

A DIRECT PROBABILISTIC GLOBAL SEARCH METHOD FOR THE  
SOLUTION OF CONSTRAINED OPTIMAL CONTROL PROBLEMS

AKBAR BANITALEBI DEHKORDI

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*To my family*

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*Akbar Banitalebi, JB, Malaysia*

## ABSTRACT

This research focuses on the development of a new direct stochastic algorithm to address the global optimization of the constrained optimal control problem where the interaction between state and control variables is governed by a system of ordinary differential equations. The objective of this method is to localize a globally optimal control curve in the feasible control space of the problem in such a way that the performance index attains its minimum value. The stochastic methodology is used on the development of the method. Thus, the resulting method is still effective when the complexity of the arising problems prohibits applying gradient-based methods. In this approach, the aforementioned control problem has first to be transformed into a nonlinear programming problem via a suitable discretization technique. The resulting problem is then solved using a stochastic method called Probabilistic Global Search Johor (PGSJ). The idea underpinning the PGSJ is to intelligently sample among potential solutions while no recombination or mutation operator is used. The sampling procedure is performed in accordance with some probability density functions (pdf) which are first initialized uniformly and then iteratively biased towards a globally optimal solution using the information obtained by evaluating the sampling points. After the PGSJ has been successfully implemented, it is found that it is able to arrive at an acceptable solution of the applied optimal control problems. The algorithm is also furnished with some theoretical supports verifying its convergence in probabilistic sense. In addition, some existing global stochastic methods which are based on using pdf are also applied on the optimal control problems where simulations reveal that the PGSJ method is superior to its competitors in terms of computation time and solution quality. These investigations lead to the extension of PGSJ into PGSJ-LS where LS indicates a line search operator added to the original method. These are then assessed and compared by applying them to a practical problem of controlling avian influenza H5N1 where it is verified that the PGSJ-LS performs slightly better than PGSJ.

## ABSTRAK

Kajian ini memberi tumpuan kepada pembangunan algoritma langsung berstokastik baharu untuk menangani masalah pengoptimuman sejagat kawalan optimum berkekangan dengan interaksi di antara pembolehubah keadaan dan kawalan ditadbir oleh sistem persamaan terbitan biasa. Objektif kaedah ini adalah untuk membendung lengkung kawalan optimum sejagat dalam ruang kawalan tersaur sehinggakan suatu indeks prestasi mencapai nilai minimumnya. Metodologi stokastik digunakan pada pembangunan kaedah. Lantas kaedah terhasil masih berkesan walaupun kerumitan masalah yang timbul membataskan penggunaan kaedah berasaskan kecerunan. Dalam pendekatan ini, masalah kawalan tersebut perlu diubah terlebih dahulu menjadi masalah pengaturcaraan tak linear melalui teknik pendiskretan yang bersesuaian. Masalah terhasil kemudiannya diselesai menggunakan suatu kaedah stokastik dinamakan Kaedah Carian Sejagat Berkebarangkalian Johor (PGSJ). Idea asas kepada PGSJ ialah melakukan persampelan bijak di kalangan penyelesaian berpotensi dengan tiada operator penggabungan semula atau mutasi digunakan. Prosedur pensampelan dilakukan sejajar dengan beberapa fungsi kebarangkalian ketumpatan (pdf) yang diberi nilai awal secara seragam dan kemudiannya dicenderungkan secara lelaran ke arah penyelesaian optimum sejagat menggunakan maklumat yang diperolehi daripada penilaian titik pensampelan. PGSJ telah dilaksanakan dengan jayanya, didapati bahawa kaedah ini berkemampuan untuk menumpu kepada penyelesaian masalah kawalan optimum yang boleh diterima pakai. Algoritma ini juga dilengkapi dengan beberapa teori sokongan yang mengesahkan penumpuannya daripada aspek kebarangkalian. Di samping itu, beberapa kaedah stokastik sejagat sedia ada yang berasaskan fungsi ketumpatan kebarangkalian juga diguna pakai pada masalah kawalan optimum. Simulasi tersebut menunjukkan bahawa kaedah PGSJ adalah lebih baik berbanding pesaing-pesaing lain dari segi masa pengiraan dan kualiti penyelesaian. Kajian ini menjurus kepada perlanjutan kaedah PGSJ kepada kaedah PGSJ-LS dengan LS mewakili operator gelintaran garis yang telah ditambah kepada kaedah asal. Kaedah-kaedah ini ditaksir berbanding satu sama lain dengan menggunakan masalah praktikal pengawalan selesema burung H5N1 di mana kajian ini mengesahkan bahawa kaedah PGSJ-LS berprestasi lebih baik daripada PGSJ.