

# IP Solutions for International Kidney Exchange Programmes

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**Abstract.** In kidney exchange programmes patients with end-stage renal failure may exchange their willing, but incompatible living donors among each other. National kidney exchange programmes are in operation in ten European countries, and some of them have already conducted international exchanges through regulated collaborations. The exchanges are selected by conducting regular matching runs (typically every three months) according to well-defined constraints and optimisation criteria, which may differ across countries. In this work we give integer programming formulations for solving international kidney exchange problems, where the optimisation goals and constraints may be different in the participating countries and various feasibility criteria may apply for the international cycles and chains. We also conduct simulations showing the long-run effects of international collaborations for different pools and under various national restrictions and objectives.

**Keywords:** integer programming · kidney exchanges · simulations.

## 1 Introduction

When an end-stage kidney patient has a willing, but incompatible living donor, then in many countries this patient can exchange his/her donor for a compatible one in a so-called kidney exchange programme (KEP). The first national kidney exchange programme was established in 2004 in the Netherlands in Europe [9]. Currently there are ten countries with operating programmes in Europe [6], the largest being the UK programme [11].

Typically the matching runs are conducted in every three months on pools with around 50-300 patient-donor pairs. The so-called virtual compatibility graph represents the patient-donor pairs with nodes and an arc represents a possible donation between the corresponding donor and patient, that is found compatible in a virtual crossmatch test. The exchange cycles are selected by well-defined optimisation rules, that can vary across countries. The most important constraints are the upper limits on the length of exchange cycles, for examples, two

in France, three in the UK and Spain, and four in the Netherlands [6]. The main goal of the optimisation in Europe is to facilitate as many transplants as possible, i.e. to maximise the number of nodes covered in the compatibility graph by independent cycles. The corresponding computational problem for cycle-length limits three or more is NP-hard, and the standard solution technique used is integer programming [1].

International kidney exchanges have already started in Europe between Austria and Czech Republic [7] since 2016, between Portugal, Spain and Italy since summer 2018, and between Sweden, Norway and Denmark in the Scandiatriansplant programme (STEP), built on the Swedish initiative [2]. The above mentioned first two collaborations are organised in a sequential fashion, first the national runs are conducted and then the international exchanges are sought for the remaining patient-donor pairs. A related game-theoretical model has been studied in [8]. In the Scandinavian programme, however, the protocol proposed is to find an overall optimum for the joint pool. In the latter situation, the fairness of the solution for the countries involved can be seen as an important requirement, which was studied in [10] with extensive long-term simulations by proposing the usage of a compensation scheme among the countries.<sup>5</sup>

In this study we will compare the sequential and the joint pool scenarios in our simulations. We will not consider compensations, or any strategic issues, but we will allow the countries to have different constraints and goals with regard to the cycles and chains they may be involved in. In particular, we will compare the benefits of the countries from international collaborations when they have different upper bounds on their national cycles, and thus also possible different constraints on the segments of the international cycles they are participating in. As an example, we mention the Austro-Czech cooperation, where Austria requires on having all exchanges simultaneously, so they allow short national cycles and short segments only, whilst in Czech Republic the longer non-simultaneous cycles and chains are also allowed. We formulate novel IP models for dealing with potentially diverse constraints and goals in international kidney exchange programmes and we test two-country cooperations under different assumptions over their constraints, the possibility of having chains triggered by altruistic donors, and the sizes and compositions of their pools.

## 2 Model of international kidney exchanges

In a standard kidney exchange problem, we are given a directed *compatibility graph*  $D(V, A)$ , where the nodes  $V = \{1, 2, \dots, n\}$  correspond to patient-donor pairs and there is an arc  $(i, j)$  if the donor of pair  $i$  is compatible with the patient of pair  $j$ . Furthermore we have a non-negative weight function  $w$  on the arcs, where  $w_{i,j}$  denotes the weight of arc  $(i, j)$ , representing the value of the transplantation. (In most applications the primary concern is to save as many patients as possible, so the value is simply equal to one.)

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<sup>5</sup> Similar situation arises in the US kidney exchange problem, where the transplant centres are the strategic agents [4, 5, 3, 13].

Let  $\mathcal{C}$  denote the set of cycles allowed in  $D$ , which are typically to be of length at most  $K$ . The solution of a classical kidney exchange problem is a set of disjoint cycles of  $\mathcal{C}$ , i.e. a cycle-packing in  $D$ . For cycle  $c \in \mathcal{C}$ , let  $A(c)$  denote the set of arcs in  $c$  and  $V(c)$  denote the set of nodes covered by  $c$ .<sup>6</sup>

In an international kidney exchange programme multiple countries ( $N$ ) are involved in the exchange, so the set of nodes is partitioned into  $V = V^1 \cup V^2 \cup \dots \cup V^N$ , where  $V^k$  is the set of patient-donor pairs in country  $k$ . We have the following modifications of the classical problem. Let  $A^k$  denote the arcs pointing to  $V^k$  (so the donations to patients in country  $k$ ). Note that  $A = A^1 \cup A^2 \cup \dots \cup A^N$ . The weights of the arcs in  $A^k$  should reflect the preferences of country  $k$ . (We may assume that these are scaled, e.g. by having the same average score in order not to bias the overall optimal solution towards some countries.) Finally, let  $A^{\mathcal{N}}$  and  $A^{\mathcal{I}}$  denote the national and international donations, i.e.  $A = A^{\mathcal{N}} \cup A^{\mathcal{I}}$ .

In a global optimal solution, small cycles within the countries can have different requirement than international cycles. Therefore, we separate the two sets of cycles into  $\mathcal{C} = \mathcal{C}^{\mathcal{N}} \cup \mathcal{C}^{\mathcal{I}}$ , where  $\mathcal{C}^{\mathcal{N}}$  is the set of national cycles and  $\mathcal{C}^{\mathcal{I}}$  is the set of international cycles. We call the national parts of an international cycle *segments*, where a segment is a path within a country, and the segments are linked by international arcs. A  $l$ -segment is a path of length  $l - 1$ , with all the  $l$  nodes belonging to the same country. Let  $\mathcal{S}$  denote the set of all possible segments, and let  $\mathcal{S}^k$  denote the set of segments allowed in country  $k$ . For  $s \in \mathcal{S}$ , let  $A(s)$  denote the set of (national) arcs and let  $V(s)$  denote the set of nodes covered (in the same country). Note that a segment may also consist of a single node, which corresponds to the case when an international donation is immediately followed by another international donation. We can have the following natural restrictions on the national and international cycles:<sup>7</sup>

1. different limits on the length of national cycles for each country;
2. different limits on the length of segments in international cycles for each country;
3. limit on the total length of an international cycle;
4. limit on the number of countries involved in one cycle;
5. limit on the number of patient-donor pairs from a country in one cycle;
6. limit on the number of segments in a country within one cycle.

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<sup>6</sup> In addition, we can also consider *altruistic donors*, in which case we separate the node set into patient-donor pairs  $V_p$  and altruistic donors  $V_a$ , so  $V = V_p \cup V_a$ . The solution may contain exchange cycles and chains triggered by altruistic donors. The latter can be conducted non-simultaneously, so different restrictions may apply for them. In this paper we focus on cycles, but we note that one can reduce the problem of finding chains to the problem of finding cycles by adding artificial patients to the altruistic donors, who are compatible with all donors.

<sup>7</sup> We can also have different constraints for altruistic chains, and we may require that an international chain may have to end in the same country where it started.

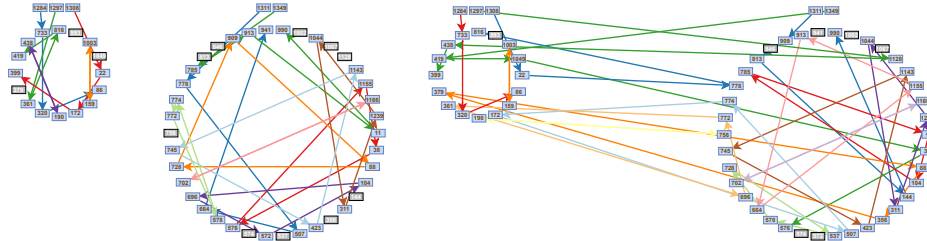
**Integer programming formulations and simulation plan:** We propose new integer programming formulations, where besides the standard edge- and cycle-formulations [1], we introduce new variables for country segments. We defer this part, together with the description of the simulation plan, to the journal version of the paper. Below we only present one simulation.

### 3 A simulation example

To determine the benefits of international kidney exchange programmes (KEPs) we conducted a case study involving two countries which aim to develop a joint KEP and are concerned about the advantages and disadvantages of cooperation between their KEPs. We compare the individual benefits from the no cooperation case to the sequential matchings and merged pool scenarios.

The simulation involves 20 instances each containing the compatibility information for 1000 patient-donor pairs. We assume that an extra 10% of this amount are altruist donors. The length of the considered time-frame in the simulated kidney exchange schemes is 5 years with matching runs occurring every 3 months for each instance, as in [12]. Each agent is assigned an uniformly distributed arrival time, and the patient-donor pairs stay in the KEP for a maximum of 1 year (or 4 matching runs) after which they leave the programme (which means that they opt for an alternative solution, such as having a direct transplant after desensitisation or getting an organ from a deceased donor).

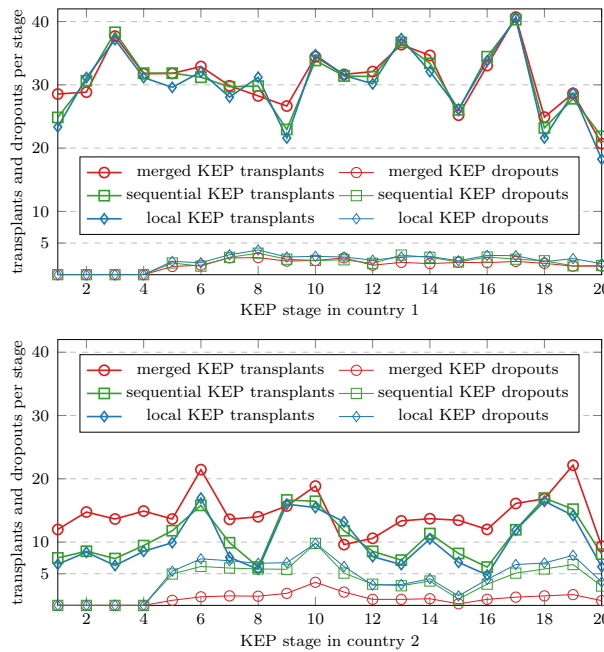
**Fig. 1.** Graphic representation of the first KEP stage in one of the instances: altruist donors are at the top, patient-donor pairs form circles for each country and arcs represent transplants. Left side, individual KEPs: 13/16 patients receive transplants in the small country, 28/38 patients in the large country are transplanted. Right side, merged KEP: the numbers are 15/16 for the small country, and 32/38 for the large one.



In the illustrative simulation we have two countries. Country 1 is twice the size of Country 2, meaning that it will have roughly twice as many patient-donor pairs, as well as altruist donors. On average, each month will mean the arrival of 33.33 patients to Country 1 and 16.66 to Country 2. Country 2 runs a smaller programme and allows only 2-cycles and 3-chains, while country 1 allows for 3-cycles and 4-chains. When they collaborate, they allow international 2-cycles, international 3-cycles where there is only one patient-donor pair involved from Country 2 and chains that must end in the same country where the altruist

donor is coming from. The objective is simply to maximise the number of transplants. There are three settings for collaboration: no cooperation (i.e. separate KEPs, baseline scenario), sequential matchings (each country runs a local KEP optimisation and then the remaining patient pools enter a joint KEP) and full collaboration (a single KEP for both countries). We present our findings in Fig. 1, Fig. 2 and Table 1.

**Fig. 2.** Average across 20 instances of transplants and dropouts from the KEP recorded after each of the 20 stages of the KEP.



**Table 1.** Average total transplants (tr.) and total patients who drop out (d.o.) of the programme across all 20 instances after 5 years.

	tr.	d.o.
C1 local	600.2	42.15
C2 local	199.45	90.2
C1 seq.	611.45	36.35
C2 seq.	214.85	78.35
C1 joint	618.7	30.85
C2 joint	289.7	22.15

We observe firstly that the size of the KEP donor pool is important to increase the number of compatible transplants: the smaller country struggles with a lower rate of transplants than the larger one, and has a significant 31% drop out rate. While the larger country does not see much benefit from entering a joining KEP with the smaller country, we can observe that its number of transplants does not decrease when international collaboration increases. However, the smaller country benefits greatly, especially in the case of merged KEPs, where the drop out rate is significantly lowered to about 7%, a value similar to the drop out rate of the single larger country’s individual KEP scenario. This information suggests that newly developing and smaller KEPs have much to gain from full collaboration with a larger KEP. On the other hand, the improvement in the sequential KEP scenario is much less than that of the fully joint KEP for the smaller country.

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