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A mixed-integer linear programming approach to the optimization of event-bus schedules: a scheduling application in the tourism sector

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Abstract This paper deals with "The Enchanted Journey," which is a daily event tour booked by Bollywood-film fans. During the tour, the participants visit original sites of famous Bollywood films at various locations in Switzerland; moreover, the tour includes stops for lunch and shopping. Each day, up to five buses operate the tour. For operational reasons, however, two or more buses cannot stay at the same location simultaneously. Further operative constraints include time windows for all activities and precedence constraints between some activities. The planning problem is how to compute a feasible schedule for each bus. We implement a two-step hierarchical approach. In the first step, we minimize the total waiting time; in the second step, we minimize the total travel time of all buses. We present a basic formulation of this problem as a mixed-integer linear program. We enhance this basic formulation by symmetry-breaking constraints, which reduces the search space without loss of generality. We report on computational results obtained with the Gurobi Solver. Our numerical results show that all relevant problem instances can be solved using the basic formulation within reasonable CPU time, and that the symmetry-breaking constraints reduce that CPU time considerably.

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1 Introduction

The tourism sector plays a major role in the economies of many countries, for example, in Switzerland, where in 2005 this sector added a gross value of 12.6 billion Swiss francs (Baumann and Schiess 2008). Offers and demand in this sector are strongly influenced by the ongoing change from a leisure-based society to an experience-based society. Today, there is an increasing demand for holiday packages with event-type elements (Müller 2001). To satisfy this demand, various travel agencies are developing services which contain such elements. An example for such services are event tours, which allow the participants to transform a passive visit into an experienced reality.

With the growing expectations of customers for quality and for convenience, and the rising amount of competition, a detailed planning of the operations is vital in such a tour. Here, we consider the event tour "The Enchanted Journey." This tour leads the participants through famous sites of Bollywood movies, which were filmed in the mountainous region of Switzerland. The transportation of the participants between the sites is done by bus. Depending on the demand, up to five buses operate on the same day; however, there must not be two or more buses at the same location simultaneously. The problem discussed in this paper is how to compute a schedule for each bus operating on a given day such that the total waiting time (primary objective) and the total travel time (secondary objective) are minimized. The tour operator contacted us in the design phase of the tour; optimal or at least good feasible solutions to this problem were required as a proof-of-concept for the negotiations with the respective marketing organizations.

This problem is related to the well-known multiple traveling salesman problem (mTSP) (see, e.g., Bektas 2006; Kara and Bektas 2006) and the open-shop scheduling problem (OSP) (see, e.g., Graves 1981; Yu et al. 2011). The major difference to the mTSP is that in the problem at hand, each site is to be visited by each bus, whereas in the mTSP, each site is to be visited by exactly one salesperson. Further differences are the application-specific objective function and several application-specific constraints, e.g., time windows (see, e.g., Solomon 1987) or prescribed precedence constraints and maximum time lags between the visits of some sites. With respect to the OSP, the sites of the tour may be interpreted as the processors and the buses as the jobs of an OSP. The main differences to the OSP, however, are the application-specific objective function and several application-specific constraints, e.g., the transportation times between the sites, prescribed precedence constraints and maximum time lags between the visit of certain sites, and time windows for the visits of certain sites. Because of these differences, a straightforward adaption of known solution approaches is not possible.

We formulate the scheduling problem at hand as a mixed-integer linear program (MILP). This approach offers a substantial flexibility with respect to changes of the problem specification, i.e., the design of the tour, which was required by the tour operator. Moreover, we enhance the MILP with symmetry-breaking constraints, i.e., we exclude solutions which differ only by the numbering of the buses. Our computational results show that all relevant problem instances can be solved with the MILP without symmetry-breaking constraints in a reasonable amount of CPU time. By introducing the symmetry-breaking constraints, we strongly reduce the required CPU time for all relevant instances.

The purpose of this paper is threefold. First, we present a real-world bus-scheduling problem which is interesting from an optimization point of view. Second, we show how to apply mixed-integer linear programming to this problem, and we discuss the results achieved. Third, we contribute to the growing field of efficient formulations of real-world scheduling problems as MILPs (cf., e.g., Brandimarte 2013; Goel 2012; Goel and Rousseau 2012; Simpson and Abakarov 2013), in particular by enhancing our basic model formulation by symmetry-breaking constraints.

The remainder of this paper is structured as follows. In Sect. 2, we present the event tour "The Enchanted Journey" in more detail, and we state the arising scheduling problem. In Sect. 3, we develop the MILP formulation and the symmetry-breaking constraints. In Sect. 4, we report on our numerical results. Concluding remarks and suggestions for further research follow in Sect. 5.

2 Real-world application and scheduling problem

In Sect. 2.1, we present the real-world application that we address in this paper. In Sect. 2.2, we state the scheduling problem arising from this application.

2.1 Real-world application

The problem refers to a bus tour that is offered daily in Switzerland and is booked by Bollywood-film fans. During the tour, the participants visit seven original sites of various Bollywood films. These sites are located in the city of Bern and in the surrounding mountains. At each site, the participants are photographed while imitating famous scenes from these films. Moreover, each tour comprises a lunch break and a guided shopping tour in Bern. In total, the tour consists of ten real activities, whose prescribed durations vary between 15 and 30 min (cf. Table 1).

The town Engelberg in central Switzerland is the starting point as well as the ending point of the tour; therefore, we consider it as two fictitious activities. The sequence of the remaining activities of the tour is not pre-determined. However, Bern must always be either the first or the last location. Moreover, for the entire tour and for each activity, a time window is prescribed that arises from, for example, the service hours for lunch or the shop-opening hours; in Table 1, the beginning and the completion of this time window is denoted by ES_k and LS_k, respectively. In addition, the two respective activities in Bern, Saanen, and Rougemont must be performed in a row.

Each day, up to five buses operate this tour. There are two alternative locations (L1 and L2) for lunch, and every bus must stop at exactly one of them. All other locations must be part of every tour, and each can have not more than one bus at a time; otherwise, the participants could not be photographed individually. For each location, a 5-min buffer time between succeeding buses is prescribed.

Due to the geographical location of the sites, the transfer between the locations is carried out by buses. The corresponding travel times are given in Table 2. Because of laws and regulations in Switzerland, the total travel time of any bus may not exceed 540 min. A shortest route of a bus corresponds to a total travel time of 413 min. The total duration of the tour may not exceed 810 min.

As the primary objective, the tour operator wishes to perform the tours of each day such that the total waiting time of all buses is minimized. As the secondary objective, the tour operator aims at minimizing the total travel time of the buses. This sequence of objectives is chosen, since a small waiting time can be regarded as a key performance indicator of customer satisfaction, whereas a longer travel time will cause less dissatisfaction of the participants.

Table 1 Locations, types, durations, and time windows of the tour activities

Activity k	Location	Abbreviation	Type	Duration (min)	ES_k (min)	LS_k (min)
1	Engelberg	ENG	Start	0	390	600
2	Bern	BRN	Photo	30	540	1’110
3	Bern	BRN	Shopping	30	540	1’110
4	Zweisimmen	ZWS	Break	20	540	1’140
5	Montbovon	MBV	Photo	30	600	1’140
6	Rossinière	ROS	Photo	30	600	1’140
7	Saanen	SAA	Photo	30	600	1’140
8	Saanen	SAA	Photo	30	600	1’140
9	Gstaad	GST	Photo	15	600	1’140
10	Rougemont	ROU	Photo	30	690	840
11/12	Rougemont	ROU	Lunch L1/L2	30	720	810
13	Engelberg	ENG	End	0	1’080	1’290

Table 2 Travel times between the locations (min)

From/to	ENG	BRN	ZWS	MBV	ROS	SAA	GST	ROU
ENG	–	125	161	196	196	178	183	184
BRN	125	–	81	71	82	98	103	94
ZWS	161	81	–	46	35	17	22	23
MBV	196	71	46	–	11	29	34	23
ROS	196	82	35	11	–	18	23	12
SAA	178	98	17	29	18	–	05	06
GST	183	103	22	34	23	05	–	11
ROU	184	94	23	23	12	06	11	–

2.2 Scheduling problem

Given the number of buses to be planned, the scheduling problem discussed in this paper is as follows. An individual schedule for each bus is sought such that all organizational and logistic constraints mentioned in Sect. 2.1 are fulfilled, and the total waiting time of all buses (primary objective) and the total travel time (secondary objective) are minimized.

3 The MILP formulation

In this section, we formulate the scheduling problem introduced in Sect. 2.2 as a MILP. First, we present a basic model formulation. Then, we propose a symmetry-breaking approach in order to reduce the search space w.l.o.g.

In Sect. 3.1, we introduce the decision variables used in our basic model formulation. In Sects. 3.2 and 3.3, we formulate the objective functions and the constraints, respectively. In Sect. 3.4, we deal with the symmetry-breaking constraints.

3.1 Decision variables

Similar to the machine-scheduling models presented by Chen (2002) and Wagner (1959), we use the following decision variables. Let $s_{ik} \geq 0$ denote the start time of activity k for bus i. The sequence of activities of bus i is controlled by the binary variable x_{ijk} with $x_{ijk} = 1$ if activity k is scheduled at position j for bus i, and $x_{ijk} = 0$ otherwise. The bus sequence of activity k is modeled by the binary variable $z_{ii'k}$ with $z_{ii'k} = 1$ if bus i visits activity k later than bus $i' \neq i$, and $z_{ii'k} = 0$ otherwise. Moreover, let $W_{ij} \geq 0$ and $T_{ij} \geq 0$ denote the waiting time and the travel time, respectively, of bus i before the activity at position j.

3.2 Objective functions

For modeling reasons, we introduce two fictitious activities $k = 1$ and $k = 13$ representing the start and the end, respectively, of the tour. Recall that every bus stops at only one of the two places where lunch is served; we represent this by two respective activities ($k = 11$ and $k = 12$). Thus, in total there are $m = 13$ activities, and the tour of each bus tour consists of $N = 12$ activities. By n, we denote the number of buses. Furthermore, let d_k be the duration of activity k, and let $t_{kk'}$ be the travel time from activity k to activity k'.

For the waiting time W_{ij} of bus i before the activity at position j constraint (1) must hold.

$$\begin{aligned} W_{ij} &\geq s_{ik'} - (s_{ik} + d_k + t_{kk'}) \\ &\quad - (LS_{k'} - d_k - t_{kk'})(2 - x_{ijk'} - x_{i,j-1,k}) \\ &\quad (i = 1,\ldots,n;\ j = 2,\ldots,N;\ k,k' = 1,\ldots,m;\ k \neq k'). \end{aligned} \tag{1}$$

LS_k denotes the prescribed latest possible starting time for activity k.

For the travel time T_{ij}, constraint (2) must be fulfilled.

$$\begin{aligned} T_{ij} &\geq (x_{ijk'} + x_{i,j-1,k} - 1)t_{kk'} \\ &\quad (i = 1,\ldots,n;\ j = 2,\ldots,N;\ k,k' = 1,\ldots,m;\ k \neq k'). \end{aligned} \tag{2}$$

To account for the two-level planning objective, we apply a two-step hierarchical approach as follows. First, we minimize the total waiting time:

$$\text{Min.} \quad \sum_{i=1}^{n}\sum_{j=2}^{N} W_{ij}.$$

Let W^* denote the resulting total waiting time from the first step. Then, in a second step, we minimize the total travel time:

$$\text{Min.} \quad \sum_{i=1}^{n}\sum_{j=2}^{N} T_{ij}$$

subject to the additional constraint

$$\sum_{i=1}^{n}\sum_{j=2}^{N} W_{ij} \leq W^*.$$

3.3 Constraints

We link the variables s_{ik} and x_{ijk} by

$$s_{ik} \leq LS_k \sum_{j=1}^{N} x_{ijk} \qquad (i = 1,\ldots,n;\ k = 1,\ldots,m). \tag{3}$$

The tour of each bus consists of the prescribed set of activities:

$$\sum_{j=1}^{N} x_{ijk} = 1 \quad (i = 1,\ldots,n;\ k = 1,\ldots,m;\ k \neq 11,12), \tag{4}$$

$$\sum_{k=1}^{m} x_{ijk} = 1 \quad (i = 1,\ldots,n;\ j = 1,\ldots,N). \tag{5}$$

Note that constraint (4) does not need to be fulfilled for activities $k = 11$ and $k = 12$, which represent the lunch activities; lunch takes place either at activity $k = 11$ or at activity $k = 12$:

$$\sum_{j=1}^{N}(x_{ij,11} + x_{ij,12}) = 1 \qquad (i = 1,\ldots,n). \tag{6}$$

The time lag between the start of activity k and the start of the subsequent activity k' must not be smaller than the duration d_k of activity k plus the travel time $t_{kk'}$, i.e.,

$$\begin{aligned} &s_{ik} + d_k + t_{kk'} \leq s_{ik'} + (LS_k + d_k + t_{kk'})(2 - x_{ijk} - x_{i,j+1,k'}) \\ &(i = 1,\ldots,n;\ j = 1,\ldots,N-1;\ k,k' = 1,\ldots,m;\ k \neq k'). \end{aligned} \tag{7}$$

For each activity k, the sequence of the buses must be determined:

$$\begin{aligned} &z_{ii'k} + z_{i'ik} = 1 \\ &\quad (i,i' = 1,\ldots,n;\ i \neq i';\ k = 1,\ldots,m). \end{aligned} \tag{8}$$

The time lag between the start of activity k in the tour of bus i' and the tour of a subsequent bus i must not be smaller than the duration of activity k plus the buffer time b, i.e.,

$$\begin{aligned} &s_{ik} - s_{i'k} \geq d_k + b - (LS_k + d_k + b)(2 - x_{ijk} - x_{i'j'k}) \\ &\quad - (LS_k + d_k + b)(1 - z_{ii'k}) \\ &\quad (i,i'{=}1,\ldots,n;\ i \neq i';\ j,j'{=}1,\ldots,N;\ k{=}1,\ldots,m). \end{aligned} \tag{9}$$

We formulate the further organizational and logistic constraints mentioned in Sect. 2.1 as follows. Each bus tour starts and ends in Engelberg, i.e.,

$$x_{i,1,1} = 1 \qquad (i = 1,\ldots,n) \tag{10}$$

$$x_{i,N,13} = 1 \qquad (i = 1,\ldots,n). \tag{11}$$

Each activity k must be started within its time window $[ES_k, LS_k]$:

$$ES_k \sum_{j=1}^{N} x_{ijk} \leq s_{ik} \qquad (i = 1,\ldots,n;\ k = 1,\ldots,m). \tag{12}$$

The latest possible starting time LS_k is captured in constraint (3).

Constraints (13)–(15) ensure that the two activities $k = 2$ and $k = 3$ in Bern are the first or the last activities in a tour and are visited directly after each other

$$\sum_{j=2}^{3}(x_{ij2}+x_{ij3})+\sum_{j=10}^{11}(x_{ij2}+x_{ij3})=2(i=1,\ldots,n) \quad (13)$$

$$x_{ij2}\leq x_{i,j-1,3}+x_{i,j+1,3}(i=1,\ldots,n;\;\; j=2,\ldots,N-1) \quad (14)$$

$$x_{ij3}\leq x_{i,j-1,2}+x_{i,j+1,2}(i=1,\ldots,n;\;\; j=2,\ldots,N-1). \quad (15)$$

Both activities in Saanen as well as both activities in Rougemont have to be visited directly after each other, i.e.,

$$x_{ij7}\leq x_{i,j-1,8}+x_{i,j+1,8} \quad (i=1,\ldots,n;\;\; j=2,\ldots,N-1) \quad (16)$$

$$x_{ij8}\leq x_{i,j-1,7}+x_{i,j+1,7} \quad (i=1,\ldots,n;\;\; j=2,\ldots,N-1) \quad (17)$$

$$x_{i,j,10}\leq x_{i,j-1,11}+x_{i,j+1,11}+x_{i,j-1,12}+x_{i,j+1,12} \quad (i=1,\ldots,n;\;\; j=2,\ldots,N-1) \quad (18)$$

$$x_{i,j,11}+x_{i,j,12}\leq x_{i,j-1,10}+x_{i,j+1,10} \quad (i=1,\ldots,n;\;\; j=2,\ldots,N-1). \quad (19)$$

The total travel time of bus i cannot be smaller than the total travel time in a shortest route, i.e., smaller than 413 min, and may not exceed 540 min:

$$413\leq\sum_{j=2}^{N}T_{ij}\leq 540 \qquad (i=1,\ldots,n), \quad (20)$$

and the total duration of any tour may not exceed 810 min

$$s_{i,13}-s_{i,1}\leq 810 \qquad (i=1,\ldots,n). \quad (21)$$

Recall that in the second step of the optimization approach, we minimize the total travel time $\sum_{i=1}^{n}\sum_{j=2}^{N}T_{ij}$. Constraint (20) imposes a lower and an upper bound on this objective function. In contrast, in the first step of the optimization approach, we minimize the total waiting time $\sum_{i=1}^{n}\sum_{j=2}^{N}W_{ij}$. For the latter objective function, the model does not impose a lower or an upper bound.

3.4 Symmetry-breaking constraints

Symmetries in an optimization problem can be used in order to reduce the search space of the solution algorithm. In the problem at hand, all buses are identical except their index (cf. Sect. 2). Thus, there are schedules which are identical, except the index of the buses. This is illustrated in Fig. 1 for activity Rougement-Photo, where the indexes of buses 1 and 2 are interchanged.

In order to reduce the search space, we exclude such identical schedules without loss of generality by adding some symmetry-breaking constants to the model. Basically, these constraints may be imposed for any arbitrary chosen activity k^*. We formulate the symmetry-breaking constraints as follows.

$$ES_{k^*}+(i-1)(d_{k^*}+b)\leq s_{ik^*} \qquad (i=1,\ldots,n) \quad (22)$$

$$LS_{k^*}-(n-i)(d_{k^*}+b)\geq s_{ik^*} \qquad (i=1,\ldots,n). \quad (23)$$

These pairs of constraints reduce the width of the time windows for each bus at the selected activity k^*. Here we assume the sequence $1,\ldots,n$ for the buses. Consider the following example with five buses and the activity $k^*=10$. Since bus 1 is the first bus in the sequence, its earliest starting time for activity 10 is at 690 min after the start of the tours. The earliest starting time of Bus 2 for this activity is then $690+d_{10}+b=725$. The same holds for the remaining buses. Since bus 5 is the last bus in the sequence, its latest starting time is at 840 min after the start of the tours. If bus 5 starts as late as possible, then bus 4 will not be able to start after $840-d_{10}-b=805$ min. The analogous constraint hold for the remaining buses. By introducing the symmetry-breaking constraints for an activity with a tight time window, we are able to exclude symmetric schedules by predetermining the sequence of the buses. Note that these symmetry-breaking constraints do not change the lower bounds on the objective function value mentioned in Sect. 3.3.

4 Computational results

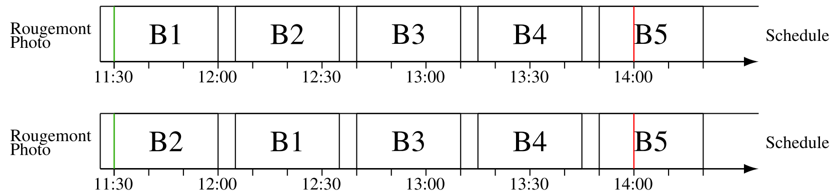
In this section, we report our computational results for the basic model formulation and for the symmetry-breaking constraints. In Sect. 4.1, we deal with the problem instance which corresponds to the real-world data provided by the tour operator. Using this instance, we illustrate the impact of the symmetry-breaking constraints introduced in Sect. 3.4. In Sect. 4.2, we analyze the model performance for various modifications of the problem data.

We have implemented the optimization models in AMPL, and we have applied version 5.0.1 of the Gurobi Solver on a standard workstation with 2 Intel Xeon CPU with 2.66GHz clock speed and 24GB RAM. For each MBLP model to be solved, we have prescribed a CPU time limit of 10'800 s. In the tables of this section, the entry NA indicates that no feasible solution has been found within the prescribed time limit.

4.1 Real-world problem instance

Table 3 shows the impact of the symmetry-breaking constraints introduced in Sect. 3.4 on the CPU times and the objective function values if introduced for the individual activities k. For each activity, the table indicates the CPU time used, the relative integrality gap for the minimization

Fig. 1 Symmetric schedules



of the total waiting time (t_W^{CPU}) and the total travel time (t_T^{CPU}), respectively, and the values of the objective function (OFV). The last column indicates the CPU time needed in total (t^{CPU}). The results without symmetry-breaking constraints are listed in the first row.

The results displayed in Table 3 indicate that the additional constraints are most effective where the time window is relatively tight, i.e., at the activity Rougemont Photo. With the symmetry-breaking constraints for this activity, the CPU time required for devising an optimal solution reduces by 68 %, respectively.

Figure 2 shows an optimal solution for the instance with five buses. In this solution, the total waiting time is zero, and the total travel time of each bus coincides with the total travel time on a shortest route; in other words, in this optimal solution the lower bounds of 0 for the total waiting time and of 2'065 for the total travel time of the five buses, respectively, are met.

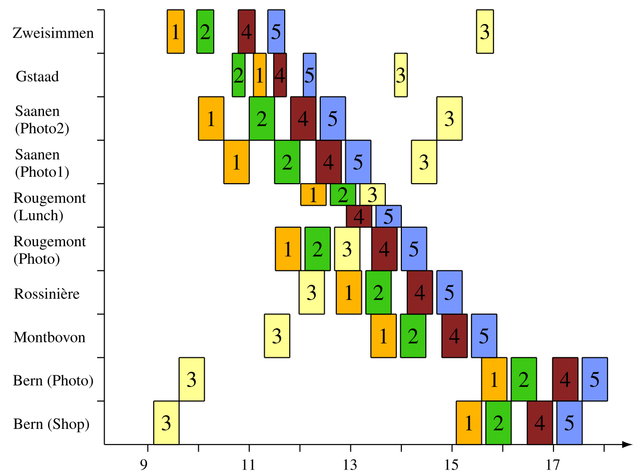


Fig. 2 Optimal schedule for 5 buses

4.2 Parameter variation

In this section, we consider the following modifications of the real-world data.

1. We vary the number of buses between 4 and 6.
2. We reduce (Act−) the number of activities to 10 by excluding one of the activities in Saanen and Berne.
3. We decrease (Trav−) or we increase (Trav+) the travel times between activities by 10 %.
4. We decrease (Dur−) or we increase (Dur+) the durations of the activities by 10 %.
5. We decrease (TW−) or we increase (TW+) the width of the time windows by 10 %.

Except the number of buses, we analyze these modifications separately. Each instance is solved with (SYM) and without (WO) the symmetry-breaking constraints for the

Table 3 Real-world data: impact of the symmetry-breaking constraints

	t_W^{CPU} (s)	MIP gap (%)	OFV	t_T^{CPU} (s)	MIP gap (%)	OFV	t^{CPU} (s)
WO	172	0	0	10'800	5.23	2'179	10'972
Sym ROU_Ph	1'692	0	0	1'827	0	2'065	3'519
Sym ENG_St	911	0	0	5'628	0	2'065	6'539
Sym BRN_Ph	594	0	0	7'446	0	2'065	8'040
Sym BRN_Sh	1'194	0	0	10'800	3.91	2'149	11'994
Sym GST_Ph	774	0	0	10'800	3.82	2'147	11'574
Sym ROS_Ph	987	0	0	10'800	3.82	2'147	11'787
Sym MBV_Ph	1'684	0	0	10'800	1.62	2'099	12'484
Sym SAA_Ph1	1'129	0	0	10'800	7.15	2'224	11'929
Sym SAA_Ph2	1'726	0	0	10'800	1.15	2'089	12'526
Sym ZWS_Br	1'637	0	0	10'800	1.15	2'089	12'437
Sym ENG_End	1'029	0	0	10'800	3.19	2'133	11'829

Table 4 Results for 4 buses and variations

	t_W^{CPU} (s)	MIP gap (%)	OFV	t_T^{CPU} (s)	MIP gap (%)	OFV	t^{CPU} (s)
WO	11	0	0	110	0	1'652	121
Sym	191	0	0	3'879	0	1'652	4'070
WO Act−	11	0	0	240	0	1'652	251
Sym Act−	6	0	0	117	0	1'652	123
WO Trav−	14	0	0	683	0	1'487	697
Sym Trav−	10	0	0	179	0	1'487	189
WO Trav+	147	0	0	4	0	2'116	151
Sym Trav+	85	0	0	3	0	2'116	88
WO Dur−	16	0	0	248	0	1'652	264
Sym Dur−	105	0	0	328	0	1'652	433
WO Dur+	9	0	0	2	0	1'652	11
Sym Dur+	294	0	0	269	0	1'652	563
WO TW−	182	0	0	10'800	0.72	1'664	10'982
Sym TW−	315	0	0	10'800	0.72	1'664	11'115
WO TW+	2	0	0	6	0	1'652	8
Sym TW+	13	0	0	165	0	1'652	178

Table 5 Results for 5 buses and variations

	t_W^{CPU} (s)	MIP gap (%)	OFV	t_T^{CPU} (s)	MIP gap (%)	OFV	t^{CPU} (s)
WO	388	0	0	8'099	0	2'065	8'487
Sym	517	0	0	5'802	0	2'065	6'319
WO Act−	134	0	0	10'800	1.71	2'101	10'934
Sym Act−	83	0	0	10'800	1.14	2'101	10'883
WO Trav−	388	0	0	10'800	1.62	1'889	11'188
Sym Trav−	336	0	0	10'800	2.82	1'913	11'136
WO Trav+	1'480	0	0	34	0	2'646	1'514
Sym Trav+	3'009	0	0	101	0	2'646	3'110
WO Dur−	30	0	0	10'800	0.59	2'077	10'830
Sym Dur−	2'145	0	0	8'946	0	2'065	11'091
WO Dur+	1'991	0	0	10'800	2.18	2'111	12'791
Sym Dur+	3'136	0	0	10'800	7.69	2'237	13'936
WO TW−	10'800	100.00	10	10'800	10.70	2'312	21'600
Sym TW−	10'800	100.00	55	10'800	1.71	2'101	21'600
WO TW+	38	0	0	2'550	0	2'065	2'588
Sym TW+	113	0	0	2'425	0	2'065	2'538

activity Rougemont Photo (cf. Sects. 3.4, 4.1). This results in 48 combinations for the variations described above. In order to obtain feasible solutions when considering each modification on its own we need to adapt the real-world data: we increase the time window at the activity Rougemont Photo, which is the tightest time window, and we increase the allowed total duration of a tour for one bus from 810 to 900 min. The lower bounds for the lengths of a tour are also adapted accordingly when the travel times are modified.

Tables 4, 5, and 6 list the results for these variations. The results indicate that the CPU times vary depending on the modification considered. For a considerable number of instances, the CPU times or the integrality gap for the minimization of the travel times can be reduced by including the symmetry-breaking constraints. For most instances, less CPU time is needed to minimize the total waiting time than to minimize the total travel time. The influence of the symmetry-breaking constraints on the CPU times is more favorable for the instances with more buses. Often, the model with symmetry-breaking constraints requires more CPU time for minimizing the total waiting time than the model without. The reason for that might be that the minimization of the total

Table 6 Results for 6 buses and variations

	t_W^{CPU} (s)	MIP gap (%)	OFV	t_T^{CPU} (s)	MIP gap (%)	OFV	t^{CPU} (s)
WO	10'800	100.00	48	10'800	12.50	2'833	21'600
Sym	10'800	100.00	45	10'800	13.40	2'863	21'600
WO Act−	10'800	100.00	25	10'800	14.40	2'894	21'600
Sym Act−	10'800	100.00	26	10'800	13.10	2'850	21'600
WO Trav−	10'800	100.00	35.5	10'800	18.80	2'748	21'600
Sym Trav−	10'800	100.00	25	10'800	15.00	2'624	21'600
WO Trav+	10'800	NA	NA	NA	NA	NA	NA
Sym Trav+	10'800	NA	NA	NA	NA	NA	NA
WO Dur−	10'800	100.00	16	10'800	12.30	2'826	21'600
Sym Dur−	10'800	100.00	19.5	10'800	13.80	2'875	21'600
WO Dur+	10'800	NA	NA	NA	NA	NA	NA
Sym Dur+	10'800	NA	NA	NA	NA	NA	NA
WO TW−	10'800	NA	NA	NA	NA	NA	NA
Sym TW−	10'800	NA	NA	NA	NA	NA	NA
WO TW+	10'800	100.00	7	10'800	9.50	2'738	21'600
Sym TW+	10'800	100.00	13.5	10'800	11.60	2'802	21'600

waiting time poses a rather easy problem, but the additional constraints increase the CPU time required. Nevertheless, we include the symmetry-breaking constraints in the first step, since we use this solution as initial solution for the second step. The results also indicate that the symmetry-breaking constraints have a larger effect when the number of buses is higher.

From the results indicated in Tables 4, 5, and 6, we can draw the following conclusions. For the instances with 4 buses, optimal or near-optimal solutions can be devised within short CPU time with both model formulations; only for the case of tighter time windows, the CPU time requirement grows significantly. For the instances with 5 buses, near-optimal solutions are obtained within reasonable CPU time. For the instances with 6 buses, the minimization of the total waiting time gets already a difficult task; however, all feasible solutions obtained for the minimization of the total travel time have a reasonable MIP gap.

For the instance with less activities (Act−) and 5 buses, different MIP gaps result although the same objective function value is obtained by the basic model formulation and by the model with symmetry-breaking constraints. The reason for this difference is that for the case with symmetry-breaking constraints, all nodes that were unexploited when the CPU time limit was reached have a higher lower bound value than the root node.

5 Summary and outlook

In this paper we have been concerned with a real-world scheduling application in the event-tourism sector. We have formulated the scheduling problem as a MILP. Furthermore, we have enhanced this formulations with symmetry-breaking constraints. We have analyzed the performance of the model and the impact of the symmetry-breaking constraints for various complexity scenarios. The results, which we have obtained using AMPL and the Gurobi Solver 5.0.1, demonstrate that the CPU times are reduced considerably with the introduction of the symmetry-breaking constraints. The approach presented in this paper has been applied in practice for the planning of "The Enchanted Journey." In particular, the computed schedules were successfully used as a proof-of-concept in the negotiations with the respective marketing organizations.

The MILP formulation presented in this paper contribute to the development of efficient formulations of practical routing and scheduling problems as MILPs, and provide insight into the performance of recent MIP solvers for practical scheduling problems. The model will help to formulate exact models for related applications such as those discussed, e.g., in Yu et al. (2010). Moreover, it should be investigated whether providing a feasible solution obtained by an appropriate heuristic would speed up the solution process.

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