



Liu, Y., Dong, X., Ren, Z., & Cooper, J. (2019). Fault-tolerant control for commercial aircraft with actuator faults and constraints. *Journal of the Franklin Institute*, 356(7), 3849-3868.
<https://doi.org/10.1016/j.jfranklin.2018.11.043>

Peer reviewed version

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Link to published version (if available):
[10.1016/j.jfranklin.2018.11.043](https://doi.org/10.1016/j.jfranklin.2018.11.043)

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Integrated Strategy for Commercial Aircraft Fault Tolerant Control

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Nomenclature

A	= system matrix
B	= input matrix
B_r	= plant reference matrix
b_i	= the i th column of B
C	= output matrix
d	= external disturbance
E	= disturbance matrix
e	= error
F	= additive fault matrix
\bar{f}_a	= additive fault value
\bar{f}_m	= fictitious multiplicative fault value
G	= observer input matrix of u
H	= observer feedforward matrix
h	= altitude, ft
K	= gain
L	= indication matrix
l_i	= indication coefficient

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M	= observer system matrix
N	= observer input matrix of y
q_1	= pitch rate, deg/s
r	= command, deg
r_{po}	= residual between the plant and observer
u	= plant input vector
u_1	= axial velocity, m/s
V_l	= Lyapunov function
V_T	= true airspeed, Ma
w_1	= longitudinal velocity, m/s
x	= state vector
y	= measured output vector
α_i, β_i	= adaptation rate
δ_{ei}	= the i th elevator angle, deg
Θ_d	= disturbance
θ_1	= pitch angle, deg/s

I.Introduction

WITH the ongoing development of commercial aircraft, flight control systems are becoming much more complex, and the requirement for safety, reliability, maintainability, and survivability has increased. Under such a background, the integrated strategy of fault tolerant control (FTC) has been widely applied and developed [1, 2]. As it is becoming increasingly easy to obtain fault information, the integrated fault estimation (FE) and reconfigurable control (RC) have attracted greater attention in recent decades [3–7]. The concept of FE intrinsically includes both fault detection and fault isolation roles, and these types of control systems are often known as fault tolerant control systems (FTCS) which are control systems that possess the ability to accommodate component faults automatically, and have the capability to maintain overall system stability and acceptable performance in the post-fault system.

Generally speaking, FTCS can be classified into passive fault tolerant control systems (PFTCS) and active fault tolerant control systems (AFTCS) [2]. The controllers in the PFTCS are designed to be robust against a class of presumed faults and so are regarded as an extension of robust control [3]. Although such as approach doesn't need FE or RC, it has limited fault tolerant capabilities. However, the research on the AFTCS has developed rapidly for structures which contain both FE and RC, bringing significant convenience and potential applications [8–14].

In some cases the FE was treated as a parallel path with the FTC [8, 9]. The general process of this class of approach is to precompute the control law of a set of fault systems which have been predicted and then let the fault system match the correct fault controller with the FE information and a switch mechanism. These approaches have obvious advantages in practice as the FE module will guide the fault system to match the predesigned fault controller quickly when a fault occurs. Furthermore, some modified FTC in this type have solved the existed problems using either robust or adaptive methods and the fault information from the FE was used in the switch mechanism [10, 11]. In other approaches, the fault was regarded as an additional part of the motion equation and some assumptions were made so that the FE module can provide accurate information. In [12], a robust fault tolerant control scheme was proposed to process actuator faults for uncertain nonlinear systems with zero dynamics, whereas in [13], adaptive control techniques were used to help obtain a faster and more accurate compensation of failure and uncertainty for discrete-time systems, solved a problem of fault tolerant controller design in the presence of partial loss of actuator effectiveness faults and the structural parameter uncertainties were assumed to be matched. A novel aircraft trajectory controller with the incremental nonlinear dynamic inversion to achieve fault tolerant trajectory control was proposed in [14]. The above methods used the fault information to direct reconfigure the controller of the post-fault system so that the robust behavior was improved.

However, the FE techniques described above were developed as a monitoring tool or diagnostic, rather than as an integral part of FTCS. As a result, some existing FE methods may not satisfy the need for controller reconfiguration [15–18]; furthermore, most of the research assumed that the FE has a perfect availability. Little attention has been paid to the analysis and design with the integration of both FE and RC. To overcome the above difficulties and to design a practical AFTCS, it is necessary to develop new techniques that can integrate the FE scheme, and reconfigurable control design smoothly, without any pre-assumption on the knowledge of the post-fault system model. Unfortunately, the available literature for the integrated design of FTC is limited. Reference [5] demonstrated the development of a robust state space observer to estimate the system states and the fault signal simultaneously,

determining an efficient fault tolerant control scheme using both the estimated states and faults in a nonlinear system. Lan and Patton also developed a robust strategy of integrated FTC for both linear and nonlinear systems [3, 4]. These approaches all extended the states of both observer and controller and designed the integrated FTC with the robust strategy. The underlying theory in these works is mathematically correct, but there are some flaws when they are applied in practice, especially when used in a flight control system. For example, some coefficients are assumed to be equal to zero when using this method [3]; furthermore, the proposed FTC strategy for the multiplicative fault case cannot obtain the estimate of real multiplicative faults. In order to achieve the integrated design of FE and RC, we must determine: (1) what are the needs and requirements of the FE for the design of RC? (2) what information can be provided by the existing FE techniques for RC designs? and (3) how to analyze systematically the interaction between FE and RC?

In this article, the main contribution is to show the advantages of using an integrated FTC and how this improves upon the flaws in the existing literature. To achieve these goals, we (1) modify a robust unknown input observer (UIO) for all kinds of actuator faults to provide the required information for the RC, including an estimate of the additive actuator faults as well as the real multiplicative ones. This integrated strategy removes the rank restriction for the UIO in the other literature so that the observer is most suitable for real-life systems. We then (2) use the fictitious multiplicative fault in the integrated RC to design the control law and process the additive part as a disturbance due to the feature of the general fault model built in this paper. Examples then demonstrate the capabilities of the new approach.

The paper is organized as follows. Sec. II gives the problem formulation, analyses and builds a new general post-fault aircraft model. The integrated FE/FTC design with the new fault model is considered in Sec. III. Sec. IV provides an illustrative example for a rigid commercial aircraft, and Sec. V concludes the study.

II. Problem Formulation

The longitudinal motion equation of an aircraft without actuator fault can be described as a linear time invariant system such that

$$\dot{x}(t) = Ax(t) + Bu(t) + Ed(t) \tag{1a}$$

$$y(t) = Cx(t) \tag{1b}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^p$ is the plant input vector, $y(t) \in R^m$ is the measured output vector, $d(t) \in R^q$ is the external disturbance (such as a gust). A , B , C , and E are the known constant matrices with appropriate dimensions and the disturbance matrix E is a full-column rank matrix, which is usually satisfied. The detailed derivations for Eq. (1) are provided in [9].

As in the previous literature, the actuator fault can be categorised as one of the following categories [15]:

Lock in place (LIP). The actuator is locked in a certain position and does not respond to subsequent commands, $u_i(t) = const$.

Float / Outage. The actuator produces zero force and moment, $u_i(t) = 0$ (special case of LIP).

Runaway / Hard over fault (HOF). The actuator locks in the maximum or minimum place, $u_i(t) = u_{max} / u_{min}$ (special case of LIP).

Loss of effectiveness. A decrease in the actuator gain that results in a deflection that is smaller than the commanded position, $u_i(t) = l_i u_{ci}(t)$, $0 < l_i < 1$.

Bias. There exists a constant bias between the ideal actuator output and the one that occurs in practice. $u_i(t) = u_{ci}(t) + const$.

In most of the literature [3–18], the post-fault model of the system (1) is described as the following two forms

$$\dot{x}(t) = Ax(t) + BLu(t) + b_i \bar{u}_i(t) + Ed(t) \quad (2a)$$

$$y(t) = Cx(t) \quad (2b)$$

or

$$\dot{x}(t) = Ax(t) + Bu(t) + \bar{F} \bar{f}_a(t) + Ed(t) \quad (3a)$$

$$y(t) = Cx(t) \quad (3b)$$

The state space model (2) is always used in the design of FE with multiple-model method; it can describe all of the actuator faults discussed above, but it cannot be directly used to design the control law. Consequently, many other works process the FTC problem using the form of equation (3), where $\bar{f}_a(t)$ is always considered to act as a generalized fault input, so that it decouples with the control input $u(t)$ but can't describe a multiplicative fault. Thus, a modified post-fault state space for (1) that combines with (2) and (3) will be proposed.

In order to illustrate the concept of the integrated FTC in a simple and effective way and process both additive and multiplicative actuator faults, we assume that there is a fault in the i th actuator and rewrite (1) under the multiple-model structure as

$$\dot{x}(t) = Ax(t) + BLu(t) + b_i \bar{u}_i(t) + Ed(t) = Ax(t) + Bu(t) + B\bar{f}_m(t) + b_i \bar{u}_i(t) + Ed(t) \quad (4a)$$

$$y(t) = Cx(t) \quad (4b)$$

where $B = (b_1, b_2, \dots, b_p)$, $L = \text{diag}[l_1, l_2, \dots, l_p]$ is an indication matrix, $l_i \in [0, 1]$, b_i denotes the i th column of B , $\bar{u}_i(t)$ is the additive fault value of the i th actuator, $\bar{f}_m(t) = (L - I)u(t) = \bar{L}u(t)$ is the so-called fictitious multiplicative fault value, $\bar{L} = \text{diag}[\bar{l}_1, \bar{l}_2, \dots, \bar{l}_p]$, $\bar{l}_i \in [-1, 0]$.

In the LIP case, $l_i = 0$, $\bar{u}_i(t) = \text{const}$; in the Float case, $l_i = 0$, $\bar{u}_i(t) = 0$; in the HOF case, $l_i = 0$, $\bar{u}_i(t) = u_{\max} / u_{\min}$; in the LOE case, $l_i \in (0, 1)$, $\bar{u}_i(t) = 0$; and in the Bias case, $l_i = 1$, $\bar{u}_i(t) = \text{const}$.

III. Integration of FTC strategy

To integrate an active FTCS, it is important to consider the plant, observer and the controller together to ensure that the augmented system will work stably and without bias. To be more precise, from the viewpoint of RC, it is necessary to get the fault signal as a component of the control law from the FE while from a FE standpoint, one need to know which kind of FE method can provide the information that the RC needs. The overall system may not function as expected if the demand and supply between these two subsystems cannot be matched. Furthermore, a FE with bias may not only result in loss of performance itself, but may also affect the overall system. A RC mechanism will lead to undesirable behavior with incorrect fault information. There exists a bi-directional interaction in FTCS which puts forward the necessity and importance of the integrated design of FTC.

To overcome this problem, an integrated strategy of FTC based on UIO and model reference adaptive control (MRAC) is shown in Fig. 1. The basic controller dominates an ideal system under normal conditions; if there are actuator faults in the system, the UIO will provide the exactly fault signal to RC to make sure that the post-fault system recovers the original system performance or to accept some degree of performance degradation.

Firstly, a basic controller is designed for the normal system, so that the output of the aircraft can follow the flight command without steady-state error such that

$$\lim_{t \rightarrow \infty} e(t) = \lim_{t \rightarrow \infty} (r(t) - y(t)) = 0 \quad (5)$$

where $r(t)$ is the command and $e(t)$ is the error.

The normal system with a basic controller will act as a reference model. Moreover, a reconfigurable fault tolerant controller with the estimation of the fault signal will be designed to compensate the fault actuator.

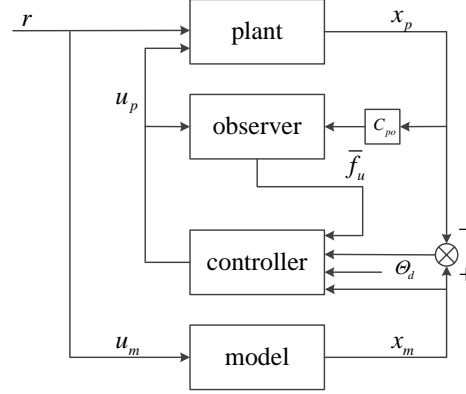


Fig. 1 Mechanism of the integrated design FTC

A. Design of the basic controller

When there is no actuator fault in the system, the controller of the plant (1) is set to be

$$\dot{x}_c = A_c x_c + B_c (r - y) \quad (6)$$

where $x_c \in R^{n_c}$ is the state of the controller. By combining (1) and (6), an augmented system of the plant yields and the control variable system can be obtained as

$$\begin{bmatrix} \dot{x} \\ \dot{x}_c \end{bmatrix} = \begin{bmatrix} A & 0 \\ -B_c C & A_c \end{bmatrix} \begin{bmatrix} x \\ x_c \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} E d \\ B_c r \end{bmatrix} \quad (7a)$$

$$y = [C \quad 0] \begin{bmatrix} x \\ x_c \end{bmatrix} \quad (7b)$$

Defining the new matrices $A_n = \begin{bmatrix} A & 0 \\ -B_c C & A_c \end{bmatrix}$, $B_n = \begin{bmatrix} B \\ 0 \end{bmatrix}$, $C_n = [C \quad 0]$.

According to [8] and [9], the augmented system (7) is controllable. Therefore, a control law can be designed as

$$u = K_x x + K_c x_c \quad (8)$$

We should make the system stable by ensuring that all the closed-loop poles of the system have a negative real part.

Thus, the closed-loop system (7) can be rewritten as

$$\begin{bmatrix} \dot{x} \\ \dot{x}_c \end{bmatrix} = \begin{bmatrix} A+BK_x & BK_c \\ -B_c C & A_c \end{bmatrix} \begin{bmatrix} x \\ x_c \end{bmatrix} + \begin{bmatrix} Ed \\ B_c r \end{bmatrix} \quad (9a)$$

$$y = [C \quad 0] \begin{bmatrix} x \\ x_c \end{bmatrix} \quad (9b)$$

Moreover, since the plant output is the controlled object variable vector and the plant follows the control command vector, then the matrices of the control variable system can be chosen as $A_c = 0$ and $B_c = I$. That is, the control law (8) becomes

$$u = K_x x + K_c \int (r - y) dt \quad (10)$$

With the control law (8), the system (1) can track the command r without any steady error. The LQR technique was applied in this study to choose the applicable gain K_x and K_c so that the system could have a better performance.

B. Design of the unknown input observer

When the aircraft suffers from an actuator fault, the plant input u will change, as discussed in the previous section. Since the models in the LIP, LOE, and Bias faults are different, without loss of generality, we don't restrict the amount and the type of fault which the aircraft has suffered from and rewrite the post-fault system (4) as

$$\dot{x}(t) = Ax(t) + BLu(t) + F\bar{f}_a(t) + Ed(t) = Ax(t) + Bu(t) + B\bar{f}_m(t) + F\bar{f}_a(t) + Ed(t) \quad (11a)$$

$$y(t) = Cx(t) \quad (11b)$$

where $F = [b_i, \dots]$ denotes there are faults in the i th, \dots actuators, $\bar{f}_a(t) = [\bar{u}_i(t), \dots]$ denotes the additive value of the i th, \dots actuators.

Note that (11) contains the unknown fault signal which should be provided by the UIO, and hence the fault signal will be a component of the control law. Thus integrated FTC method based on MRAC and multiple-model based adaptive UIO will be applied in this paper.

In this fault tolerant strategy, the reference model contains the normal close-loop system with the corresponding basic controller which can be expressed in the form as

$$\dot{x}_m = A_m x_m + B_m u_m \quad (12a)$$

$$y_m = C_m x_m \quad (12b)$$

where we choose the close-loop normal system as the reference model which is derived from (7), such that

$$A_m = \begin{bmatrix} A+BK_x & BK_c \\ -C & 0 \end{bmatrix}, B_m = \begin{bmatrix} 0 \\ I \end{bmatrix}, C_m = [C \ 0], x_m = [x \ x_c]^T, u_m = r, y_m = y. \text{ Note that the reference model is}$$

controlled by the baseline controller.

In this fault tolerant strategy, the plant must have the same dimension, so a control variable $\dot{x}_c = r - y$ is introduced and thus the plant model is obtained as

$$\begin{aligned} \dot{x}_p &= A_p x_p + B_p L u_p + F_p \bar{f}_a + E_p d + B_r r \\ &= A_p x_p + B_p u_p + B_p \bar{f}_m + F_p \bar{f}_a + E_p d + B_r r \end{aligned} \quad (13a)$$

$$y_p = C_p y_{po} = C_p C_{po} x_p \quad (13b)$$

where $A_p = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}$, $B_p = \begin{bmatrix} B \\ 0 \end{bmatrix}$, $F_p = \begin{bmatrix} F \\ 0 \end{bmatrix}$, $E_p = \begin{bmatrix} E \\ 0 \end{bmatrix}$, $B_r = \begin{bmatrix} 0 \\ I \end{bmatrix}$, $C_p = [C \ 0]$, C_{po} is the plant measurable output matrix which is used to design the UIO, $x_p = [x \ x_c]^T$, $y_p = y$.

Since the plant model and the corresponding reference model have the same dimension, the model reference adaptive control method can be used here. In the integrated strategy, the controller and the observer should be designed together so that the augmented closed-loop system consisting of (12), (13), and a multiple-model based adaptive UIO for the plant is

$$\dot{x}_m = A_m x_m + B_m u_m \quad (14a)$$

$$y_m = C_m x_m \quad (14b)$$

$$\begin{aligned} \dot{x}_p &= A_p x_p + B_p L u_p + F_p \bar{f}_a + E_p d + B_r r \\ &= A_p x_p + B_p u_p + B_p \bar{f}_m + F_p \bar{f}_a + E_p d + B_r r \end{aligned} \quad (14c)$$

$$y_p = C_p C_{po} x_p \quad (14d)$$

$$\dot{x}_o = M x_o + G B_p \hat{L} u_p + G F_p \hat{f}_a + N y_{po} \quad (14e)$$

$$\hat{x}_p = x_o + H y_{po} \quad (14f)$$

where $x_o \in R^{n+n_c}$, $\hat{x}_p \in R^{n+n_c}$ denote the observer state vector and the estimation of the plant state vector respectively,

$\hat{L} = \text{diag}[\hat{l}_1, \hat{l}_2, \dots, \hat{l}_p]$ is the estimation of L , $\hat{l}_i \in [0,1]$, M , G , N , and H are the matrices to be designed.

Moreover, let $N = N_1 + N_2$, N_1 and N_2 to be determined, and defined estimation error $e_{po}(t) = x_p(t) - \hat{x}_p(t)$.

Substitute (14) into the error yields

$$\begin{aligned} \dot{e}_{po} = & (A_p - HC_{po}A_p - N_1C_{po})e_{po} + [(A_p - HC_{po}A_p - N_1C_{po}) - M]x_o \\ & + [(A_p - HC_{po}A_p - N_1C_{po})H - N_2]y_{po} + (I - HC_{po})Ed + (I - HC_{po})B_r r \\ & + [(I - HC_{po})B_p Lu_p - GB_p \hat{L}u_p] + [(I - HC_{po})F_p \bar{f}_a - GF_p \hat{f}_a] \end{aligned} \quad (15)$$

If it holds that

$$M \text{ is Hurwitz} \quad (16)$$

$$M = A_p - HC_{po}A_p - N_1C_{po} \quad (17)$$

$$N_2 = MH \quad (18)$$

$$(I - HC_{po})E = 0 \quad (19)$$

$$(I - HC_{po})B_r = 0 \quad (20)$$

$$G = I - HC_{po} \quad (21)$$

then the state estimation error will be

$$\begin{aligned} \dot{e}_{po} = & Me_{po} + GB_p(L - \hat{L})u_p + GF_p(\bar{f}_a - \hat{f}_a) \\ = & Me_{po} + \sum_{i=1}^{p_f} Gb_{pi}(l_i - \hat{l}_i)u_{pi} + \sum_{i=1}^{p_f} Gb_{pi}(\bar{f}_{ai} - \hat{f}_{ai}) \\ = & Me_{po} + \sum_{i=1}^{p_f} Gb_{pi}\Delta_{l_i}u_{pi} + \sum_{i=1}^{p_f} Gb_{pi}\Delta_{\bar{f}_{ai}} \end{aligned} \quad (22)$$

Since M is Hurwitz, when $l_i = \hat{l}_i$, $\bar{f}_{ai} = \hat{f}_{ai}$, $\lim_{t \rightarrow \infty} e_{po}(t) = 0$, so the UIO is not affected by the unknown disturbance.

Note that the purpose of FE is to obtain the fault information \hat{l}_i and \hat{f}_{ai} for the integrated design of FTC so that an adaptive law is developed to adjust \hat{l}_i and \hat{f}_{ai} to guarantee the convergence of the estimation. \hat{l}_i and \hat{f}_{ai} are adjusted by the adaptive algorithm

$$\begin{cases} \hat{l}_i(t) = \int_0^t \alpha_i r_{po}^T(\tau) P_{po} C_{po} Gb_{pi} u_{pi} d\tau + l_i(0) \\ \hat{\bar{l}}_i(t) = \hat{l}_i(t) - 1 \\ \hat{f}_{ai}(t) = \int_0^t \beta_i r_{po}^T(\tau) P_{po} C_{po} Gb_{pi} d\tau + \bar{f}_{ai}(0) \end{cases} \quad (23)$$

where α_i and β_i are the adaptation rates which can be adjusted and may affect the convergence rate of the adaptive estimation. P_{po} is a symmetric positive definite matrix which is the unique solution of the Lyapunov matrix equation

$$M^T P_{po} + P_{po} M = -Q_{po} \quad (24)$$

where Q_{po} is an arbitrary symmetric positive definite matrix. The residual

$$r_{po}(t) = y_{po}(t) - \hat{y}_{po}(t) = C_{po}[x_p(t) - \hat{x}_p(t)] = C_{po}e_{po}(t) \quad (25)$$

The stability of the overall system in (14) is guaranteed by using the theorem in Appendix A. It can also show that the proposed observer is robust to the disturbance.

C. Design of the fault tolerant controller

In the augmented closed-loop system (14), the error between the reference model and the plant model can be defined as $e_{mp}(t) = x_m(t) - x_p(t)$, and a control law is defined in the form as

$$u_p = K_e e_{mp} + K_m x_m + K_f \bar{f}_m + \Theta_d \quad (26)$$

to achieve the input of the plant model. Substitute (12) and (13) into the error expression yields

$$\begin{aligned} \dot{e}_{mp} &= \dot{x}_m - \dot{x}_p = A_m x_m + B_m u_m - A_p x_p - B_p u_p - B_p \bar{f}_m - F_p \bar{f}_a - E_p d - B_r r \\ &= (A_p - B_p K_e) e_{mp} + (A_m - A_p - B_p K_m) x_m - (B_p K_f + B_p) \bar{f}_m - (B_p \Theta_d + F_p \bar{f}_a - E_p d) \end{aligned} \quad (27)$$

Choosing the appropriate gain K_m , K_f , and Θ_d to make sure that $A_m - A_p - B_p K_m = 0$, $B_p K_f + B_p = 0$, and $B_p \Theta_d + F_p \bar{f}_a - E_p d = 0$ respectively, we can get $\dot{e}_{mp}(t) = (A_p - B_p K_e) e_{mp}$, and in order to make $(A_p - B_p K_e)$ stable, the gain K_e can be obtained.

However, since there exists an unknown external disturbance such as a gust, Θ_d cannot be obtained by using $B_p \Theta_d + F_p \bar{f}_a - E_p d = 0$ so that we should integrated design the control law with an adaptive law combined with the UIO proposed here. Substituting $\tau = 0$ and $\tau = t$ into (27) respectively yields

$$\begin{aligned} \dot{e}_{mp} &= [A_p - B_p K_e(t)] e_{mp} + [A_m - A_p - B_p K_m(t)] x_m \\ &\quad - [B_p K_f(t) + B_p] \bar{f}_m - [B_p \Theta_d(t) + F_p \bar{f}_a - E_p d] \\ &= \bar{A}_p e_{mp} - B_p [K_e(t) - K_e(0)] e_{mp} - B_p [K_m(t) - K_m(0)] x_m \\ &\quad - B_p [K_f(t) - K_f(0)] \bar{f}_m - B_p [\Theta_d(t) - \Theta_d(0)] \\ &= \bar{A}_p e_{mp} - B_p \Phi e_{mp} - B_p \Psi x_m - B_p \Lambda \bar{f}_m - B_p \xi \end{aligned} \quad (28)$$

The adaptive law can now be obtained as

$$\begin{cases} K_e(t) = \int_0^t (e_{mp} e_{mp}^T P_{mp} B_p \Gamma_1)^T d\tau + K_e(0) \\ K_m(t) = \int_0^t (x_m e_{mp}^T P_{mp} B_p \Gamma_2)^T d\tau + K_m(0) \\ K_f(t) = \int_0^t (\bar{f}_m e_{mp}^T P_{mp} B_p \Gamma_3)^T d\tau + K_f(0) \\ \Theta_d(t) = \int_0^t (e_{mp}^T P_{mp} B_p \Gamma_4)^T d\tau + \Theta_d(0) \end{cases} \quad (29)$$

where P_{mp} , Γ_1 , Γ_2 , Γ_3 , and Γ_4 are all symmetric positive definite matrixes. $K_e(0)$, $K_m(0)$, $K_f(0)$, and $\Theta_d(0)$ are the initial values which can be obtained from the previous analysis. $K_e(t)$, $K_m(t)$, $K_f(t)$, and $\Theta_d(t)$ are the current values obtained through the designed adaptive law.

There are several restrictions in the traditional model following control or model reference adaptive control: (1) These control methods in the previous literature build the fault aircraft model before designing the control law and don't use the fault information in it, so these fault models can only describe one special kind of fault which the aircraft has suffered from. (2) The FE module was just used to help the system switch to the designed reconfigurable controller through a reconfigurable mechanism module, not to directly send the fault signal to reconfigure the fault tolerant controller. Furthermore, most of the research of the fault tolerant control always regards FE and RC as two separate entities. These restrictions are the motivation of this paper, to develop an integrated strategy of FTC in which the plant contains the fault information and the FE will send the fault estimation to RC.

IV. Application example

A. Initializing the simulation

The integrated strategy is implemented on a high-fidelity simulation model on the B747-200, and the aircraft model is trimmed at straight and level flight with a flight condition of $V_T = 0.8Ma$ and the altitude is $h = 40000ft$. We assume that there are four independent elevators and only consider the longitudinal stability in this paper [9].

The detailed data of the rigid linear model derives from NASA report CR-2144 [19].

$$A = \begin{bmatrix} -0.00276 & 0.0389 & 0 & -32.02 \\ -0.0650 & -0.317 & 771 & -2.8 \\ 0.000193 & -0.00105 & -0.42878 & 0.0003248 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.36 & 0.36 & 0.36 & 0.36 \\ -4.47 & -4.47 & -4.47 & -4.47 \\ -0.29 & -0.29 & -0.29 & -0.29 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C = [0 \ 0 \ 0 \ 1]$$

$$E = [0.0481 \ -0.9568 \ 0.0046 \ 0]^T$$

$x = [u_1 \ w_1 \ q_1 \ \theta_1]^T$ denotes the longitudinal states, u_1 is the axial velocity, w_1 is the longitudinal velocity, q_1 is the pitch rate and θ_1 is the pitch angle. $u_1 = [\delta_{e1} \ \delta_{e2} \ \delta_{e3} \ \delta_{e4}]^T$ is the longitudinal control input, δ_{ei} denotes the i th elevator angle. $y = \theta_1$ is the longitudinal output. d is the uniform 1-cos vertical gust, the max velocity is $10m/s$.

In the simulation, we will divide the response of the system into three distinct time zones: fault estimation, reconfigurable control, and final stable condition. There are several key time points (The command is given at t_c . t_{fe} denotes the faults have been estimated without bias, and t_{rc} stands for that all FTC processes are finished) in the simulation and they play an important role in the analysis of the integrated design so that we will pay more attention to them in the following subsections.

B. Simulation of the LIP fault

In the LIP case, we assume that $r = 5^\circ$, $t_c = 0s$, δ_{e1} locked in 5° and δ_{e2} locked in -5° . The true outputs of the four elevators were shown in Fig. 2. δ_{e1} and δ_{e2} locked in 5° and -5° respectively when the surfaces are intended to stabilise the system after a command was given. In most of the present FTC literature, the actuator fault information is always considered as a simple step signal which cannot reflect the true situation of the fault so that Fig. 2 used here will improve the reliability of the simulations. Fig. 3 shows that the proposed integrated designed UIO in this study will estimate the LIP fault steadily without bias in about 1.5s (t_{fe}). We can conclude from Fig. 4 that the basic controller will degrade the control performance in the post-fault system; meanwhile the integrated FTC strategy will nearly recover the performance of the original system. In the LIP case, \bar{f}_m and \bar{f}_a were processed as input compensation and a disturbance respectively. The controller will re-stabilise the system with an actuator fault in about 4s ($t_{rc} - t_c$).

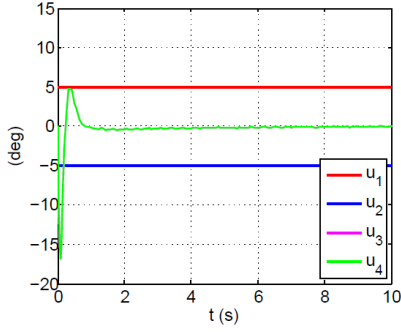


Fig. 2 Control surface deflections.

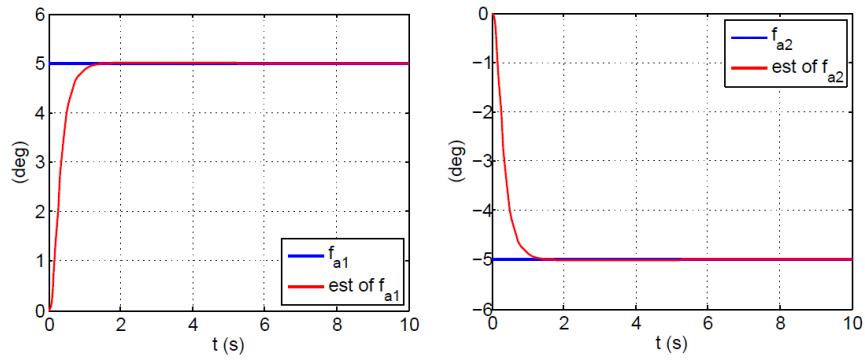


Fig. 3 Comparisons between the additive faults and their estimations.

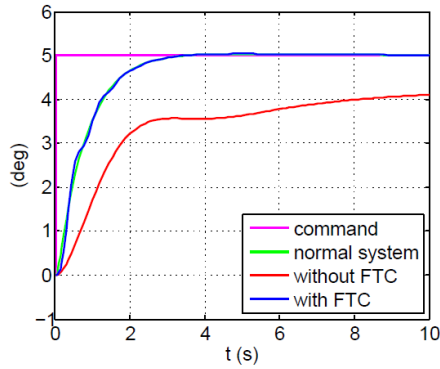


Fig. 4 Closed-loop responses under FTC controller and nominal controller.

C. Simulation of the LOE fault

In the LOE case, we assume that $r = 5^\circ$, $t_c = 0s$, δ_{e1} and δ_{e2} lost 50% and 25% of the effectiveness respectively during the whole simulation, i.e., $\bar{l}_1 = -0.5$ and $\bar{l}_2 = -0.25$. The true outputs of the four elevators were shown in Fig.

5. The effectiveness of δ_{e1} and δ_{e2} decreased to 50% and 75% respectively. Fig. 6 shows that the proposed integrated designed UIO in this study will estimate the LOE indication unit steadily without bias in about 1.5s (t_{fe}). From Fig. 7, we can get the same result as in the LIP case. In the LOE case, $\bar{f}_a = 0$, \bar{f}_m was processed as input compensation. The controller will re-stable the system with an actuator fault in about 4s ($t_{rc} - t_c$).

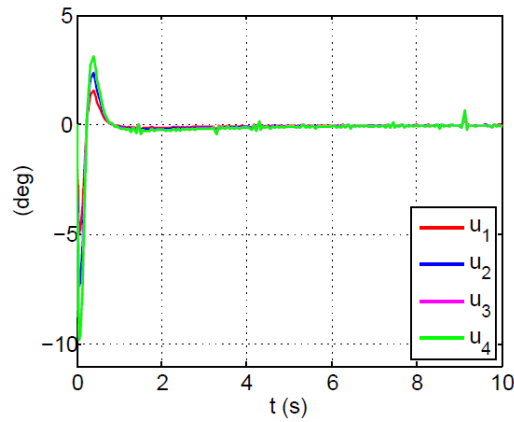


Fig. 5 Control surface deflections.

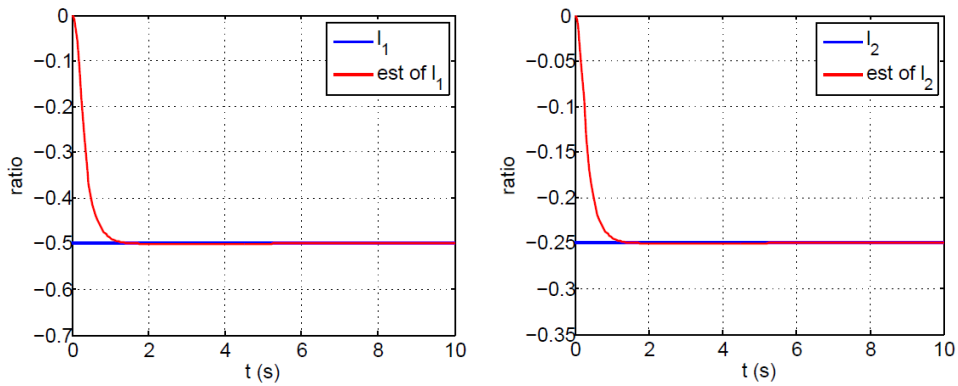


Fig. 6 Comparisons between the indication coefficient and their estimations.

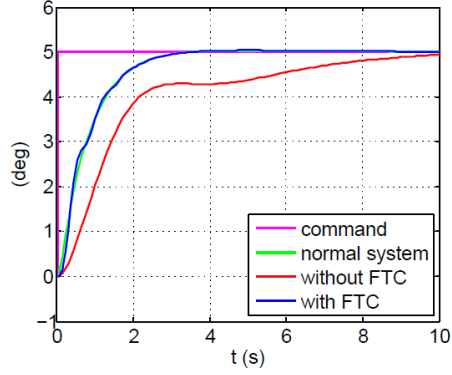


Fig. 7 Closed-loop responses under FTC controller and nominal controller.

D. Simulation of the Bias fault

In the Bias case, we assume that $r = 5^\circ$, $t_c = 0s$, δ_{e1} and δ_{e2} had constant bias 1° and -1° respectively during the whole simulation. The true outputs of the four elevators were shown in Fig. 8. The bias of δ_{e1} and δ_{e2} were to 1° and -1° respectively. Fig. 9 shows that the proposed integrated designed UIO in this study will estimate the Bias actuator fault steadily without bias in about 1.5s (t_{fe}). We can conclude from Fig. 10 that the post-fault system with the integrated FTC strategy will nearly recover the original system performance but the basic controller will not. In the Bias case, $\bar{f}_m = 0$, \bar{f}_a was processed as a disturbance. The controller re-stables the system with an actuator fault in about 4s ($t_{re} - t_c$).

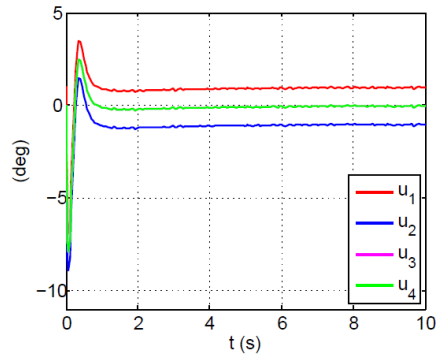


Fig. 8 Control surface deflections.

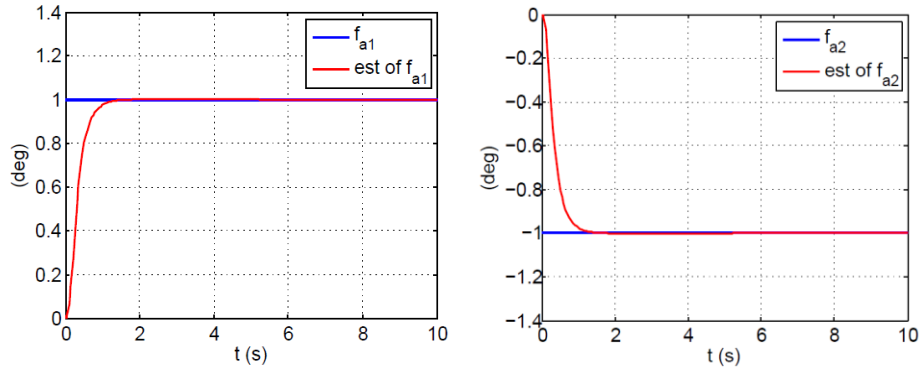


Fig. 9 Comparisons between the indication coefficient and their estimations.

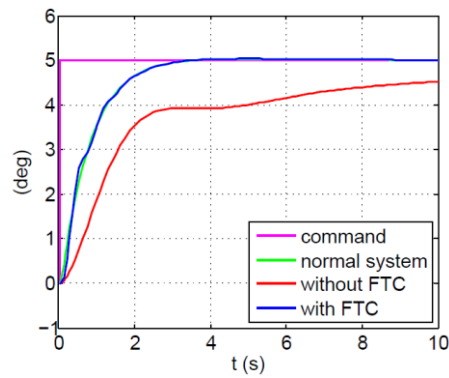


Fig. 10 Closed-loop responses under FTC controller and nominal controller.

V. Conclusion

A new integrated strategy of fault tolerant control (FTC) for linear system with actuator faults and disturbance is proposed. The presented approach is to design the fault estimation (FE) and reconfigurable control (RC) together; using a multiple-model based adaptive unknown input observer and model reference adaptive control method. All of the three classes of actuator faults are discussed. Simulation and comparison of the longitudinal attitude control for a commercial aircraft B747-200 model shows that the proposed integrated design approach leads to an ideal FE and RC performance.

The limitations of this paper are (1) the proposed design was based on a rigid aircraft model; we could use a more exact flexible aeroelastic model to integrate the FE and RC and it might bring a series of new problems, particularly if the flexible modes are close to the rigid body modes, but this wouldn't change the underlying methodology, and (2) it is important to employ actuator constraints such as amplitude and rate saturation when designing FTC controllers

in the real-world because the magnitude of control surface deflections is physically constrained. Thus, it remains an open question as to how to develop alternative strategies to compensate both the actuator faults and saturations as well as to handle systems with an aeroelastic aircraft model.

Appendix A

Theorem 1: There exists a multiple-model based adaptive UIO for the plant model in the augmented system (14) and a control law (26) to make the post-fault system (11) stable and unbiased, if the integrate designed adaptive law for (14) holds that (23) and (29).

Proof: In the integrated design of the FTC, let $\chi = [r_{po}^T, e_{mp}^T]^T$, define a Lyapunov function for the augmented closed-loop system (14) as

$$V_l = \frac{1}{2}(\chi^T P_l \chi + \Xi) \quad (A1)$$

where $\Xi = \sum_{i=1}^{P_f} \Delta_{l_i}^2 / \alpha_i + \sum_{i=1}^{P_f} \Delta_{f_{ai}}^2 / \beta_i + \text{tr}(\Phi^T \Gamma_1^{-1} \Phi + \Psi^T \Gamma_2^{-1} \Psi + \Lambda^T \Gamma_3^{-1} \Lambda + \xi^T \Gamma_4^{-1} \xi)$, $P_l = \text{diag}(P_{po}, P_{mp})$ is positive definite, $\Phi = K_e(t) - K_e(0)$, $\Psi = K_m(t) - K_m(0)$, $\Lambda = K_f(t) - K_f(0)$, and $\xi = \Theta_d(t) - \Theta_d(0)$. $\alpha_i > 0$, $\beta_i > 0$, Γ_1 , Γ_2 , Γ_3 , and Γ_4 are positive definite so that $V > 0$. The first derivative of V can be derived as

$$\dot{V}_l = \frac{1}{2}(\dot{\chi}^T P_l \chi + \chi^T P_l \dot{\chi} + \dot{\Xi}) \quad (A2)$$

where

$$\begin{aligned} \dot{\chi}^T P_l \chi + \chi^T P_l \dot{\chi} &= (\dot{r}_{po}^T, \dot{e}_{mp}^T) P_l (r_{po}^T, e_{mp}^T)^T + (r_{po}^T, e_{mp}^T) P_l (\dot{r}_{po}^T, \dot{e}_{mp}^T)^T \\ &= e_{po}^T (M^T C_{po}^T P_{po} C_{po} + C_{po}^T P_{po} C_{po} M) e_{po} + e_{mp}^T (\bar{A}_p^T P_{mp} + P_{mp} \bar{A}_p) e_{mp} \\ &\quad + 2 \sum_{i=1}^{P_f} e_{po}^T C_{po}^T P_{po} C_{po} G b_{pi} \Delta_{l_i} u_{pi} + 2 \sum_{i=1}^{P_f} e_{po}^T C_{po}^T P_{po} C_{po} G b_{pi} \Delta_{f_{ai}} \\ &\quad - 2 e_{mp}^T P_{mp} B_p \Phi e_{mp} - 2 e_{mp}^T P_{mp} B_p \Psi x_m - 2 e_{mp}^T P_{mp} B_p \Lambda f_m - 2 e_{mp}^T P_{mp} B_p \xi \end{aligned} \quad (A3)$$

$$\begin{aligned} \dot{\Xi} &= 2 \sum_{i=1}^{P_f} \Delta_{l_i} \dot{\Delta}_{l_i} / \alpha_i + 2 \sum_{i=1}^{P_f} \Delta_{f_{ai}} \dot{\Delta}_{f_{ai}} / \beta_i + \text{tr}(\dot{\Phi}^T \Gamma_1^{-1} \Phi + \Phi^T \Gamma_1^{-1} \dot{\Phi} + \dot{\Psi}^T \Gamma_2^{-1} \Psi + \Psi^T \Gamma_2^{-1} \dot{\Psi} \\ &\quad + \dot{\Lambda}^T \Gamma_3^{-1} \Lambda + \Lambda^T \Gamma_3^{-1} \dot{\Lambda} + \dot{\xi}^T \Gamma_4^{-1} \xi + \xi^T \Gamma_4^{-1} \dot{\xi}) \\ &= -2 \sum_{i=1}^{P_f} \Delta_{l_i} \dot{\Delta}_{l_i} / \alpha_i - 2 \sum_{i=1}^{P_f} \Delta_{f_{ai}} \dot{\Delta}_{f_{ai}} / \beta_i + 2 \text{tr}[\dot{K}_e^T(t) \Gamma_1^{-1} \Phi + \dot{K}_m^T(t) \Gamma_2^{-1} \Psi + \dot{K}_f^T(t) \Gamma_3^{-1} \Lambda + \dot{\Theta}_d^T(t) \Gamma_4^{-1} \xi] \end{aligned} \quad (A4)$$

Substituting (A3) and (A4) into (A2) yields

$$\begin{aligned}
\dot{V}_I = & \frac{1}{2} [e_{po}^T (M^T C_{po}^T P_{po} C_{po} + C_{po}^T P_{po} C_{po} M) e_{po} + e_{mp}^T (\bar{A}_p^T P_{mp} + P_{mp} \bar{A}_p) e_{mp} \\
& + 2 \sum_{i=1}^{p_f} e_{po}^T C_{po}^T P_{po} C_{po} G b_{pi} \Delta_i u_{pi} + 2 \sum_{i=1}^{p_f} e_{po}^T C_{po}^T P_{po} C_{po} G b_{pi} \Delta_{\bar{f}_{ai}} \\
& - 2 e_{mp}^T P_{mp} B_p \Phi e_{mp} - 2 e_{mp}^T P_{mp} B_p \Psi x_m - 2 e_{mp}^T P_{mp} B_p \Lambda \bar{f}_m - 2 e_{mp}^T P_{mp} B_p \xi \\
& - 2 \sum_{i=1}^{p_f} \Delta_i \dot{l}_i / \alpha_i - 2 \sum_{i=1}^{p_f} \Delta_{\bar{f}_{ai}} \dot{f}_{ai} / \beta_i + 2 \text{tr} [\dot{K}_e^T(t) \Gamma_1^{-1} \Phi + \dot{K}_m^T(t) \Gamma_2^{-1} \Psi + \dot{K}_f^T(t) \Gamma_3^{-1} \Lambda + \dot{\Theta}_d(t) \Gamma_4^{-1} \xi]
\end{aligned} \tag{A5}$$

In most of the literature, C_{po} was always assumed to be a full rank matrix so that $P_{po}^* = C_{po}^T P_{po} C_{po}$ is still a symmetric positive definite matrix which is restrictive and often cannot be satisfied. However, for the integration of FTC, we don't have this restriction for the UIO. That is, $P_{po}^* = C_{po}^T P_{po} C_{po}$ is a symmetric positive semidefinite matrix. Since M is Hurwitz, there exists a symmetric positive semidefinite matrix Q_{po}^* so that

$$M^T P_{po}^* + P_{po}^* M = -Q_{po}^* \leq 0. \text{ See Appendix B.}$$

Since \bar{A}_p is Hurwitz, a symmetry positive definite matrix Q_{mp} satisfied the Lyapunov equation

$$\bar{A}_p^T P_{mp} + P_{mp} \bar{A}_p = -Q_{mp} < 0. \text{ Moreover, } e_{po}^T (M^T P_{po}^* + P_{po}^* M) e_{po} + e_{mp}^T (\bar{A}_p^T P_{mp} + P_{mp} \bar{A}_p) e_{mp} < 0. \text{ Thus, only to design the}$$

last items of (34) as zero will ensure $\dot{V} < 0$. It follows that

$$\begin{cases}
e_{po}^T C_{po}^T P_{po} C_{po} G b_{pi} \Delta_i u_{pi} = \Delta_i \dot{l}_i / \alpha_i \\
e_{po}^T C_{po}^T P_{po} C_{po} G b_{pi} \Delta_{\bar{f}_{ai}} = \Delta_{\bar{f}_{ai}} \dot{f}_{ai} / \beta_i \\
e_{mp}^T P_{mp} B_p \Phi e_{mp} = \dot{K}_e^T(t) \Gamma_1^{-1} \Phi \\
e_{mp}^T P_{mp} B_p \Psi x_m = \dot{K}_m^T(t) \Gamma_2^{-1} \Psi \\
e_{mp}^T P_{mp} B_p \Lambda \bar{f}_m = \dot{K}_f^T(t) \Gamma_3^{-1} \Lambda \\
e_{mp}^T P_{mp} B_p \xi = \dot{\Theta}_d(t) \Gamma_4^{-1} \xi
\end{cases} \tag{A6}$$

and rearranging (A6) we can obtain the adaptive law (23) and (29).

Appendix B

Theorem 2: A sufficient condition of a stable linear time invariant system $\dot{e}_{pw} = M e_{pw}$ is that for an arbitrary symmetric positive semidefinite matrix Q_{pw}^* , there is only one symmetric positive semidefinite matrix P_{pw}^* that holds

$$M^T P_{pw}^* + P_{pw}^* M = -Q_{pw}^*.$$

Proof:

Existence: As M is Hurwitz, consider the matrix equation

$$\dot{e}_{pw} = M^T \dot{e}_{pw} + \dot{e}_{pw} M, \quad e_{pw}(0) = Q_{pw}^* \quad (\text{B1})$$

It is easy to get the solution of (B1) that $e_{pw} = e^{M^T t} Q_{pw}^* e^{Mt}$, integrating (B1) yields

$$e_{pw}(\infty) - e_{pw}(0) = M^T \left(\int_0^\infty e_{pw} dt \right) + \left(\int_0^\infty e_{pw} dt \right) M \quad (\text{B2})$$

Due to M is asymptotic stable, $e_{pw}(\infty) = 0$, so that

$$-Q_{pw}^* = M^T \left(\int_0^\infty e^{M^T t} Q_{pw}^* e^{Mt} dt \right) + \left(\int_0^\infty e^{M^T t} Q_{pw}^* e^{Mt} dt \right) M \quad (\text{B3})$$

Denote $P_{pw}^* = \int_0^\infty e^{M^T t} Q_{pw}^* e^{Mt} dt$, P_{pw}^* satisfies $M^T P_{pw}^* + P_{pw}^* M = -Q_{pw}^*$. Moreover, $P_{pw}^{*T} = P_{pw}^*$,

$e_{pw}^T P_{pw}^* e_{pw} = \int_0^\infty (e^{Mt} e_{pw})^T Q_{pw}^* (e^{Mt} e_{pw}) dt \geq 0$. Thus, P_{pw}^* is a symmetric positive semidefinite matrix.

Uniqueness: Assume that both P_{pw1}^* and P_{pw2}^* are the solutions of $M^T P_{pw}^* + P_{pw}^* M = -Q_{pw}^*$, so that

$$M^T (P_{pw1}^* - P_{pw2}^*) + (P_{pw1}^* - P_{pw2}^*) M = 0 \quad (\text{B4})$$

Pre-multiplying (B4) by $e^{M^T t}$ and Post-multiplying (B4) by e^{Mt} yields

$$e^{M^T t} M^T (P_{pw1}^* - P_{pw2}^*) e^{Mt} + e^{M^T t} (P_{pw1}^* - P_{pw2}^*) M e^{Mt} = \frac{d}{dt} [e^{M^T t} (P_{pw1}^* - P_{pw2}^*) e^{Mt}] = 0 \quad (\text{B5})$$

That is, $e^{M^T t} (P_{pw1}^* - P_{pw2}^*) e^{Mt}$ is a scalar matrix. Substituting $t = 0$ into it yields

$$P_{pw1}^* - P_{pw2}^* = e^{M^T t} (P_{pw1}^* - P_{pw2}^*) e^{Mt} \quad (\text{B6})$$

Due to M is asymptotic stable, when $t \rightarrow \infty$, $P_{pw1}^* - P_{pw2}^* = 0$, $P_{pw1}^* = P_{pw2}^*$.

Acknowledgments

The work reported in this paper is partially supported by China Scholarship Council (CSC).

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