm$ 0.2 + i (0.35 $^{+0.09}_{-0.07}$)). Assuming the existence of a $\bar{K}N$ I = 0 bound state lying below the $\Sigma\pi$ threshold and using the T^{-1} matrix formalism[103] we can explain the smallness of Re Ao. Such a bound state with binding energy much greater than Y*. has been postulated by Sakurai and by Gell-Mann[108].

Let us denote the $\bar{K}N$ channel by '1' and the $\Sigma\pi$ channel by '2'. Let us suppose, W = a is the position of this bound state. We next write down two subtracted dispersion relations[109] for the elements of T^{-1} matrix with the subtraction point at W = a. These relations can be put down in the form

$$T^{-1} = A^{-1} + \alpha - ik, \tag{IV.10}$$

where

$$\alpha - ik = \begin{pmatrix} \alpha_1 - ik_1 & o \\ o & \alpha_2 - ik_2 \end{pmatrix}$$

$$\alpha_1 = -\frac{(W-a)^2}{\pi} \int_{W_1} \frac{k_1' \, dW'}{(W'-W)(W'-a)^2} , \tag{IV.11a}$$

$$\alpha_2 = -\frac{(W-a)^2}{\pi} \int_{W_2} \frac{k'_2 \, dW'}{(W'-W)(W'-a)^2} , \tag{IV.11b}$$

w_1 and w_2 are the thresholds of $\bar{K}N$ and $\Sigma\pi$ channels and A^{-1} matrix elements contain the lefthand contributions, T^{-1} matrix elements at the subtraction point and derivative terms. Time reversal invariance and unitarity require A^{-1} to be a symmetric real matrix.

Equation (10) can be written in the following form

$$AT^{-1} = 1 + A\alpha - iAk \qquad \text{(IV.12)}$$

which is exactly the BT^{-1} dispersion formulation of Feldman Matthews and Salam[29], if A can be approximated by the Born matrix. We note that since the subtraction point is far below the $\bar{K}N$ threshold, so the energy dependence of A^{-1} and α may be neglected in the low energy K^-p scattering region and we shall have the zero range theory, as seen below.

For our following discussion, we use equation (12). Then

$$T_{11} = \left\{ \left[1 + A\,(\alpha - ik) \right]^{-1} A \right\}_{11} \qquad \text{(IV.13a)}$$

Again

$$T_{11} = \frac{1}{\frac{1}{a_o + ib_o} - ik_K} = \frac{1}{x_o - iy_o - ik_K} \qquad \text{(IV.13b)}$$

$$\text{where} \quad \frac{1}{a_o + ib_o} = x_o - iy_o$$

From (13a) and (13b), we get,

$$x^o = \frac{(1 + \alpha_2 A_{22})(A_{11} + \alpha_2\Delta) + k_\Sigma^2 A_{22}\Delta}{(A_{11} + \alpha_2\Delta)^2 + (k_\Sigma \Delta)^2} + \alpha_1 \qquad \text{(IV.14a)}$$

$$y^o = k_\Sigma \frac{A_{22}\,A_{11} - \Delta}{(A_{11} + \alpha_2\,\Delta)^2 + (k_\Sigma \Delta)^2} \quad (\Delta \equiv \text{Det } A) \qquad \text{(IV.14b)}$$

From (14a) and 14b) we can obtain the following relation

$$\Delta\beta = (A_{22} - \frac{y_o}{k_\Sigma} A_{11}) \qquad \text{(IV.14c)}$$

where $\beta \equiv (x_o - \alpha_1 + \frac{y_o}{k_\Sigma}\alpha_2)$.

The $\bar{K}N$ and $\Sigma\pi$ amplitudes should have a pole corresponding to our $\bar{K}N$ I = 0 bound state. Usually, one expects that when the T-matrix is diagonalized, this pole should occur in one of the diagonal elements[110]. In other words, we should be able to write the T-matrix in the following form

$$T = U^{-1} \begin{pmatrix} \frac{R}{W - a} & \\ & e \end{pmatrix} U \qquad U = \begin{pmatrix} \cos\epsilon & \sin\epsilon \\ -\sin\epsilon & \cos\epsilon \end{pmatrix}$$

where the pole terms correspond to the Born amplitudes for the different processes and the terms involving 'e' represent contributions from other singularities. The Born matrix is given by $B = U^{-1}$ $\begin{pmatrix} \frac{R}{W - a} & \\ & o \end{pmatrix} U$, so that we have $\text{Det } B = 0$. (IV.15).

Again, in a situation like this where the important lefthand contributions may be thought of coming from the pole term, the FMS dispersion formulation[29] is applicable and we can approximate our 'A' by B. Equation (15) then, at once, leads to

$$\text{Det } A \approx 0 \qquad . \tag{IV.16}$$

Using (16) in (14a), we get,

$$x_o \approx \frac{1}{A_{11}} + \alpha_2 \frac{y_o}{k_\Sigma} + \alpha_1 \tag{IV.17}$$

From (10), $\text{ReT}^{-1} = A^{-1} + \alpha$, so that taking determinant on both sides,

$$\text{Det ReT}^{-1} = \frac{1 + A_{11}\alpha_1 + A_{22}\alpha_2 + \alpha_1\alpha_2\Delta}{\Delta} \quad (\Delta = \text{Det } A)$$

If now, Det ReT^{-1}($= \frac{1}{\text{Det } K}$ where K is the K-matrix) is not very large[111], then putting $\Delta \approx 0$, gives

$$1 + A_{11}\alpha_1 + A_{22}\alpha_2 \approx 0 \quad .$$

From (14c), we have for $\Delta \approx 0$, $A_{22} \approx \frac{y_o}{k_\Sigma} A_{11}$, so that using this in the above relation, we get

$$\frac{1}{A_{11}} + \alpha_1 + \alpha_2 \frac{y_o}{k_\Sigma} \approx 0 \quad . \tag{IV.18}$$

Comparing (17) and (18), we expect x_o to be small, which is, of course, indicated by our parameters.

The $\Sigma\pi$ elastic scattering amplitude below the $\bar{K}N$ threshold is given by

$$T_{22} = \frac{(T^{-1})_{11}}{(T^{-1})_{22}} T_{11} = \frac{A_{22} + (\alpha_1 + |k_K|)\Delta}{A_{11} + (\alpha_2 - ik_\Sigma)\Delta} \times \frac{1}{(x_o + |k_K|) - iy_o}$$

$$\approx \frac{1}{\frac{k_\Sigma}{y_o}(x_o + |k_K|) - ik_\Sigma} \quad (\text{using } \Delta \approx 0) \tag{IV.19}$$

Again, $T_{22} = \dfrac{1}{k_{\Sigma} \operatorname{Cot} \delta_{\Sigma\pi} - ik_{\Sigma}}$ (IV.20)

Therefore from (19) and (20), we find that if x_o is small andnegative, then just below the $\bar{K}N$ threshold, $k_{\Sigma} \operatorname{Cot} \delta_{\Sigma\pi} = 0$ or the elastic phase shift passes through $\pi/2$.

This at once leads to an important result. Since the phase of the non-diagonal amplitude is sum of the phases of the diagonal amplitudes[112] from unitarity, so the Σ-production amplitude in $I = 0$ state should have a phase shift as large as $\frac{\pi}{2} + n\pi$ $(n = 0, \pm 1)$. This will provide an explanation[113] why this relative phase of Σ-production amplitudes in $I = 0$ and in $I = 1$ states is so large[96] $(\sim \pm 60^o)$.

Equation (19) can be written in the following way,

$$T_{22} = \frac{(|k_K| - x_o)}{\frac{k_{\Sigma}}{y_o}\left(|k_K|^2 - x_o^2\right) - i\left(|k_K| - x_o\right) k_{\Sigma}} \qquad \text{(IV.21a)}$$

$$= \frac{1}{2k_{\Sigma}} \frac{\Gamma}{(E_o - E) - i\Gamma/2} \qquad (E < o)$$

where $$\frac{\Gamma}{2} = \frac{|x_o|\, y_o}{\mu} \qquad \text{(IV.21b)}$$

$$E = -\frac{|k_K|^2}{2\mu}, \quad E_o = -\frac{|x_o|^2}{2\mu} \approx 0 , \qquad \text{(IV.21c)}$$

which shows that the elastic $\Sigma\pi$ amplitude below $\bar{K}N$ threshold in $I = 0$ state has a Breit-Wigner resonance form with a narrow width ($|x_o|$ being small).

The elastic $\Sigma\pi$ scattering amplitude above $\bar{K}N$ threshold in I = 0 state is given by

$$T_{22} = \frac{1}{\frac{k_\Sigma x_o}{y_o} - ik_\Sigma\left(1 + \frac{k_K}{y_o}\right)}$$

IV.4 Conclusion

We conclude that the low energy K^-p scattering data is consistent with a large effective range for I = 1 state which gives positive Coulomb-nuclear interference. The real part of the zero range scattering length in I = 0 state is, in our opinion, small and negative[115]. The smallness can be accounted for by assuming the existence of a $\bar{K}N$ I = 0 bound state lying below $\Sigma\pi$ threshold. If, further, Re A_0 is negative, then we get a narrow $\Sigma\pi$ resonance[116,117] just below $\bar{K}N$ threshold, which will provide an explanation of the large relative phase of Σ - production amplitudes at $\bar{K}N$ threshold.

Besides a complex scattering length, we have a complex effective range for I = 1 state. This means we have two more parameters than that of Dalitz-Tuan. For this reason, we needed two more pieces of data to fix the parameters (viz. the mass and the width of Y*) while they can predict these two things with their (a-) solution. The main justification of our approach is that experiments may well confirm the Coulomb-nuclear interference in low energy K^-p scattering to be constructive[118] and it is worthwhile to consider this possibility.

FIGURE CAPTIONS

Fig.1 The total elastic scattering cross-section as a function of the laboratory momentum. The continuous curve corresponds to nuclear interaction only, normalized so as to agree with the 172 MeV/c data. The dotted curve corresponds to nuclear plus Coulomb effect, with the cut-off angle determined by the criterion that the recoil proton have a laboratory momentum of at least 30 MeV/c. The experimental data are the bubble chamber results of ref. (96) and the emulsion results of ref. (97).

Fig. 2 Variation of the function $|T|^2$ with laboratory momentum of the incident K-meson. $|T|^2$ represents the charge exchange cross-section without the kinematic factor $4\pi\frac{k'}{k}$. The experimental data are that of ref. (96).

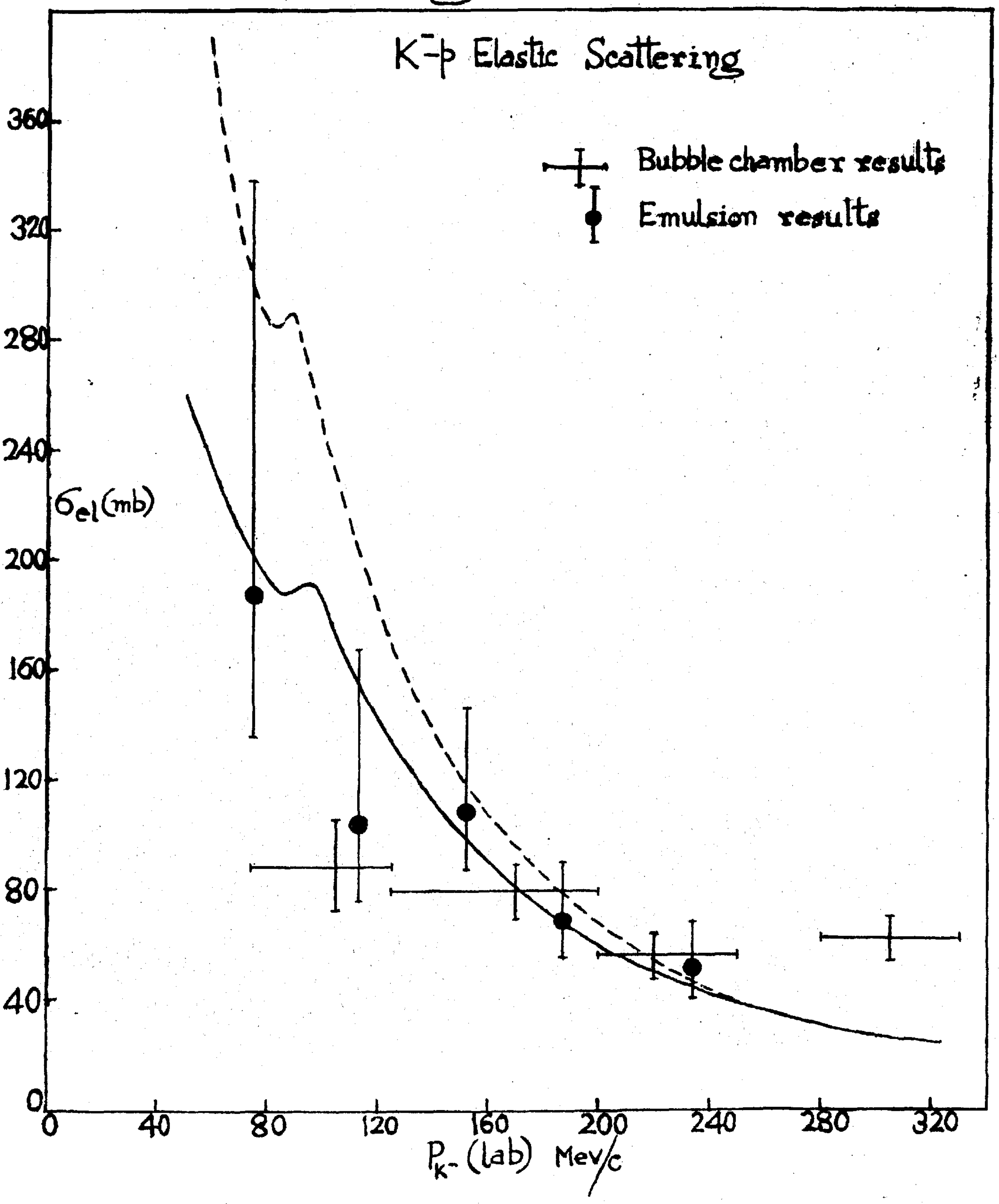
Fig. 3 $(\Sigma^- + \Sigma^+)$ production cross-section plotted against K-meson laboratory momentum. The experimental data ate of ref. (96).

Fig. 4
$$\begin{aligned} \mathrm{Re}\, T^{-1} &= k \cot\delta \quad (k^2 > 0) \\ &= |k| \cot\delta' \quad (k^2 < 0) \end{aligned}$$
plotted against k^2 ($\equiv$ E') for the two cases:

(a) when scattering length is positive,

(b) when it is negative.

Fig. 1


K⁻-p Elastic Scattering
Bubble chamber results
Emulsion results
σ_{el}(mb)
0
40
80
120
160
200
240
280
320
360
0
40
80
120
160
200
240
280
320
P_{K^-} (lab) Mev/c

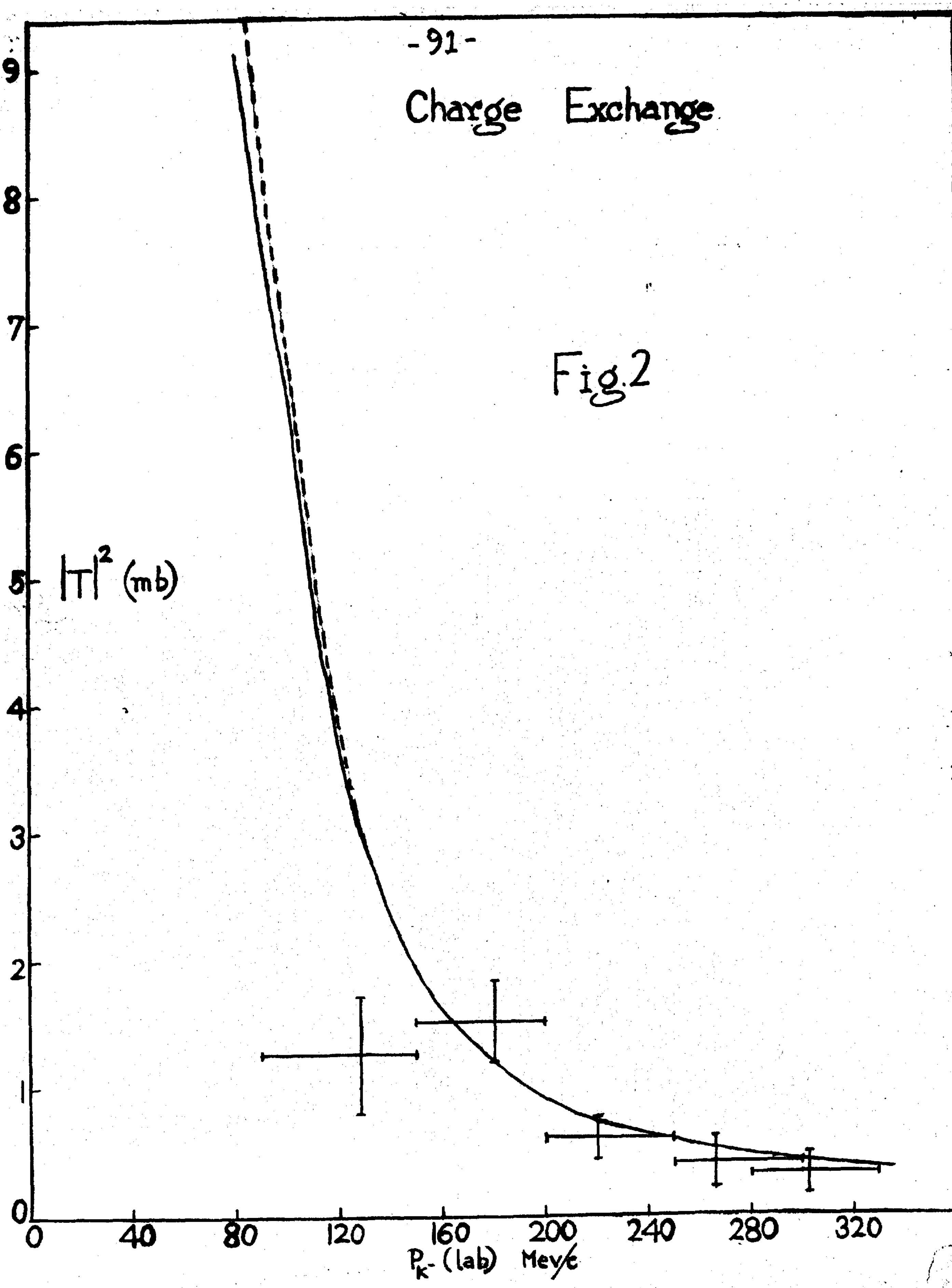
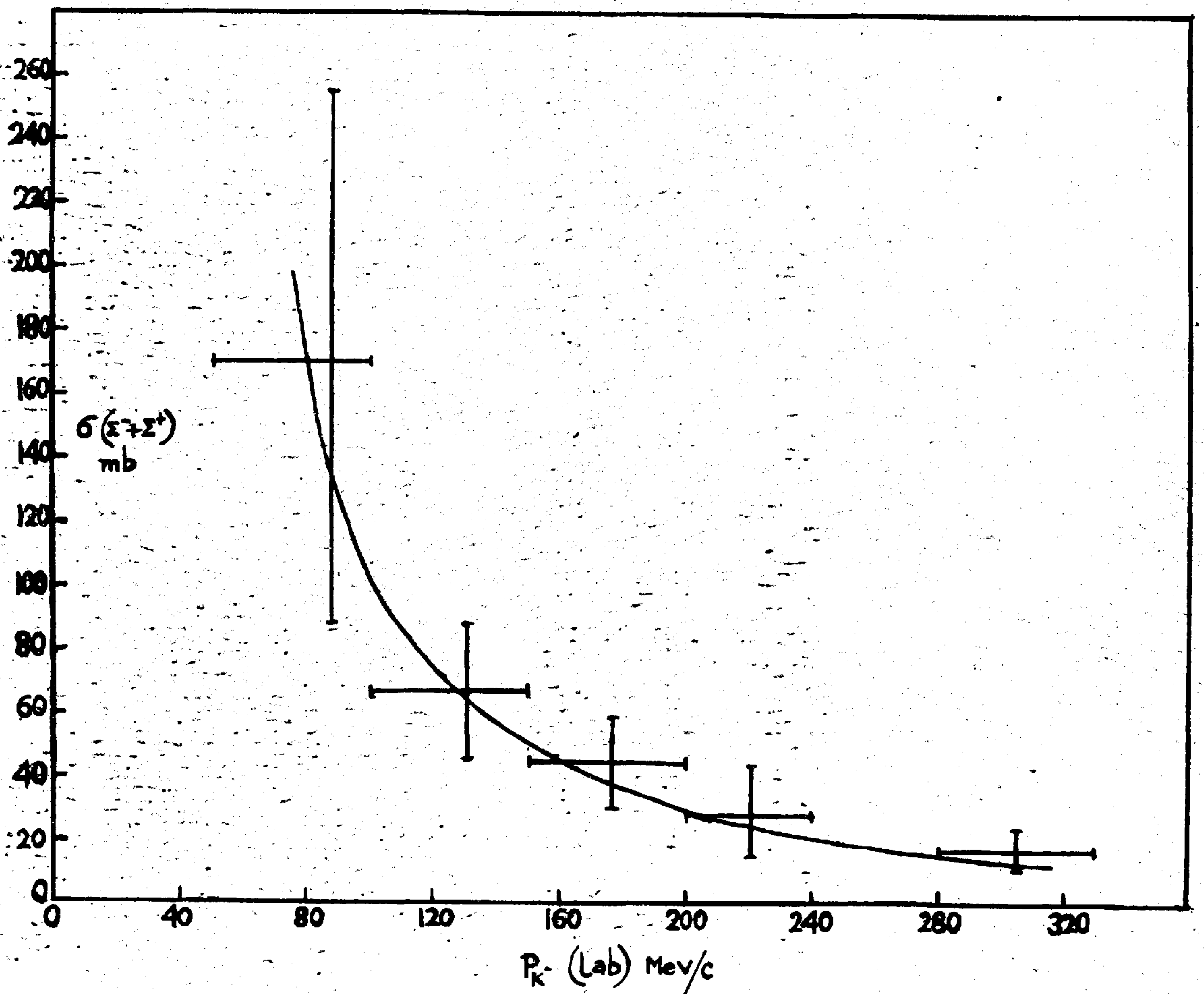
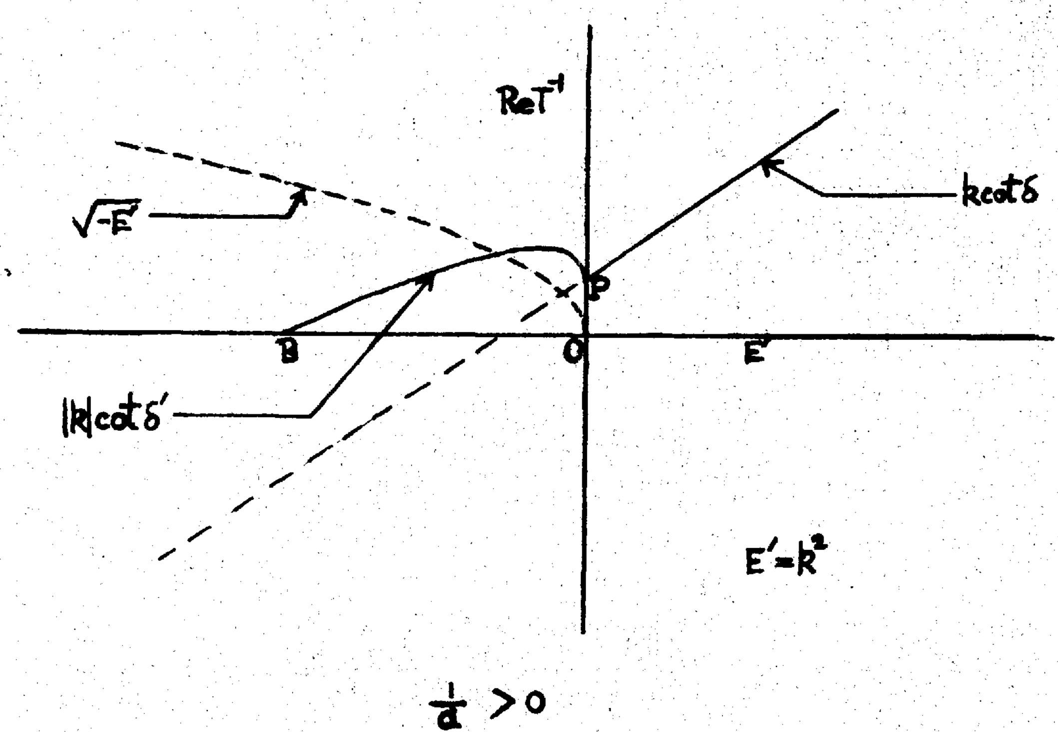

Charge Exchange
Fig.2
$|T|^2$ (mb)
0
1
2
3
4
5
6
7
8
9
0
40
80
120
160
200
240
280
320
P_{K^-} (lab) Mev/c

Fig. 3

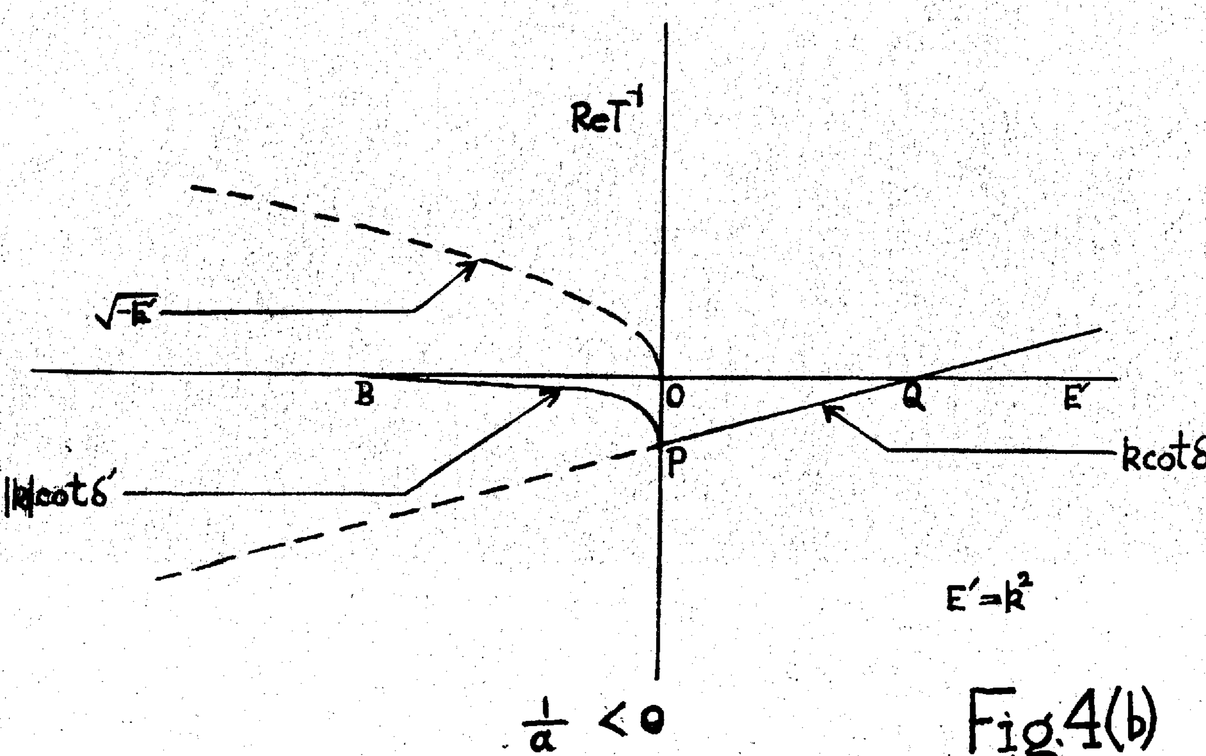


σ(Σ⁻+Σ⁺) mb
0
20
40
60
80
100
120
140
160
180
200
220
240
260
0
40
80
120
160
200
240
280
320
Pₖ₋ (Lab) Mev/c



$Re T^{-1}$
$\sqrt{-E'}$
$k\cot\delta$
P
B
O
E'
$|k|\cot\delta'$
$E'=k^2$
$\frac{1}{a}>0$

Fig. 4(a)



$Re T^{-1}$
$\sqrt{-E'}$
B
O
Q
E'
P
$k\cot\delta$
$|k|\cot\delta'$
$E'=k^2$
$\frac{1}{a}<0$

Fig. 4(b)

APPENDIX 1

In this appendix, we re-examine our investigation on K-meson parity, given in Chapter I, using the new experimental data available in the high energy region[119, 120], and the theory put forward by us in Chapter IV on low energy K^-p scattering.

In evaluating the dispersion integral in Chapter I, we took σ^+ to be constant at a value 15 mb all throughout and σ^- at a value 40 mb above K-meson kinetic energy 120 MeV. We used a cut-off 5 m_k following the data of Burrowes et al[7]. The present high energy data extends up to K-meson mementum 8 GeV/c. We shall therefore use a cut-off 16 m_k in our new evaluation. We shall take σ^+ to be constant at a value 15 mb as before, but σ^- at a value 30 mb above K-meson K.E. 120 MeV. To evaluate the dispersion integrals in the unphysical region and in the low energy K^-p scattering region up to K-meson lab. energy 120 MeV (i.e. up to K-meson lab. momentum 344 MeV/c), we use the effective range and the scattering lengths obtained by us in Chapter IV which fit low energy K^-p scattering data. The imaginary part of the forward scattering amplitude in terms of these parameters is given by

$$A^- = \frac{W}{N}\,\frac{1}{2}\left[\frac{q+y_1-sq^2}{(x_1+rq^2)^2+(q+y_1-sq^2)^2} + \frac{q+y_0}{x_0^2+(q+y_0)^2}\right] \quad (q^2>0) \tag{A1.1}$$

$$= \frac{W}{N}\frac{1}{2}\left[\frac{y_1 + s|q|^2}{(x_1+|q|-r|q|^2)^2 + (y_1+s|q|^2)^2} + \frac{y_0}{(x_0+|q|)^2+y_0^2}\right] \quad (q^2<0) \tag{A1.2}$$

where q = c.m. momentum, $\frac{1}{2}R = r + is$, $\frac{1}{A_1} = -\gamma + \frac{1}{2}R\gamma^2 = x_1 - iy_1$ and $\frac{1}{a_o + ib_o} = x_o - iy_o$. Using equations (A1.1) and (A1.2), we can evaluate the dispersion integrals numerically.

For Matthews-Salam dispersion relation, we obtain

$$\frac{1}{6\pi^2}\int_{W_{N\pi}}^{1+t} \mathrm{Im}\,M^-\left(\frac{1}{w'-1} - \frac{1}{w'+1}\right)dw' = 1.07 \tag{A1.3a}$$

$$\frac{\sigma^-}{6\pi^2}\int_{1+t}^{16} k'\left(\frac{1}{w'-1} - \frac{1}{w'+1}\right)dw' = 1.79 \tag{A1.3b}$$

$$\left(\mathrm{Im}\,M^- = k\,\sigma_T,\ \sigma^- = 30\text{ mb} = 18.75\frac{1}{K^2}\right)$$

$$\frac{\sigma^+}{6\pi^2}\int_{1}^{16} k'\left(\frac{1}{w'-1} - \frac{1}{w'+1}\right)dw' = 1.10 \tag{A1.3c}$$

$$\left(\sigma^+ = 15\text{ mb} = 9.38\frac{1}{K^2}\right)$$

Further, $b = \frac{N}{N+1} D^-(1) = \mathrm{Re}\,A_1$

$$= 4.30 \tag{A1.4}$$

Thus M-S dispersion relation gives, instead of eqn. (1.23),

$$4.30 + 0.86 = 1.07 + 1.79 - 1.10 + [B\,S] \tag{A1.5}$$

so that $[B\,S] = 3.40$ (A1.6)

For Igi's dispersion relation, we obtain

$$\int_{w_{\Lambda\pi}}^{1+t} \frac{\mathrm{Im}\, M^- dw'}{k'^2 (w'+w)} = 13.88,$$

$$\sigma^- \int_{1+t}^{\infty} \frac{dw'}{k'(w'+w)} = 12.57,$$

so that instead of eqn. (I.29), we have

$$4.30 + 0.86 + 0.16 = \frac{0.66}{\pi^2} (- 938 + 13.88 + 12.57) + \frac{4M_p}{M_p + 1} .2F \qquad (A1.7)$$

$$\text{Therefore, } \frac{4M_p}{M_p + 1} .2F = 4.18 \qquad (A1.8)$$

The positive sign of the bound state term in equations (A1.6) and (A1.8) shows that K-meson cannot be scalar relative to both the hyperons. Comparing (A1.5) with equation (I.23), we find that the contribution from the high energy region has not changed appreciably. The only term in (A1.5) or in (A1.7) which considerably differs from the corresponding term in (I.23,29) is the magnitude of b (b = 4.30, in the present case, is to be compared with b = 1.68 taken before). The larger value of b has increased the value of the bound state term and therefore gives much larger coupling constants. However, we must not take the value of b literally, because our parameters fitting the low energy K^-p data are quite rough. The extreme variation of

σ^+ which was considered in Chapter I as a possible case, seems to be ruled out by present experimental data.

Our only important conclusion is, therefore, that K is pseudoscalar at least relative to one of the hyperons, and the pseudoscalar coupling constant is comparable to π- N coupling constant.

APPENDIX 2

For two channel case, we may write[121]

$$T^{-1} = \begin{pmatrix} a - ik_1 & c \\ c & b - ik_2 \end{pmatrix} \tag{A2.1}$$

when both the channels are open. The K-matrix, in this case, is given by

$$k^{1/2} K k^{1/2} = \begin{pmatrix} a & c \\ c & b \end{pmatrix}. \tag{A2.2}$$

If we now define the condition for resonance as the vanishing of the denominator of K-matrix elements, as has been used by ASV, we have

$$ab - c^2 = 0 \quad . \tag{A2.3}$$

However, there are two other ways of defining the condition for resonance. One is to take

$\mathrm{Re}\,(\mathrm{Det}\ T^{-1}) = 0$, which gives

$$ab - c^2 - k_1 k_2 = 0 \quad , \tag{A2.4}$$

while the other is

$$Re \left(\frac{1}{T_{ij}}\right) = 0 \qquad , \qquad (A2.5)$$

which gives the position of resonance for '11' process as

$$(ab - c^2)b + k_2^2\, a = 0 \ , \qquad (i) \qquad (A2.6)$$

for '12' or '21' process as

$$ab - c^2 - k_1 k_2 = 0 \qquad , \qquad (ii)$$

and for '22' process as

$$(ab - c^2)\, a + k^2 b = 0 \ . \qquad (iii)$$

If we neglect the momentum dependent terms, we find that (A2.3), (A2.4) and (A2.6) are the same.

Let us now suppose that channel 2 is closed. Then, if we completely ignore this channel, the condition for resonance for the '11' process is

$$a = 0 \qquad . \qquad (A2.7)$$

If, on the other hand, we takeinto account the presence of channel 2, by making the continuation $k_1 \rightarrow i\,|k_2|$, the condition for resonance becomes, in the K-matrix formalism,

$$a(b + |k_2|\,) - c^2 = 0 \qquad . \qquad (A2.8)$$

If we neglect the momentum dependent term, we get,

$$ab - c^2 = 0 \qquad . \qquad (A2.9)$$

If the coupling between channel 1 and 2 is weak, then 'c' is small, so that (A2.8) or (A2.9) gives essentially $a \approx 0$ as the condition for resonance. For strongly coupled channels, however, we can expect considerable deviation from $a \approx 0$.

APPENDIX 3

In Chapter III, we used the principle of crossing relation which essentially states that the three processes:

I $K + N \rightarrow K + N$

$(q_1 + p_1 \rightarrow - q_2 - p_2)$ (A3.1)

II $\bar{K} + N \rightarrow \bar{K} + N$ $(q_2 + p_1 \rightarrow - q_1 - p_2)$

III $K + \bar{K} \rightarrow N + \bar{N}$ $(q_1 + q_2 \rightarrow - p_1 - p_2)$

have the same scattering matrix element. We shall show this using the Lehmann, Syma nzik, Zimmerman technique.

The nucleon field in terms of plane wave states is

$$\psi(x) = \frac{1}{(2\pi)^{3/2}} \sum_{r,\lambda} \int d^3p \left(\frac{m}{p_0}\right)^{1/2} \left[c_{\lambda r}(\underline{p}) e^{ip.x} \xi_\lambda u_r(p) + d^+_{\lambda r}(\underline{p}) e^{-ip.x} \xi_\lambda v_r(p) \right] \tag{A3.2}$$

where λ is the isotopic spin index and r is the spin index;

λ = 1 for p and $\bar{p}$, 2 for n and $\bar{n}$; $\xi_1 = \binom{1}{0}$, $\xi_2 = \binom{0}{1}$; $p.x = \underline{p}.\underline{x} - p_0 x_0$.

Defining, $h_{\lambda r}(p,x) = \frac{1}{(2\pi)^{3/2}} \left(\frac{m}{p_0}\right)^{1/2} e^{ip.x} u_r(p) \xi_\lambda$

(A3.3)

and $u_{-r}(-p) = v_r(p)$ $(r = 1, 2)$,

we can write the nucleon field as

$$\psi(x) = \sum_{\lambda, r} \int d^3p \left[c_{\lambda r} h_{\lambda, r}(p,x) + d_{\lambda r} h_{\lambda, -r}(-p,x) \right] \tag{A3.4}$$

The creation and destruction operators for nucleons and anti-nucleons are

$$c_{\lambda r}(\underline{p}) = \int d^3x \; \bar{h}_{\lambda, r}(p,x)\, \gamma_4 \, \psi(x) \qquad \text{(A3.5)}$$

$$c^{+}_{\lambda r}(\underline{p}) = \int d^3x \; \bar{\psi}(x)\, \gamma_4 \, h_{\lambda, r}(p,x)$$

$$d_{\lambda r}(\underline{p}) = \int d^3x \; \bar{\psi}(x)\, \gamma_4 \, h_{\lambda, -r}(-p,x)$$

$$d^{+}_{\lambda r}(\underline{p}) = \int d^3x \; \bar{h}_{\lambda, -r}(-p,x)\, \gamma_4 \, \psi(x)$$

The K-meson field in terms of plane wave states is

$$\phi(x) = \sum_{\nu=1,2} \int d^3p \left[a_{\underline{p},\nu} f_{p,\nu}(x) + b^{+}_{\underline{p},\nu} f_{-p,\nu}(x) \right] \qquad \text{(A3.6)}$$

where 'ν' is the isospin index (ν = 1 for K^+ and K^-; ν = 2 for K^0 and $\bar{K}^0$) and

$$f_{p,\nu}(x) = \frac{1}{(2\pi)^{3/2}} \frac{e^{ip.x}}{\sqrt{2p_0}} \eta_\nu ,$$

η_ν being the isospin wave function.

The creation and destruction operators for kaons and anti-kaons are

$$a_{\underline{p},\nu} = i \int f^{+}_{p,\nu}(x) \overleftrightarrow{\partial_0} \phi(x) \, d^3x \qquad \text{(A3.7)}$$

$$a^{+}_{\underline{p},\nu} = i \int \phi^{+}(x) \overleftrightarrow{\partial_0} f_{p,\nu}(x) \, d^3x$$

$$b_{\underline{p},\nu} = -i \int \phi^{+}(x) \overleftrightarrow{\partial_0} f_{-p,\nu}(x) \, d^3x$$

$$b^{+}_{\underline{p},\nu} = -i \int f^{+}_{-p,\nu}(x) \overleftrightarrow{\partial_0} \phi(x) \, d^3x$$

Let us consider

$$\langle N(-p_2, j, s), K(-q_2, l) \underset{out}{} \,|\, N(p_1, i, r), K(q_1, k) \underset{in}{} \rangle$$

$$= \delta_{fi} + i\int d^4z \frac{\partial}{\partial z_0} f^+_{-q_2,l}(z) \overleftrightarrow{\frac{\partial}{\partial z_0}} \langle N(-p_2,j,s)|\,\phi(z)\,|N(p_1,i,r),K(q_1,k)\rangle_{in}$$

$$= \delta_{fi} + i\int d^4z\, f^+_{-q_2,l}(z)\,(\mu^2 - \overrightarrow{\Box}_z^2)\langle N(-p_2,j,s)|\,\phi(z)\,|N(p_1,i,r),K(q_1,k)\rangle_{in}$$

$$= \delta_{fi} - i^2\int d^4z\, d^4w \frac{\partial}{\partial w_0} f^+_{-q_2,l}(z)\,(\mu^2 - \overrightarrow{\Box}_z^2)\langle N(-p_2,j,s)|\,T(\phi(z)\phi^+(w))\,|N(p_1,i,r)\rangle$$
$$\times \overleftrightarrow{\frac{\partial}{\partial w_0}} f_{q_1,k}(w)$$

$$= \delta_{fi} + i^2\int d^4z\, d^4w\, f^+_{-q_2,l}(z)\,(\mu^2 - \overrightarrow{\Box}_z^2)\langle N(-p_2,j,s)|\,T(\phi(z)\phi^+(w))\,|N(p_1,i,r)\rangle$$
$$\times (\mu^2 - \overleftarrow{\Box}_w^2) f_{+q_1,k}(w) \quad \text{(A3.8)}$$

On the other hand,

$$\langle N(-p_2,j,s),\bar{K}(-q_1,k)_{out}|N(p_1,i,r),\bar{K}(q_2,l)\rangle_{in}$$

$$= \delta_{fi} + \int d^4w \frac{\partial}{\partial w_0} -i\langle N(-p_2,j,s)|\,\phi^+(w)\,|N(p_1,i,r),\bar{K}(q_2,l)\rangle_{in} \overleftrightarrow{\frac{\partial}{\partial z_0}} f_{q_1,k}(w)$$

$$= \delta_{fi} + i\int d^4w \langle N(-p_2,j,s)|\,\phi^+(w)\,|N(p_1,i,r),\bar{K}(q_2,l)\rangle_{in} (\mu^2 - \overleftarrow{\Box}_w^2) f_{q_1,k}(w)$$

$$= \delta_{fi} + i^2\int d^4w\, d^4z\, f^+_{-q_2,l}(z)\,(\mu^2 - \overrightarrow{\Box}_z^2)\langle N(-p_2,j,s)|\,T(\phi^+(w)\phi(z))\,|N(p_1,i,r)\rangle$$
$$\times (\mu^2 - \overleftarrow{\Box}_w^2) f_{q_1,k}(w)$$

(A3.9)

Comparing (A3.8) and (A3.9), we find that processes I and II in (A3.1) have the same scattering amplitude.

Let us now consider the matrix element for the process $K + \bar{K} \rightarrow N + \bar{N}$

$$\langle N(-p_2,j,s),\bar{N}(-p_1,i,r)_{out}|\bar{K}(q_2,l),K(q_1,k)\rangle_{in}$$

$$= \int d^4x \, \frac{\partial}{\partial x_0} \bar{h}_{j,s}(-p_2,x)\, \gamma_4 \langle \bar{N}(-p_1,i,r) | \psi(x) | \bar{K}(q_2,l), K(q_1,k) \rangle_{in}$$

$$= i \int d^4x \, \bar{h}_{j,s}(-p_2,x)\, \vec{D}_x \langle \bar{N}(-p_1,i,r) | \psi(x) | \bar{K}(q_2,l), K(q_1,k) \rangle_{in}$$

$$= i \int d^4x \, d^4y \, \bar{h}_{j,s}(-p_2,x)\, \vec{D}_x \frac{\partial}{\partial y_0} \langle 0 | T(\bar{\psi}(y)\, \psi(x)) | \bar{K}(q_2,l), K(q_1,k) \rangle_{in} \gamma_4 h_{i,-r}(p_1,y)$$

$$= i^2 \int d^4x \, d^4y \, \bar{h}_{j,s}(-p_2,x)\, \vec{D}_x \langle 0 | T(\bar{\psi}(y)\, \psi(x)) | \bar{K}(q_2,l), K(q_1,k) \rangle \overleftarrow{D}_y h_{i,-r}(p_1,y)$$

where $\vec{D}_x = \left(\gamma_\mu \frac{\vec{\partial}}{\partial x_\mu} + m \right)$, $\overleftarrow{D}_y = \left(-\frac{\overleftarrow{\partial}}{\partial y_\mu} \gamma_\mu + m \right)$.

If we now take out the K, $\bar{K}$ states, then we get ,

$$\langle N(-p_2,j,s), \bar{N}(-p_1,i,r) |_{out} \bar{K}(q_2,l), K(q_1,k) \rangle_{in}$$

$$= -i^4 \int d^4y \, d^4x \, d^4z \, d^4w \, \bar{h}_{j,s}(-p_2,x)\, \vec{D}_x f^{+}_{-q_2,l}(z)\, \vec{K}_z$$
$$\langle 0 | T(\bar{\psi}(y)\, \psi(x)\, \phi(z)\, \phi^{+}(w)) | 0 \rangle \overleftarrow{K}_w \overleftarrow{D}_y \qquad \text{(A3.10)}$$
$$\times f_{q_1,k}(w)\, h_{i,-r}(p_1,y)$$

where $\vec{K}_z = \left(\mu^2 - \vec{\Box}_z^2 \right)$, $\overleftarrow{K}_w = \left(\mu^2 - \overleftarrow{\Box}_w^2 \right)$.

Proceeding in the same way, we can obtain

$$\langle N(-p_2,j,s), K(-q_2,l) |_{out} K(q_1,k), N(p_1,i,r) \rangle_{in}$$

$$= \delta_{fi} + i^4 \int d^4x \, d^4y \, d^4z \, d^4w \, \bar{h}_{j,s}(-p_2,x)\, f^{+}_{-q_2,l}(z)\, \vec{D}_x \vec{K}_z$$
$$\langle 0 | T(\phi(z)\, \psi(x)\, \bar{\psi}(y)\, \phi^{+}(w)) | 0 \rangle \overleftarrow{K}_w \overleftarrow{D}_y$$
$$\times f_{q_1,k}(w)\, h_{i,r}(p_1,y) \qquad \text{(A3.11)}$$

Comparing (A3.10), (A3.11) and remembering that the fermion fields anti-commute, we find that the amplitudes for the processes I and III in (A1.1) are the same, if we separate out the Dirac spinors. This proves our crossing relation.

APPENDIX 4

In this Appendix, we discuss the relation between phase shift, bound state and resonance. We assume a single channel with a bound state. Then, we have

$$\begin{aligned} \mathrm{Re}\, T^{-1} &= k \cot \delta \qquad (k^2 > 0) \\ &= \frac{1}{a} + \frac{1}{2} r k^2 \end{aligned} \tag{A4.1}$$

The S-wave scattering amplitude is given by

$$T = \frac{1}{k \cot \delta - ik} \tag{A4.2}$$

and the correct continuation of $\mathrm{Re}\, T^{-1}$ below threshold $(k^2 < 0)$ is well known to be $k \rightarrow i\,|k|$. Let us define a phase shift by

$$|k| \cot \delta' \equiv \mathrm{Re}\, T^{-1} \qquad (k^2 < 0),$$

then

$$|k| \operatorname{Cot} \delta' = \frac{1}{a} - \frac{1}{2} r\, |k|^2 + |k| \tag{A4.3}$$

The bound state corresponds to

$$|k| \operatorname{Cot} \delta' = 0 \tag{A4.4}$$

and from (A.3), we then get the relation,

$$\frac{1}{a} = -\gamma + \frac{1}{2} r \gamma^2 \tag{A4.5}$$

where $\gamma^2 = 2\mu B$, B is the binding energy and μ is the reduced mass.

From (A4.1) and (A4.3) we find that

$$\lim_{k \to 0} k \cot \delta = \frac{1}{a} = \lim_{|k| \to 0} |k| \cot \delta'$$

Since $\frac{1}{a}$ is a constant, this, therefore, means that δ and δ' are either o or π at the threshold.

First of all, let us consider $\frac{1}{a}$ positive. From (A4.5) we find, in this case, $\frac{1}{2}r > \frac{1}{\gamma}$. We plot Re T^{-1} against $E' \equiv k^2$, as in Fig. 4 (a).

(See Fig. 4(a); Page 93)

At the point B, $|k| \cot \delta' = 0$ and this corresponds to the bound state. We take $\delta' = \pi/2$ at the bound state. Since $|k| \cot \delta'$ is always positive as we go from the bound state to the threshold, so δ' must fall from $\pi/2$ to 0 at the threshold. The phase shift δ begins from 0 in this case and increases.

Next we consider $\frac{1}{a}$ negative; then from (A4.5) $\frac{1}{2} r < \frac{1}{\gamma}$. In Fig. 4 (b) we have plotted Re T^{-1} against E'. Again, the point B where $|k| \cot \delta' = 0$, corresponds to bound state and $\delta' = \pi/2$.

Since $|k| \cot \delta'$ is always negative, therefore, δ' must increase from $\pi/2$ to π, as we go from the bound state to the threshold. The phase shift δ begins at π and gradually decreases and falls through $\pi/2$ at the point Q.

(See Fig. 4(b); Page 93)

This happens in neutron-proton triplet scattering where we have the deutron bound state. This falling of δ through $\pi/2$ is not called a resonance[122], since it does not give rise to a resonance peak. Let us examine this point.

From (A4.1) and (A4.2),

$$T = \frac{1}{\frac{1}{a} + \frac{1}{2} rk^2 - ik} = \frac{1}{\frac{1}{2} r(k^2 - k_o^2) - ik} = \frac{1}{2k} \frac{\Gamma}{(E - E_o) - i\frac{\Gamma}{2}} \quad \text{(A4.6)}$$

$$\text{where } \frac{1}{2} rk_o^2 = -\frac{1}{a}, \quad E = \frac{k^2}{2\mu} \text{ and } \frac{\Gamma}{2} = \frac{k}{\mu r} \quad \text{(A4.7)}$$

Obviously, (A4.6) is a Breit-Wigner type of resonance amplitude. However, the concept of a resonance is only useful when $\Gamma/2$ is small, so that there is a peak in $|T|^2$. If the effective range is small, (i.e. if the phase shift goes through $\pi/2$ very slowly) then the width will be very large (equation A4.7) and we can no longer talk about a resonance. This is what happens in $3S_1$ scattering of neutron-proton ($\frac{\Gamma}{2} = \frac{k_o}{\mu r} = 23.4$ MeV whereas $E_o = 9.3$ MeV). However, it is not inconceivable for r to be large so that the phase shift falls through $\pi/2$ rapidly; and we should then call it a resonance.

The extreme case, r = 0 gives $k \cot \delta = -\gamma$. This is the zero range theory with a bound state and corresponds to the Dalitz-Tuan bound state theory of Y*.

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31. We illustrate this by showing how the Chew-Low equations for all the p-wave amplitudes can be written down.

The Born amplitudes for the p-wave π-N scattering, in the static limit, are

$$B_{11} = \lambda_{11} \frac{k^2}{w} \qquad \lambda_{11} = -\frac{8}{3} f_N^2$$

$$B_{13} = B_{31} = \lambda_{13} \frac{k^2}{w} \qquad \lambda_{13} = -\frac{2}{3} f_N^2$$

$$B_{33} = \lambda_{33} \frac{k^2}{w} \qquad \lambda_{33} = \frac{4}{3} f_N^2$$

$$\left(f_N^2 = \frac{1}{4 m_N^2} \cdot \frac{g^2_{\pi N}}{4\pi} \right)$$

Also, $\mathrm{Re}\, T^{-1}_{1\pm} = k \cot \delta_{1\pm}$ (for single channel case)

From equation (I.1), we at once get[34],

$$\lambda_\alpha \frac{k^3 \cot\delta_\alpha}{w} = 1 - w r_\alpha \quad \text{where} \quad r_\alpha = \frac{\lambda_\alpha}{\pi} \int \frac{k'^3 dw'}{w'^2 (w' - w)}$$

Also, we find,

$r_{11} = -2\, r_{33}$, $r_{13} = r_{31} = -\frac{1}{2}\, r_{33}$ (no crossing term included).

This is to be compared with that of CG LN[25]

$r_{11} \cong r_{13} = r_{31} \cong -r_{33}$.

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Let us first suppose we are interested in the energy region near the threshold of our channel '2' (i.e. $\Sigma\pi$ channel) and that the bound state lies close to this threshold. We, then, expect A^{-1} to be much more important than α (physically, this means that the contribution from the Born term is much more important than the contribution from the unitarity cut). Obviously, in this situation, we cannot assume Det Re T^{-1} to be not very large. On the other hand, in our present case, we are interested in the energy region near the threshold of our channel '1' (i.e. the $\bar{K}N$ channel), which is far away from the bound state pole. We can now expect that the contribution from the unitarity cut to be quite important, so that Re T^{-1} is considerably different from A^{-1}. In this situation, our assumption that Det Re T^{-1} is not very large, appears to be justified.

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