Opportunities to acquire foundational number sense: A quantitative comparison of popular English and Swedish textbooks

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Anna Löwenhielm[1], Gosia Marschall[1], Judy Sayers[1] and Paul Andrews[1]

[1]Stockholm University, Sweden; anna.lowenhielm@mnd.su.se, gosia.marschall@mnd.su.se, judy.sayers@mnd.su.se and paul.andrews@mnd.su.se

In this paper, we present analyses of popular grade one textbooks, one from each of England and Sweden. Focused on Foundational Number Sense, we examine how each book's tasks facilitate children's learning of those number-related competences that require instruction and which underpin later mathematical learning. Analyses identified both similarities and differences. Similarities lay in books' extensive opportunities for children to recognise and write numbers and undertake simple arithmetical operations. However, neither offered more than a few tasks related to estimation or simple number patterns. Differences lay in the Swedish book's greater emphases on different representations of number, quantity discrimination and relating numbers to quantity, highlighting conceptual emphases on number. The English book offers substantially more opportunity for students to count systematically, highlighting procedural emphases.

Keywords: Foundational number sense, mathematics textbooks, England, Sweden, grade one.

Introduction

In this paper we offer a comparative analysis of how commonly used textbooks, one from each of England and Sweden, enable year one pupils' acquisition of foundational number sense (FoNS). FoNS, which has been discussed in earlier CERME papers (Back, Sayers & Andrews, 2013; Andrews, Sayers & Marschall, 2015; Sayers & Andrews, 2015), comprises those number-related competences that underpin later mathematical learning, both in the short and the long term, and require instruction. Derived from a systematic review of the literature (Andrews & Sayers, 2015), FoNS comprises the eight broad categories shown in Table 1. Focused on the FoNS-related opportunities initiated during whole class teaching, the framework has structured analyses of grade one lessons in various European countries (Back et al., 2013; Andrews et al., 2015; Sayers, Andrews & Björklund Boistrup, 2016) and identified didactical emphases commensurate with earlier research undertaken in the same countries.

Until now, we have not examined the framework's effectiveness with respect to identifying FoNS-related opportunities in textbooks. This is a significant omission, particularly as both textbook production and deployment are unregulated in England and Sweden. This significance is heightened by uncertainty with respect to pre-school students' likely FoNS-related experiences. On the one hand, the English pre-school curriculum specifies that children should "count reliably with numbers from 1 to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer" (Department for Education, 2014, p.11). On the other hand, the Swedish pre-school curriculum, which specifies no such detail, expects children to develop an understanding of the basic properties of quantity, number and number concepts (Skolverket, 2016). Thus, while there are no explicit FoNS-related expectations in the Swedish pre-school curriculum, a number, but not all, are addressed in the English.

FoNS Characteristic	Learners are encouraged to
Number recognition	Identify, name and write particular number symbol
Systematic counting	Count systematically, forwards and backwards, from arbitrary starting points
Number and quantity	Understand the one-to-one correspondence between number and quantity
Quantity discrimination	Compare magnitudes and deploy language like 'bigger than' or 'smaller than'
Different representations	Recognise and make connections between different representations of number
Estimation	Estimate, whether it be the size of a set or an object
Simple arithmetic	Perform simple addition and subtraction operations
Number patterns	Recognise and extend number patterns, identify a missing number

Table 1: Summaries of the eight FoNS categories

Of particular interest to this paper is Bierhoff's (1996) comparison of the number-related opportunities offered in commonly used English, German and Swiss textbooks. Focused on the transition from "working with numbers up to 20… to working with two-digit numbers" (p. 143), she found that English textbooks were the least coherently structured. Also, students were expected to calculate with large numbers before consolidating their understanding of the integers up to 20, a situation made problematic by the English overemphasis on place value. Turning more explicitly to studies focused solely on English textbooks, Newton and Newton (2007), in an evaluation of the professional support school textbooks might afford primary teachers, examined eighteen textbooks written for use with English 7-11 students. They found few tasks that would facilitate mathematical reasoning, being primarily focused on skills acquisition.

With respect to Sweden, as in England, the production of textbooks has been unregulated since 1991 (Ahl, 2016) and several recent studies have examined Swedish mathematics textbooks against various criteria. For example, at the university level, Lithner (2004) found tasks typically promoting low levels of imitative reasoning. At the upper secondary, or post-compulsory, level Nordström and Löfwall (2005) analysed the extent to which students were offered opportunities to engage with proof in two commonly used sets of textbooks. They found little evidence of proof in any of their examined topics, although there were many implicit opportunities in many of the tasks analysed. In similar vein, Lundberg (2011) compared three of the most commonly used textbooks from the perspective of proportional reasoning and found not only that direct proportion dominated but also that while both dynamic and static notions of proportion were present in all three textbooks, justifications were rare. With respect to the final years of compulsory school, Ahl (2016) examined the proportional reasoning in two popular textbooks. She found that "the impact of research findings on the representation of proportional reasoning is scant" in both (Ahl, 2016, p. 198) and that the books failed to encourage learners to understand the distinction between additive and multiplicative situations. In short, the limited available evidence indicates that textbooks written for older Swedish students present few opportunities for them to make mathematical connections or engage in mathematical reasoning.

However, little is yet known about the ways in which textbooks written for young children present mathematical ideas.

This study is a first attempt to evaluate the FoNS framework as a tool for analysing grade one textbooks. Thus, while it is not an explicit attempt to evaluate the content of the books themselves, it is an important first comparison of textbooks from the two countries. In making this comparison, we acknowledge Rezat's (2006, p. 482) position that the mathematics textbook "can be regarded as an artefact in the broad sense of the term. It is historically developed, culturally formed, produced for certain ends and used with particular intentions". In other words, comparative analyses of this nature highlight well cultural differences in expected learning outcomes.

Methods

Two popular textbooks, one from England and one from Sweden, were identified for analytical purposes. In focusing on popular textbooks, we believed we would gain insight into not only how a reasonably high proportion of children in both countries experiences FoNS but also what teachers and schools value in their choice of textbooks. Before formal analyses were undertaken, all four authors met for two days to discuss and evaluate a range of textbook tasks in order to operationalise the FoNS categories. Drawing on the studies of Li (2000) and others, only those tasks explicitly addressed to the student were analysed. For example, both of the examined textbooks included instructions or suggested activities that teachers might use. However, these were not analysed as they did not explicitly address the learner and typically included too little detail to show how they might have been used with children. For similar reasons, since tasks included in teacher guides were not focused directly on students, teacher guides were not included in the analyses. After this first pass, each of the first two authors took responsibility for analyses of the Swedish and English textbooks respectively. In these roles, each was supported by the third and fourth authors with respect to ambiguous or difficult to interpret tasks. In addition, random exercises from each textbook were also coded by both the third and fourth authors as part of a moderation process.

Operationalising the codes

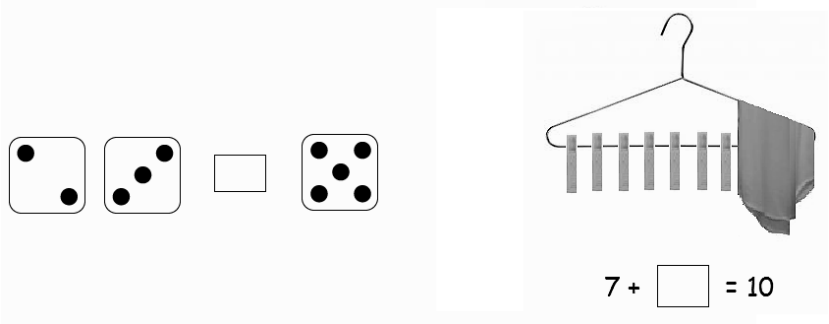

Figure 1: Additive tasks from the Swedish and English textbooks respectively

Figure 1 shows one example from each of the textbooks, Swedish on the left and English on the right. In one of several similar tasks in one exercise, Swedish students were asked to "compare the number of dots" and then "write either = or ≠" in the box. This particular task, which occurred before the introduction of addition, was thought to encourage completion by counting and coded for *systematic counting*. The expectation that students would address issues of equality or inequality led to its also being coded for *quantity discrimination*. In addition, the dot patterns not only offered *different*

representations of number but allowed for subitising and an *awareness of the relationship between number and quantity*. The goal of the English task, based on a coat hanger with ten pegs of which some of which had been covered with a cloth, was to identify the number of hidden pegs. The way in which the task was presented explicitly involved *number recognition*, while its focus was on *simple arithmetic*. In addition, its allusion to cardinality led to its being coded for *awareness of the relationship between number and quantity*. In short, many tasks attracted multiple codes.

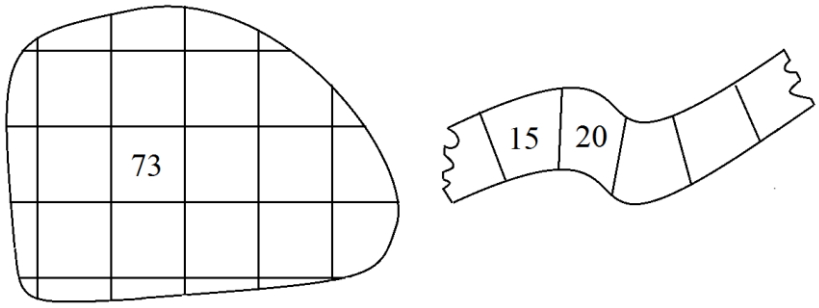

Figure 2: Number patterns tasks from the Swedish and English textbooks respectively

Some FoNS categories, as shown in Table 3, were rare in both textbooks. In this respect, Figure 2 shows tasks, one from each textbook, with explicit foci on *number patterns*. The Swedish task on the left was based on a section of a hundred square, with students being expected to complete the missing values. In addition to being coded for *number patterns*, the explicit focus of the task, it was also coded for *systematic counting*, *number recognition* and, implicitly, *simple arithmetical operations*. These decisions drew on the facts that the task required students to count on, recognise numbers and, in moving from one row to another, add or subtract ten. The English task on the right was one of several based around a section of a multiplication table torn from a longer strip of paper that invited students to count on in fives and enter the missing numbers. In addition to being coded for *number patterns*, these tasks were also coded for *number recognition*, *systematic counting* and *simple arithmetical operations*.

Results

Below we present two analyses offering similar but importantly different perspectives on the data. The first is based on frequencies and the second on proportions.

A frequency analysis

The figures of Table 2 show the distribution of the eight FoNS categories across the two textbooks, one from England and one from Sweden. The first thing to notice, acknowledging that both books are intended to provide the complete learning experience for year one students, is that the Swedish book offered 444 tasks appropriate for FoNS coding, while the English only 257. That is, while both figures represented similar proportions of the totality of tasks within their respective books, the Swedish textbook comprised 187 (73%) more FoNS-related tasks than the English. Table 2 also shows that of the eight FoNS categories, number recognition was the most frequently observed, with 532 out of 691 tasks providing opportunities for learners to recognise, write and say numbers. In similar vein, *simple arithmetical operations* were common occurrences throughout both books. Neither of these results, we suggest, is surprising as arithmetical competence is an unequivocal curricular goal, which

relies extensively on number recognition. The least commonly observed FoNS category was estimation, with just 18 occurrences.

Category present in task (444 Swedish tasks and 257 English tasks)

	No	Yes	No	Yes	No	Yes
	Number recognition		Systematic counting		Number and quantity	
England	29	228	145	112	194	63
Sweden	130	304	354	80	259	175
	159	532	499	192	453	238
	$\chi^2 = 31.8$ (A)		$\chi^2 = 50.9$ (A)		$\chi^2 = 17.9$ (A)	
	Quantity discrimination		Different representations		Estimation	
England	237	20	202	55	250	7
Sweden	370	64	181	253	423	11
	607	84	383	308	672	18
	$\chi^2 = 7.33$ (B)		$\chi^2 = 88.9$ (A)		$\chi^2 = 0.03$ (C)	
	Simple arithmetic		Number patterns			
England	154	103	232	25		
Sweden	232	202	406	28		
	386	305	638	53		
	$\chi^2 = 2.74$ (C)		$\chi^2 = 2.45$ (C)			

Chi squares marked A yielded $p<0.0005$, B yielded $p<0.01$ and C were not significant

Table 2: Frequencies and chi square tests for each category for each country.

When data are compared, some interesting results emerge. On the one hand the English books comprised significantly higher proportions of tasks involving *number recognition* (χ^2=31.8, $p<0.0005$) and *systematic counting* (χ^2=50.9, $p<0.0005$) than the Swedish. On the other hand, the Swedish books offered significantly higher proportions of tasks involving opportunities for students to *relate numbers to quantity* (χ^2=17.9, $p<0.0005$), engage in *quantity discrimination* (χ^2=7.33, $p=0.007$) and experience *different representations of number* (χ^2=88.9, $p<0.0005$). Proportionally, the figures of Table 2 show no significant differences with respect to *estimation*, *simple arithmetical operations* or *number patterns*. These results take us to the second step of the analysis.

A proportional analysis

A second perspective on the data can be seen in Table 3. Firstly, several FoNS categories were found in similar proportions in both textbooks. These included relatively high occurrences of *simple arithmetical operations*, implicated in just under half of all tasks in both textbooks. In smaller proportions, around a quarter of all tasks in both books, were opportunities for students to *relate number to quantities*. In very small proportions in both books, were found *number patterns* and *estimation*. Secondly, several categories distinguished the expectations found in one book from the other. On the one hand, the English textbook comprised a significantly higher percentage of number recognition tasks (89%) than the Swedish (70%) ($t=6.31$, $p<0.0005$). Also, almost half of all English tasks involved systematic counting in comparison with less than a fifth in the Swedish ($t=6.95$, $p<0.0005$). Alternatively, the Swedish textbook comprised nearly three times as many tasks involving different representations of number as the English ($t=10.57$, $p<0.0005$), twice as many tasks focused

on quantity discrimination (t=2.92, p=0.004) and almost twice as many tasks relating numbers to quantity (t=4.42, p<0.0005). Finally, Table 3 shows that the percentage of tasks coded for *estimation*, *simple arithmetical operations* and *number patterns* were comparable in both books, confirming that the two analyses, one essentially parametric and the other non-parametric, yielded equivalent results.

	E%	S%	t	p
Number recognition	89	70	6.31	0.000
Systematic counting	44	18	6.95	0.000
Relating number to quantity	25	40	-4.42	0.000
Quantity discrimination	8	15	-2.92	0.004
Different representations	21	58	-10.57	0.000
Estimation	3	3	0.16	0.874
Simple arithmetical operations	40	47	-1.66	0.097
Number patterns	10	6	1.49	0.136

Table 3: Percentage of all tasks coded for each FoNS category along with t-tests

Discussion

In this paper our objective was to examine the efficacy of the FoNS framework as tool for evaluating the learning opportunities embedded in commonly used textbooks and to undertake a comparative analysis to determine the framework's sensitivity to different cultural expectations. In both cases, we believe the study to have been successful. For example, with respect to the identification of the different FoNS categories, very few tasks were identified with an emphasis on *estimation*, a finding resonating closely with earlier classroom observations showing no evidence of teachers in England, Hungary, Poland, Russia or Sweden emphasising it in their teaching (Back et al., 2013; Andrews et al., 2015; Sayers et al., 2016). This, it seems to us, is an issue of some concern and the basis of further systematic inquiry. Indeed, acknowledging that estimation skills are important indicators of later mathematical competence (Booth & Sigler, 2006), that both older students (Sowder & Wheeler, 1989) and many otherwise competent adults (Hanson & Hogan, 2000) are uncomfortable with estimation tasks, it seems sensible to ask; why does estimation play such a lowly role in the classroom practice and textbooks of these two countries? This, we argue, is particularly pertinent in light of evidence from other countries that teachers see little relevance in teaching estimation (Alajmi, 2009). Furthermore, the similar frequencies of other FoNS categories are unsurprising. For example, it is reasonable to assume that the relative lack, in both textbooks, of tasks focused on *number patterns* may be explained by the fact that most year one curricular goals emphasise learners' *number recognition, relating number to quantity* and the beginnings of *arithmetic*. In other words, while number patterns are important in preparing students for later mathematical learning (Lembke & Foegen, 2009), they may be subordinated in children's early number experiences to more pressing developmental needs.

With respect to cultural sensitivity the data yielded several hitherto uncovered insights. For example, on the one hand, the higher proportions of Swedish tasks coded for *different representations of number*, *relating number to quantity* and *quantity discrimination* allude to a book focused on conceptual understanding. On the other hand, the apparent lack of a conceptual emphasis in the English book finds further support in the high proportions of tasks coded for *systematic counting* and extremely high proportions of tasks addressing *number recognition*, which tend to suggest a book

focused on the development of procedural knowledge commensurate with the low levels of mathematical challenge found in earlier studies of English textbooks (Bierhoff, 1999; Haggarty & Pepin, 2002; Newton & Newton, 2007). However, the conceptual emphasis found in the Swedish textbooks seemed not to match the generally negative findings of earlier Swedish studies (Ahl, 2016; Lundberg, 2011; Nordström & Löfwall, 2005). In this respect, it is not improbable that these differences may be because these earlier studies addressed textbooks for students in grades 7 and upward rather than on those for young children. Finally, drawing on Bernstein's (1990) notion of curricular framing, it is interesting to note that the weakly framed Swedish pre-school curriculum seems to have prompted a conceptually focused textbook, while the strongly framed English pre-school curriculum seems to have precipitated a procedurally focused textbook. Such matters allude to research beyond the scope of this paper but which will form a key aspect of any further analyses we make.

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