COMPATIBLE PAIRS OF ACTIONS FOR FINITE CYCLIC 2-GROUPS AND THE ASSOCIATED COMPATIBLE ACTION GRAPHS

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Doctor of Philosophy

UNIVERSITI MALAYSIA PAHANG

SUPERVISOR'S DECLARATION

We hereby declare that We have checked this thesis and in our opinion, this thesis is adequate in terms of scope and quality for the award of the degree of Doctor of Philosophy

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STUDENT'S DECLARATION

I hereby declare that the work in this thesis is based on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at Universiti Malaysia Pahang or any other institutions.

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Thesis submitted in fulfillment of the requirements
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My humble effort I dedicate to my sweet and loving

Father who could not see this thesis completed
Mother
Siblings and spouses
Two beautiful nieces

Whose affection, love, encouragement and prays of day and night make me able to get such success and honor.

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1	identity element
a mod b	a modulo b
a^{-1}	inverse of a
$\langle a \rangle$	cyclic subgroup generated by a
$[a, b]$	commutator of a and b
Aut (G)	automorphism group of group G
C_n	the cyclic group of order n
$\deg^+(v)$	the out-degree of a vertex v
$\deg^-(v)$	the in-degree of a vertex v
D_n	the dihedral group of order $2n$
$E(\Gamma_{G\otimes H})$	the set of edges of the compatible action graph $\Gamma_{G\otimes H}$
(g^k, h^l)	the compatible pair of actions
G	a finite group
$\lvert G \rvert$	the order of G
$G \otimes H$	the nonabelian tensor product of the groups G and H
$G \cong H$	the groups G and H are isomorphic
${}^g h$	action of g on h
$H \leq G$	H is a subgroup of G
$L \wedge K$	the nonabelian tensor exterior product
$L \square N$	the diagonal ideal
$\mathbb{N}$	the set of natural numbers
Q_n	the quaternion group of order $2n$
t | s	t divides s
$V(\Gamma_{G\otimes H})$	the set of vertices of the compatible action graph $\Gamma_{G\otimes H}$
S_n	the symmetric group of degree n
SO_n	the special orthogonal group of order n
$Z(G)$	the center of G
$\mathbb{Z}$	the set of integers
$\in$	element of
$\Gamma_{G\otimes H}$	the compatible action graph of $G \otimes H$
$\Gamma_{C_{2^{m-i}} \otimes C_{2^{n-i}}}$	the subgraph of the compatible action graph for the groups $C_{\rho^{m-i}}$ and $C_{\rho^{n-i}}$
$\square$	end of proof