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NEWCASTLE

Optimal Power Flow Problem Considering Multiple-Fuel Options and Disjoint Operating Zones: A Solver-friendly MINLP Model

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Abstract**:** Optimal power flow is inherently a very complex nonlinear and nonconvex problem. Considering practical logic-based constraints, namely multiple-fuel option (MFO) and prohibited operating zones (POZ), jointly with the nonsmooth terms such as valve-point effect results in even more difficulties in finding a high-quality solution. Moreover, most of the existing commercial solvers either fail in handling the original logic-based models or show intractability in solving the equivalent mixed integer nonlinear programming (MINLP) models. This paper proposes a solver-friendly MINLP (SF-MINLP) model to fill the existing gap in handling the MFO and POZ simultaneously in OPF problems. To obtain the most adaptable model with the existing MINLP commercial solvers and due to the actions done in the pre-solve step, some primary integer decision variables are melted into the objective function. The pre-solve step, via pre-processing and probing techniques, reduces the model complexity and then the simplified model is handled via the most appropriate optimization algorithms. For the verification and didactical purposes, the proposed SF-MINLP model is applied to the IEEE 30-bus system under two different loading conditions, namely normal and increased, and details are provided. The model is also tested on the IEEE 118-

bus system to reveal its effectiveness and applicability in larger-scale systems. Results show the effectiveness and tractability of the model in finding a high-quality solution with high computational efficiency.

***Keywords*:** Mixed-integer nonlinear programming, multiple fuel option, non-smooth terms, optimal power flow, prohibited operating zones.

NOMENCLATURE

a) Indices and Sets

i, j	bus indices
k	index for disjoint operating zones
m	index for fuel type m
ij	index for the transmission line or transformer between bus i and j
d	index for direct power flow
r	index for reverse power flow
Ω_b	set of buses.
Ω_g	set of generating units.
Ω_l	set of transmission elements.

b) Variables and Functions

$F_i(\cdot)$	fuel cost function of unit i
$F_{i,m}(\cdot)$	fuel cost function of unit i using fuel m
$F_{ik,m}(\cdot)$	fuel cost function of unit i operating at zone k using fuel m
fl_{ij}	power flow at branch ij
P_{g_i}	active power generation of unit i
P_{ik}	active power corresponding to the operating zone k of unit i
$P_{ik,m}$	active power corresponding to unit i while using fuel m and operating at zone k
p_{ij}^d / p_{ij}^r	direct/reverse active power between bus i and bus j of branch ij
q_{ij}^d / q_{ij}^r	direct/reverse reactive power between bus i and bus j of branch ij

Q_{g_i}	reactive power generation of unit *i*
tp_{ij}	transformer tap of branch *ij*
u_{ik}	binary decision-making variables corresponding to unit *i* operating at zone *k*
$u_{ik,m}$	binary decision-making variables corresponding to unit *i* while using fuel *m* and operating at zone *k*
v_i	voltage magnitude at bus *i*
δ_i	voltage angle of bus *i*
θ_{ij}	voltage angle difference between bus *i* and *j*, $\theta_{ij} = \delta_i - \delta_j$.

c) Parameters

a_i, b_i, c_i	Cost coefficients of unit *i*.
$a_{i,m}, b_{i,m}, c_{i,m}$	Cost coefficients corresponding to fuel *m* of unit *i*.
a'_i, b'_i, c'_i	Cost coefficients of unit *i*, used in the MINLP model
b_{ij}^{ch}	Charging susceptance of branch *ij*
b_i^{sh}	Shunt susceptance of bus *i* (□)
b_{ij}	Susceptance of branch *ij* (□)
e_i, f_i	Valve-point cost coefficients of unit *i*
$e_{i,m}$	Valve-point cost coefficients of fuel *m* for unit *i*
e'_i	Valve-point cost coefficients of unit *i* used in MINLP model
$\overline{fl}_{ij}$	maximum power flow of branch *ij*
g_{ij}	conductance of branch *ij* (Ω)
g_i^{sh}	shunt conductance of bus *i* (Ω)
P_{D_i}	active power demand at bus *i*
$\underline{P}_{g_i}$, $\overline{P}_{g_i}$	minimum and maximum active power generation limits of unit *i*, respectively

$\underline{P}_{ik}$, $\overline{P}_{ik}$ minimum and maximum active power limits correspond to unit *i* operating at zone *k*, respectively.

$\underline{P}_{ik,m}$, $\overline{P}_{ik,m}$ minimum and maximum active power limits corresponding to unit *i* while using fuel *m* and operating at zone *k*, respectively.

Q_{D_i} reactive power demand at bus *i*

$\underline{Q}_{g_i}$, $\overline{Q}_{g_i}$ minimum and maximum reactive power generation limits of unit *i*, respectively

r_{ij} resistance of branch *ij* (Ω)

$\underline{tp}_{ij}$, $\overline{tp}_{ji}$ minimum and maximum limits of transformer tap of branch *ij*, respectively

$\underline{v}_i$, $\overline{v}_i$ minimum and maximum voltage magnitude limits of bus *i*, respectively

z_i number of operating zones for unit *i*

I. INTRODUCTION

In power system problems, economic aspects are the primary concerns to be addressed. In power system analysis, economic dispatch (ED) and optimal power flow (OPF) are the most widely used tool to address economic concerns. However, compared to ED problem, the OPF not only considers the economic aspects but also thoroughly takes into account the operational and technical constraints [1, 2]. The conventional OPF problem, which disregards many practical constraints, is a very nonlinear and nonconvex problem and finding a high-quality solution has always been a challenging issue [3]. Considering practical constraints and terms such as prohibited operating zones (POZs), multiple-fuel options (MFO), and valve-point effects (VPEs) bring even more difficulties to the problem and may result in intractability or failure in finding an optimal solution. On the other hand, the logical constraints such as POZs and MFOs, due to disjoint characteristics cannot be handled by existing commercial solvers, while considering the VPEs simultaneously with the logical constraints increases the degree of nonconvexity and nonlinearity that causes severe problems in finding an optimal solution. Therefore, more often than not, for the sake of simplicity, the aforementioned constraints and terms are overlooked. Disregarding such formidable hurdles is a great help for the existing commercial solvers, however, the model becomes unrealistic and the results are not trustworthy, i.e., although the simplified model may easily find a near-optimal or even the global solution, there is no guarantee that the obtained solution be also a feasible solution of the original real-world problem. This paper aims at addressing the aforementioned shortcomings by introducing new recast strategies.

In the real world, due to social welfare importance and environmental issues, power generation units might use multiple fuels that can play an important role by not only reducing the electricity bills but also mitigating the emission by selecting the most economical-environmental fuel at a certain hour. Moreover, the fuel prices fluctuate, and the emission policies frequently change while, on the other hand, the demand is also another source of uncertainty, therefore, the units with the capability of switching to other fuels bring more flexibility to the power plant in addressing such uncertainties. This shows the importance of considering generating units with multiple fuels options in the OPF problems. In a multiple fuel-based system, the generation cost function is represented as a segmented piecewise quadratic function; therefore, determining the most appropriate fuel to satisfy the constraints and the objectives is a complicated task and considering these kinds of generators makes the OPF problem even a more complex problem. On the other hand, a typical thermal unit might have a discontinuous fuel cost characteristics which are mostly due to either the vibrations in the shaft bearing or the effects of auxiliary equipment such as boiler or feed pumps [4]. Therefore, the generating units are subject to POZs, and violation this constraint may cause undeniable issues. Similar to the OPF problem at the presence of MFO, considering POZs also makes hurdles to the solution approach in finding a high-quality solution.

In the literature, most of the works consider either the conventional OPF or only one logic constraint at a time is taken into account. To solve OPF problems with disjoint terms, almost all of the works took the advantages of heuristic-based techniques as the existing commercial solvers for nonlinear models cannot adequately handle the logic constraints. A hybrid algorithm based on the particle swarm optimization and the shuffle frog leaping algorithms has been proposed in [5] to handle the OPF problem with MFO. This proposed approach was also applied to OPF problems with POZs. The authors in [6] proposed a teaching learning based (TLB) algorithm to solve several types of OPF problem including the MFO-based OPF. Likewise, in [7] similar OPF models were studied via gravitational search algorithm. In [8], an imperialist competitive algorithm (ICA) has been used to handle the OPF problem with MFO. In this work, to address the convergence issues, the ICA was empowered by a TLB algorithm. Results confirmed the enhancement of the algorithm in convergence and achieving a higher-quality solution. In [9], a Gbest guided artificial bee colony optimization algorithm was used to solve OPF considering MFO at the presence of wind power. The authors handled the stochastic characteristic of wind speed via Weibull probability density function. A Moth Swarm Algorithm (MSA) was used in [10] to handle fourteen different models of OPF problems, including the OPF with MFOs. In [11], a bacterial search algorithm was proposed to solve both the single- and multi-objective OPF problems while considering the MFO. A particle swarm optimization with differentially perturbed velocity hybrid algorithm jointly with an adaptive acceleration coefficient was proposed in [12] to solve MFO-based OPF, while later the same authors in [13] proposed a Genetic evolving ant direction hybrid differential evolution to solve a similar problem. A robust differential evolution algorithm that use a new recombination operator was proposed in [14] two handle both the logic-based OPF model, namely POZ- and MFO-based OPF. To solve a dynamic reserve-constrained OPF taking into account the POZs, VPEs, and MFOs separately, a charged system search algorithm enhanced by a novel mutation strategy was proposed in [15]. A high performance heuristic algorithm, namely social spider optimization, was proposed in [16], to address different single objective while considering POZs and MFO separately. The authors in [17] proposed an improved artificial bee colony algorithm based on Pareto optimization to solve a dynamic OPF problem considering the POZs as discontinuous constraints to test the effectiveness of the approach. The OPF problems with VPEs and POZs was studied in [18] where to solve the models in an efficient way, three different heuristic-based approaches, were proposed. In [19], a Shuffle Frog Leaping Algorithm (SFLA) and Simulated Annealing (SA) approach was proposed to address the difficulties of solving the OPF problem with non-smooth and non-convex generator fuel cost, while in [20], the hybrid model of the same problem was studied via an efficient evolutionary algorithm that works based on sensitivity and heuristic approach principles. Unlike the above studies, the authors in [21] performed an effective mixed-integer nonlinear model to solve the OPF problems at the presence of VPEs and POZs. In this work, the authors recast the logic terms in a way that the existing commercial solvers can handle them.

Table 1 summarizes the existing works that addressed the discontinuous OPF problems. The aforementioned recently published and in-press works in this area reveals 1) the necessity and importance of considering the multiple fuel options in OPF problems, and 2) the lack of a model that can be solved via commercial solvers. However, neither the solver-based model in [21] nor the heuristic-based approaches did not handle two logic constraints, POZs and MFOs, simultaneously. The main obstacle in considering more than one logic constraint simultaneously is the overlapped areas in which both the logic constraints are active.

Table 1. Comparison of Works with Discontinuous OPF Model.

Work	POZ	Multiple Fuel	Valve Point	Approach
[5–16]		✓		Heuristic-based
[5, 14–18]	✓			Heuristic-based
[8, 18–20]	✓		✓	Heuristic-based
[21]	✓		✓	Solver-based
Proposed SF-MINLP	✓	✓	✓	Solver-based

Comparing with the existing models in the literature, the main contributions of this paper are twofold.

1) Addressing OPF problems with more than one logical constraints simultaneously. Until now, to the best of our knowledge, due to tractability issues, only one logical constraints was considered in the existing models, either MFOs or POZs, while in this paper, both the logical constraints are considered simultaneously. Moreover, to have a more practical model, the valve-point effect is also taken into account.
2) Finding a solver-friendly model that is adaptable to the pre-solving and solving process of existing commercial solvers. To this end, first, the original model is recast into an equivalent MINLP model, and then, to facilitate the pre-processing and probing techniques, the model is adapted to the nature of commercial MINLP solvers via some innovative recast techniques that transfer the primary decision variables from constraints into the objective function.

II. OPF Problem Considering Multiple Fuel Options and Prohibited Operating Zones

In this section, first, the general formulation of OPF problem considering multiple fuel options is presented, and then, the general formulation of OPF problem with multiple fuel options at the presence of disjoint operating zones is considered.

A. *OPF problem with multiple fuel options*

The OPF problem with considering multiple fuel option units is formulated as follows.

$$\min \sum_{i \in \Omega_g} F_i(P_{g_i}) \tag{1}$$

$$s.t.$$

$$P_{g_i} - P_{D_i} - g_i^{sh} v_i^2 - \sum_{ij \in \Omega_l} p_{ij}^d - \sum_{ji \in \Omega_l} p_{ji}^r = 0, i \in \Omega_b \tag{2}$$

$$Q_{g_i} - Q_{D_i} + b_i^{sh} v_i^2 - \sum_{ij \in \Omega_l} q_{ij}^d - \sum_{ji \in \Omega_l} q_{ji}^r = 0, i \in \Omega_b \tag{3}$$

$$\underline{P_{g_i}} \le P_{g_i} \le \overline{P}_{g_i}, i \in \Omega_g \tag{4}$$

$$\underline{Q_{g_i}} \le Q_{g_i} \le \overline{Q}_{g_i}, i \in \Omega_g \tag{5}$$

$$\underline{v_i} \le v_i \le \overline{v}_i, i \in \Omega_b \tag{6}$$

$$\underline{tp_{ij}} \le tp_{ij} \le \overline{tp_{ij}},\ ij \in \Omega_l \tag{7}$$

$$\left| fl_{ij}(v, \theta, tp) \right| \le \overline{fl}_{ij}, ij \in \Omega_l \tag{8}$$

where (1) stand for the objective function, which is usually the costs corresponding with the fossil fuel costs; (2) and (3) stand for the active and reactive equality constraints; active power output of a generating unit is limited by its upper and lower limits in (4), while the reactive power output can vary between its lower and upper limits in (5); voltage magnitude at each bus should be remained between a predefined limit, as (6); (7) stand for the transformer upper and lower limits; and (8) presents the transmission line limit in which usually is the active or the apparent powers.

In OPF problems, more often than not, the fuel cost of a generating unit, $F_i(\cdot)$, is approximated by a quadratic function, as (9).

$$F_i(P_{g_i}) = \mathrm{a}_i P_{g_i}^2 + b_i P_{g_i} + c_i \tag{9}$$

However, in practice, multiple valves result in the ripples and therefore considering the valve point effects in cost function is inevitable [22]. The valve point effect is models as a rectified sinusoidal term as (10).

$$F_i(P_{g_i}) = a_i (P_{g_i})^2 + b_i P_{g_i} + c_i + \left| e_i \sin(f_i (\underline{P}_{g_i} - P_{g_i})) \right| \tag{10}$$

where e_i and f_i are the cost coefficients corresponding to the valve point effect; and $\underline{P}_{g_i}$ is the minimum generation capacity of the unit i.

However, the cost function of a unit with multi-fuel options and without valve-point effects is modeled as (11), and it shows that for each operating zone only one fuel can be used.

$$F_i(P_{g_i})=\begin{cases} a_{i,1}P_{g_i}^2+b_{i,1}P_{g_i}+c_{i,1}, \text{for fuel type 1, } \underline{P}_{g_i}=\underline{P}_{i,1}\le P_{g_i}\le \overline{P}_{i,1}, \text{or} \\ a_{i,2}P_{g_i}^2+b_{i,2}P_{g_i}+c_{i,2}, \text{for fuel type 2, } \overline{P}_{i,1}=\underline{P}_{i,2}\le P_{g_i}\le \overline{P}_{i,2}, \text{ or} \\ \vdots \\ a_{i,m-1}P_{g_i}^2+b_{i,m-1}P_{g_i}+c_{i,m-1}, \text{for fuel type m-1, } \overline{P}_{i,m-2}=\underline{P}_{i,m-1}\le P_{g_i}\le \overline{P}_{i,m-1}, \text{ or} \\ a_{i,m}P_{g_i}^2+b_{i,m}P_{g_i}+c_{i,m}, \text{for fuel type m, } \overline{P}_{i,m-1}=\underline{P}_{i,m}\le P_{g_i}\le \overline{P}_{i,m}=\overline{P}_{g_i} \end{cases} \tag{11}$$

where $a_{i,m}$, $b_{i,m}$, and $c_{i,m}$ are the quadratic, linear, and constant cost coefficients of unit i working with fuel *m*.

By considering valve point, (11) is modified to (12).

$$F_i(P_{g_i})=\begin{cases} a_{i,1}P_{g_i}^2+b_{i,1}P_{g_i}+c_{i,1}+\left|e_{i,1}\sin(f_i(\underline{P}_{g_i}-P_{g_i}))\right|, \text{fuel type 1, } \underline{P}_{g_i}=\underline{P}_{i,1}\le P_{g_i}\le \overline{P}_{i,1}, \text{or} \\ a_{i,2}P_{g_i}^2+b_{i,2}P_{g_i}+c_{i,2}+\left|e_{i,2}\sin(f_i(\underline{P}_{g_i}-P_{g_i}))\right|, \text{fuel type 2, } \overline{P}_{i,1}=\underline{P}_{i,2}\le P_{g_i}\le \overline{P}_{i,2}, \text{ or} \\ \vdots \\ a_{i,m-1}P_{g_i}^2+b_{i,m-1}P_{g_i}+c_{i,m-1}+\left|e_{i,m-1}\sin(f_i(\underline{P}_{g_i}-P_{g_i}))\right|, \text{fuel type m-1, } \overline{P}_{i,m-2}=\underline{P}_{i,m-1}\le P_{g_i}\le \overline{P}_{i,m-1}, \text{ or} \\ a_{i,m}P_{g_i}^2+b_{i,m}P_{g_i}+c_{i,m}+\left|e_{i,m}\sin(f_i(\underline{P}_{g_i}-P_{g_i}))\right|, \text{fuel type m, } \overline{P}_{i,m-1}=\underline{P}_{i,m}\le P_{g_i}\le \overline{P}_{i,m}=\overline{P}_{g_i} \end{cases} \tag{12}$$

where $e_{i,m}$ and $f_{i,m}$ are the cost coefficients corresponding to the valve point effects of unit *i* working with fuel *m*.

In (2) and (3), direct and reverse active and reactive powers at each bus are calculated as follows.

$$p_{ij}^d=(tp_{ij}v_i)^2 g_{ij}-(tp_{ij}v_i)v_j[g_{ij}\cos(\theta_{ij})+b_{ij}\sin(\theta_{ij})] \tag{13}$$

$$p_{ij}^r=v_j^2 g_{ij}-(tp_{ij}v_i)v_j[g_{ij}\cos(\theta_{ij})-b_{ij}\sin(\theta_{ij})] \tag{14}$$

$$q_{ij}^d=-(tp_{ij}v_i)^2(b_{ij}+\frac{b_{ij}^{ch}}{2})-v_iv_j[g_{ij}\sin(\theta_{ij})-b_{ij}\cos(\theta_{ij})] \tag{15}$$

$$q_{ij}^{r}=-v_j^2(b_{ij}+\frac{b_{ij}^{ch}}{2})+(tp_{ij}v_i)v_j[g_{ij}\sin(\theta_{ij})+b_{ij}\cos(\theta_{ij})] \tag{16}$$

B. OPF problem with multiple fuel options and POZs

Considering multi-fuel option simultaneously with POZs brings lots of difficulties in solving the problem. The first problem is that by considering these logic constraints simultaneously the degree of nonlinearity and nonconvexity goes higher, and the second issue is related to the overlapping the operating zones with the fuel type operating zones. The mathematical formulation of the OPF problem with POZs and multi-fuel option is presented as follows.

$$\min \sum_{i\in\Omega_g} F_i(P_{g_i}) \tag{17}$$

$$s.t.$$

$$P_{g_i}-P_{D_i}-g_i^{sh}v_i^2-\sum_{ij\in\Omega_l}p_{ij}-\sum_{ji\in\Omega_l}p_{ji}=0, i\in\Omega_b \tag{18}$$

$$Q_{g_i}-Q_{D_i}+b_i^{sh}v_i^2-\sum_{ij\in\Omega_l}q_{ij}-\sum_{ji\in\Omega_l}q_{ji}=0, i\in\Omega_b \tag{19}$$

$$\underline{P}_{g_i}\le P_{g_i}\le\overline{P}_{g_i}, i\in\Omega_g \tag{20}$$

$$\underline{Q}_{g_i}\le Q_{g_i}\le\overline{Q}_{g_i}, i\in\Omega_g \tag{21}$$

$$\underline{v}_i\le v_i\le\overline{v}_i, i\in\Omega_b \tag{22}$$

$$\left|fl_{ij}(v,\theta,tp)\right|\le\overline{fl}_{ij}, ij\in\Omega_l \tag{23}$$

The active power limits (20) for a unit with disjoint operating zone and multi-fuel options are modified as follows.

$$\begin{cases}\begin{cases}\underline{P}_{g_i}=\underline{P}_{i1,1}\le P_{g_i}\le\overline{P}_{i1,1}, \text{or}\\ \underline{P}_{ik,1}\le P_{g_i}\le\overline{P}_{ik,1}, \quad 2\le k\le(z_i-1), \text{ for fuel type 1or}\\ \underline{P}_{iz_i,1}\le P_{g_i}\le\overline{P}_{iz_i,1}\end{cases}\\ \vdots\\ \begin{cases}\underline{P}_{i1,m}\le P_{g_i}\le\overline{P}_{i1,m}, \text{or}\\ \underline{P}_{ik,m}\le P_{g_i}\le\overline{P}_{ik,m}, \qquad 2\le k\le(z_{im}-1), \text{ for fuel type } m\\ \underline{P}_{iz_{im},m}\le P_{g_i}\le\overline{P}_{iz_{im},m}=\overline{P}_{g_i}\end{cases}\end{cases} \tag{24}$$

As can be seen from (24), each fuel's cost function may cover one or more POZs, therefore, practically for each fuel type, several disjoint operating zones may occur.

On the other hand, depending on the unit's active power generation a unit with multiple fuel options may change its fuel, and consequently, depending on the type of cost function, with or without valve point effect, one of the cost functions (11) or (12) are taken into account, respectively.

Comparing with the mathematical formulation presented in section A, this formulation is much more complicated to be solved, even with heuristic-based approaches. Moreover, considering the valve point effect makes the model even more complicated

C. Mixed integer nonlinear programming (MINLP) model

In this section, a mixed integer nonlinear programming model for OPF problems considering POZs and multiple fuel options is proposed. In this regard, the constraints related to the POZs and multiple fuel options are recast into the mixed integer terms of the objective function. First, we consider a unit with multiple fuel options, and then the POZs are taken into account.

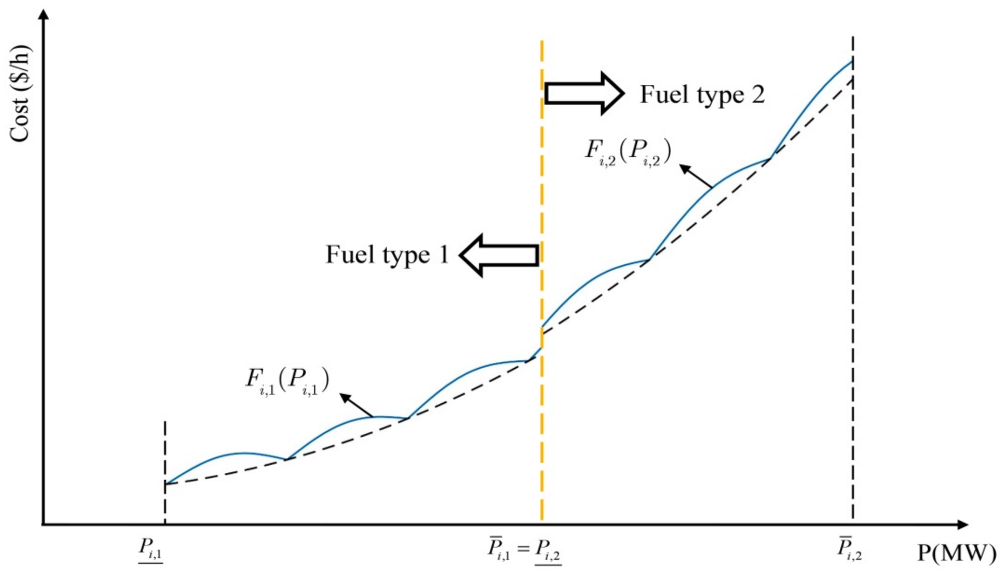

Figure 1. The input-output curve of unit *i* considering multiple fuel option with (—) and without (--) valve point effect.

Figure 1 demonstrates a generating unit with two different fuel types. As can be seen, the generating unit before reaching the upper limit of unit *i*, $\overline{P}_{i,1}$, works on fuel 1 and afterward it switches to the second fuel; this point is called the switching point, therefore the upper limit of unit i using fuel 1 is equal to the initial point or lower limit of unit i using fuel 2, $\underline{P}_{i,2}$. In this figure, the same as a practical situation, the fuels are incrementally ordered with their corresponding costs, e.g., fuel type 1 is cheaper than the fuel type 2. This situation is called logical condition, due to fuel cost discontinuity, and cannot be handled by classical approaches or commercial nonlinear solvers. In Figure 1, the unit's cost function is divided into two sub-functions, which are indexed by the unit and its corresponding fuel types. For example, $F_{i,m}$ stand for the cost function corresponding to the active power generated by unit i working with fuel m. Finding an equivalent for the multi-fuel-based generating units requires appropriate recast techniques, which is not an easy task. However, the main obstacle is revealed when a unit is subject to the MFOs and POZs simultaneously, specifically when there is an overlap between their regions.

The input-output curve of a unit with two fuels and three disjoint operating zones, two POZs, is depicted in Figure 2. As can be seen, in this figure, the second POZ is overlapped with the region that the unit should switch the fuel. It is worth mentioning that if there is no overlapped region between the operating zones and this fuel types, the concept of the model remains unchanged.

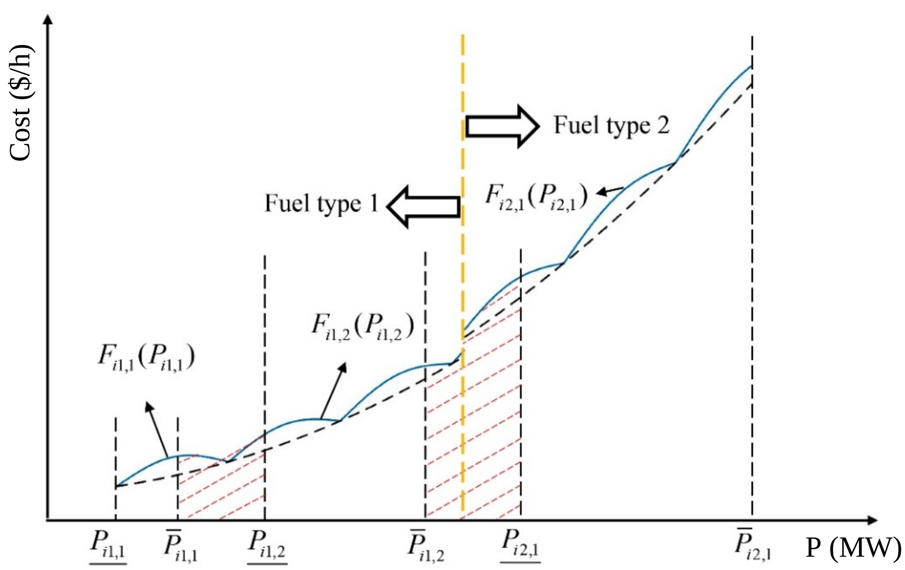

Figure 2. The input-output curve of unit i considering multiple fuel options and POZs with (—) and without (--) valve point effect.

To obtain the proposed model, first, the cost function of each generating unit is indexed with its corresponding operating region and corresponding fuel; $F_{iz_i,m}$ means that the cost function corresponding to the active power generated by unit *i* at operating zone z_i and fuel *m*. Then, the following formulation is proposed to consider the POZs and multiple fuel options jointly.

$$\begin{cases} \text{fuel type 1} \begin{cases} F_{i1,1}(P_{i1,1}) = a_{i,1}P_{i1,1}^2 + b_{i,1}P_{i1,1} + c_{i,1}, \text{or} \\ F_{ik,1}(P_{ik,1}) = a_{i,1}P_{ik,1}^2 + b_{i,1}P_{ik,1} + c_{i,1}, \ \forall 2 \le k \le (z_{i,1} - 1), \ \text{or} \\ F_{iz_{i,1},1}(P_{iz_{i,1},1}) = a_{i,1}P_{iz_{i,1},1}^2 + b_{i,1}P_{iz_{i,1},1} + c_{i,1} \end{cases} \\ \vdots \\ \text{fuel type } m \begin{cases} F_{i1,m}(P_{i1,m}) = a_{i,m}P_{i1,m}^2 + b_{i,m}P_{i1,m} + c_{i,m}, \text{or} \\ F_{ik,m}(P_{ik,m}) = a_{i,m}P_{ik,m}^2 + b_{i,m}P_{ik,m} + c_{i,m}, \ \forall 2 \le k \le (z_{i,m} - 1), \ \text{or} \\ F_{iz_{im},m}(P_{iz_{im},m}) = a_{i,m}P_{iz_{im},m}^2 + b_{i,m}P_{iz_{im},m} + c_{i,m} \end{cases} \end{cases} \tag{25}$$

And for the case with VPEs, (25) is modified as follows, (26).

$$\begin{cases} \text{fuel type 1} \begin{cases} F_{i1,1}(P_{i1,1}) = a_{i,1}P_{i1,1}^2 + b_{i,1}P_{i1,1} + c_{i,1} + \left| e_{i,1}\sin(f_i(\underline{P}_{g_i} - P_{i1,1})) \right|, \text{or} \\ F_{ik,1}(P_{ik,1}) = a_{i,1}P_{ik,1}^2 + b_{i,1}P_{ik,1} + c_{i,1} + \left| e_{i,1}\sin(f_i(\underline{P}_{g_i} - P_{ik,1})) \right|, \forall 2 \le k \le (z_{i,1} - 1), \ \text{or} \\ F_{iz_{i,1},1}(P_{iz_{i,1},1}) = a_{i,1}P_{iz_{i,1},1}^2 + b_{i,1}P_{iz_{i,1},1} + c_{i,1} + \left| e_{i,1}\sin(f_i(\underline{P}_{g_i} - P_{iz_{i,1},1})) \right| \end{cases} \\ \vdots \\ \text{fuel type } m \begin{cases} F_{i1,m}(P_{i1,m}) = a_{i,m}P_{i1,m}^2 + b_{i,m}P_{i1,m} + c_{i,m} + \left| e_{i,m}\sin(f_i(\underline{P}_{g_i} - P_{i1,m})) \right|, \text{or} \\ F_{ik,m}(P_{ik,m}) = a_{i,m}P_{ik,m}^2 + b_{i,m}P_{ik,m} + c_{i,m} + \left| e_{i,m}\sin(f_i(\underline{P}_{g_i} - P_{ik,m})) \right|, \forall 2 \le k \le (z_{i,m} - 1), \ \text{or} \\ F_{iz_{im},m}(P_{iz_{im},m}) = a_{i,m}P_{iz_{im},m}^2 + b_{i,m}P_{iz_{im},m} + c_{i,m} + \left| e_{i,m}\sin(f_i(\underline{P}_{g_i} - P_{iz_{im},m})) \right| \end{cases} \end{cases} \tag{26}$$

From this fact that only one fuel and only one of its operating zones can be selected, the following MINLP model is proposed.

We consider an integer variable to decide which fuel and which operating zone to be selected. Therefore, the following recasts are made for units without and with VPEs, cost function (27) and (28) are considered, respectively.

$$F_i(P_{g_i}) = \sum_{k=1}^{z_i} \sum_{m} (a_i' P_{ik,m}^2 + b_i' P_{ik,m} + c_i') u_{ik,m} \tag{27}$$

$$F_i(P_{g_i}) = \sum_{k=1}^{z_i} \sum_{m} (a_i' P_{ik,m}^2 + b_i' P_{ik,m} + c_i' + \left| e_i' \sin(f_i(\underline{P}_{g_i} - P_{ik,m})) \right|) u_{ik,m} \tag{28}$$

As can be seen from this equation, the coefficients are different from the coefficients of the fuel cost functions, instead, auxiliary variables have been defined to select the most appropriate fuel by selecting its corresponding cost coefficients. Therefore, by taking into account the following equations, (29)-(33), are applied.

$$\sum_{k=1}^{z_i}\sum_{m} a_{i,m}u_{ik,m} = a_i', \forall i \in \Omega_g \tag{29}$$

$$\sum_{k=1}^{z_i}\sum_{m} b_{i,m}u_{ik,m} = b_i', \forall i \in \Omega_g \tag{30}$$

$$\sum_{k=1}^{z_i}\sum_{m} c_{i,m}u_{ik,m} = c_i', \forall i \in \Omega_g \tag{31}$$

$$\sum_{k=1}^{z_i}\sum_{m} e_{i,m}u_{ik,m} = e_i', \forall i \in \Omega_g \tag{32}$$

$$\sum_{k=1}^{z_i}\sum_{m} u_{ik,m} = 1, \forall i \in \Omega_g \tag{33}$$

where (33) guarantees that only one of the fuels and its corresponding operating region can be selected.

In order to consider these equations in more detail, (29) related to Figure 2, can be expanded as follows, (34).

$$a_{i,1}u_{i1,1} + a_{i,1}u_{i2,1} + a_{i,2}u_{i1,2} = a_i' \tag{34}$$

And by considering (33), $u_{i1,1} + u_{i2,1} + u_{i1,2} = 1$, either one of the first terms (related to two different operating zones of the first fuel), or the third term (related to the one operating zone of the second fuel) is selected. Consequently, the other coefficients are selected as well.

However, to obtain the mixed-integer programming model, some more definition such as (35) is necessary while the P_{g_i} in (18) must be replaced by (36).

$$\underline{P_{\mathrm{ik,m}}} \le P_{\mathrm{ik,m}} \le \overline{P}_{\mathrm{ik,m}}; \ \forall i \in \Omega_g, k \in \{1, \cdots, z_i\}, m \tag{35}$$

$$P_{g_i} = \sum_{k=1}^{z_i}\sum_{m} P_{ik,m}u_{ik,m} \tag{36}$$

Therefore, by making some modifications in the mathematical formulations presented in subsection B, the proposed model is obtained. These modifications are 1) replacing the functions in (17) with a sub-functions (27) or (28), 2) by putting (36) in (18), and 3) considering (29)-(33), (35), and (36).

III. CASE STUDIES AND RESULTS

In order to validate the proposed models and for didactical purposes, the commonly used IEEE 30-bus system, which is the only system that the data related to the VPEs, POZs, and MFOs are available in the literature, is thoroughly studied. It is worth mentioning that although the data of both the logical constraints have been derived from the published papers, to the best of the authors' knowledge, until now, none of the existing works in the literature has considered the POZs and VPEs simultaneously. This system is studied under two loading conditions, normal and 17.5% increased load for each load bus, and each condition is tested with two cost functions, without and with valve-point effects. However, to show the effectiveness of the model, IEEE 118-bus system is considered under normal and critical loading conditions. Details of finding the critical loading condition is explained below in subsection B.

In this work, to implement the proposed models, a modeling language for mathematical programming (AMPL) [23] is used and to solve the proposed mixed integer nonlinear programming models via a 2.67-GHz computer with 3 GB of RAM, the nonlinear commercial solvers KNITRO [24] is used.

A. IEEE 30-bus test system

The IEEE 30-bus system, among the others, is a commonly used system in the literature. This system consists of 30 buses (9 buses with shunt VAR compensator), 41 branches (4 branches with transformer) and 6 generators [25]. For this system, four different cases are considered. The data of POZs are obtained from [18], while the data of multiple fuel options are obtained from [26]. However, as a quick reference, the fuel options and POZs are presented in Table VI. The minimum and maximum bounds of voltages at load buses are 0.95 p.u. and 1.05 p.u. while for the generator buses are 0.95 p.u. and 1.1 p.u., respectively. The maximum and minimum bounds for the transformer tap setting are 0.9 p.u. and 1.1 p.u., respectively.

a) IEEE 30-bus under normal loading condition

In order to show the effectiveness of the proposed model, the IEEE 30-bus system without and with valve-point effects is tested under normal loading condition, while the POZs and multiple fuel options are considered simultaneously.

Table II. Outputs Results of Proposed Model considering POZs and Multiple Fuel Option with and without Valve Point Effect— IEEE 30-Bus System Under Normal Loading Condition

Optimal Output	Without Valve Point	With Valve Point
P_{g1} (MW)	140.00	140.00
P_{g2} (MW)	45.00	45.00
P_{g5} (MW)	25.8299	25.8162
P_{g8} (MW)	35.00	35.00
P_{g11} (MW)	22.5162	23.0777
P_{g13} (MW)	21.4713	20.9169
tp_{6-9} (p.u.)	0.95001	0.95
tp_{6-10} (p.u.)	1.09163	1.09127
tp_{4-12} (p.u.)	1.02786	1.02810
tp_{28-27} (p.u.)	1.02729	1.02736
Cost ($/h)	672.3587	715.7730
Loss (MW)	6.42	6.41
Time (s)	0.217	0.304
# of nodes	4	5
# of subproblems	4	6

Table III. Fuel Type and Operating Zones of the Generating Units Corresponding to the Cases with and Without Valve Point Effects— IEEE 30-Bus System Under Normal Loading Condition

Without Valve Point					With Valve Point				
Unit # (Bus #)	Fuel type 1			Fuel type 2	Unit # (Bus #)	Fuel type 1			Fuel type 2
	OZ 1	OZ 2	OZ 3			OZ 1	OZ 2	OZ 3	
1 (1)			*		1 (1)			*	
2 (2)			*		2 (2)		*		
3 (5)	*				3 (5)	*			
4 (8)		*			4 (8)		*		
5 (11)	*				5 (11)	*			
6 (13)	*				6 (13)	*			

Table II presents the optimal solution of IEEE 30-bus system considering POZs and multiple fuel options via the proposed model. As can be seen from this table, the proposed model is very fast; this is mainly because of the adaptability of the model with the pre-solving and solving process of the commercial solver Knitro. The adaptability of the model can be seen from the number of nodes to be probed, subproblems to be handled, and the execution time. While for the case without valve point effect, only 4 nodes have been probed and 4 subproblems have been solved, for the case with valve-point effects that is more complicated, 5 nodes and 6 subproblems have been handled. Moreover, the execution time for both cases is less than one second. Results show that the cost of the case with valve-point effect is higher than the case without valve point effect; this is not only the effect of the valve point but also can be the effect of POZs and multiple fuel options.

Table III presents the binary decision variables related to the operating zones (OZ), and multiple fuel option. As can be seen from this table, for none of the cases, without or with VPEs, under normal loading condition the second fuel has been selected. However, the pattern of units' operating zone has been changed. Without considering valve point effect, unit 2 is operated in the second operating zone (OZ 2), while with considering valve point effect, its operating zone has changed to the third one (OZ 3).

b) IEEE 30-bus under increased loading condition

Since in subsection a) the IEEE 30-bus system only worked with the first fuel, therefore, in this subsection, this system is tested under increased loading condition to show the potential of the proposed model in choosing among multiple fuels.

Table IV. Outputs Results of Proposed Model considering POZs and Multiple Fuel Option with and without Valve Point Effect— IEEE 30-Bus System under Increased Loading Condition

Optimal Output	Without Valve Point	With Valve Point
P_{g1} (MW)	140.00	140.00
P_{g2} (MW)	80.00	80.00
P_{g5} (MW)	28.2441	27.7562
P_{g8} (MW)	35.00	35.00
P_{g11} (MW)	28.0468	28.5491
P_{g13} (MW)	30.00	30.00
tp_{6-9} (p.u.)	0.95	0.95
tp_{6-10} (p.u.)	1.08236	1.0819
tp_{4-12} (p.u.)	1.03178	1.03205
tp_{28-27} (p.u.)	1.03209	1.03211
Cost ($/h)	925.3260	976.4617
Loss (MW)	8.30	8.31
Time (s)	0.718	0.595
# of nodes	21	13
# of subproblems	23	15

Table V. Fuel Type and Operating Zones of the Generating Units Corresponding to the Cases with and Without Valve Point Effects— IEEE 30-Bus System Under Increased Loading Condition

Without Valve Point					With Valve Point				
Unit # (Bus #)	Fuel type 1			Fuel type 2	Unit # (Bus #)	Fuel type 1			Fuel type 2
	OZ 1	OZ 2	OZ 3			OZ 1	OZ 2	OZ 3	
1 (1)			*		1 (1)			*	
2 (2)				*	2 (2)				*
3 (5)	*				3 (5)	*			
4 (8)		*			4 (8)		*		
5 (11)		*			5 (11)		*		
6 (13)		*			6 (13)		*		

Table IV presents the optimal solution of IEEE 30-bus system considering POZs and multiple fuel options via the proposed model. As can be seen from this table, even under increased load condition in which the system operation is critical, the proposed model is very fast. Compared to the case in subsection a), even by increasing the number of nodes to be probed and the number of subproblems to be handled, the execution time is still less than one second for both cases, with and without valve-point effects. This proves the adaptability of the proposed model with the pre-solving and solving process of the commercial solver Knitro. Results show that the cost of the case with valve-point effect is higher than the case without valve point effect; this is not only the effect of the valve point but also can be the effect of POZs and multiple fuel options.

Table V presents the binary decision variables related to the operating zones (OZ), and multiple fuel option. As can be seen from this table, in both cases, without or with VPEs, under increased loading condition, the second fuel has been selected by unit 2. Although the binary decision variables in both cases are similar, from Table IV the generation pattern by generating units at buses 5 and 11 are different; such difference is mainly the result of valve point effect of generating units.

B. IEEE 118-bus test system

This system consists of 118 buses, 186 branches, and 54 generators where 20 out of 54 generating units have a total 42 prohibited operating zones, and nine out of these 20 units work with two or three different fuel types. The system data and information related to the POZs are obtained from [27] and [21], respectively, while the data of multiple fuel option are provided in the appendix, Table VII. It is worth mentioning that the cost coefficients related to fuel types 2 and 3 are considered to be 10% and 15% higher than the first fuel type, respectively. This system is tested under two loading conditions, namely normal and critical. To obtain the critical loading conditions, first the maximum loading point (MLP) of this system is obtained [28] and then to obtain more practical results 90% of the obtained MLP is considered as the critical active load. For validity purposes, the data related to the critical loading condition is provided in Appendix, Table X.

Table VI. Results of the Proposed Model considering POZs and Multiple Fuel Option Under Normal and Critical Loading Conditions— IEEE 118-Bus System

Optimal Output	Normal Loading	Critical Loading
P_{g1} (MW)	36.989	60.000
P_{g4} (MW)	0.000	60.000
P_{g6} (MW)	0.000	60.000
P_{g8} (MW)	0.000	100.000
P_{g10} (MW)	336.498	395.000
P_{g12} (MW)	87.168	185.000
P_{g15} (MW)	20.000	60.000
P_{g18} (MW)	20.325	100.000
P_{g19} (MW)	28.290	100.000
P_{g24} (MW)	0.000	100.000
P_{g25} (MW)	190.000	320.000
P_{g26} (MW)	283.787	414.000
P_{g27} (MW)	16.340	100.000
P_{g31} (MW)	7.301	107.000
P_{g32} (MW)	21.249	100.000
P_{g34} (MW)	30.000	60.000
P_{g36} (MW)	13.115	100.000
P_{g40} (MW)	45.000	100.000
P_{g42} (MW)	45.000	100.000
P_{g46} (MW)	19.199	119.000
P_{g49} (MW)	200.00	304.000
P_{g54} (MW)	49.827	148.000
P_{g55} (MW)	37.452	100.000
P_{g56} (MW)	38.169	100.000
P_{g59} (MW)	140.000	140.000
P_{g61} (MW)	145.000	210.000
P_{g62} (MW)	0.000	100.000
P_{g65} (MW)	350.000	350.000
P_{g66} (MW)	353.937	492.000
P_{g69} (MW)	460.545	805.200
P_{g70} (MW)	0.000	60.000
P_{g72} (MW)	0.000	100.000
P_{g73} (MW)	0.308	100.000
P_{g74} (MW)	24.173	100.000
P_{g76} (MW)	30.819	100.000
P_{g77} (MW)	0.000	100.000
P_{g80} (MW)	439.942	577.000
P_{g85} (MW)	0.000	100.000
P_{g87} (MW)	3.929	102.268
P_{g89} (MW)	410.000	410.000
P_{g90} (MW)	0.745	100.000
P_{g91} (MW)	0.000	100.000
P_{g92} (MW)	0.000	100.000
P_{g99} (MW)	0.000	100.000
P_{g100} (MW)	238.722	352.000
P_{g103} (MW)	39.022	140.000
P_{g104} (MW)	7.216	100.000
P_{g105} (MW)	15.725	100.000
P_{g107} (MW)	34.729	100.000
P_{g110} (MW)	13.534	100.000
P_{g111} (MW)	35.477	136.000
P_{g112} (MW)	39.704	100.000
P_{g113} (MW)	0.000	100.000
P_{g116} (MW)	0.000	100.000
Cost ($/h)	131984.681	379644.983
Loss (MW)	67.24	111.02
Time (s)	12.030	154.226
# of nodes	96	971
# of subproblems	197	1072

Table VI presents the proposed model under two loading conditions. As can be seen from this table, under the critical condition, 42 out of 54 units are generating at their maximum capacity, while under normal loading condition the system has more degree of freedom and none of the units are operating at its maximum limit. Results also show that the model complexity is increased as the loading condition changes; for the normal loading condition, only 96 nodes have been explored, and 197 sub-problems have been solved, while for the critical loading condition, 971 nodes have been explored and 1072 sub-problems have been solved. Although under critical condition, compared to the normal loading condition, 875 more sub-problems have been solved, the CPU time has been increased by only 142.194 seconds, which accounts for 0.162 seconds per sub-problem. This shows the high computational efficiency of the proposed model in solving OPF problems under critical condition.

Table VII shows the operating pattern of the generating unit with POZs and MFOs. As can be seen, under the critical condition, the units work with their second fuel option that generates more power. The only unit with three fuel option is at bus 89 that never uses its third fuel option due to the economic priorities.

Table VII. Fuel Type and Operating Zones of the Generating Units Corresponding to the Normal and Critical Loading Conditions—IEEE 118-Bus System

Unit at Bus #	Normal Loading Condition							Critical Loading Condition						
	Fuel type 1			Fuel type 2		Fuel type 3		Fuel type 1			Fuel type 2		Fuel type 3	
	OZ 1	OZ 2	OZ 3	OZ 1	OZ 2	OZ 1	OZ 2	OZ 1	OZ 2	OZ 3	OZ 1	OZ 2	OZ 1	OZ 2
1		*									*			
4	*										*			
6	*										*			
10				*							*			
15	*										*			
25		*								*				
26			*							*				
34		*									*			
40		*								*				
42		*								*				
49			*								*			
59		*							*					
61	*									*				
65			*						*					
85	*									*				
89				*							*			
99	*									*				
104	*									*				
116	*									*				

IV. CONCLUDING REMARKS

The importance of optimal power flow in power system operating and planning problems especially in recently developed technologies yielded to draw more attention. On the other hand, due to the robustness of commercial solvers, the generation and transmission companies are more interested in solver-based models. This paper, by taking advantage of preprocessing and probing techniques of the existing commercial solvers, has proposed a solver-friendly mixed-integer nonlinear programming (SF-MINLP) model. This model uses innovative recasting techniques to transform the binary decision variables, existing in the constraints, into the objective function. The model has been tested on two IEEE systems, 30- and 118-bus. Results show that the recast techniques in SF-MINLP model not only supports the commercial solver in finding a high-quality solution but also increases the computational efficiency.

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Appendix

Table VIII. POZs and Fuel Types of IEEE 30-bus System

Bus #	POZs	Fuel						
		Type	Range	Cost Coefficient				
				a	*b*	*c*	*e*	*f*
1	(55 66), (80 120)	1	[50 140]	0.0050	0.70	55.0	16.5	0.037
		2	(140 200]	0.0075	1.05	82.5	18.0	0.037
2	(21 24) (45 55)	1	[20 55]	0.0100	0.3	40.0	14.75	0.038
		2	(55 80]	0.0200	0.6	80.0	16.0	0.038
5	(30 36)	1	[15 50]	0.0625	1.0	0.0	14.0	0.040
8	(25 30)	1	[10 35]	0.0083	3.25	0.0	12.0	0.045
11	(25 28)	1	[10 30]	0.0250	3.0	0.0	13.0	0.042
13	(24 30)	1	[12 40]	0.0250	3.0	0.0	13.5	0.041

Table IX. POZs and Fuel Types of IEEE 118-bus System

Buses with POZ	POZs	Fuel	
		Type	Range
1, 4, 6, 15, 34, 70	(20 30) (60 85)	1	[0 50]
		2	(50 100]
10	(15 45) (165 200) (395 410)	1	[0 175]
		2	(175 550]
25	(40 65) (190 200)	1	[0 320]
26	(75 95) (260 280)	1	[0 414]
40,42,85,99,104,116	(20 30) (45 55)	1	[0 100]
49	(45 60) (185 200)	1	[0 210]
		2	(210 304]
59	(95 105) (140 155)	1	[0 255]
61	(145 155) (210 230)	1	[0 260]
65	(180 200) (350 360)	1	[0 491]
89	(120 145) (410 460) (500 525)	1	[0 150]
		2	(150 445]
		3	(445 707]

Table X. Critical Loading Condition— IEEE 118-bus System

Bus #	Demand	Bus #	Demand	Bus #	Demand	Bus #	Demand
1	45.9	31	38.7	59	249.3	92	58.5
2	18.0063	32	53.1	60	70.20018	93	84.12696
3	35.10117	33	20.70009	62	69.3	94	101.7423
4	35.1	34	53.1	64	390.3876	95	57.609
5	27.72045	35	29.70063	66	35.1	96	34.20009
6	46.8	36	27.9	67	47.89251	97	16.72533
7	42.20559	38	74.07963	68	1196.982	98	55.21113
8	25.2	39	24.30009	70	59.4	99	37.8
9	520.1226	40	59.4	71	177.4116	100	33.3
11	63.00018	41	33.30009	72	10.8	101	88.97247
12	42.3	42	86.4	73	5.4	102	172.9386
13	30.60009	43	16.20009	74	61.2	103	20.7
14	12.60036	44	14.86692	75	42.30036	104	34.2
15	81	45	47.70009	76	61.2	105	27.9
16	22.50018	46	25.2	77	54.9	106	197.3871
17	9.90081	47	79.40115	78	63.90009	107	45
18	54	48	66.16494	79	59.46741	108	62.52399
19	40.5	49	78.3	80	117	109	166.1013
20	16.20009	50	32.15151	81	324.423	110	35.1
21	14.86692	51	15.30009	82	50.17563	112	61.2
22	9.29196	52	16.20009	83	18.58365	113	5.4
23	308.9367	53	20.70009	84	30.43458	114	38.20905
24	11.7	54	101.7	85	21.6	115	52.56108
27	63.9	55	56.7	86	124.668	116	165.6
28	45.31536	56	75.6	88	325.2321	117	18.00009
29	46.08378	57	10.80063	90	146.7	118	29.70018
30	494.4654	58	10.80018	91	9	Others	0.00

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