UDK 620.1:666.97
Original scientific article/Izvirni znanstveni članek

ISSN 1580-2949
MTAEC9, 53(2)233(2019)

MIXED-MODE FRACTURE ANALYSIS IN HIGH-PERFORMANCE CONCRETE USING A BRAZILIAN DISC TEST

ANALIZA MEŠANEGA PRELOMA VISOKOKAKOVOSTNIH BETONOV Z BRAZILSKIM PREIZKUSOM

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Prejem rokopisa – received: 2018-07-24; sprejem za objavo – accepted for publication: 2018-11-06

doi:10.17222/mit.2018.161

This paper reports and discusses the results of an experimental investigation into the fracture mechanical properties of high-performance concrete (HPC) with a compressive strength higher than 100 MPa. In total, 6 specimens for a three-point bending test and 12 specimens of a Brazilian disc under a mixed-mode load were included in the experimental measurements. The fracture toughness was evaluated for mode I in the case of a three-point bend test and a Brazilian disc test, allowing a correlation between two geometrically different specimens. The fracture resistance under mixed mode I/II was analysed using the Brazilian disc test specimen with a central notch. To evaluate the fracture resistance under the mixed mode a generalised maximum tangential stress criterion was employed and the value of the critical distance r_C is discussed as an appropriate HPC material characteristic. The last objective of this study was to provide some experimental data that can be useful in engineering practice for calibrating numerical constitutive models.

Keywords BDCN, 3PBT, fracture mechanics, GMTS, HPC, mixed mode, critical distance

Avtorji poročajo in razpravljajo o eksperimentalnih raziskavah lomno-mehanskih lastnosti visokokakovostnega betona (HPC; angl.: High Performance Concrete) s tlačno trdnostjo nad 100 MPa. V kompletu so izvedli tritočkovne upogibne preizkuse na šestih (6) vzorcih in na dvanajstih (12) vzorcih v obliki diskov so izvedli brazilski preizkus, ki predstavlja mešano (natezno/tlačno) obremenitev. Določili so lomno žilavost za način I v obeh primerih; to je tritočkovnega upogibnega preizkusa in brazilskega testa, upoštevajoč geometrijske lastnosti obeh vrst vzorcev. Odpornost proti lomu v mešanem načinu I/II so analizirali s pomočjo brazilskega preizkusa na vzorcih (diskih) s centralno zarezo. Da bi ovrednotili odpornost proti lomu v mešanem načinu, so uporabili posplošeni kriterij maksimalne tangencialne napetosti in ocenili, da je vrednost kritične razdalje primeren kriterij za oceno lastnosti HPC. V zadnjem delu te študije so podali nekaj eksperimentalnih podatkov, ki bi lahko bili uporabni v inženirski praksi za kalibracijo numeričnih konstitutivnih modelov.

Ključne besede: BDCN, 3PBT, lomna mehanika, GMTS, HPC, mešani način, kritična razdalja

1 INTRODUCTION

The sustainability of concrete structures assumes the optimised use of concrete. Instead of using high volumes of lower-strength concrete, a high-performance concrete (HPC) is considered as a sustainable solution to design subtle structures (thin elements) and therefore reduce the total consumption of material. The production of HPC consumes much less natural resources, i.e., raw materials for cement, aggregates, water, etc. To reduce the consumption of natural resources and to improve its mechanical characteristics the secondary pozzolans like silica fume, slag, fly ash, etc, are commonly used as a mineral admixtures into the HPC mixture.

HPC is used where a weight reduction of structure is important or where architectural design demands smaller and thinner bearing structural elements for a required esthetical value. The use of optimized structural elements made from HPC leads to a reduction in the total amount of transported material, which results in an overall cost reduction for the structure.

An experimental study of the structural behaviour of HPC and very-high-performance concrete is presented in[1]; however, the HPC has a compressive strength limited to 84 MPa. A pilot study with a focus on the fracture mechanical parameters (specific fracture energy) of five different kinds of HPC is mentioned in[2], but the compressive strength is only 88 MPa of all the tested HPCs. The influence of various coarse aggregates for high-strength concrete on the fracture energy and strength properties is shown in[3], but the compressive strength of the studied HPC was only 90 MPa.

The aim of long-term research is to design and prepare concrete mixtures suitable for precast elements, which result to total weight reduction. Typically produced precast elements are light frames and beams of small-span bridges. These subtle structural elements have to be designed with a greater attention to durability, fatigue behaviour and fracture behaviour. This is one of the reasons why fracture mechanics is employed in this

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contribution, especially for the fracture behaviour of HPC under the mixed-mode I/II loading conditions. All recent studies are focused on HPC with a compressive strength of less than 100 MPa, thus it is useful to see how HPC behaves when the compressive strength is higher than 100 MPa.

This contribution aims to introduce the mechanical characteristics and the fracture resistance of HPC under the mixed mode I/II using a Brazilian disc test with a central notch (BDCN)[4]. To evaluate the fracture resistance under the mixed mode a generalised maximum tangential stress (GMTS)[5] criterion was employed. The discussion of an appropriate value for the critical distance r_C with use as a material characteristic is presented.

2 THEORETICAL BACKGROUND

This contribution is based on linear elastic fracture mechanics. The linear elastic fracture mechanics concept uses the stress field in the close vicinity of the crack tip described by the Williams' expansion.[6] This expansion is an infinite power series, originally derived for a homogenous elastic isotropic cracked body. The stress field for mode I and mode II can be described with the following equation:

$$\sigma_{ij} = \frac{K_{\mathrm{I}}}{\sqrt{2\pi r}} f^{\mathrm{I}}_{i,j}(\theta) + \frac{K_{\mathrm{II}}}{\sqrt{2\pi r}} f^{\mathrm{I}}_{i,j}(\theta) + T + O_{i,j}(r,\theta) \quad (1)$$

where σ_{ij} represents the stress tensor components, K_{I}, K_{II} are the stress-intensity factors (SIFs) for mode I and mode II, respectively, $f^{\mathrm{I}}_{i,j}(\theta)$, $f^{\mathrm{II}}_{i,j}(\theta)$ are known shape functions for mode I and mode II, usually written for BDCN[7] as Y_{I} and Y_{II}, T (or T-stress) represents the second term independent of r, O_{ij} represents the higher-order terms, and r, θ are the polar coordinates (with the origin at the crack tip; the crack faces lie along the x-axis).

2.1 Brazilian disc test with a central notch (BDCN)

The BDCN is a specimen with a circular cross-section made from a cylinder with a notch in the middle of the specimen (**Figure 1a**). The test performed on the BDCN specimen is carried out under relatively simple experimental conditions (**Figure 1b**), using only the testing press with a sufficient load capacity. The evaluation of the fracture parameters for modes I, II and mixed mode I/II is made by inclining the notch at an angle α against the load position (**Figure 1a**).

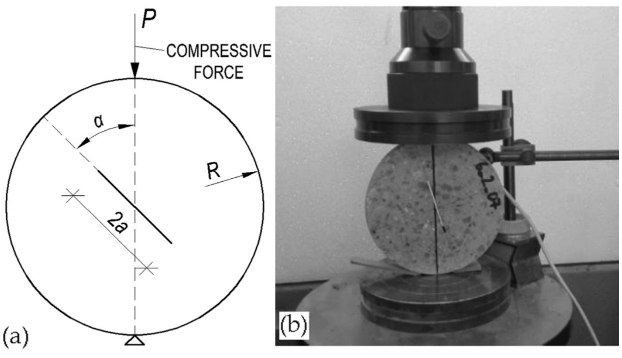

Figure 1: Brazilian disc with central notch: a) principle of testing and b) actual test setup

The SIFs for a finite specimen in shape of a BDCN and the polar angle $\theta = 0°$ can be calculated by following Equations (2–3):[8]

$$K_{\mathrm{I}} = \frac{P\sqrt{a}}{RB\sqrt{\pi}} \frac{1}{\sqrt{1-\frac{a}{R}}} Y_{\mathrm{I}}(a/R,\alpha) \quad (2)$$

$$K_{\mathrm{I}} = \frac{P\sqrt{a}}{RB\sqrt{\pi}} \frac{1}{\sqrt{1-\frac{a}{R}}} Y_{\mathrm{I}}(a/R,\alpha) \quad (3)$$

where P is the compressive load, a is the initial notch length, R is the radius of the disc ($D/2$), B is the disc's thickness, α is the inclination angle and $Y_{\mathrm{I}}(a/R,\alpha)$, $Y_{\mathrm{II}}(a/R,\alpha)$ are the dimensionless shape functions for mode I and mode II, respectively. The geometry functions Y_{I} and Y_{II} used in Equation (2) and (3) can be found in[7,9]. To calculate the T-stress, a direct extrapolation method[10] was used, for the polar angle $\theta = 0°$ the following Equation (4) was used:

$$T = \lim_{r\to 0}(\sigma_{xx} - \sigma_{yy}) \quad (4)$$

where σ_{xx} and σ_{yy} are the stress components in front of the crack tip in the direction for $\theta = 0°$.

2.2 GMTS Criterion

There are several criteria for predicting the onset of the mixed mode I/II fracture of a brittle material, such as the strain energy density (SED) criterion,[11] the averaged strain energy (ASED) criterion[12] and the extended maximum tangential strain (EMTSN) criterion.[13] A commonly used criterion on the BDCN specimen is the maximum tangential stress (MTS) criterion.[14] However, this criterion conservatively predicts the onset of mixed mode I/II fracture. This disadvantage leads to the development of the GMTS criterion. The GMTS criterion has been recently used with an accurate prediction of fracture resistance by Aliha et al. for PMMA,[15] Seitl et al. for concrete C 50/60[9,16] and Hou et al.[17] for mortar and concrete.

$$\sigma_{\theta\theta} = \frac{1}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left[K_{\mathrm{I}} \cos^2\frac{\theta}{2} - \frac{3}{2} K_{\mathrm{I}} \sin\theta\right] + \quad (5)$$
$$+ T \sin^2\theta + O(r^{1/2})$$

The GMTS criterion uses two terms of the Williams' expansion (SIFs and T-stress) in the series for $\sigma_{\theta\theta}$. The higher-order terms $O(r^{1/2})$ are often negligible near the crack tip and only three terms K_{I}, K_{II} and the T-stress are used in the further analysis. According to the GMTS criterion, the onset of fracture is the angle of the maximum tangential stress θ_0 and can be determined from:

$$\left.\frac{\partial \sigma_{\theta\theta}}{\partial \theta}\right|_{\theta=\theta_0} = 0 \text{ and } \frac{\partial^2 \sigma_{\theta\theta}}{\partial^2 \theta} < 0 \quad (6)$$

The assumption mentioned in Equation (6) leads into:

$$\left[K_{\mathrm{I}} \sin\theta + K_{\mathrm{II}}(3\cos\theta_0 - 1)\right] - -\frac{16T}{3}\sqrt{2\pi r_{\mathrm{c}}}\cos\theta_0 \sin\frac{\theta_0}{2} = 0 \quad (7)$$

The crack-initiation angle θ_0 is then used for the evaluation of the beginning of the mixed mode I/II on the BDCN specimen. While in the MTS criterion the crack-initiation angle θ_0 is a function of only SIFs, in the case of GMTS the angle θ_0 depends on SIFs, T-stress and the critical distance r_{C}, where the GMTS criterion is applied.[15] The critical distance r_{C} and the angle θ_0 are then considered as a material constant.

2.3 Application of the GMTS on THE Brazilian Disc Specimen

Pure mode I fracture initiation appears when $K_{\mathrm{I}} = K_{\mathrm{IC}}$ (fracture toughness), $K_{\mathrm{II}} = 0$ and $\theta_0 = 0°$. The fracture toughness K_{IC} can be expressed as $K_{\mathrm{IC}} = \sqrt{2\pi r_{\mathrm{c}}}\sigma_{\theta\theta}$ and Equation (7) leads into Equation (8):

$$K_{\mathrm{IC}} = \cos\frac{\theta_0}{2}\left[K_{\mathrm{I}}\cos^2\frac{\theta_0}{2} - \frac{3}{2}K_{\mathrm{II}}\sin\theta_0\right] + +\sqrt{2\pi r_{\mathrm{c}}}\,Tc\sin^2\theta_0 \quad (8)$$

where K_{IC} is the materials' fracture toughness, Equation 8 shows that the angle θ_0 for any combination of modes I and II depends on K_{I}, K_{II}, T, and r_{C}. The critical distance r_{C} can be evaluated from the recommended Equations (9) for plane stress and plane strain, respectively[8]. Equations (9) give the value of r_{C} for predefined boundary conditions, hence the material behaviour could be different.

$$r_{\mathrm{c}} = \frac{1}{2\pi}\left(\frac{K_{\mathrm{IC}}}{\sigma_t}\right)^2, \quad r_{\mathrm{c}} = \frac{1}{6\pi}\left(\frac{K_{\mathrm{IC}}}{\sigma_t}\right)^2 \quad (9)$$

where K_{IC} is the fracture toughness and σ_{t} is the tensile strength.

3 MATERIALS AND METHODS

The studied material was kind of high-performance concrete (HPC) that was designed with the intent to produce subtle elements. The maximum size of the aggregate was chosen to be 8 mm. The aggregates were composed of natural sand 0/4 mm and crushed high-quality granite 4/8 mm. Portland cement CEM I 42.5 R was used with three mineral admixtures. The first, was metakaolin, with strong pozzolanic properties.[18] The second and third admixture were chosen to reach synergy in ternary binders based on experiments, see, for example.[19] Generally, the binder consist 81 % of CEM I 42.5 R, 9.5 % of metakaolin, 7.5 % of granulated blast-furnace slag (GBFS) and 2.5 % of limestone. The water-to-binder ratio was w/b = 0.22 with drinkable water. A polycarboxylate-based superplasticizer was selected based on its compatibility with cement. The concrete was mixed in a volume of 0.7 m^3 and poured into moulds.

3.1 Mechanical characteristics of the HPC

The concrete was tested at ages of 1 d and 28 d, some other characteristics – especially non-destructive – were tested in a later age. Due to mixing the concrete in an industrial mixer, the mechanical properties, especially the compressive strengths, were lower compared to the laboratory tests. The measured mechanical properties from the tests were the 150-mm-cube compressive strength was 65.2±1.4 MPa at the age of 24 h and 106.2±2.5 MPa at the age of 28 d, the flexural strength at the age of 28 d was 9.3±0.9 MPa[20] and static modulus of elasticity measured on cylinders 150 mm × 300 mm was 41.0±0.6 GPa.[21] This is in accordance with Carrasquillo's et al. formula:[21]

$$E = 3320\, f_{\mathrm{c}}^{1/2} + 6900 \text{ [MPa]} \quad (10)$$

where E is the Young's modulus and f_{c} is the compressive strength.

The Young's modulus was obtained from Equation (10) by using f_{c} = 106.2 MPa is E = 40.91 GPa. However, the measured/calculated values of E were lower than expected with respect to compressive strength according to[3] – an absence of the coarse aggregates could be a reason. The indirect tensile strength on an unnotched Brazilian disc test[4] was measured using three samples. During the indirect tensile-strength test, the sample is attached between two load stripes and loaded with a speed of the upper support equal to 0.025 mm/s. The maximum load at the fracture was measured. The data were evaluated by following Equation (11):

$$f_{\mathrm{t}} = \frac{2P_{\mathrm{c}}}{\pi BD} \text{ [MPa]} \quad (11)$$

where P_{C} is the maximum compressive load, D is the diameter of the disc and B is the disc's thickness. The introduced mechanical parameters ($f_{\mathrm{c,cube}}$, f_{ct}, E, f_{t}) of the HPC are listed with a standard deviation in **Table 1**.

Table 1: Mechanical parameters with a standard deviation of the tested concrete at 28 days.

Compressive strength $f_{\mathrm{c,cube}}$ (MPa)	106.2 ± 2.5
Young's modulus E (GPa)	41.0 ± 0.6
Flexural strength f_{ct} (MPa)	9.3 ± 0.9
Indirect tensile strength f_{t} (MPa)	6.43 ± 0.3

3.4 Fracture mechanical parameters for the normal mode

Firstly, the fracture mechanical properties (FMPs) were tested on a notched beam with dimensions of

80 mm × 80 mm × 480 mm, in accordance with Karihaloo's and Nallathambi's effective crack model[22] and afterwards on a BDCN specimen. The FMPs measured from the beams were: modulus of elasticity E = 46.5±1.2 GPa, which is similar to the values from the mechanical measurement, see **Table 1**, effective crack length a_{ef} = 15.5±2.8 mm, effective fracture toughness K_{ICe} = 1.74±0.18 MPam$^{1/2}$ and fracture energy G_C = 65±11 Jm^{-2}.

3.4 Method of measurement of fracture mechanical parameters for mixed mode

The machine for the BDCN tests has a maximum loading capacity of 200 kN, and the speed of the induced displacement of the upper support was equal to 0.025 mm/s. The BDCN specimens were prepared from standardized cylinders with diameter D = 150 mm, normally used for testing material for compressive strength. The initial notches were prepared with a water-jet cutter; this technique provides a straight through notch. The initial notch length was selected as $2a$ = 60 mm (to have a length of the ligament area five times the size of the maximum aggregate in front of each crack tip). The BDCN specimens with an average relative notch length a/R = 0.4 were inclined against the loading positions under the selected angles α = {0°; 5°; 10°; 15°; 20°; 25.25°}. **Table 2** gives an overview of the mean values of the specimen dimensions.

Table 2: Dimensions of the tested BDCN specimens

Specimen no.	Inclination angle α (°)	Diameter D (mm)	Thickness B (mm)	Notch length $2a$ (mm)	a/R (-)
6_2_02	0	149.09	29.43	59.70	0.400
6_2_01	0	149.15	29.99	59.44	0.399
6_2_05	5	149.23	28.35	59.91	0.401
6_2_10	10	149.32	28.48	59.27	0.396
6_2_11	10	149.01	27.57	60.13	0.403
6_2_08	15	149.18	28.09	60.06	0.403
6_2_09	15	149.28	28.70	59.96	0.402
6_2_06	20	149.21	28.33	60.01	0.402
6_2_07	20	149.12	28.45	60.03	0.403
6_2_03	25.2	149.18	28.45	59.81	0.400
6_2_04	25.2	149.23	28.96	59.93	0.402

4 MIXED-MODE RESISTANCE CURVES AND DISSCUSION

The evaluated value of the fracture toughness from the BDCN specimen test is $K_{IC,BDCN}$ = 1.106±0.06 MPa m$^{1/2}$. This result is lower by 45% than the K_{IC} measured on notched beams geometry and the difference is cause by the geometry effect. This geometry effect was recently discussed for lime stone,[23] for C 50/60,[16] for normal strength concrete (NSC) and high-strength concrete (HSC).[24]

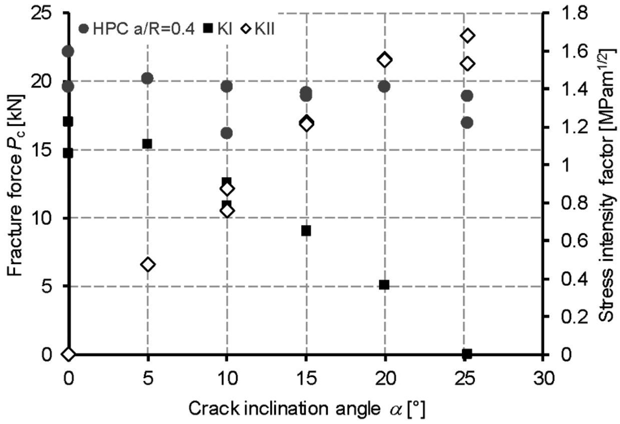

Figure 2: Fracture forces and values of SIFs for selected angles α used for the BDCN measurement for a/R = 0.4

To evaluate the fracture resistance curve of the HPC under the mixed I/II, BDCN specimens were tested with various inclination angles α. The measured values of the fracture force Pc and the corresponding evaluated values of SIFs for mode I – K_I and for mode II – K_{II} are presented in **Figure 2**.

The fracture resistance for both modes is expressed with the ratio K_I/K_{IC} and K_{II}/K_{IC}. This ratio is obtained from Equation (8) by dividing the whole expression by K_I and K_{II}, respectively. **Figure 3** gives an overview for the mixed mode I/II fracture resistance of the HPC for various critical distances r_C. The critical distance used for the evaluation were: MTS – r_C = 0 mm, GMTS plane strain – r_C = 1.56 mm, GMTS plane stress – r_C = 4.67 mm and maximum fine aggregate size – r_C = 4 mm.

From **Figure 3** it is clear that the r_C for the plane-strain boundary condition gives reliable results, where the K_I is dominant in the fracture process, i.e., r_C = 1.56 mm, while r_C for a maximum aggregate size of 4 mm gives reliable results, when the K_{II} is dominant.

Recently, it was shown by researchers in[16] that the r_C for C 50/60 could also be used for plane-strain boundary

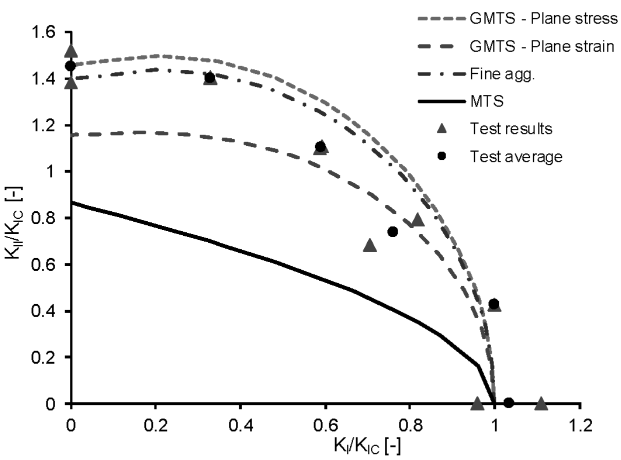

Figure 3: Mixed-mode I/II fracture-toughness diagram for the relative crack length a/R = 0.4

conditions in whole fracture process, i.e., $r_C = 1.559$ mm. In the work[17], the r_C was 1.38 mm and 2.54 mm for the low-strength mortar and concrete, respectively. The critical distance for two types of limestone was presented in[23] and r_C varies from 2.6 mm to 5.2 mm.

The experimental work performed on the Brazilian disc with a central notch is verified by a numerical simulation using a concrete damaged plasticity model.[25]

5 CONCLUSIONS

In this contribution a general overview of the mechanical and fracture properties of HPC are presented and discussed. The main focus was given to the fracture resistance under the mixed mode I/II by using the GMTS criterion. The influence of various critical distances r_C was studied, and it can be concluded that the r_C for the plane-strain boundary condition $r_C = 1.56$ mm gives more reasonable results for K_I dominance, i.e., for the ratio K_I/K_{IC} from 0.6 to 1.2. However, where the K_{II} is dominant, i.e., for a K_I/K_{IC} ratio lower than 0.4, r_C should be used with respect to the maximum aggregate size, i.e., 4 mm.

As the fracture-resistance curves for the HPC material are governed by the critical distance and the loading conditions, it can be concluded that these parameters should be included in the evaluation of the fracture in mixed mode I/II by the GMTS criterion.

Acknowledgment

The research is partially supported by the project FAST-J-18-5164. The first author is Brno Ph.D. Talent Scholarship Holder – Funded by the Brno City Municipality. The second author would like to acknowledge the support of the project no. 7AMB187AT – Failure initiation and fracture of quasi-brittle building materials. This paper has been worked out under the "National Sustainability Programme I" project "AdMaS UP – Advanced Materials, Structures and Technologies" (no. LO1408) supported by the Ministry of Education, Youth and Sports of the Czech Republic.

6 REFERENCES

[1] J. Pereiro-Barceló, J. L. Bonet, S. Gomez-Portillo, C. Castro-Bugallo, Ductility of high-performance concrete and very-high-performance concrete elements with Ni-Ti reinforcements, Constr. and Buil. Mat. 175 (**2018**), 531–551, doi:10.1016/j.conbuildmat.2018.04.172

[2] R. A. Einsfeld, M. S. L. Velasco, Fracture parameters for high-performance concrete, Cem. and Conc. Res., 36 (**2006**) 3, 579–583, doi:10.1016/j.cemconres.2005.09.004

[3] K. P. Vishalakshi, V. Revathi, S. S. Reddy, Effect of type of coarse aggregate on the strength properties and fracture energy of normal and high strength concrete, Eng. Fract. Mech. 194 (**2018**), 52–60, doi:10.1016/j.engfracmech.2018.02.029

[4] D. Li, L. N. Y. Wong, The brazilian disc test for rock mechanics applications: Review and new insights, Rock Mech. and Rock Eng., 46 (**2013**), 269–287, doi:10.1007/s00603-012-0257-7

[5] D. J. Smith, M. R. Ayatollahi, M. J. Pavier, The role of T-stress in brittle fracture for linear elastic materials under mixed-mode loading, Fat. & Fract. of Eng. Mat. & Struc., 24 (**2001**), 137–150, doi:10.1046/j.1460-2695.2001.00377.x

[6] M. L. Williams, On the Stress distribution at the base of a stationary crack, J. of App. Mech.s, 24 (**1956**) 6, 109–114

[7] M. R. Ayatollahi, M. R. M. Aliha, On the use of Brazilian disc specimen for calculating mixed mode I-II fracture toughness of rock materials, Eng. Frac. Mech., 75 (**2008**), 4631–4641, doi:10.1016/j.engfracmech.2008.06.018

[8] T. L. Anderson, Fracture mechanics: fundamentals and applications, 5th ed., CRC press, London 2017, 640

[9] S. Seitl, P. Miarka, Evaluation of mixed mode I/II fracture toughness of C 50/60 from Brazilian disc test, Fratt. ed Integ. Strutt., 11 (**2017**), 119–127, doi:10.3221/IGF-ESIS.42.13

[10] Y. J. Chao, X. H. Zhang, Constraint effect in brittle fracture, in: Fat., and Frac., Mech., 27 (**1997**), doi:10.1520/STP16227S

[11] G. C. Sih, Strain-energy-density factor applied to mixed mode crack problems, Int. J. of Frac., 10 (**1974**), 305–321, doi:10.1007/BF00035493

[12] P. Lazzarin, R. Zambardi, A finite-volume-energy based approach to predict the static and fatigue behavior of components with sharp V-shaped notches, Int. J. of Frac., 112 (**2001**), 275–298, doi:10.1023/A:1013595930617

[13] M. M. Mirsayar, F. Berto, M. R. M. Aliha, P. Park, Strain-based criteria for mixed-mode fracture of polycrystalline graphite, Eng. Frac. Mech., 156 (**2016**), 114–123, doi:10.1016/j.engfracmech.2016.02.011

[14] F. Erdogan, G. C. Sih, On the crack extension in plates under plane loading and transverse shear, J. of Basic Eng., 85 (**1963**), 519–525, doi:10.1115/1.3656897

[15] M. R. M. Aliha, A. Bahmani, S. Akhondi, Mixed mode fracture toughness testing of PMMA with different three-point bend type specimens, Eu. J. of Mech. Sol., 58 (**2016**), 148–162, doi:10.1016/j.euromechsol.2016.01.012

[16] S. Seitl, P. Miarka, V. Bílek, The mixed-mode fracture resistance of C 50/60 and its suitability for use in precast elements as determined by the Brazilian disc test and three-point bending specimens, Theor. and App. Frac. Mech., 97 (**2018**), 108–119, doi:10.1016/j.tafmec.2018.08.003

[17] C. Hou, Z. Wang, W. Liang, J. Li, Determination of fracture parameters in center cracked circular discs of concrete under diametral loading: A numerical analysis and experimental results, Theo. and App. Frac. Mech., 85 (**2016**), 355–366, doi:10.1016/j.tafmec.2016.04.006

[18] S. Adu-Amankwah, M. Zajac, C. Stabler, B. Lothenbach, Influence of limestone on the hydration of ternary slag cements. Cem. and Con. Res. 100 (**2017**), 96–109, doi:10.1016/j.cemconres.2017.05.013

[19] V. Bilek, D. Pytlik, M. Bambuchova, High-performance concrete with ternary binders, Key Eng. Mat., 761 (**2018**) 761, 120–123, doi:10.4028/www.scientific.net/KEM.761.120

[20] RILEM, Recommendation, Determination of the fracture energy of mortar and concrete by means of three-point bend tests on notched beams, Mat. and struc., 18 (**1985**) 285–290, doi:10.1007/BF02498757

[21] ISO 1920-10:2010 Testing of concrete – Part 10: Determination of static modulus of elasticity in compression, International Organisation for Standardization (TC 71), Geneve

[22] B. L. Karihaloo, P. Nallathambi, Effective crack model for the determination of fracture toughness (KICe) of concrete. Eng. Fract. Mech., 35 (**1990**), 4–5, 637–645, doi:10.1016/0013-7944(90)90146-8

[23] M. R. M. Aliha, M. Sistaninia, D. J. Smith, M. J. Pavier, M. R. Ayatollahi, Geometry effects and statistical analysis of mode I fracture in guiting limestone, Int. J. of Rock Mech. and Min. Sci., 51 (**2012**), 128–135, doi:10.1016/j.ijrmms.2012.01.017

[24] S. Seitl, J. D. Ríos, H. Cifuentes, Comparison of fracture toughness values of normal and high strength concrete determined by three point bend and modified disk-shaped compact tension specimens, Fratt. ed Int. Strutt., 11 (**2017**), 56–65, doi:10.3221/IGF-ESIS.42.07

[25] P. Miarka, S. Seitl, W. De Corte, Numerical analysis of the failure behaviour of a C50/60 Brazilian disc test specimen with a central notch, Key Eng. Mat., 774 (**2018**), 570–575, doi:10.4028/www.scientific.net/KEM.774.570