

COMPATIBLE PAIRS OF ACTIONS FOR FINITE CYCLIC 2-GROUPS AND THE ASSOCIATED COMPATIBLE ACTION GRAPHS

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We hereby declare that We have checked this thesis and in our opinion, this thesis is adequate in terms of scope and quality for the award of the degree of Doctor of Philosophy

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I hereby declare that the work in this thesis is based on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at Universiti Malaysia Pahang or any other institutions.

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My humble effort I dedicate to my sweet and loving

Father who could not see this thesis completed

Mother

Siblings and spouses

Two beautiful nieces

Whose affection, love, encouragement and prays of day and night

make me able to get such success and honor.

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TABLE OF CONTENTS

DECLARATION	
TITLE PAGE	
ACKNOWLEDGEMENTS	iii
ABSTRAK	iv
ABSTRACT	v
TABLE OF CONTENTS	vi
LIST OF TABLES	ix
LIST OF SYMBOLS	x
CHAPTER 1 INTRODUCTION	1
1.1 An Overview	1
1.2 Research Background	1
1.3 Problem Statement	3
1.4 Research Objectives	3
1.5 Research Scope	4
1.6 Research Significant	4
1.7 Thesis Organization	4
CHAPTER 2 LITERATURE REVIEW	6
2.1 Introduction	6
2.2 The Nonabelian Tensor Product	6
2.3 A View of Graph Theory	11
2.4 Conclusion	12

CHAPTER 3 PRELIMINARY RESULTS	13
3.1 Introduction	13
3.2 Automorphism Groups	13
3.3 Compatible Actions	15
3.4 Graph Theory	17
3.5 The GAP Programmes for Compatibility	19
3.6 Conclusion	22
CHAPTER 4 THE EXACT NUMBER OF THE COMPATIBLE PAIR OF ACTIONS FOR CYCLIC 2-GROUPS	23
4.1 Introduction	23
4.2 The Compatible Pairs of Actions with Specific Order for Cyclic 2-Groups	23
4.3 The Total Number of the Compatible Pairs of Actions for Cyclic 2-Groups	26
4.4 Compatible Pairs of Actions for Cyclic 2-Groups of the Same Order	29
4.5 Conclusion	34
CHAPTER 5 THE COMPATIBLE ACTION GRAPH	35
5.1 Introduction	35
5.2 Motivation of Compatible Action Graph	35
5.3 Properties of Compatible Action Graph	37
5.4 Types of Compatible Action Graphs	42
5.5 Conclusion	44
CHAPTER 6 SOME SUBGRAPHS OF THE COMPATIBLE ACTION GRAPH	45
6.1 Introduction	45
6.2 Compatibility Condition for Subgroup of Cyclic 2-Groups	45

6.3	Properties of a Subgraph of the Compatible Action Graph	47
6.4	Conclusion	54
CHAPTER 7 SUMMARY AND CONCLUSION		55
7.1	Summary of the Research	55
7.2	Recommendation for Future Research	56
REFERENCES		57
APPENDIX A		61
APPENDIX B		72

LIST OF TABLES

Table 3.1	The Number of Compatible Pairs of Actions for $C_{2^m} \otimes C_{2^n}$	21
Table 4.1	Compatible Pairs of Actions for $C_{2^4} \otimes C_{2^6}$	29
Table 6.1	The intersection between two compatible action graphs for $\deg^+(v) = 4$	50

LIST OF SYMBOLS

1	identity element
$a \bmod b$	a modulo b
a^{-1}	inverse of a
$\langle a \rangle$	cyclic subgroup generated by a
$[a, b]$	commutator of a and b
$\text{Aut}(G)$	automorphism group of group G
C_n	the cyclic group of order n
$\text{deg}^+(v)$	the out-degree of a vertex v
$\text{deg}^-(v)$	the in-degree of a vertex v
D_n	the dihedral group of order $2n$
$E(\Gamma_{G \otimes H})$	the set of edges of the compatible action graph $\Gamma_{G \otimes H}$
(g^k, h^l)	the compatible pair of actions
G	a finite group
$ G $	the order of G
$G \otimes H$	the nonabelian tensor product of the groups G and H
$G \cong H$	the groups G and H are isomorphic
${}^g h$	action of g on h
$H \leq G$	H is a subgroup of G
$L \wedge K$	the nonabelian tensor exterior product
$L \square N$	the diagonal ideal
\mathbb{N}	the set of natural numbers
Q_n	the quaternion group of order $2n$
$t s$	t divides s
$V(\Gamma_{G \otimes H})$	the set of vertices of the compatible action graph $\Gamma_{G \otimes H}$
S_n	the symmetric group of degree n
SO_n	the special orthogonal group of order n
$Z(G)$	the center of G
\mathbb{Z}	the set of integers
\in	element of
$\Gamma_{G \otimes H}$	the compatible action graph of $G \otimes H$
$\Gamma_{C_{2^{m-i}} \otimes C_{2^{n-i}}}$	the subgraph of the compatible action graph for the groups $C_{2^{m-i}}$ and $C_{2^{n-i}}$
\square	end of proof