MATEMATIKA, 2010, Volume 26, Number 2, 137–151

On Selection of Probability Distributions of Annual Maximum Daily Rainfalls Using TL-Moments

[1]**Ani Shabri and** [2]**Noratiqah Mohd Ariff**
[1] Universiti Teknologi Malaysia
Department of Mathematics, Faculty of Science, UTM, 81310, Skudai Johor
[2] Universiti Kebangsaan Malaysia
School of Mathematical Science, Faculty of Science and Technology, UKM, 43600 Bangi Selangor.
e-mail: [1]ani@utm.my

Abstract Knowledge related to distributions of rainfall amounts are of great important for designs of water related structures. The greater problem facing hydrologists and engineering is the identification the best distribution form for regional data. The main goal of the study is to perform regional frequency analysis of maximum daily rainfalls selected each year among daily rainfalls measured over stations in Selangor and Kuala Lumpur by using the method of TL-moment. Several distributions were taken into account in this study which include two-parameter normal (NOM), lognormal (LN2), three-parameter lognormal (LN3), logistic (LOG), generalized logistic (GLO), extreme value type I (EV1), generalized extreme value (GEV) and generalized Pareto (GPA) distribution. The most suitable distribution among the selected distributions was determined according to the mean absolute deviation index (MADI), mean square deviation index (MSDI) and the L-moment ratio diagram. The result of this study showed that the GLO distribution is the most suitable distribution to fit the data of maximum daily rainfalls for stations in Selangor and Kuala Lumpur.

Keywords L-moment; L-moment ratio; TL-moments; Annual maximum daily rainfalls

1 Introduction

Flood or also known as a deluge is an overflow of an expense of water that submerges land. Flood is a type of extreme environmental event that causes a lot of damage to life and property of human society. Knowledge related to distributions of rainfall amounts is of great importance for the design of water-related structure. Identification of the true statistical distributions for various hydrologic data sets to be a major problem facing engineers. Hence, the main objective is to specify the most suitable statistical distribution fit to the observations. In other words, the main purpose is to relate the magnitude of these extreme events to their frequency occurrence through the use of probability distributions. However, a reliable design quantile estimate is commonly impossible. Estimating the frequencies of extreme environmental events such as flood is difficult because extreme events are rare and the relevant data record is often short. Furthermore, there is no universal distribution to fit all the maximum daily rainfalls of any region.

The method of L-moments has been used increasingly by hydrologists Chen et al. [1]. For example, the L-moment method was used for analyzing the flood frequency analysis in Malaysia; Lim and Lye [2], Shabri and Ariff [3], Zin et al. [4], Zalina et al. [5], New Zealand; Pearson [6], Canada; Rao et al. [7], Glaves et al. [8], Yue and Wang [9], USA; Vogel et al. [10], India; Parida et al. [11], Kumar et al. [12], Iran; Chebana and Ouarda [13], Sicily, Italy; Eslamian and Feizi [14], Noto et al. [15], Pakistan; Hussain and Pasha [16], China; Chen et al. [1] and Turkey; Saf [17].

Probability distributions are used to analyze data in many disciplines and are often complicated by certain characteristics such as large range, variation or skewness. Hence, outliers or highly influential values are common Asquith [18]. Outliers can have undue influence on standard estimation methods Elamir and Seheult [19]. According to Elamir and Seheult [19], if there is a concern about extreme observations having undue influence, a robust method of estimation which is developed to reduce the said influence of outliers on the final estimates should be preferable. TL-moments are derived from L-moments and might have additional robust properties compared to L-moments. In other words, TL-moments are claimed to be more robust than the L-moment. Hence, for extreme data, TL-moments are also considered for estimating the parameters of the selected probability distributions.

Thus, this study focused on identifying a suitable probability distribution, including normal (NORM), logistic (LOG), generalized logistic (GLO), extreme value type I (EV1), generalized extreme value (GEV) and generalized Pareto (GPA) distributions by using TL-moments method for annual maximum daily rainfalls over the regions in Selangor and Kuala Lumpur, Malaysia. The TL-moments for all the said distributions were derived in order to be able to fit the rainfall data to the probability distributions. In the case of TL-moments which are symmetrically trimmed by one conceptual sample value for NOM and GP distributions, the TL-moments and their parameter estimates were computed and checked with those obtained by Elamir and Seheult [19]. Meanwhile, the TL-moments and their parameter estimates for GLO, EV1 and GEV distributions were derived since none had been done before. The results were then compared with those obtained using the method of L-moments similar to the previous study by Shabri and Ariff [3].

2 L-Moments

Hosking [20] developed the L-moment theory based on order statistic. The first four L-moment are defined as:

$$\begin{aligned}
\lambda_1 &= \beta_0 \\
\lambda_2 &= 2\beta_1 - \beta_0 \\
\lambda_3 &= 6\beta_2 - 6\beta_1 + \beta_0 \\
\lambda_4 &= 20\beta_3 - 30\beta_2 + 12\beta_1 - \beta_0
\end{aligned}$$

where β_r are PWMs defined as (Greenwood et al. [21])

$$\beta_r = \int_0^1 Q(f)F^r dF$$

F being nonexceedance probability. The L-moment ratios are calculated as:

$$\tau_2 = \frac{\lambda_2}{\lambda_1}, \quad \tau_3 = \frac{\lambda_3}{\lambda_2}, \quad \tau_4 = \frac{\lambda_4}{\lambda_2}$$

where τ_2,τ_3 and τ_4 are L-coefficients of variance (LCv), L-skewness (LCs) and L-kurtosis (LCk), respectively. Given a ranked sample $x_1 \leqslant x_2 \leqslant ... \leqslant 2_n$, an unbiased estimate of sample PWMs can be written as Hosking [20].

$$\hat{\beta}_r = \frac{1}{n}\sum_{i-1}^{n}\frac{(i-1)(i-2)...(i-r)}{(n-1)(n-2)...(n-r)}x_{(i)} = b_r$$

Therefore, the unbiased estimators for the λ_r are given by

$$\begin{aligned} l_1 &= b_0, \\ l_2 &= 2b_1 - b_0, \\ l_3 &= 6b_2 - 6b_1 + b_0, \\ l_4 &= 20b_3 - 30b_2 + 12b_1 - b_0. \end{aligned}$$

Sample estimates for τ are $t_r = l_r/l_2$ for $r = 3, 4$ and $t_2 = l_2/l_1$.

Parameters belonging to statistical distributions used in the study, connection of these parameters to the L-moments can be found in numerous literatures such as Hosking [20]; Hosking and Wallis [22]; Sankarasubramanian and Srinivasan [23].

3 TL-Moments

The fundamental concepts of TL-moments are essentially the same as L-moments. Elamir and Seheult [19] defined TL-moments as

$$\lambda_r^{(1)} = \frac{1}{r}\sum_{k=0}^{r-1}(-1)^k \begin{pmatrix} r-1 \\ k \end{pmatrix} E\left(Y_{r+1-k:r+2}\right)$$

where

$$E\left(Y_{i:r}\right) = \frac{r!}{(i-1)!(r-i)!}\int_0^1 x(F)F^{i-1}\left(1-F\right)^{r-1}dF$$

For $t = 0$, TL-moments yields the original L-moments defined by Hosking [20]. For $t = 1$, the first four TL-moments are expressed as

$$\begin{aligned} \lambda_1^{(1)} &= E\left(X_{2:3}\right) = 6\beta_1 - 6\beta_2 \\ \lambda_2^{(1)} &= \frac{1}{2}E\left(X_{3:4} - X_{2:4}\right) = 6(-2\beta_3 + 3\beta_2 - \beta_1) \\ \lambda_3^{(1)} &= \frac{1}{3}E\left(X_{4:5} - 2X_{3:5} + X_{2:5}\right) = \frac{20}{3}(-5\beta_4 + 10\beta_3 - 6\beta_2 + \beta_1) \\ \lambda_4^{(1)} &= \frac{1}{4}E\left(X_{5:6} - 3X_{4:6} + 3X_{3:6} - X_{2:6}\right) = \frac{15}{2}(-14\beta_5 + 35\beta_4 - 30\beta_3 + 10\beta_2 - \beta_1) \end{aligned}$$

The TL-moment ratios: TL-coefficient of variation (TL-Cv, $\tau_2^{(1)}$), TL-coefficient of skewness (L-Cs,$\tau_3^{(1)}$) and TL-coefficient of kurtosis (TL-Ck,$\tau_4^{(1)}$) are defined as

$$\tau_2^{(1)} = \frac{\lambda_2^{(1)}}{\lambda_1^{(1)}},\ \tau_3^{(1)} = \frac{\lambda_3^{(1)}}{\lambda_2^{(1)}},\ \text{and}\ \tau_4^{(1)} = \frac{\lambda_4^{(1)}}{\lambda_2^{(1)}}.$$

Parameters belonging to statistical distributions used in the study, connection of these parameters to the TL-moments are given in the Table 1.

4 Mean Absolute Deviation Index (MADI) and Mean Square Deviation Index (MSDI)

For comparison among the probability distributions for fitting the data used in the study, two indices (mean absolute deviation index and mean square deviation index), which were proposed by Jain and Sing [24], were taken into account to measure the relative goodness of fit. The mean absolute deviation index (MADI) and mean square deviation index (MSDI) can be calculated by :

$$MADI = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{x_i - z_i}{x_i}\right| \tag{1}$$

$$MSDI = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{x_i - z_i}{x_i}\right)^2 \tag{2}$$

where x_i are observed flows whereas z_i are predicted flows respectively for successive values of empirical probability of exceedence given by Gringorten plotting position formula. Jain and Singh [24] claimed and believed that Gringorten formula ensures to maintain unbiasedness for different distributions. Hence, they suggest this plotting position formula for comparison of the probability distributions of fitting the data. The formula for F (cummulative probability of non-exceedence) in the Gringorten plotting position is given by :

$$F = \frac{i - 0.44}{N + 0.12} \tag{3}$$

with i is the rank in ascending order, and N is the number of observations.

The smaller the value obtained for the mean absolute deviation index (MADI) and mean square deviation index (MSDI) of a given distribution shows that the distribution is more fitted for the actual data. Hence the distribution with the smallest value implies that the particular distribution is the most fitted whereas the largest shows that it is the least fitted to present the observed data.

5 L-moment and TL-moment Ratio Diagrams

The L-moment and TL-moment ratio diagrams are based on relationships between the L-moment and TL-moment ratios respectively. The ratio diagrams are based on unbiased sample quantities and the sample L-moment or TL-moment ratios plot as fairly well separated group. Thus, this permits better discrimination between the distributions. Hence, the identification of a parent distribution can be achieved.

Hosking [20] develop the relationships between τ_3 and τ_4 for the GEV, GPA, GLO, EV1 and NORM distributions. In this research, the newly TL-moment ratio diagrams is developed for the GEV, GPA, GLO, EV1 and NORM distributions. Table 2 shows the equation coefficients for the GPA, GEV and GLO distributions. The sample L-moment and TL-moment ratios for each distribution is taken for the range $-1 \leq \tau_3 \leq 1$.

Table 1: Parameter Development for the NORM, LOG, EV1, GLO, GEV and GPA Distributions Using TL-moments Method

Distribution	TL-Moments	Parameter Estimate
NORM	$\lambda_1^1 = \mu$; $\lambda_2^{(1)} = 0.2965\sigma$; $\lambda_3^{(1)} = 0$; $\lambda_4^{(1)} = 0.019\sigma$	$\mu = l_1^{(1)}$; $\hat{\sigma} = \frac{l_2^{(1)}}{0.2965}$
LOG	$\lambda_1^{(1)} = \xi$; $\lambda_2^{(1)} = 0.5\sigma$; $\lambda_3^{(1)} = 0$; $\lambda_4^{(1)} = 0.042\sigma$	$\xi = l_1^{(1)}$ $\hat{\alpha} = \frac{l_2^{(1)}}{0.5}$;
GLO	$\lambda_1^{(1)} = \frac{\alpha}{k} + \xi - \frac{\pi\alpha(1-k^2)}{sin(\pi k)}$ $\lambda_2^{(1)} = -\frac{\pi\alpha k(k^2-1)}{2sin(\pi k)}$ $\lambda_3^{(1)} = \frac{5\pi\alpha k^2(k^2-1)}{18sin(\pi k)}$ $\lambda_4^{(1)} = \frac{5\pi\alpha k^2 - 7\pi\alpha k^5 + 2\pi\alpha k}{48sin(\pi k)}$	$k = \frac{-9t_3^{(1)}}{15}$; $\hat{\alpha} = \frac{2l_2^{(1)}sin(\pi\hat{k})}{\pi\alpha\hat{k}(\hat{k}^2-1)}$; $\hat{\xi} = l_1^{(1)} + \frac{\pi\hat{\alpha}(1-\hat{k}^2)}{sin(\pi\hat{k})} - \frac{\hat{\alpha}}{\hat{k}}$
EV1	$\lambda_1^{(1)} = \xi + 0.459\alpha$; $\lambda_2^{(1)} = 0.3533\alpha$; $\lambda_3^{(1)} = 0.0376\alpha$; $\lambda_4^{(1)} = 0.0266\alpha$	$\hat{\alpha} = 2.83l_2^1$; $\xi = l_1^{(1)} - 0.459\hat{\alpha}$
GEV	$\lambda_1^{(1)} = \xi + \frac{\alpha}{k}\left(1 - \Gamma(1+k)\left(\frac{3}{2^k} - \frac{2}{3^k}\right)\right)$; $\lambda_2^{(1)} = 6\alpha\Gamma(k)\left(\frac{1}{2}\left(\frac{1}{4^k}\right) - \frac{1}{3^k} + \frac{1}{2}\left(\frac{1}{2^k}\right)\right)$; $\lambda_3^{(1)} = \frac{20\alpha}{3}\Gamma(k)\left(\frac{1}{5^k} - \frac{5}{2(4^k)} + \frac{2}{3^k} - \frac{1}{2(2^k)}\right)$; $\lambda_4^{(1)} = \frac{15\alpha}{2}\Gamma(k)\left(\frac{7}{3(6^k)} - \frac{7}{5^k} + \frac{15}{2(4^k)} - \frac{10}{3^{k+1}} \frac{1}{2^{k+1}}\right)$	$\hat{k} = 0.2816 - 2.8825t_3^{(1)} + 1.3744(t_3^{(1)})^2 - 0.8462(t_3^{(1)})^3$; $\hat{\alpha} = \frac{l_2^{(1)}}{\pi\alpha\hat{k}(\hat{k}^2-1)}$; $\hat{\xi} = l_1^{(1)} - \frac{\hat{\alpha}}{\hat{k}}\left(1 - \Gamma(1+\hat{k})\left(\frac{3}{2^{\hat{k}}} - \frac{2}{3^{\hat{k}}}\right)\right)$
GP	$\lambda_1^{(1)} = \xi + \frac{\alpha(k-5)}{(k-2)(k-3}$; $\lambda_2^{(1)} = 6\frac{6\alpha}{(k+2)(k+3)(k+4)}$; $\lambda_3^{(1)} = \frac{20\alpha(1-k)}{3(k+2)(k+3)(k+4)(k=5)}$; $\lambda_4^{(1)} = \frac{15\alpha(k-1)(k-2)}{2(k+2)(k+3)(k+4)(k+5)(k+6)}$	$\hat{k} = \frac{10-45t_3^{(1)}}{10+9t_3^{(1)}}$; $\hat{\alpha} = \frac{l_2^{(1)}}{6}(\hat{k}+2)(\hat{k}+3)(\hat{k}+4)$; $\hat{\xi} = l_1^{(1)} - \frac{(\hat{k}-5)\alpha}{(\hat{k}+2)(\hat{k}+3)}$

Table 2: Equation coefficient of TL-moments ratio for the GEV, GPA and GLO distributions

Equation: $\tau_4^{DIS} = a_0 + a_1 t_3^{(1)} + a_2(t_3^{(1)})^2 + a_3(t_3^{(1)})^3 + a_4(t_3^{(1)})^4$					
Distribution	a_0	a_1	a_2	a_3	a_4
GEV	0.0576	0.0942	0.9183	−0.0745	0.0373
GPA	0	0.1610	0.9904	−0.1295	0.0184
GLO	0.0833	0	0.9450	0	0

For this interval, the values for t_4 are counted for all the distribution using their relationships with t_3. Then the average values for the sample L-moment and TL-moment ratios were calculated as points in the diagram $(\hat{\tau}_3, \hat{\tau}_4)$ and $(\hat{\tau}_3^1, \hat{\tau}_4^1)$.

The distributions which have L-moment or TL-moment ratios that are nearest to the average sample values of sample ratios are considered good distributions for fitting the observed data. Otherwise, they are taken as unsuitable distribution to represent the data.

6 Data

The data of daily rainfalls for stations in Selangor and Kuala Lumpur was collected and taken from Department of Irrigation and Drainage, Malaysia. The data contains measurements of daily rainfalls in millimeters from the year 1971 until 2007. The data is listed in Table 3 including informations on the data. The maximum rainfalls of each month were identified followed by the maximum of each year (1971 − 2007). This is done to all the 55 stations in Selangor and Kuala Lumpur.

L-Moments and TL-Moments for all stations in Selangor and Kuala Lumpur were calculated using the MathCAD program. These values were used in the calculation of quantile function for each distribution using the L-Moment method.The summary statistics of the data are given in Table 2 including informations on the data. The data from 55 stations in Selangor and Kuala Lumpur have the latitude that ranges from 26^0 up to 38^0 while a longitude from 8^0 to 18^0 (Figure 1).

7 Results

All the maximum values of daily rainfalls for each year for the 55 stations were analyzed using MathCAD. The case of $t = 0$ are actually the L-moment method. Meanwhile, $t = 1$ referred to TL-moment which was symmetrically trimmed for one conceptual sample value. Then, their distributions for each case were compared using mean absolute deviation index (MADI) and mean square deviation index (MSDI). For better view, the ratio diagrams were constructed for each case.

Table 3: Name and Characteristics of Maximum Daily Rainfalls for all stations in Selangor and Kuala Lumpur

NO.	STATIONS	STATS. NO	n	Mean	Std Dev	Kurt	Skew
1	LDG. BATU UNTONG	2615131	37	132.760	35.639	-0.540	0.282
2	LDG. TELOK MERBAU	2616135	37	105.900	34.983	2.144	1.025
3	LDG. SEPANG	2617134	35	103.863	31.949	-0.072	0.881
4	LDG. BUTE	2717114	37	95.908	26.609	-0.230	0.275
5	PEJABAT JPS. SG. MANGG	2815001	37	88.657	26.783	1.647	0.794
6	LDG. BROOKLANDS	2815115	35	88.143	24.513	2.747	1.031
7	SMK. BDR TASIK KESUMA	2818110	34	117.171	56.947	0.972	1.547
8	P.KWLN P.S TELOK GONG	2913001	33	119.788	76.604	21.328	4.246
9	LDG. WEST	2913121	37	108.327	39.884	1.292	0.817
10	JPS. PULAU LUMUT	2913122	37	98.030	34.085	0.902	0.882
11	LDG. BKT. CHEEDING	2915116	38	90.261	29.214	3.140	0.868
12	PEJABAT JPS. KLANG	3014084	36	86.814	26.327	2.314	1.512
13	LDG. DOMINION	3018107	38	96.540	26.420	4.681	0.896
14	LDG. BUKIT KERAYONG	3113059	37	108.587	99.032	31.629	5.450
15	LDG. SG. KAPAR	3113087	37	105.114	30.082	-0.315	0.502
16	LDG. NORTH HUMMOCK	3114085	36	153.053	207.288	11.178	3.422
17	LDG. HARPENDEN	3114086	27	93.067	26.253	-0.048	0.559
18	LDG. ELMINA	3115053	38	108.724	56.631	6.876	2.283
19	SG. BULOH	3115079	38	94.097	29.121	0.175	0.414
20	LDG. EDINBURGH SITE 2	3116006	31	95.326	22.588	-0.658	0.432
21	JPS AMPANG	3117070	37	106.727	25.776	0.475	0.434
22	PEMASOKAN AMPANG	3118069	22	103.014	35.893	0.988	0.314
23	SEK.KEB.KG.LUI	3118102	37	114.654	61.209	2.426	1.599
24	LDG. BRAUNSTON	3213057	34	91.832	33.297	0.026	0.769
25	LDG. BKT. CHERAKAH	3213058	38	96.982	69.306	27.097	4.830
26	LDG. TUAN MEE	3214054	36	83.956	25.161	0.724	0.811
27	LDG. BKT. IJOK	3214055	35	106.971	46.239	1.697	1.486
28	KG. SG. TUA	3216001	36	98.225	29.775	0.807	1.259
29	KEPONG (SEMAIAN)	3216002	7	70.357	38.812	-0.961	-0.717
30	IBU BEKALAN KM. 16	3217001	36	97.258	21.489	1.339	0.401
31	EMPANGAN GENTING KLANG	3217002	36	100.306	38.420	11.265	2.716
32	IBU BEKALAN KM. 11	3217003	9	114.444	28.815	-0.304	0.550
33	STN. JENALETRIK LLN.	3218101	37	108.257	56.682	2.921	1.636
34	LDG. BKT. BELIMBING	3312042	36	97.828	40.173	3.722	1.918
35	JLN. KELANG	3312045	37	96.560	32.381	7.144	2.201
36	LDG. BKT. TALANG	3313040	35	98.646	48.726	6.764	2.433
37	LDG. KUALA SELANGOR	3313043	37	100.597	40.463	0.561	0.843
38	LDG. SG. BULOH	3313060	38	93.242	31.410	1.437	0.987
39	RMH PAM JPS JAYA SETIA	3314001	36	108.400	75.241	21.781	4.255
40	LDG. SG. GAPI	3316028	35	113.891	30.993	1.490	1.282
41	AIR TERJUN SG BATU	3317001	23	97.957	25.656	3.435	1.542
42	GENTING SEMPAH	3317004	34	105.991	125.596	30.707	5.429
43	PARIT 1 SG. BURONG	3411016	36	102.839	31.911	0.556	0.440
44	IBU BEKALAN SG. TENGKI	3412001	33	91.267	29.351	0.457	0.961
45	LDG. RAJA MUSA	3412041	37	93.049	40.909	4.914	2.020
46	LDG. SG. TINGGI	3414029	36	98.956	58.200	9.454	2.279
47	LDG. HOPEFUL	3414030	35	109.254	36.913	0.145	0.940
48	FDC. SEKICHAN	3510001	33	87.985	30.241	2.247	1.354
49	PARIT 1 SG. BESAR	3609012	36	93.628	23.950	-0.109	0.559
50	SG. NIPAH	3610014	33	83.906	33.314	0.049	-0.228
51	LDG. SG. GUMUT	3615002	7	109.443	17.123	-0.510	-0.298
52	RMH PAM JPS BGN TERAP	3710006	37	86.527	22.678	0.044	0.364
53	PARIT 6 SG. BESAR	3710011	37	88.024	23.485	2.911	1.103
54	PARIT SALIRAN SG. AIR TAWAR	3808001	33	89.924	39.671	2.672	1.538
55	LDG SG. BERNAM	3809009	37	91.654	30.064	1.468	1.168

Figure 1: Location Maps of Rainfall Gauge Stations in Selangor and Kuala Lumpur

Each MADI and MSDI for all 55 stations were calculated for each distribution which includes normal (N), logistic (LOG), generalized logistic (GLO), extreme value type I (EV1), generalized extreme value type I (GEV) and generalized Pareto (GPA). Then, the distributions were ranked according to their MADI and MSDI from the best distribution that fits the data to the least. The number of times each distribution obtains a given rank were then calculated and tabulated.

The results obtained were tabulated in Table 4 and 5, respectively. For each station, their MADI and MSDI were then ranked with the smallest value as the distribution which best fit the data and so on. From all stations, the number of times the distribution obtained a given rank was summed up and the totals for each rank were also put into a table. The results were then listed and tabulated in Table 6 and 7.

From both Table 6 and Table 7, for L-moments method, it was observed that the GLO distribution ranked first most of the time compared to the other distributions. This was followed closely by the GEV distribution. Next, the GPA obtained the third rank the most. Meanwhile, EV1 ranked fourth the most and the NORM distribution was frequently ranked fifth. Lastly, the LOG distribution was the most often to rank last.

For TL-moments method, the GEV was the distribution with the most number of times to be ranked first followed by EV1 based on MADI and the EV1 was the most often ranked as first in the calculations followed by GEV based on MSDI. However, for the rest of the rankings from the third to the sixth (the last rank), both MADI and MSDI showed the same results. The GLO was the most often to be ranked third. The most frequent to rank fourth was the GPA distribution, followed by the NORM in the fifth rank and LOG distribution in the last rank.

The results obtained from using the L-moment method were quite precise. It gave almost the same results in all the methods of goodness-of-fit test used in this study which were MADI and MSDI. The GLO distributions were also deemed suitable. This can be seen clearly in the L-moment ratio diagrams (Figure 2) show that the GLO distribution was the closest to the average of the sample L-moment ratios, $\bar{\tau}_3$ and $\bar{\tau}_4$ followed by the GEV for the 55 stations analysis.

However, the case of using the TL-moment method, the GEV and EV1 distributions were deemed able to fit the actual data properly or as good compared to all the other distributions considered in this study. The TL-moment ratio diagrams (Figure 3) constructed proved the results of the data analysis since the average of the sample TL-moment ratios, $\bar{\tau}_3^{(1)}$ and $\bar{\tau}_4^{(1)}$, for the calculations of the 55 stations were nearest to those TL-moment ratios of the EV1 and GEV distributions. Meanwhile, the furthest were those from the normal and logistic distributions.

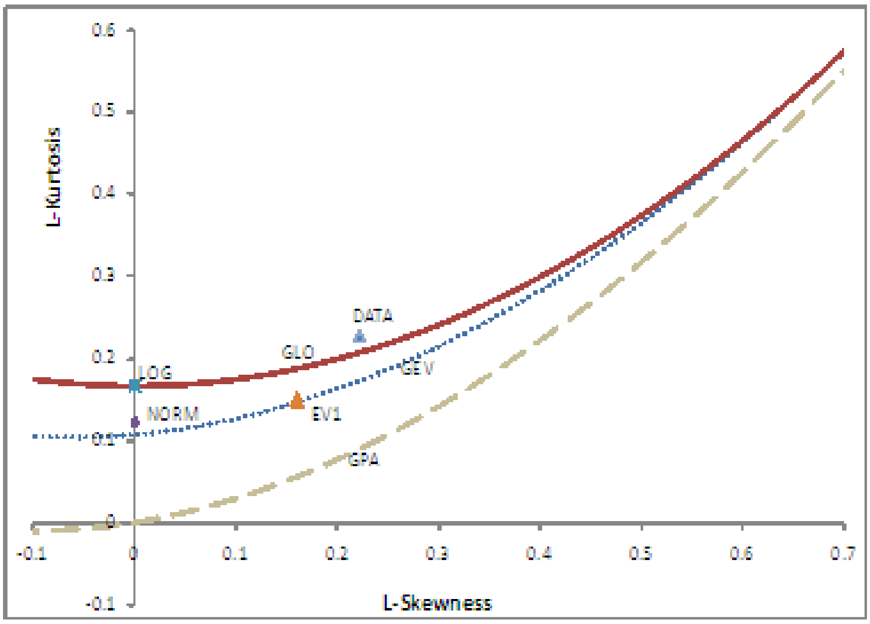

Figure 2: L-moments ratio diagram of L-kurtosis versus L-skewness for annual maximum rainfall of the Selangor and Kuala Lumpur region

8 Conclusions

Overall, the extreme value type I (EV1), generalized extreme value (GEV) and generalized logistic (GLO) distributions were good distributions to represent the actual maximum daily

Table 4: Mean Absolute Deviation Index (MADI) for L-moments and TL-moments Method

No	L-Moment						TL-Moment					
	NORM	EV1	GEV	LOG	GLO	GPA	NORM	EV1	GEV	LOG	GLO	GPA
1	0.03	0.032	**0.018**	0.038	0.029	0.03	0.03	0.027	**0.019**	0.038	0.028	0.03
2	0.053	0.039	**0.038**	0.062	0.044	0.046	0.052	0.039	0.042	0.060	0.048	**0.037**
3	0.082	0.040	**0.032**	0.084	0.038	0.032	0.084	0.046	0.039	0.087	0.042	**0.037**
4	0.030	0.039	**0.024**	0.037	0.034	0.037	0.03	0.036	**0.026**	0.04	0.037	0.031
5	0.036	0.037	**0.029**	0.042	0.034	0.045	0.033	0.038	**0.03**	0.041	0.037	0.036
6	0.045	0.044	0.042	0.043	**0.036**	0.064	0.037	0.046	**0.037**	0.037	0.039	0.050
7	0.237	0.166	**0.091**	0.237	0.094	0.091	0.205	0.153	0.115	0.210	0.116	**0.114**
8	0.185	0.12	0.05	0.185	**0.046**	0.068	0.082	0.044	0.040	0.088	0.045	**0.036**
9	0.063	0.047	0.047	0.066	**0.038**	0.087	0.054	0.052	0.053	0.057	**0.048**	0.081
10	0.061	0.028	**0.028**	0.061	0.03	0.045	0.054	**0.028**	0.031	0.060	0.032	0.046
11	0.067	0.08	0.073	**0.051**	0.057	0.111	0.065	0.088	0.075	**0.052**	0.061	0.105
12	0.089	0.052	0.037	0.091	**0.037**	0.042	0.076	0.049	**0.037**	0.078	0.038	0.041
13	0.052	0.072	0.059	**0.039**	0.044	0.092	0.05	0.073	0.045	0.041	**0.031**	0.072
14	0.289	0.216	0.067	0.286	**0.066**	0.083	0.082	0.059	0.047	0.088	**0.046**	0.056
15	0.049	0.031	**0.029**	0.05	0.033	0.041	0.051	**0.031**	0.039	0.057	0.039	0.045
16	0.791	0.623	**0.112**	0.779	0.113	0.123	0.273	0.22	**0.099**	0.28	0.103	0.101
17	0.043	0.031	**0.027**	0.045	0.031	0.040	0.04	0.031	**0.03**	0.043	0.03	0.045
18	0.175	0.093	**0.047**	0.180	0.048	0.056	0.134	0.076	**0.045**	0.143	0.047	0.057
19	0.043	0.041	**0.035**	0.048	0.037	0.055	0.044	**0.038**	0.040	0.054	0.042	0.049
20	0.036	0.026	**0.023**	0.043	0.029	0.023	0.038	0.026	0.026	0.044	0.030	**0.025**
21	0.030	0.036	0.028	0.030	**0.024**	0.05	**0.027**	0.043	0.029	0.030	0.031	0.049
22	0.117	0.129	0.125	**0.103**	0.115	0.159	0.123	0.139	0.179	**0.107**	0.181	0.182
23	0.214	0.116	0.085	0.220	**0.084**	0.102	**0.060**	0.017	0.079	0.100	0.085	0.094
24	0.08	0.038	**0.038**	0.081	0.044	0.044	0.083	**0.042**	0.038	0.091	0.043	0.044
25	0.193	0.125	0.090	0.190	**0.082**	0.120	0.057	0.053	0.047	0.066	**0.042**	0.070
26	0.048	0.027	0.027	0.047	**0.025**	0.053	0.038	0.030	0.028	0.041	**0.025**	0.051
27	0.157	0.087	**0.052**	0.159	0.053	0.059	0.128	0.075	**0.055**	0.135	0.057	0.057
28	0.095	0.053	**0.039**	0.097	0.040	0.044	0.089	0.055	**0.048**	0.09	0.049	0.049
29	**0.191**	0.309	0.273	0.218	0.236	0.283	**0.220**	0.370	0.955	0.232	0.875	0.891
30	0.030	0.035	0.029	0.025	**0.022**	0.053	0.029	0.032	0.031	**0.023**	0.026	0.045
31	0.082	0.046	0.040	0.083	**0.039**	0.059	0.049	**0.037**	0.044	0.055	0.051	0.036
32	0.042	0.023	**0.023**	0.042	0.026	0.032	0.047	**0.026**	0.032	0.049	0.031	0.035
33	0.175	0.085	0.069	0.173	**0.061**	0.102	0.116	**0.065**	0.084	0.131	0.082	0.105
34	0.141	0.081	0.036	0.140	**0.036**	0.045	0.104	0.062	**0.039**	0.108	0.040	0.043
35	0.086	0.042	0.027	0.085	**0.025**	0.044	0.061	0.034	0.029	0.064	**0.028**	0.044
36	0.158	0.091	0.035	0.157	**0.03**	0.057	0.094	0.051	0.033	0.101	**0.031**	0.047
37	0.093	0.053	0.054	0.095	**0.043**	0.09	0.072	**0.055**	0.070	0.083	0.067	0.093
38	0.077	0.055	0.054	0.075	**0.048**	0.075	0.062	**0.049**	0.054	0.058	0.052	0.067
39	0.232	0.157	0.105	0.235	**0.103**	0.125	0.12	0.111	**0.104**	0.114	0.105	0.113
40	0.068	0.03	**0.022**	0.068	0.026	0.023	0.063	0.032	**0.023**	0.066	0.024	0.026
41	0.057	0.028	0.025	0.055	**0.022**	0.042	0.037	**0.025**	0.027	0.038	0.026	0.038
42	0.405	0.312	0.064	0.398	**0.063**	0.08	0.107	0.068	0.044	0.112	**0.043**	0.053
43	0.036	0.045	0.032	0.043	**0.032**	0.051	0.034	0.043	**0.031**	0.045	0.044	0.042
44	0.075	0.035	0.033	0.077	0.039	**0.03**	0.074	0.04	**0.034**	0.08	0.040	0.029
45	0.129	0.067	**0.026**	0.129	0.027	0.041	0.094	0.048	0.028	0.100	**0.028**	0.037
46	0.189	0.205	0.208	0.193	**0.173**	0.292	0.205	0.296	0.197	**0.17**	0.174	0.301
47	0.09	0.043	0.036	0.091	0.045	**0.031**	0.088	0.048	0.038	0.094	0.042	**0.034**
48	0.085	0.031	**0.02**	0.087	0.024	0.031	0.07	0.031	**0.022**	0.076	0.025	0.030
49	0.061	0.052	0.051	0.061	**0.050**	0.059	0.063	**0.052**	0.069	0.065	0.072	0.067
50	0.102	0.221	0.104	0.073	**0.073**	0.206	0.120	0.231	0.204	**0.071**	0.172	0.286
51	0.017	0.035	**0.017**	0.019	0.018	0.027	0.021	0.034	0.03	**0.021**	0.03	0.031
52	0.232	0.157	0.105	0.235	**0.103**	0.125	0.035	**0.03**	0.031	0.04	0.033	0.034
53	0.042	0.038	0.034	0.039	**0.028**	0.056	0.032	0.042	0.035	0.032	**0.03**	0.053
54	0.126	0.055	0.033	0.125	**0.032**	0.057	0.100	0.044	0.039	0.108	**0.038**	0.055
55	0.078	0.027	**0.020**	0.079	0.026	0.030	0.066	0.027	**0.021**	0.071	0.028	0.030

Table 5: Mean Square Deviation Index (MSDI) for L-moments and TL-Moments method

No	L-Moments						TL-Moments					
	NORM	EV1	GEV	LOG	GLO	GPA	NORM	EV1	GEV	LOG	GLO	GPA
1	0.002	0.002	**0.001**	0.004	0.002	0.002	0.003	0.002	**0.001**	0.007	0.003	0.003
2	0.009	**0.003**	0.003	0.004	0.012	0.003	0.010	**0.003**	0.004	0.017	0.008	0.003
3	0.011	0.003	0.002	0.014	0.003	**0.002**	0.015	0.004	0.006	0.021	0.009	**0.003**
4	0.002	0.003	**0.001**	0.004	0.002	0.003	0.003	0.003	**0.002**	0.007	0.006	0.003
5	0.004	0.002	**0.002**	0.006	0.003	0.004	0.004	0.003	**0.003**	0.008	0.006	0.004
6	0.004	0.004	0.003	0.005	**0.003**	0.006	**0.004**	0.005	0.005	0.005	0.006	0.007
7	0.078	0.037	0.015	0.082	0.016	**0.015**	0.065	**0.032**	0.092	0.076	0.089	0.081
8	0.062	0.026	0.007	0.067	**0.007**	0.010	0.028	0.013	**0.011**	0.036	0.011	0.012
9	**0.006**	0.017	0.015	0.009	0.008	0.042	**0.005**	0.025	0.030	0.007	0.021	0.069
10	0.010	**0.001**	0.002	0.014	0.003	0.003	0.010	**0.001**	0.002	0.018	0.004	0.004
11	0.008	0.030	0.019	**0.004**	0.010	0.045	0.024	0.057	0.036	**0.012**	0.021	0.078
12	0.012	0.004	0.003	0.013	**0.003**	0.005	0.010	**0.004**	0.006	0.012	0.007	0.006
13	0.010	0.028	0.015	**0.005**	0.008	0.035	0.022	0.045	0.016	0.013	**0.005**	0.047
14	0.123	0.065	0.014	0.130	**0.014**	0.019	0.025	0.019	0.017	0.027	**0.016**	0.022
15	0.004	0.002	**0.001**	0.006	0.002	0.003	0.005	**0.002**	0.004	0.009	0.005	0.006
16	0.902	0.531	0.030	0.926	**0.030**	0.031	0.143	0.087	**0.025**	0.166	0.026	0.025
17	0.004	0.002	**0.001**	0.005	0.002	0.003	0.004	0.002	**0.001**	0.006	0.002	0.004
18	0.061	0.015	0.005	0.073	**0.005**	0.010	0.040	0.011	**0.008**	0.058	0.008	0.014
19	0.004	0.010	0.005	0.004	**0.003**	0.017	**0.004**	0.009	0.010	0.009	0.008	0.026
20	0.003	0.001	**0.001**	0.004	0.002	0.008	0.004	0.002	0.002	0.007	0.003	**0.001**
21	0.001	0.003	0.001	0.002	**0.001**	0.005	**0.001**	0.004	0.002	0.002	0.002	0.006
22	0.039	0.112	0.098	**0.026**	0.077	0.160	0.079	0.173	0.334	**0.049**	0.338	0.376
23	0.088	**0.019**	0.027	0.118	0.024	0.050	0.060	**0.017**	0.079	0.100	0.085	0.094
24	0.014	0.002	**0.002**	0.019	0.003	0.003	0.019	**0.003**	0.003	0.031	0.005	0.004
25	0.067	0.026	0.020	0.079	**0.017**	0.032	0.016	0.015	0.013	0.021	**0.011**	0.026
26	0.005	0.001	**0.001**	0.007	0.001	0.004	0.004	0.002	0.002	0.007	**0.001**	0.006
27	0.037	0.010	**0.005**	0.043	0.005	0.008	0.032	**0.009**	0.013	0.043	0.015	0.013
28	0.014	0.004	0.003	0.016	0.003	**0.003**	0.014	**0.004**	0.009	0.017	0.010	0.005
29	**0.107**	0.156	0.197	0.134	0.212	0.203	**0.135**	0.275	4.026	0.148	3.119	3.729
30	0.002	0.005	0.003	**0.001**	0.002	0.008	0.003	0.007	0.005	**0.001**	0.003	0.012
31	0.015	0.005	0.004	0.018	**0.003**	0.007	0.011	**0.004**	0.008	0.014	0.011	0.008
32	0.003	0.009	0.009	0.003	**0.001**	0.002	0.004	**0.001**	0.002	0.005	0.002	0.002
33	0.062	0.010	0.010	0.084	**0.007**	0.026	0.034	**0.008**	0.024	0.061	0.022	0.047
34	0.031	0.010	**0.003**	0.034	0.003	0.005	0.022	0.008	**0.005**	0.028	0.005	0.006
35	0.013	0.003	0.001	0.014	**0.001**	0.003	0.010	0.003	0.002	0.012	**0.002**	0.006
36	0.044	0.014	0.003	0.049	**0.003**	0.006	0.028	0.009	0.004	0.037	**0.004**	0.008
37	0.013	0.008	0.008	0.021	**0.004**	0.028	**0.009**	0.012	0.026	0.020	0.022	0.055
38	0.009	0.014	0.017	**0.009**	0.013	0.033	0.008	0.019	0.030	**0.008**	0.027	0.049
39	0.093	**0.037**	0.073	0.116	0.071	0.097	0.030	0.051	0.112	**0.028**	0.115	0.137
40	0.008	0.002	**0.001**	0.009	0.001	0.001	0.009	0.002	0.002	0.011	0.002	**0.001**
41	0.006	0.002	0.001	0.006	**0.001**	0.003	0.005	**0.002**	0.003	0.005	0.002	0.005
42	0.238	0.133	0.014	0.247	**0.014**	0.017	0.036	0.023	0.017	0.042	**0.016**	0.019
43	0.003	0.007	0.002	0.004	**0.002**	0.010	0.003	0.007	**0.003**	0.007	0.007	0.011
44	0.012	0.002	0.002	0.014	0.003	**0.001**	0.015	0.003	0.002	0.021	0.004	**0.001**
45	0.032	0.008	**0.002**	0.037	0.002	0.003	0.026	0.007	0.002	0.035	0.002	0.004
46	0.101	0.118	0.125	0.179	**0.097**	0.258	0.194	0.498	0.174	0.144	**0.129**	0.481
47	0.015	0.003	0.002	0.018	0.003	**0.002**	0.020	0.004	0.006	0.029	0.009	**0.004**
48	0.014	0.002	**0.001**	0.016	0.001	0.002	0.014	0.002	**0.001**	0.020	0.001	0.002
49	0.005	0.004	0.004	0.006	**0.004**	0.006	0.006	**0.004**	0.014	0.007	0.016	0.013
50	0.053	0.465	0.057	0.022	**0.020**	0.388	0.095	0.632	0.476	0.020	0.278	1.148
51	0.001	0.002	0.000	0.001	**0.000**	0.001	0.005	0.002	0.002	0.005	**0.001**	0.002
52	0.002	0.005	0.003	0.002	**0.002**	0.008	**0.002**	0.005	0.006	0.004	0.005	0.013
53	0.003	0.003	0.002	0.003	**0.001**	0.006	0.003	0.004	0.003	0.003	**0.002**	0.008
54	0.031	0.005	0.002	0.037	**0.002**	0.006	0.024	0.004	0.003	0.036	**0.003**	0.008
55	0.011	0.001	0.007	0.014	**0.001**	0.002	0.012	**0.002**	0.008	0.018	0.002	0.002

Table 6: Ranks of Mean Absolute Deviation Index (MADI) for each distribution

	L-Moments						**TL-Moments**					
Distribution	**1**	**2**	**3**	**4**	**5**	**6**	**1**	**2**	**3**	**4**	**5**	**6**
Normal	2	2	8	3	29	11	4	8	2	11	29	1
EV1	3	6	14	21	7	4	11	7	10	17	8	2
GEV	25	22	2	5	1	0	18	18	12	5	1	1
LOG	4	2	3	6	0	20	7	3	2	1	7	35
GLO	27	14	11	3	0	0	12	17	13	8	4	1
GPA	5	1	18	15	3	13	8	3	12	12	6	14

Table 7: Ranks of Mean Square Deviation Index (MSDI) for each distribution

	L-Moments						**TL-Moments**					
Distribution	**1**	**2**	**3**	**4**	**5**	**6**	**1**	**2**	**3**	**4**	**5**	**6**
Normal	2	7	6	7	33	0	7	10	2	9	27	0
EV1	4	7	15	16	10	3	15	7	15	11	6	1
GEV	18	21	3	11	2	0	11	18	11	7	6	2
LOG	5	2	4	4	6	34	6	6	6	0	2	35
GLO	26	11	12	4	0	2	12	9	9	18	6	1
GPA	5	2	15	13	4	16	6	4	11	11	7	16

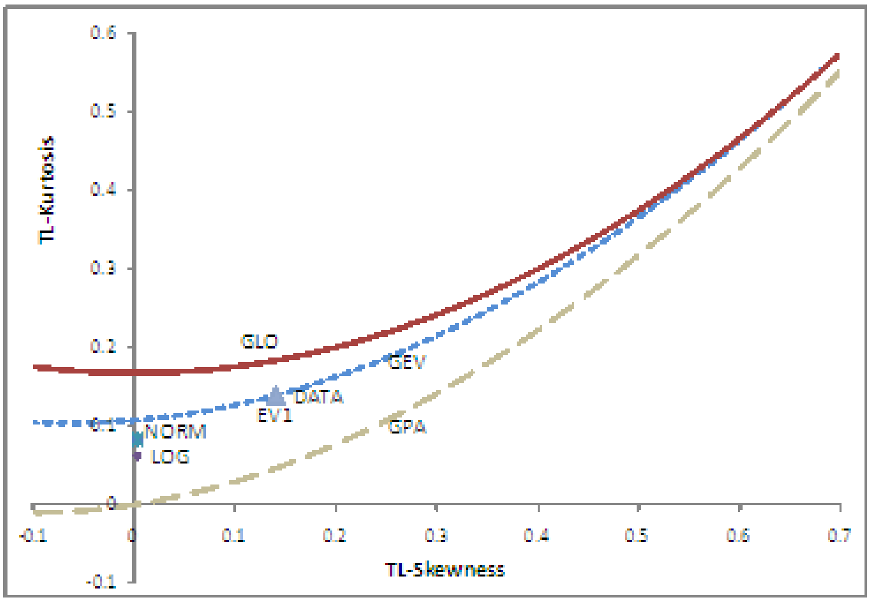

Figure 3: TL-moments ratio diagram of TL-kurtosis versus TL-skewness for annual maximum rainfall of the Selangor and Kuala Lumpur region

rainfalls of stations in Selangor and Kuala Lumpur. The L-moment method gave a more precise result and showed that the generalized logistic (GLO) distribution was the best distribution to fit the data independent on any goodness-of-fit test used MADI and MSDI in analyses of the 55 stations. Meanwhile, the TL-moment, method, had a wider spread answer and showed that the extreme value type I (EV1) and generalized extreme value (GEV) distributions were the most suitable distributions. Extreme value type I (EV1) distribution is a special case of the generalized extreme value (GEV) distribution. Hence, both distributions are similar.

However, bear in mind that the TL-moment method had trimmed the actual data symmetrically by one conceptual sample values. Thus, the results obtained from using this method did not represent the whole observed data but only those that remained after trimming. Meanwhile, the L-moment method is a special case of the TL-moment method with $t = 0$ which implies no trimming is done on the actual data. However, in accordance with most flood frequency analysis, the extreme value type I (EV1), generalized extreme value (GEV) and generalized logistic (GLO) distributions were proven as good distributions to fit the maximum daily rainfalls data.

Normal and logistic (LOG) distributions were also shown that both were not suitable distributions to present the actual data in all the goodness-of- fit test used in this research.

References

[1] Chen, Y., Huang, G., Shao, Q. and Xu, C. Regional analysis of low flow using l-moments for dongjiang basin, south china. *Hydrological Sciences-Journal-des Sciences*

Hydrologiques. 2006. 51(6): 1051–1064.

[2] Lim, Y. and Lye, L. Regional flood estimation for ungauged basins in sarawak, malaysia. *Hydrological Sciences Journal*. 2003. 48(1): 79–94.

[3] Shabri, A. and Ariff, N. Frequency analysis of maximum daily rainfalls via l-moment approach. *Sains Malaysiana*. 1999. 38(2): 149–158.

[4] Zin, W., Jemain, A. and Ibrahim, K. The best fitting distribution of annual maximum rainfall in peninsular malaysia based on methods of l-moment and lq-moment. *Theoretical Applied Climatology*. 2009. 96: 337–344.

[5] Zalina, M., Nguyen, V., M.K.Amir and Nor, M. M. Selecting a probability distribution for extreme rainfall series in malaysia. *Water Science Technology Journal*. 2002. 45(2): 63–68.

[6] Pearson, C. Regional flood frequency analysis for small new zealand: Flood frequency groups. *Journal of Hydrology (New Zealand)*. 1991. 30(2): 77–92.

[7] Rao, A. and Hamed, K. Regional frequency analysis of wabash river flood data by l-moments. *Journal of Hydrologic Engineering*. 1997. 2(4): 169–179.

[8] Glaves, R. and Waylen, P. Regional flood frequency analysis in southern ontario using l-moments. *Canadian Geographer*. 1997. 41(2): 178–193.

[9] Yue, S. and Wang, C. Determination of regional probability distributions of canadian flood flows using l-moments. *Journal of Hydrology New Zealand*. 2004. 43(1): 59–73.

[10] Vogel, R. and Fennessey, N. L-moment diagrams should replace product moment diagrams. *Water Resources Research*. 1993. 29(6): 1745–1752.

[11] Parida, B., Kachroo, R. and Shrestha, D. Regional flood frequency analysis of mahisabarmati basin (subzone 3-a) using index flood procedure with l-moments. *Water Resources Management*. 1998. 12: 1–12.

[12] Kumar, R., Chatterjee, C. and Kumar, S. Regional flood formulas using l-moments for small watersheds of sone subzone of india. *Applied Engineering in Agriculture*. 2003b. 19(1): 47–53.

[13] Chebana, F. and Ouarda, T. Multivariate l-moment homogeneity test. *Water Resources Research*. 2007. 43(8): No. W08406.

[14] Eslamian, S. and Feizi, H. Maximum monthly rainfall analysis using l-moments for an arid region in isfahan province, iran. *Journal of Applied Meteorology and Climatology*. 2007. 46(4): 494–503.

[15] Noto, L. and Loggia, G. L. Use of l-moments approach for regional frequency analysis in sicily, italy. *Water Resources Management*. 2008. 2: 1–23.

[16] Hussain, Z. and Pasha, G. Regional flood frequency analysis of the seven sites of punjab, pakistan, using l-moments. *Water Resources Management*. 2009. 23(10): 1917–1933.

[17] Saf, B. Regional flood frequency analysis using l-moments for the west mediterranean region of turkey. *Water Resources Management*. 2009a. 23(3): 531–551.

[18] Asquith, W. L-moments and tl-moments of the generalized lambda distribution. *Computational Statistics and Data Analysis*. 2007. 51: 4484–4496.

[19] Elamir, E. and Seheult, A. Trimmed l-moments. *Computational Statistics and Data Analysis*. 2003. 43: 299–314.

[20] Hosking, J. L-moments: Analysis and estimation of distributions using linear combinations of order statistics. *Journal of Royal Statistical Society B*. 1990. 52: 105–124.

[21] Greenwood, J., Landwehr, J., Matalas, N. and Wallis, J. Probability weighted moments: Definition and relation to parameters of several distributions expressible in inverse form. *Water Resources Research*. 1997. 15(5): 1049–1054.

[22] Hosking, J. and Wallis, J. Some statistics useful in regional frequency analysis. *Water Resources Research*. 1993. 29(2): 271–281.

[23] Sankarasubramaniam, A. and K.Srinivasan. Investigation and comparison of sampling properties of l-moments and conventional moments. *Journal of Hydrology*. 1999. 218: 13–34.

[24] Jain, D. and Singh, V. A comparison of transformation methods for flood frequency analysis. *Water Resources Bulletin*. 1986. 22(6): 903–912.