A strain gradient plasticity based damage model for quasibrittle materials

Article *in* Journal of Structural Engineering (Madras) · July 2016

Amirtham Rajagopal
Indian Institute of Technology Hyderabad

A strain gradient plasticity based damage model for quasibrittle materials.

K. Santosh[1] and Amirtham Rajagopal*[1]

[1]Department of Civil Engineering, IIT Hyderabad, India

June 2, 2016

Abstract

Boundary value problems for a softening material suffer from loss of uniqueness in the post-peak regime. Numerical solutions to such problems shows mesh dependency due to lack of internal length scale in the formulation. A regularization method which introduces a characteristic length is required to get mesh independent results. A second gradient model introduces a characteristic length by taking into account the second gradient of the displacement in the principle of virtual work and thus regularizing the solution of the boundary value problem. In this work a second gradient finite element model has been developed. The regularization property of the method has been studied for elastoplastic and damage constitutive laws. It has been shown that mesh independent results can be achieved in this model. Even though unique solution is not achieved a finite number of solutions have been obtained from the proposed model.

Keywords Localization, Regularization Methods, Second Gradient Model, Damage Mechanics, Quasi Brittle materials

*Corresponding author email: rajagopal@iith.ac.in

1 Introduction

Strain softening is the characteristic behaviour of concrete and rock materials during which the material shows decrease in stress with increase in strain or the material tangent stiffness matrix ceases to be positive. After establishing the constitutive behaviour in thermodynamical framework the next challenge is to solve the boundary value problem to get a finite number of responses. These softening models now lead to loss of uniqueness which leads to the loss of ellipticity of the corresponding boundary value problem [1].

Conditions for loss of uniqueness are derived using bifurcation theory. In bifurcation theory we explore the existence of non-unique stress states which correspond to localised deformation in bands [2]. The loss of ellipticity of the boundary value problem can be manifested as the existence of a line in two-dimensional continuum or a plane in three-dimensional continuum in which strain is discontinuous. This leads in the context of fracture or damage mechanics to failure with zero energy dissipation which is not physical [6]. If we try to solve numerically the boundary value problem the results are mesh dependent [4]. The introduction of characteristic length will provide a numerical solution which is independent of mesh [5].

Rice and Rudnicki [2],Rice [3] studied the formation of shear bands and concluded that the material localization is an instability phenomenon owing to its constitutive behaviour which leads to mesh dependent results in a numerical analysis. The main result of their analysis is that the corresponding boundary value problem looses ellipticity and the bifurcated solution is localized in to deformation bands when,

$$det(Q) = 0 \tag{1}$$

where $Q = \bar{n}.\overset{\equiv}{\bar{C}}.\bar{n}$ is called the acoustic tensor, $\overset{\equiv}{\bar{C}}$ is the tangent stiffness matrix and $\bar{n}$ is the unit normal to the deformation band. The numerical solutions then show spurious mesh dependency and become meaningless as they imply that the material fails with out any energy dissipation [6].

In order to solve this problem Bazant [7] introduced energy based approach and later in [8] he proposed non-local models. Gradient plasticity models are also proposed by Vordoulakis and Aifantis [10], Fleck and Hutchinson

[12] refining the constitutive behaviour. Instead of refining the constitutive behaviour law another idea is to refine the kinematics of the continuum using theory of materials with micro structure Aifantis [9]. Borst [11] refined the classical continuum using Cosserat models and achieved mesh independent results. Recently a new trend in this idea was proposed by chambon et al., [13–19], Fernandes [21] called local second gradient models. The advantage of these models is that a second gradient of the displacement field is introduced in to the kinematics of the continuum but not as a thermodynamic variable as done in gradient models by Peerlings [20]. So we can use the algorithms for classical constitutive laws to model first grade part and a second gradient constitutive law to model second gradient part. [18]. Later Chambon [13] gave the closed form solutions for a one-dimensional bar in traction in a second gradient setting. The results were also compared to a finite element discretization. However, since second order derivative are present in the weak formulation of a second gradient problem, C_1 continuity elements were used. Later in [15] a 2D second gradient finite element method was developed using mixed finite elements with C^0 continuity for deformation field and its gradient. The bifurcation analysis for a second gradient model was performed by Chambon et al., [14] using random initialisation technique.

In this work we present a local second gradient finite element model. The regularization property of the method is studied for elastoplastic and damage constitutive laws. The mesh dependency is studied using the distribution of the strains in case of elastoplastic constitutive law and the damage variable for the case of damage constitutive law. The paper has been organized as follows, in section 2 we present the second gradient model. In section 3 we present the finite element implementation of the second gradient model. In section 4 we present the constitutive laws for the elastic plastic framework. In section 5 we present the numerical results obtained from the present model.

2 SECOND GRADIENT MODEL

Second gradient theory can be derived from the theory of continua with microstructure which is derived first by Mindlin [22]. Later Germain [24] developed a framework taking in to account the microstucture in the principle of virtual work. In micromorphic continuum theory the microscopic structure of the continuum is taken in to account along with the macroscopic structure thus enhancing the kinematic field. By imposing a constraint on the micro kinematic field we can derive wide subclasses of theories like Cosserat theory, second gradient theory from the theory of micromorphic media. The key feature of the second gradient theory that enables it to introduce length scale is the additional degrees freedom that are added to the classical kinematic field. Different constraints on the kinematic field are considered by different workers [for example see the works of Fernandes [21], Chambon [13]]. In this paper the work of Chambon [13] is followed and a full decoupling between classical part and the second gradient part is considered. Neglecting the double body forces the principle of virtual work as in the general case of micromorphic continuum of degree 1 with domain Ω and boundary Γ is written as,

$$\int_{\Omega}((\sigma_{ij}+s_{ij})\frac{\partial u_i^*}{\partial x_j}-s_{ij}v_{ij}^*+\Sigma_{ijk}\frac{\partial v_{ij}^*}{\partial x_k})d\Omega=\int\rho f_i u_i^* d\Omega+\int_{\Gamma}(p_i u_i^*+T_{ij}v_{ij}^*)d\Gamma \tag{2}$$

where u_i^* is the virtual classical macro deformation field, v_{ij}^* is the virtual micro deformation field σ_{ij} is the classical macro stress tensor, s_{ij} is the dual stress tensor, Σ_{ijk} is the double stress tensor, p_i classical surface traction and T_{ij} double surface traction. By considering that the micro deformation field is same as the gradient of macro deformation field, we are imposing a constraint on the kinematic field which leads to the second gradient theory as follows,

$$v_{ij}=\frac{\partial u_i}{\partial x_j} \tag{3}$$

Finally the principle of virtual work in the case of second gradient theory is written as,

$$\int_{\Omega}(\sigma_{ij}\frac{\partial u_i^*}{\partial x_j}+\Sigma_{ijk}\frac{\partial v_{ij}^*}{\partial x_k})d\Omega=\int\rho f_i u_i^* d\Omega+\int_{\Gamma}(p_i u_i^*+P_i D u_i^*)d\Gamma \tag{4}$$

where, Du_i^* is the normal derivative of u_i^* and p_i, P_i are the two variables on which the boundary conditions are imposed. Using integration by parts we obtain the strong form of the equilibrium equation as,

$$\frac{\partial(\sigma_{ij}-\frac{\partial\Sigma_{ijk}}{\partial x_k})}{\partial x_j}+\rho f_i=0 \tag{5}$$

Assuming that the boundary is regular which means existence and uniqueness of the normal for every point of the boundary, after one more integration by parts, we get boundary conditions as,

$$\sigma_{ij}n_j - n_k n_j D\Sigma_{ijk} - D_k\Sigma_{ijk}n_j - D_j\Sigma_{ijk}n_k + D_l n_l \Sigma_{ijk}n_j n_k - D_k n_j \Sigma_{ijk} = p_i \quad (6)$$

and

$$\Sigma_{ijk}n_j n_k = P_i \quad (7)$$

where, D is the normal derivative of any quantity and D_k is the tangential derivative of any quantity q is defined as,

$$D_k = \frac{\partial q}{\partial x_j} - \frac{\partial q}{\partial x_k} n_k n_j \quad (8)$$

For the case of a one dimensional bar in traction the analytical solution (as derived in [13]) contains a characteristic length defined by $l_c = \sqrt{\frac{-B}{A_2}}$ where A_2 is the softening modulus of first gradient part and B is the modulus of second gradient part.

3 SECOND GRADIENT FINITE ELEMENT FORMULATION

Chambon [13] formulated 1D finite element method using C^1 continuity elements due to the presence of second order derivatives in the weak form or principle of virtual work. But it is very difficult to formulate 2D and 3D cases using C^1 continuity elements. So the solution proposed by Matsushima [15] is to introduce the Lagrange multipliers in to the weak form and expressing the kinematic constraint in weak form.

3.1 Basic Equilibrium

Starting from the principle of virtual work for second grade micromorphic continua with virtual displacement field u_i^*,

$$\int_\Omega (\sigma_{ij}\frac{\partial u_i^*}{\partial x_j} + \Sigma_{ijk}\frac{\partial^2 u_i^*}{\partial x_j \partial x_k})d\Omega - P_e^* = 0 \quad (9)$$

where P_e^* is a virtual work by external forces (surface force and body force)

$$P_e^* = \int_\Gamma n_j \sigma_{ij} u_i^* d\Gamma + \int_\Omega \rho f_i u_i^* d\Omega + \int_\Gamma n_j (\Sigma_{ijk} \frac{\partial u_i^*}{\partial x_k} - \frac{\partial \Sigma_{ijk}}{\partial x_k} u_i^*) d\Gamma \quad (10)$$

$$= \int \rho f_i u_i^* d\Omega + \int_\Gamma n_j (\sigma_{ij} - \frac{\partial \Sigma_{ijk}}{\partial x_k}) u_i^* d\Gamma + \int_\Gamma n_j \Sigma_{ijk} \frac{\partial u_i^*}{\partial x_k} d\Gamma \quad (11)$$

Using divergence theorem and addition theorem the strong form :

$$\frac{\partial \sigma_{ij}}{\partial x_j} - \frac{\partial^2 \Sigma_{ijk}}{\partial x_j \partial x_k} + \rho f_i = 0 \quad (12)$$

We introduce Lagrange multiplier in the constraint equation to avoid C^1 continuity. Now introducing Lagrange multiplier, λ_{ij} the above equation is transformed to the following two equations:

$$\frac{\partial \sigma_{ij}}{\partial x_j} - \frac{\partial \lambda_{ij}}{\partial x_j} + \rho f_i = 0 \quad (13)$$

$$\frac{\partial \Sigma_{ijk}}{\partial x_k} - \lambda_{ij} = 0 \quad (14)$$

weak form of the above equations with the virtual displacement fields, u_i^* and v_{ij}^* :

$$\int_\Omega (\frac{\partial \sigma_{ij}}{\partial x_j} - \frac{\partial \lambda_{ij}}{\partial x_j} + \rho f_i) u_i^* + \int_\Omega (\frac{\partial \Sigma_{ijk}}{\partial x_k} - \lambda_{ij}) v_{ij}^* d\Omega = 0 \quad (15)$$

weak form of the equilibrium equations :

$$\int_\Omega (\sigma_{ij} \frac{\partial u_i^*}{\partial x_j} + \Sigma_{ijk} \frac{\partial v_{ij}^*}{\partial x_k}) d\Omega - \int_\Omega \lambda_{ij} (\frac{\partial u_i^*}{\partial x_j} - v_{ij}^*) d\Omega - P_e^* = 0 \quad (16)$$

where,

$$P_e^* = \int_\Gamma n_j \sigma_{ij} u_i^* d\Gamma + \int_\Omega \rho f_i u_i^* d\Omega + \int_\Gamma n_j (\Sigma_{ijk} \frac{\partial u_i^*}{\partial x_k} - \frac{\partial \Sigma_{ijk}}{\partial x_k} u_i^*) d\Gamma \quad (17)$$

we need the following constraint to make the perfectly equivalent form to the original one:

$$\frac{\partial u_i}{\partial x_j} - v_{ij} = 0 \tag{18}$$

weak form of the above equation:

$$\int_\Omega \lambda_{ij}^* (\frac{\partial u_i}{\partial x_j} - v_{ij}) d\Omega = 0 \tag{19}$$

In order to avoid any boundary layer effects at the surface we impose $\Sigma_{ijk} = 0$ this assumption leads to

$$\int_\Gamma n_k \Sigma_{ijk} v_{ij}^* d\Gamma = 0 \tag{20}$$

3.2 1-D Formulation

Combining the eqn.(17) and eqn.(19) we obtain,

$$\int_\Omega (\sigma_{ij} \frac{\partial u_i^*}{\partial x_j} + \Sigma_{ijk} \frac{\partial v_{ij}^*}{\partial x_k}) d\Omega - \int_\Omega \lambda_{ij} (\frac{\partial u_i^*}{\partial x_j} - v_{ij}^*) d\Omega - \int_\Omega \lambda_{ij}^* (\frac{\partial u_i}{\partial x_j} - v_{ij}) d\Omega - P_e^* = 0 \tag{21}$$

where owing to the eqn.(20),

$$P_e^* = \int_\Gamma n_j \sigma_{ij} u_i^* d\Gamma + \int_\Omega \rho f_i u_i^* d\Omega \tag{22}$$

So the variables to be considered are:

$$[U]^T \equiv [\ \frac{\partial u_1}{\partial x_1}\ \ \frac{\partial v_{11}}{\partial x_1}\ \ v_{11}\ \ \lambda_{11}\ ] \tag{23}$$

Let the constitutive relations be:

$$\sigma_{ij} = c_{ijkl} \frac{\partial u_k}{\partial x_l} \tag{24}$$

$$\Sigma_{ijk} = d_{ijklmn} \frac{\partial v_{lm}}{\partial x_n} \tag{25}$$

In the case of one dimensional problem,

$$\sigma_{11} = c \frac{\partial u_1}{\partial x_1} \tag{26}$$

$$\Sigma_{111} = d\frac{\partial v_{11}}{\partial x_1} \tag{27}$$

Hence the eqn.(21) can also be written in matrix notation as,

$$\int_\Omega [U^{*T}][E][U]d\Omega = P_e^* \tag{28}$$

where,

$$[E] = \begin{bmatrix} c & 0 & 0 & -1 \\ 0 & d & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

In order to enforce the mathematical constraint eqn.(19),penalisation parameter along with lagrange multipliers is introduced and the corresponding element stiffness is modified as below,

$$\int_\Omega (\sigma_{ij}\frac{\partial u_i^*}{\partial x_j} + \Sigma_{ijk}\frac{\partial v_{ij}^*}{\partial x_k})d\Omega - \int_\Omega \lambda_{ij}(\frac{\partial u_i^*}{\partial x_j} - v_{ij}^*)d\Omega - \int_\Omega [\lambda_{ij}^*(\frac{\partial u_i}{\partial x_j} - v_{ij}) - r(\frac{\partial u_i}{\partial x_j} - v_{ij})(\frac{\partial u_i^*}{\partial x_j} - v_{ij}^*)]d\Omega - P_e^* = 0 \tag{29}$$

with r being the penalisation constant. The corresponding $[E_{pen}]$ matrix is given by,

$$[E_{pen}] = [E] + \begin{bmatrix} r & 0 & -r & 0 \\ 0 & 0 & 0 & 0 \\ -r & 0 & r & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

3.3 Discretisation in 1-D Formulation

3.3.1 Shape function and Mapping function

We use linear elements which has 3 nodes for u_1, 2 nodes for v_{11} and one node for λ_{11}. The parent element is denoted by s(-1 $<$ s $<$ 1), and all the variables are defined as the functions of these parent coordinates.

In the parent element the different polynomial shape function is assumed for different variables considering its physical dimension.

As for u_1 we adopt the quadratic shape function,

$$u_1(s) = [\ \phi_1(s)\ \ \phi_2(s)\ \ \phi_3(s)\ ] \begin{bmatrix} u_1|_{(s=-1)} \\ u_1|_{(s=0)} \\ u_1|_{(s=1)} \end{bmatrix}$$

where,

$$\begin{aligned} \phi_1(s) &= 0.5s(s-1) \\ \phi_2(s) &= 0.5s(s+1) \\ \phi_3(s) &= 1 - s^2 \end{aligned} \tag{30}$$

As for v_{ij} we adopt the linear shape function,

$$v_{11}(s) = [\ \psi_1(s)\ \ \psi_2(s)\ ] \begin{bmatrix} v_{11}|_{(s=-1)} \\ v_{11}|_{(s=1)} \end{bmatrix}$$

where,

$$\begin{aligned} \psi_1(s) &= 0.5(1-s) \\ \psi_2(s) &= 0.5(1+s) \end{aligned} \tag{31}$$

For λ_{11}

$$\lambda_{11}(s) = \lambda_{11}|_{(s=0)}$$

As for the mapping function from x_1 to s, we assume the same function as the shape function for $u_1(s)$ that is,

$$x_1(s) = [\ \phi_1(s)\ \ \phi_2(s)\ \ \phi_3(s)\ ] \begin{bmatrix} x_1|_{(s=-1)} \\ x_1|_{(s=0)} \\ x_1|_{(s=1)} \end{bmatrix}$$

3.3.2 Transformation matrix

In order to transform the set of variables U used in discretised equation, we use the following relation considering that all the variables are the functions of the parent coordinates,

$$\frac{\partial u_1}{\partial s} = \frac{\partial u_1}{\partial x_1}\frac{\partial x_1}{\partial s} \tag{32}$$

Similarly for v_{ij},

$$\frac{\partial v_{11}}{\partial s} = \frac{\partial v_{11}}{\partial x_1}\frac{\partial x_1}{\partial s} \tag{33}$$

To connect with nodal variables,

$$[U] = [T][U_{(s)}] \tag{34}$$

$$[U_{(s)}] = [B][U_{node}] \tag{35}$$

where,

$$[T] = \begin{bmatrix} t & 0 & 0 & 0 \\ 0 & t & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and

$$t = [\frac{\partial x_1}{\partial s}]^{-1}$$

and

$$[B] = \begin{bmatrix} \frac{\partial \phi_1}{\partial x_1} & 0 & \frac{\partial \phi_2}{\partial x_1} & 0 & \frac{\partial \phi_3}{\partial x_1} & 0 \\ 0 & \frac{\partial \psi_1}{\partial x_1} & 0 & \frac{\partial \psi_2}{\partial x_1} & 0 & 0 \\ 0 & \psi_1 & 0 & \psi_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[U_{(s)}]^T \equiv [\ \frac{\partial u_1}{\partial s}\ \ \frac{\partial v_{11}}{\partial s}\ \ v_{11}\ \ \lambda_{11}\ ] \tag{36}$$

$$[U_{node}] = [\ u_{1(-1)}\ \ v_{11(-1)} \tag{37}$$

$$\lambda_{11(0)} \tag{38}$$

$$u_{1(1)}\ \ v_{11(1)}\ ] \tag{39}$$

$$\tag{40}$$

3.3.3 Element Stiffness Matrix

Using the above equations,

$$\int_{\Omega_{elem}} [U^*]^T [E_{pen}][U] d\Omega = [U^*_{node}]^T \int_{-1}^{1} \int_{-1}^{1} [B]^T [T]^T [E_{pen}][T][B] \frac{\partial x_1}{\partial s} ds [U_{node}]$$
$$= [U_{node}]^T [k][U_{node}] \qquad (41)$$

The external body force is not considered here and as the surface orientation does not change during the deformation the external surface forces are applied at each node.

4 CONSTITUTIVE LAWS

4.1 First grade part

4.1.1 Elastoplastic framework

With in the Elastoplastic framework we consider,

1. Decomposition of strain,

$$\frac{\partial \dot{u}_1}{\partial x_1} = (\frac{\partial \dot{u}_1}{\partial x_1})_e + (\frac{\partial \dot{u}_1}{\partial x_1})_p$$

2. Elastic part is defined by,

$$\dot{\sigma_{11}} = A_1 (\frac{\partial \dot{u}_1}{\partial x_1})_e$$

3. Yield condition,

$$f(\sigma) = \sigma_{11} - \sigma_{max} + (\frac{\partial u_1}{\partial x_1})_p \frac{A_1 A_2}{A_2 - A_1} = 0$$

with $\sigma_{max} = A_1 \, e_{lim}$,

4. Consistency condition,

$$\frac{\partial \dot{\sigma_{11}}}{\partial x_1} + \frac{\partial \dot{u_1}}{\partial x_1}\frac{A_1 A_2}{A_2 - A_1} = 0$$

Finally, if f(σ) <= 0 and $\dot{f}(\sigma) < 0$ then $\dot{\sigma_{11}} = A_1 \frac{\partial \dot{u_1}}{\partial x_1}$, otherwise $\dot{\sigma_{11}} = A_2 \frac{\partial \dot{u_1}}{\partial x_1}$. For small strain assumptions the constitutive equations are integrated as,

1. If $\frac{\partial u_1}{\partial x_1} < e_{lim}$

$$\sigma_{11} = A_1 \frac{\partial u_1}{\partial x_1} \tag{42}$$

2. If $e_{lim} < \frac{\partial u_1}{\partial x_1} < e_{lim}(1 - (A1/A2))$

$$\sigma_{11} = A_2 \frac{\partial u_1}{\partial x\ _1} + (A_1 - A_2) e_{lim} \tag{43}$$

3. If $\frac{\partial u_1}{\partial x_1} > e_{lim}(1 - (A1/A2))$

$$\sigma_{11} = 0 \tag{44}$$

4.1.2 Damage mechanics framework

With in the Damage mechanics frame work we consider scalar local damage model by Mazars [25], lemaitre and chaboche [26]. In this model, the material is supposed to behave elastically and to remain isotropic. The loading surface takes the following form:

$$f(\epsilon, D) = \epsilon_{eq} - \epsilon_{d0} \tag{45}$$

where ϵ_{eq} is defined as,

$$\epsilon_{eq} = \sqrt{\Sigma_1^2 < \epsilon >_+^2} \tag{46}$$

ϵ_{do} is the damage threshold. For the case of pure traction (which we consider here) the evolution law for damage in tension is given by,

$$D \ = \ 0 \ \ if \, f \ < \ 0 \tag{47}$$

$$D = 1 - \frac{\epsilon_{d0}}{\epsilon_{eq}} (1 - A) - A\, exp(-B\, (\epsilon_{eq} - \epsilon_{d0}))\ \ if\, f > 0 \tag{48}$$

A,$B$and ϵ_{d0} are material parameters. Finally the constitutive relation is given by,

$$\sigma_{11} = (1 - D)\epsilon_{11} \tag{49}$$

4.2 Second Grade Part

For the second grade part a linear elastic relationship as proposed by Mindlin [22] is assumed as,

$$\Sigma_{111} = d\ \frac{\partial^2 u_1}{\partial x_1^2} \tag{50}$$

5 NUMERICAL RESULTS

5.1 Using elastoplastic constitutive law for first grade part

5.1.1 Mesh Independency

A simple bar of length 1 m in traction is studied using second gradient model. Finite element calculations are done using MATLAB. The parameters of the constitutive equation chosen are $A_1 = 150$, $A_2 = -75$, $d = 0.8$ and $e_{lim} = 0.01$. Fig.(1) illustrates the mesh dependency of the classical finite element method. Mesh independency is studied using 50, 100, 150, 200 elements for an applied displacement U = 15.0E-3. Fig.(2) demonstrates clearly the mesh independency of the second gradient model. In all the cases an imperfection length of 0.2 m is introduced and corresponding localised solution obtained is hard-soft-hard. Fig.(3) shows the evolution of global reaction against the applied global displacement at the end of the bar.

5.1.2 Non-Unicity

Another technique to trigger localisation is random initialisation. In this technique we initialise the newton-raphson method at particular step with random solution other than the previously converged one. With different initialisations we obtain different localised solution and thus the bifurcation phenomenon is studied. Totally there are three types of solutions achievable

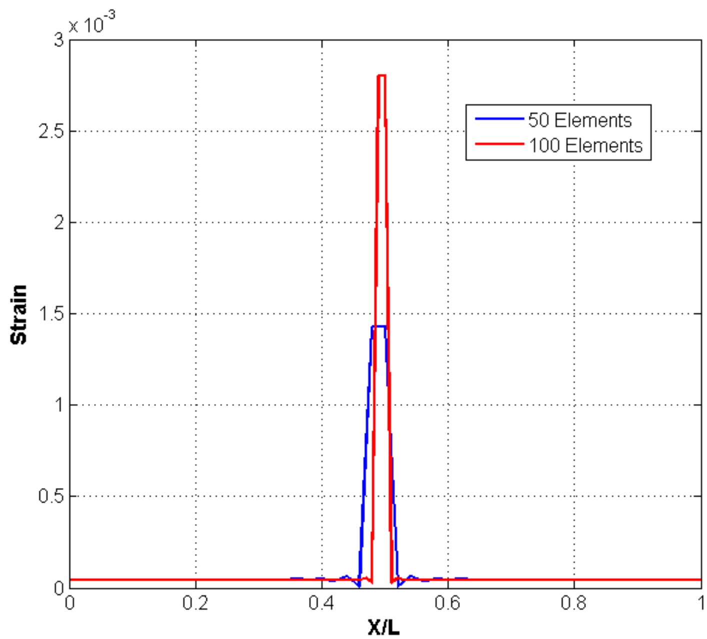

Figure 1: Mesh Dependency (U = 15.0E-3)

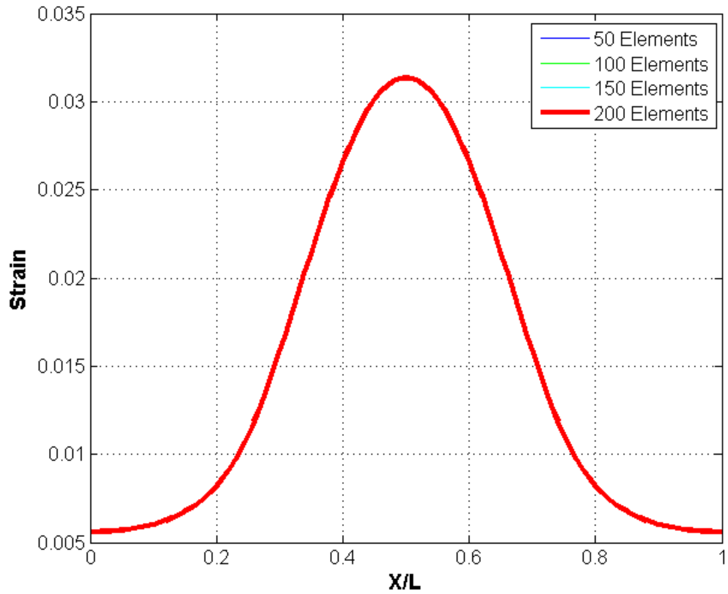

Figure 2: Mesh Independency (U = 15.0E-3)

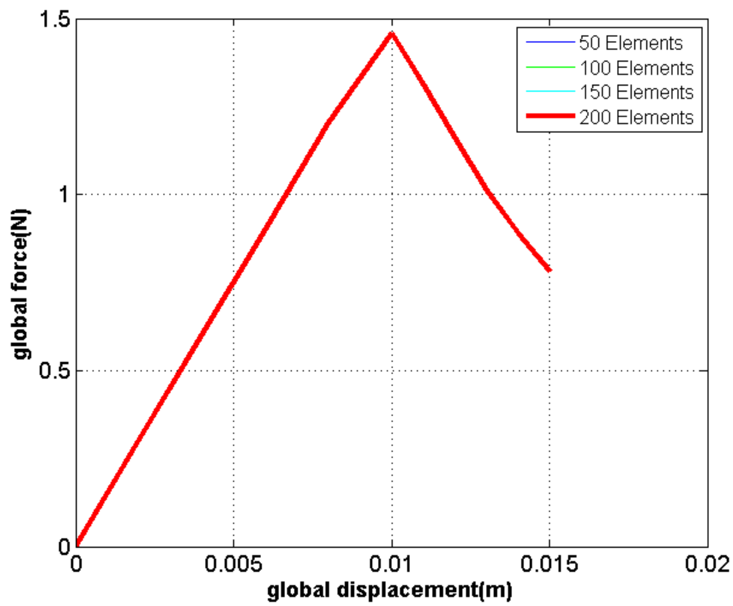

Figure 3: global reaction vs. global displacement (U = 15.0E-3)

for the considered problem. Fig.(4) demonstrates the non-unicity of solutions and Fig.(5) shows the bifurcation phenomenon.

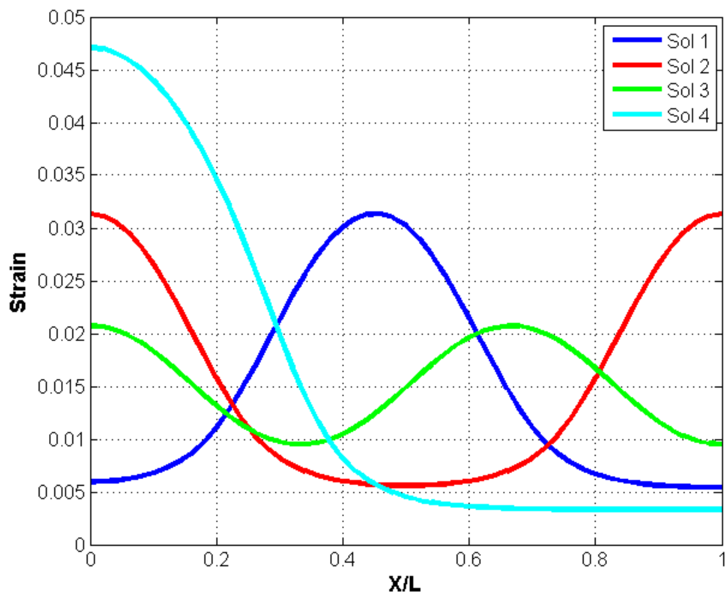

Figure 4: Non-Unicity

5.1.3 Effect of the Penalisation Term

It is observed that the fields v_{11} and $\dfrac{\partial u_1}{\partial x_1}$ are not equal along the element or in other words the constraint is not exactly imposed with out the penalisation term. For example take an arbitrary element (say tenth element), Fig.(6) shows the distribution of both the fields along the element and the penalisation term of 1.0E+04 corrects the distribution as shown in Fig.(7).

5.1.4 Using Mazars damage law for first grade part

Here Mazars law with parameters as ϵ_{d0} = 1.0e-4, A_t = 0.5,B_t = 2.0e-05 are used as constitutive driver for the first gradient part and for second gradient

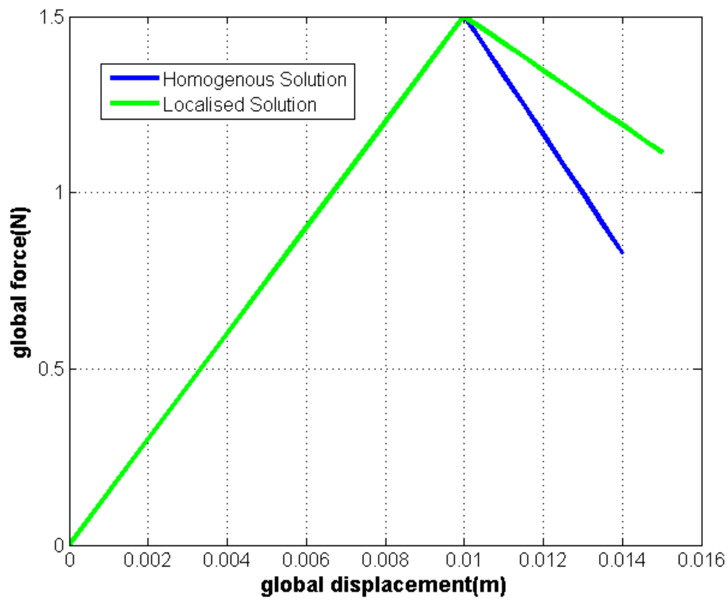

Figure 5: Bifurcation

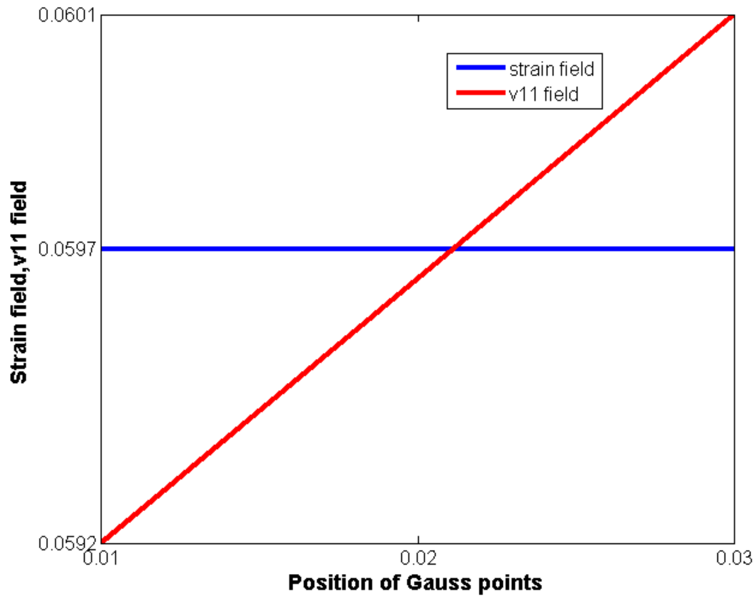

Figure 6: Constraint with out penalty

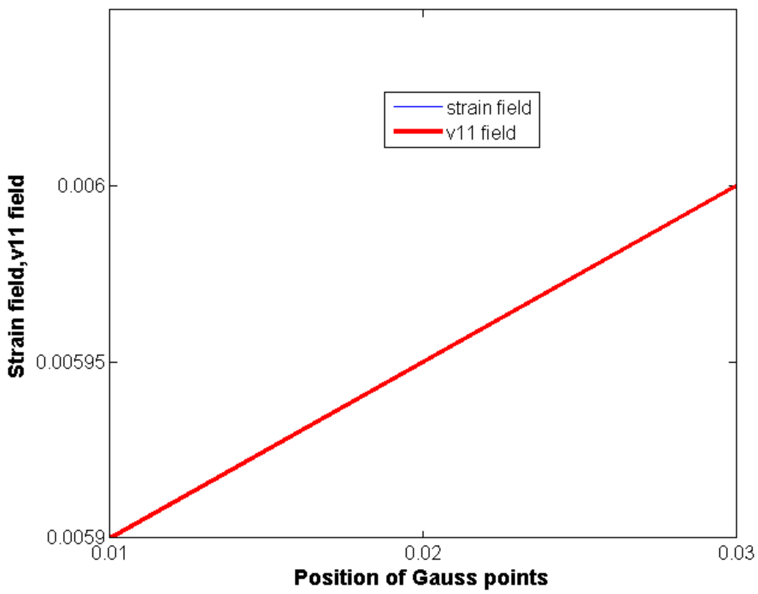

Figure 7: Constraint with penalty

part d = 200 is used . Fig.(8) shows the mesh independent damage distribution for 50 and 100 elements and Fig.(9) shows the Force vs. Displacement curve for both meshes for applied displacement U = 15E-05.

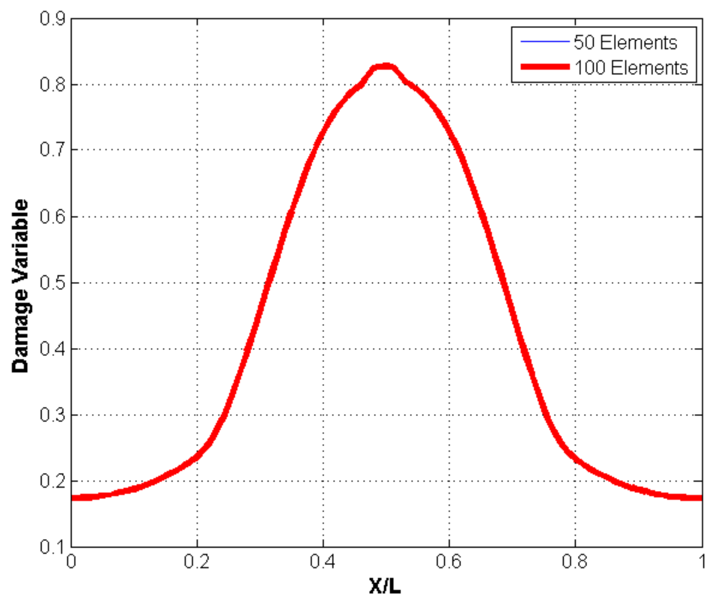

Figure 8: Damage distribution (U = 15E-05)

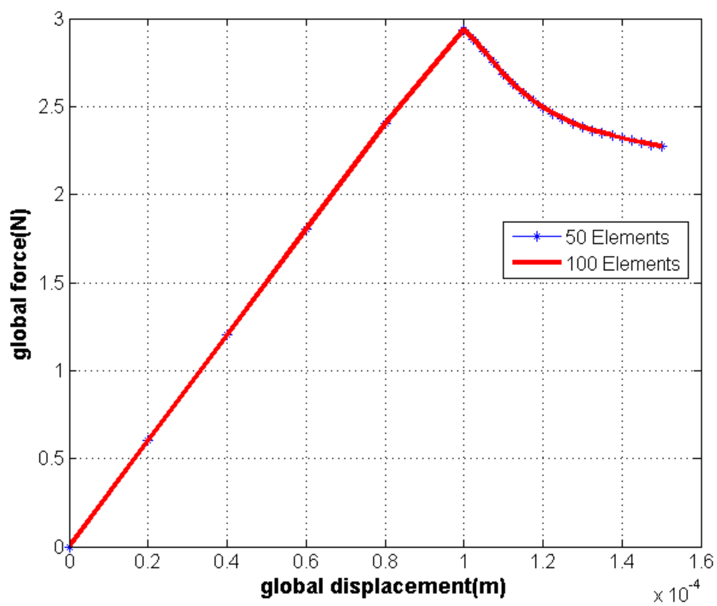

Figure 9: global reaction vs. displacement (U = 15E-05)

6 CONCLUSION

The characteristic length introduced using material properties in second gradient model can regularise the solution and mesh independency can be achieved. A second gradient model does not ensures uniqueness of the solution boundary value problem but instead of infinite number of solutions by using a classical model we get finite number of solutions for a second gradient model.

References

[1] Hill R. (1958), *A general theory of uniqueness and stability in elastic-plastic solids*, J. Mech. Phys. Solids, 1958, Vol. 6 ,pp. 236-249.

[2] Rudnicki J.W. and Rice J.R. (1975), *Conditions for the Localization of Deformation in Pressure-Sensitive Dilatant Materials.*,J. Mech. Phys. Solids, 1975, Vol. 23, pp. 371-394.

[3] Rice J.R. (1976),*The localization of plastic deformation*, pp. 207–220 in 14th Int. Cong. Theor. Appl. Mech., edited by W. T. Koiter, Theoretical and Applied Mechanics (North-Holland Pub. Comp., Delft.

[4] Needlemann A. (1988),*Material rate dependence and mesh sensitivity in localisation problems*, Comp. Meth. Appl. Mech. Eng. 1988, Vol. 67, pp. 69-86.

[5] Jirasek M. (2002), *Objective modelling of strain localization*,Revue francaise de genie civil, 2002, Vol .6, pp. 1190-1132.

[6] Pijaudier-Cabot G., Bazant ZP. (1987) *Nonlocal damage theory*, ASCE J Eng Mech, 1987, Vol. 113, pp. 1512–1533

[7] Bazant ZP. (1976), *Instability, ductility and size effect in strain softening concrete*, Journal of Eng. Mech., ASCE, 1976, Vol.102, pp. 331-344.

[8] Bazant ZP., Oh BH. (1983), *Crack band theory for fracture of concrete. Materials and Structures*, 16 RILEM : Paris, France, 1983, pp 155-177.

[9] Aifantis E. C.(1984), *On the microstructural origin of certain inelastic models.*, Journal of Engineering Materials and Technology, ASME, 1984, Vol. 106, pp. 326–330.

[10] Vardoulakis I., Aifantis E., (1991), *A gradient flow theory of plasticity for granular materials*, Acta Mech. 1991, Vol. 87, pp. 197–217.

[11] Borst R.D. (1991),*Numerical Modelling of Bifurcation and Localisation in Cohesive-Frictional Materials*, PAGEOPH, Vol. 137, pp. 367-390.

[12] Fleck N.A and Hutchinson J.W (1997),*Strain Gradient Plasticity*, Advances in Applied Mechanics, 1997, Vol 33, pp. 295-358.

[13] Chambon R., Caillerie D. and EL Hassan N. (1998), *One-dimensional localisation studied with a second grade model*,Eur. .I. Mech. A/Solids, 1998, Vol.17, pp. 637-656.

[14] Chambon R., Moullet J.C.,(2004) *Uniqueness studies in boundary value problems involving some second gradient models*,Comput. Methods Appl. Mech. Engrg. 2004, Vol. 193, pp. 2771–2796.

[15] Matsushima T., Chambon R. and Caillerie D. (2002), *Large strain finite element analysis of a local second gradient model: application to localization*,Int. J. Numer. Meth. Engng 2002, Vol. 54, pp. 499–521.

[16] Besuelle P., Chambon R. and Collin F. (2006), *Switching Deformation Modes in Post-Localization Solution with a Quasibrittle Material*, Journal of Mechanics of Materials and Structures vol. 1, pp.1115-1134

[17] Besuelle P., Chambon R., *Modelling the Post-Localisation regime with Local-Second Gradient Models: Non-uniqueness of Solutions and Non-persistent Shear Bands*,Springer Proceedings in Physics Vol.106, pp. 209-221.

[18] Kotronis P., Al Holo S., Besuelle P., Chambon R. (2008), *Shear softening and localization: Modelling the evolution of the width of the shear zone*,Acta Geotechnica, 2008, Vol. 3, pp. 85–97.

[19] Kotronis P., Chambon R., Mazars J. and Collin F., *Local Second Gradient Models and Damage Mechanics: Application to Concrete.*

[20] Peerlings R.J (1999), *Enhanced damage modelling for fracture and fatigue*, Phd Thesis.

[21] Fernandes R., Clement Chavant B, Chambon R (2008), *A simplified second gradient model for dilatant materials: Theory and numerical implementation*, International Journal of Solids and Structures, 2008, Vol. 45, pp. 5289–5307.

[22] Mindlin R.D., (1965). *Second gradient of strain and surface-tension in linear elasticity*, Int. J. Solids Struct. 1965, Vol. 1, pp. 417–738.

[23] Mindlin R.D., (1964), *Micro-structure in linear elasticity*, Arch. Ration. Mech. Anal. 1964,Vol.16, pp. 51–78.

[24] Germain (1973), *The Method of Virtual Power in Continuum Mechanics. Part 2: Microstructure*,SIAM Journal on Applied Mathematics, 1973, Vol. 25, No. 3: 556-575.

[25] Mazars J. (1986), *A description of micro and macroscale damage of concrete structures.* Engrg. Fract. Mech., Vol. 25, pp. 729-737.

[26] Lemaitre J., Chaboche J.L. (1985) *Mécanique des materiaux solides.* Dunod-Bordas Ed. Paris, France.