

ANALYTICAL SOLUTIONS OF DISSIPATIVE HEAT TRANSFER ON THE  
PERISTALTIC FLOW OF NON-NEWTONIAN FLUIDS IN ASYMMETRIC  
CHANNELS

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To my parents (late)

~ Thank you for everything

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## ABSTRACT

Peristalsis is a natural mechanism responsible for the propulsion and the segmentation of biofluids in living structures, and this mechanism is important due to its efficient pumping characteristics. An essential feature of peristalsis is dissipation, thus dissipative heat transfer must be considered in the propulsion of biofluids. Most biofluids exist with different non-Newtonian fluid characteristics and experimental investigations reveal that the physiological structures are non-uniform with asymmetric peristaltic waves. This research focuses on the development of mathematical models which take into account the dissipative heat transfer on the peristaltic flow of non-Newtonian fluids. The non-Newtonian fluids include Walter's B, fourth grade and Sisko fluids and the flow have been considered in the horizontal and inclined asymmetric channels. Governing equations are first modeled in the laboratory frame and then transformed into the wave frame. Resulting equations are non-dimensionalized and the nonlinearity has been reduced by adopting the long wavelength and small Reynolds number approximations. Explicit forms of the analytical solutions have been obtained using the regular perturbation method. Influences of various parameters such as velocity slip parameter, Sisko fluid parameter, Brinkman, Eckert, Deborah, Soret and Schmidt numbers on the flow quantities namely velocity, shear stress, pumping, trapping, temperature, concentration and heat transfer coefficients have been investigated. Results show that pumping, trapping and temperature are reduced for increasing velocity slip parameter. Temperature and heat transfer coefficients are increased with the increase of Brinkman, Eckert and Deborah numbers. Concentration decreases with the increase of Brinkman, Soret and Schmidt numbers. Comparative study amongst viscous, shear thinning and shear thickening fluids has also been presented.

## ABSTRAK

Peristalsis adalah mekanisme semula jadi yang bertanggungjawab bagi pendorongan dan penegetan biobendalir dalam struktur hidup, dan mekanisme ini adalah penting kerana ciri pengepamannya yang cekap. Suatu ciri utama dalam peristalsis adalah pelepasan, maka pemindahan haba lesapan mesti dipertimbangkan untuk pendorongan biobendalir. Kebanyakan biobendalir wujud dengan pelbagai ciri bendalir bukan Newtonian dan kajian secara eksperimen mendedahkan bahawa struktur fisiologi adalah tidak seragam dengan gelombang peristalsis tak simetri. Kajian ini memberi tumpuan kepada pembangunan model matematik dengan mengambil kira pemindahan haba lesapan pada aliran peristalsis bagi bendalir bukan Newtonian. Bendalir bukan Newtonian termasuklah bendalir Walter B, bendalir gred keempat dan bendalir Sisko, manakala aliran telah dipertimbangkan dalam saluran tak simetri mendatar dan saluran condong. Persamaan menakluk asalnya dimodelkan dalam kerangka makmal, diubah kepada kerangka gelombang. Persamaan yang terhasil adalah persamaan tanpa dimensi dan ketidaklinearan diturunkan menjadi linear dengan mengadaptasi penghampiran gelombang panjang dan nombor Reynolds yang kecil. Bentuk tak tersirat bagi penyelesaian analisis telah diperolehi dengan menggunakan kaedah usikan biasa. Pengaruh pelbagai parameter seperti parameter halaju gelinciran, parameter bendalir Sisko, nombor Brinkman, nombor Eckert, nombor Deborah, nombor Soret dan nombor Schmidt terhadap kuantiti aliran seperti halaju, tegasan ricih, keupayaan mengepam, keupayaan memerangkap, suhu, kepekatan dan pekali pemindahan haba telah dikaji. Hasil kajian menunjukkan bahawa keupayaan mengepam, keupayaan memerangkap dan suhu berkurangan dengan peningkatan parameter halaju gelinciran. Suhu dan pekali pemindahan haba meningkat dengan peningkatan nombor Brinkman, nombor Eckert dan nombor Deborah. Kepekatan menurun dengan peningkatan nombor Brinkman, nombor Soret dan nombor Schmidt. Kajian perbandingan antara bendalir likat dengan bendalir penipisan ricih dan bendalir penebalan ricih juga dibentangkan.