

Resonant Waves in the Gap between Two Advancing Barges

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Abstract

The gap resonance between two advancing rectangular barges in side-by-side arrangement is investigated using a 3-D Rankine source method. A modified Sommerfeld radiation condition accounting for Doppler shift is applied for the low forward speed problem when the scattered waves could propagate ahead of the barges. Numerical studies are conducted to investigate various factors which will influence the wave resonance in the narrow gap with particular attention paid on the forward speed effect and its coupling effects with gap width and draft. It is found that in the absence of forward speed, the trapped water surface oscillates like a flexible plate and the wave flow within the gap behaves like a standing wave. When the two barges are travelling ahead, the resonant wave patterns within the gap are reshaped. Additionally, the resonant frequencies shift to lower value and are compressed within a narrow range. Gap resonances are reduced by the augment of gap width. The effect of draft is shown to be associated with resonant modes. Draft effect becomes less pronounced at higher order resonant modes. Furthermore, both gap width and draft effects on gap resonance are found to be independent from forward speed.

Keywords: Gap resonance; forward speed; hydrodynamic interaction; Rankine source method; ship-to-ship problem

25 1. Introduction

26 Due to the increasing exploration of oil and gas in deep and ultra-deep waters, more
27 side-by-side operations have been adopted in the marine industry. Zhao et al. [1]
28 investigated the hydrodynamic interaction between a FLNG and a LNG carried in side-
29 by-side configuration. Zhao et al. [2] predicted the hydrodynamic performance of a FLNG
30 system during offloading operation. Traditional offloading operation involves ship-to-
31 ship interaction in the absence of forward speed. But recent years have seen an increasing
32 number of lightering operations, due to the restrictions during cargo vessel calling at ports
33 or navigating in shallow waters. In a typical lightering operation, the cargo vessel and
34 service vessel are moored to each other and sail at a low speed. In this specific
35 circumstance, the gap resonance can be quite complicated with the addition of forward
36 speed effect. Therefore, a reliable analysis methodology should be developed to predict
37 the wave resonance behaviour during a lightering operation.

38 Potential flow theory has been widely applied in the study of gap wave resonance
39 between two side-by-side floating bodies. Under the assumption of infinite water depth
40 and infinite body length, Molin [3] derived a quasi-analytical approximation of natural
41 modes of the inner free surfaces within a rectangular moonpool. Gap resonance between
42 two adjacent ships in close proximity is very similar to moonpool resonance. The only
43 fundamental difference is that, for the gap resonance problem (open-ended moonpool
44 problem), a Dirichlet condition rather than the Neumann condition should be imposed to
45 the open ends [4]. By applying the same approach, Molin et al. [5] expanded the analytical
46 approximation of moonpool natural modes to gap resonance between two rectangular
47 boxes. Sun et al. [6] studied the first and second order resonant waves between two
48 adjacent barges. Influence of gap width was examined. Zhao et al. [7] investigated the
49 first and higher harmonic components of gap response. Transient wave groups were used
50 to avoid the wave reflection. It was found that the viscous damping has a linear form.

51 A shortcoming of the application of potential flow theory is the over-estimation of
52 the gap resonance. Molin et al. [8] presented experimental evidence that the discrepancies
53 between measured and predicted wave elevations were mostly due to the flow separation.

54 In order to analyse floaters with different corner radius, Moradi et al. [9] took the
55 roundness factor into consideration by modifying the incident wave frequency. One of the
56 popular solutions to the flow separation problem is the introduction of an artificial
57 damping zone imposed on the water free surface. Yao and Dong [10] added an artificial
58 damping surface and successfully overcame the over-estimation in their investigation of
59 the gap resonance between two barges. An alternative approach is the application of
60 computational fluid dynamics (CFD) technique, which can fully simulate the flow
61 separation at sharp corners. Moradi et al. [11] examined the effect of water depth on gap
62 resonance of two side-by-side rectangular boxes in a 2D numerical wave flume.

63 Generally, the study of gap resonance assumes that the two floating bodies have no
64 forward speed, while investigations on forward speed problem are still inadequate.
65 Fredriksen et al. [12] investigated resonant piston-mode resonance in a moonpool at low
66 incoming current speed with both experimental and numerical methods. They found that
67 the current speed had a slightly decreasing effect on the moonpool piston-mode. Lataire
68 et al. [13] investigated the hydrodynamic forces acting on the ships during lightering
69 operations in calm water. The mathematical model of lightering manoeuvres for both
70 ships involved was derived. A typical challenge in forward speed problem is the
71 satisfaction of radiation condition for outgoing waves, especially at low frequencies. This
72 is because the waves generated ahead of a ship reflect from the outward computational
73 boundary and smear the flow around the ship. One of the valid radiation conditions
74 available for forward speed problem is the so-called upstream radiation condition [14],
75 which truncates the free water surface at some upstream points and imposes different
76 boundary conditions to the separated free water surface. Jensen et al. [15] proposed
77 another approach by moving the panels on free water surface some distance downstream,
78 namely the so-called panel shift technique. Both of the two approaches can achieve good
79 agreement with experimental measurement at $\tau > 0.25$ ($\tau = u_0\omega_0/g$, u_0 is the ship speed,
80 ω_0 is the wave frequency, and g is the gravity acceleration). However, the two approaches
81 are based on the assumption that no scattered wave travel ahead of the ship so they are no
82 longer valid for low forward speed problem at $\tau < 0.25$. Other categories of radiation

83 condition applicable to low forward speed problem have been proposed and validated
84 recently. Yasuda et al. [16] introduced Rayleigh's artificial friction in the boundary
85 condition to suppress longer wave components in a computational region apart from the
86 ship. Their simulation results showed that wave reflection from upstream boundary was
87 very limited and good agreement with measured results was obtained as a result. Das and
88 Cheung [17] proposed a modified Sommerfeld type radiation condition taking into
89 account the Doppler shift effect of the scattered waves. Yuan et al. [18] expanded the
90 radiation condition proposed by Das and Cheung [17] and investigated the hydrodynamic
91 interaction between two ships travelling or stationary in shallow water. Their simulation
92 results showed that this improved radiation condition has a better prediction capacity than
93 upstream upwind condition in the case of low forward speed.

94 The primary objective of the present study is to investigate how forward speed
95 influences the gap resonance between two barges in close proximity. Additionally, wave
96 resonance behaviours in the case of different barge drafts and gap widths will be studied
97 as well as their couplings with forward speed. A 3-D Rankine source method is applied
98 to solve the boundary value problem and the radiation condition proposed in Ref [17] is
99 used to dissipate the outgoing scattered waves.

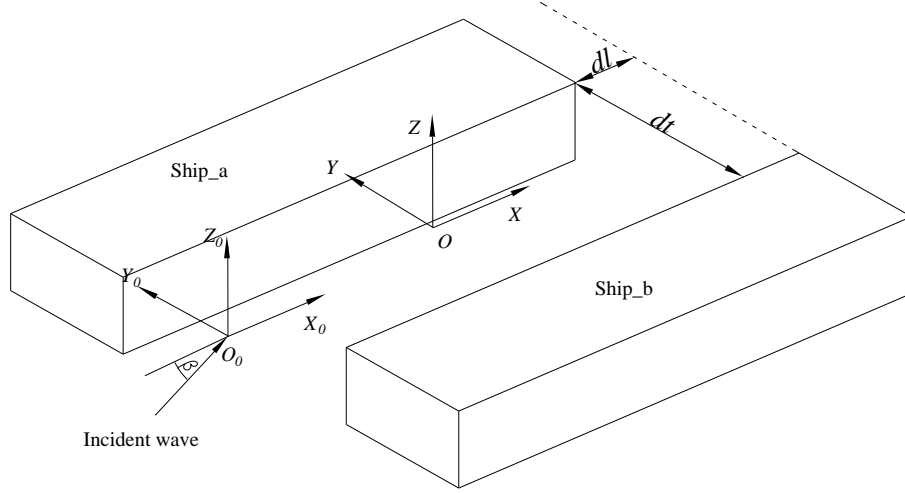
100 2. Mathematical formulations

101 2.1. *Boundary value problem*

102 Assuming the flow field is ideal, linear potential flow theory is applied to model the
103 wave resonance in the gap between two rectangular barges in lightering operation. The
104 flow within the fluid domain is described by velocity potential $\Phi(\mathbf{r}, t)$ in which $\mathbf{r} = (x, y,$
105 $z)$ denotes the position of the point concerned. $\Phi(\mathbf{r}, t)$ satisfies Laplace equation in the
106 whole fluid domain. Consequently, the calculation of $\Phi(\mathbf{r}, t)$ is converted to boundary
107 value problem.

108 The right-handed coordinate systems selected in the present study are shown in Fig.
109 1. $O_0-X_0Y_0Z_0$ is the global coordinate system. The centre O_0 is located on the mean sea

110 surface and O_0Z_0 axis positive upward. The body-fixed coordinate systems O - XYZ is fixed
 111 to the two ships, with its origin located the centre of the gap and OZ axis positive upward.
 112 dt and dl represent the transverse and longitudinal distance between the two barges
 113 respectively. In the present research, $dl = 0$ m is used. The boundary value problem of
 114 each ship is addressed in the body-fixed coordinate system.



115
 116 **Fig. 1.** Coordinate system.

117 In the present study, two rectangular barges are enforced to travel ahead together in
 118 head sea with the same forward speed and any oscillating motions are restricted.
 119 Consequently, the velocity potential within the fluid domain can be represented with Eq.
 120 (1) in the body-fixed coordinate system.

$$121 \quad \Phi(\mathbf{r}, t) = (\varphi_s(\mathbf{r}) - u_0 x) + \text{Re} \sum_{j=0}^1 [\eta_j \varphi_j(\mathbf{r}) \cdot e^{-i\omega_e t}] \quad (1)$$

122 $\varphi_s(\mathbf{r})$ is the steady wave potential produced by the advancing movement of the ship.
 123 $\varphi_0(\mathbf{r})$ is the incident wave potential and η_0 is the wave amplitude. $\varphi_1(\mathbf{r})$ is the scattered
 124 wave potential with $\eta_1 = \eta_0$. ω_e is the encounter frequency, which is given by

$$125 \quad \omega_e = \omega_0 - u_0 \frac{\omega_0^2}{g} \cos \beta \quad (2)$$

126 where ω_0 is the incident wave frequency, β is the angle of wave heading ($\beta = 180^\circ$
 127 corresponds to head sea).

128 The steady velocity potential is solved as

$$\nabla^2 \varphi_s = 0, \text{ in the fluid domain}$$

$$u_0^2 \frac{\partial^2 \varphi_s}{\partial x^2} + g \frac{\partial \varphi_s}{\partial z} = 0, \text{ on free water surface}$$

$$129 \quad \frac{\partial \varphi_s}{\partial n} = u_0 n_1, \text{ on mean wetted surface of Ship_a} \quad (3)$$

$$\frac{\partial \varphi_s}{\partial n} = u_0 n_1, \text{ on mean wetted surface of Ship_b}$$

$$\frac{\partial \varphi_s}{\partial n} = 0, \text{ on sea bed}$$

130 where $\mathbf{n} = (n_1, n_2, n_3)$ is the unit normal vector directed inward on body surface. $\mathbf{r} = (x, y,$
131 $z)$ is the position vector. The generalised normal vectors n_j are defined as

$$132 \quad n_j = \begin{cases} \mathbf{n}, & j = 1, 2, 3 \\ \mathbf{r} \times \mathbf{n}, & j = 4, 5, 6 \end{cases} \quad (4)$$

133 The scattered wave velocity potential $\varphi_1(\mathbf{r})$ is solved with Eq. (5).

$$\nabla^2 \varphi_1 = 0, \text{ in the fluid domain}$$

$$g \frac{\partial \varphi_1}{\partial z} - \omega_e^2 \varphi_1 + 2i\omega_e u_0 \frac{\partial \varphi_1}{\partial x} + u_0^2 \frac{\partial^2 \varphi_1}{\partial x^2} - i\omega_e u_0 \frac{\partial^2 \varphi_s}{\partial x^2} \varphi_1 - 2i\omega_e u_0 \frac{\partial \varphi_s}{\partial x} \frac{\partial \varphi_1}{\partial x}$$

$$= i\omega_e u_0 \frac{\partial^2 \varphi_s}{\partial x^2} \varphi_0 + 2i\omega_e u_0 \frac{\partial \varphi_s}{\partial x} \frac{\partial \varphi_0}{\partial x}, \text{ on free water surface}$$

$$134 \quad \frac{\partial \varphi_1}{\partial n} = -\frac{\partial \varphi_0}{\partial n}, \text{ on mean wetted surface of Ship_a} \quad (5)$$

$$\frac{\partial \varphi_1}{\partial n} = -\frac{\partial \varphi_0}{\partial n}, \text{ on mean wetted surface of Ship_b}$$

$$\frac{\partial \varphi_1}{\partial z} = 0, \text{ on sea bed}$$

135 The incident wave velocity potential $\varphi_0(\mathbf{r})$ is described with

$$136 \quad \varphi_0(\mathbf{r}) = \frac{g}{i\omega_0} \cdot \frac{\cosh(kz + kH)}{\cosh(kH)} \cdot e^{-ik(x \cos \beta + y \sin \beta)} \quad (6)$$

137 The above boundary value problem will be completed with the addition of a suitable
138 radiation condition imposed on the control surface and the velocity potential $\Phi(\mathbf{r}, t)$ is
139 obtained subsequently.

140 The modified Sommerfeld radiation condition proposed by Das and Cheung [17] is
141 used

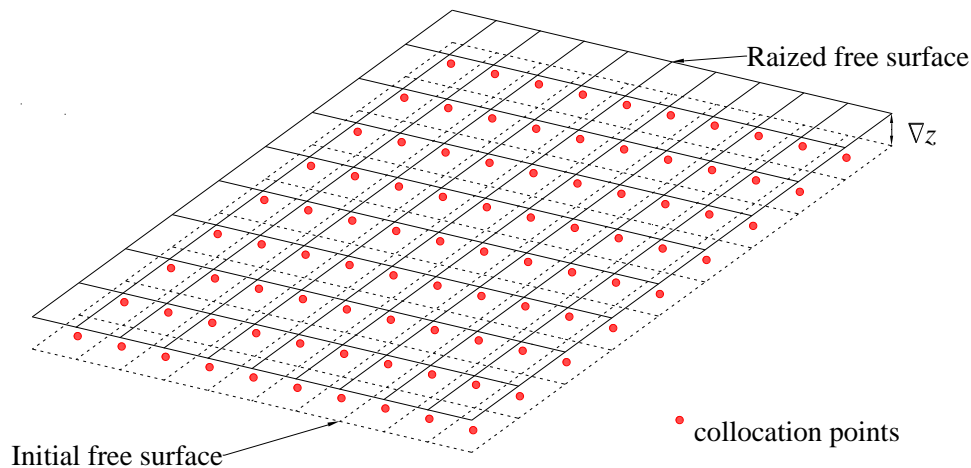
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$$\frac{\partial \varphi}{\partial \mathbf{n}} - ik_s \varphi \cos \theta = 0, \text{ on } S_1 \quad (7)$$

143
$$\nabla \varphi = 0, \text{ on } S_2$$

143 where k_s is the local wave number. S_1 is a certain area in behind the two ships and S_2 is a
 144 certain area in front of the ships. Please refer to Das and Cheung [17] for more details of
 145 the development of this radiation condition.

146 *2.2. Desingularized method*

147 The classical Rankine source method requires distributing the singularity exactly on
 148 the boundary of the fluid domain. Nevertheless, a designularied method which raises the
 149 panels on the free water surface a short distance ∇z upward (see Fig. 2) is commonly used
 150 in a forward speed problem [19]. Meanwhile, the collocation points, where the boundary
 151 condition is satisfied, still stay exactly on the free water surface. Different values of the
 152 raised distance were reported by researchers. For instance, Zhang [20] used \sqrt{S} as the
 153 raised distance (S is the area of the local mesh). In the present study, a raised distance
 154 $\nabla z = 0.1L_j$ suggested by Kim et al. [21] is selected, where L_j is the diagonal length of panel
 155 j .



156 **Fig. 2.** Illustration of the raised mesh on the free surface.

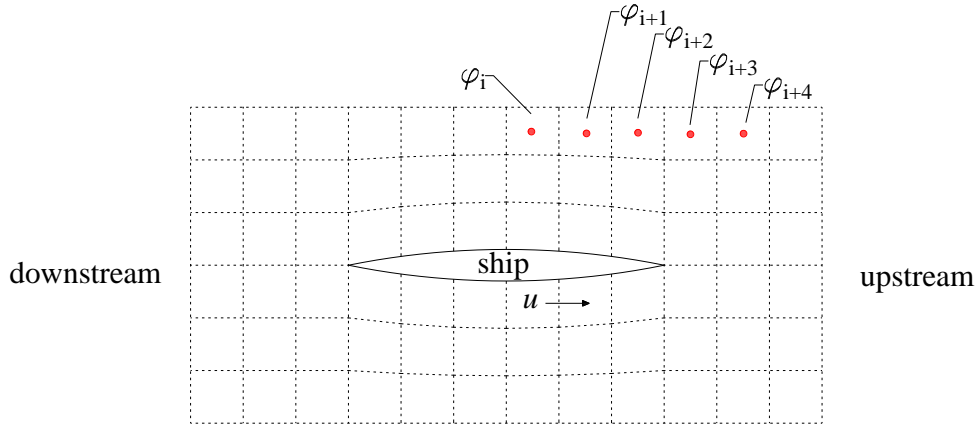
158 The appearance of the second derivative of the potential in the free surface condition
 159 of diffraction potential requires special treatment. In theory, second derivative terms can
 160 be handled with analytical approach. However, it is found that the influence matrix tends

161 to be ill-conditional when analytical representation is applied, which is likely to be caused
162 by the large diagonal value of influence matrix. This kind of numerical phenomenon was
163 firstly reported by Longuet-Higgins [22] in their time-domain study of steep surface
164 waves. They found that the wave pattern developed a saw-toothed appearance in which
165 the computed particle positions lay alternately above and below a smooth curve. Similar
166 problem has been reported by Xu and Yue [23] as well. Similar problem may also happen
167 in a frequency-domain analysis. Therefore, the present study applies a difference scheme
168 rather than an analytical formula to represent the second derivative term. Common
169 difference schemes are upwind difference scheme and central difference scheme.
170 Generally, central difference scheme is more accurate while the stability of upwind
171 difference is better. In addition, the application of upwind difference scheme enforces that
172 the wave pattern mainly depends on upstream flow, which is consistent with physical
173 observation. Due to this favorable property of upwind difference scheme, the second-
174 order upwind difference scheme proposed by Bunnik [24] (see Fig. 3) is adopted to
175 represent the second derivative of velocity potential:

$$176 \quad \frac{\partial^2 \varphi}{\partial x^2}(\mathbf{r}_i) = \frac{1}{\Delta x_i^2} \left[\frac{1}{4} \varphi(\mathbf{r}_{i+4}) - 2\varphi(\mathbf{r}_{i+3}) + \frac{11}{2} \varphi(\mathbf{r}_{i+2}) - 6\varphi(\mathbf{r}_{i+1}) + \frac{9}{4} \varphi(\mathbf{r}_i) \right], i \leq N - 4 \quad (8)$$

177 where N is the number of panels on the straight line along the ship speed. For some panels,
178 there are fewer than 5 panels in front. In that case, the following rule is used

$$179 \quad \begin{aligned} \frac{\partial^2 \varphi}{\partial x^2}(\mathbf{r}_i) &= \frac{1}{\Delta x_i^2} [\varphi(\mathbf{r}_{i+1}) - 2\varphi(\mathbf{r}_i) + \varphi(\mathbf{r}_{i-1})], N - 3 \leq i \leq N - 1 \\ \frac{\partial^2 \varphi}{\partial x^2}(\mathbf{r}_i) &= \frac{\partial^2 \varphi}{\partial x^2}(\mathbf{r}_{i-1}), i = N \end{aligned} \quad (9)$$



• collocation points

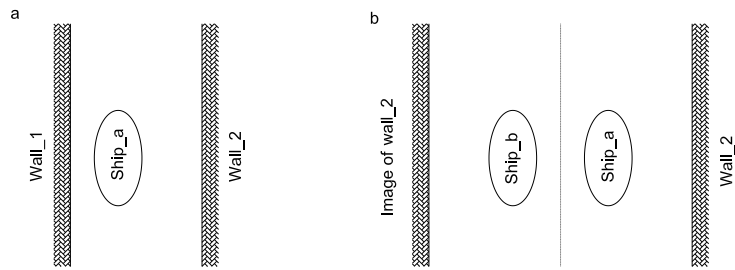
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Fig. 3. Illustration of the second-order upwind difference scheme.

182 3. Validation

183 Model test research on side wall effect performed by Kashiwagi and Ohkusu [25] is
 184 quoted to validate the numerical program. Assuming that the two barges are identical to
 185 each other and advance with the same forward speed, the single ship problem in Fig. 4 (a)
 186 is identical to the ship-to-ship problem in Fig. 4 (b) as the Wall_1 in Fig. 4 (a) plays a role of
 187 mirror. In this way, the experimental data can be used to validate the numerical program.



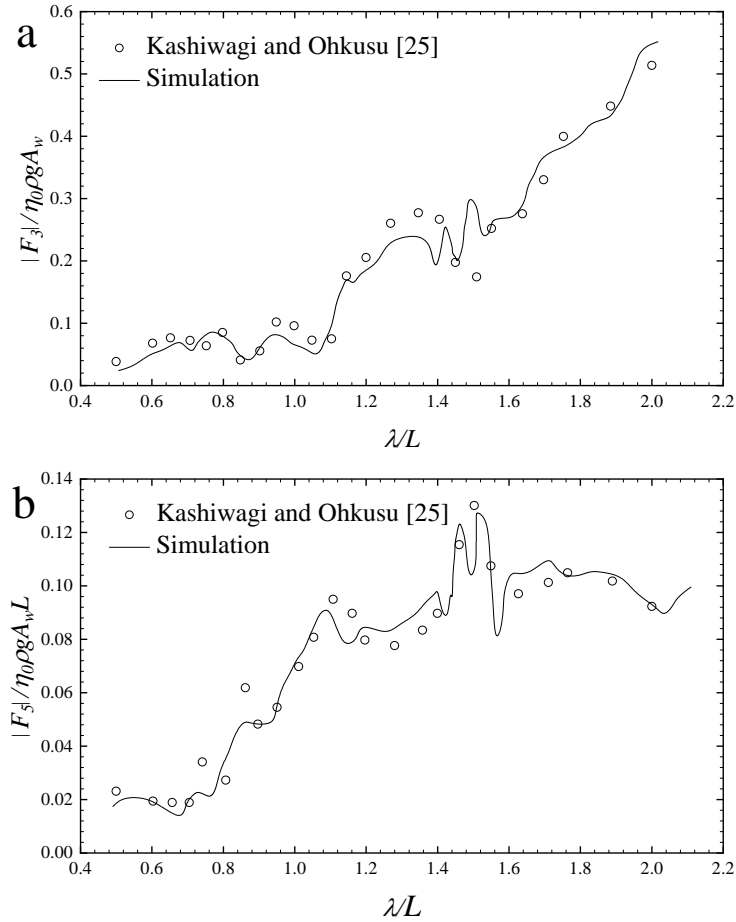
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Fig. 4. Ship advancing in narrow waterway. (a) single ship problem; (b) two ships problem.

191 The ship model in the model test was a half-immersed prolate spheroid with length
 192 $L = 2$ m and breath $B = 0.4$ m. Water depth is set to 2.3 m. The spheroid is advancing at
 193 Froude number $F_n = 0.1$. The simulated and model test measured hydrodynamic forces of
 194 the spheroid are compared in Fig. 5. In general, the predicted hydrodynamic forces agree
 195 well with experimental data.



196
 197 **Fig. 5.** Wave exciting forces of a half-immersed prolate spheroid. (a) heave wave
 198 exciting force; (b) pitch wave exciting moment. A_w is the water plane area and λ is the
 199 wave length.

200 **4. Gap resonance between two advancing rectangular barges**

201 In this section, the gap resonance between two side-by-side arranged rectangular
 202 barges during lightering operation will be investigated in terms of trapped wave elevation
 203 and transverse wave exciting force. Only head sea condition is considered in this study.
 204 The two barges are enforced to sail together with the same forward speed and any
 205 oscillating motions are restricted. Main dimensions of the barges are listed in Table 1.
 206 Water depth is set to 100 m. Simulation cases for a set of regular waves are performed to
 207 capture the gap resonance at a wide range of wave frequency. The wave amplitude is set
 208 to 1 m. Please also note that no extra damping is added in this study.

209 **Table 1**

210 Main dimensions of the rectangular barge

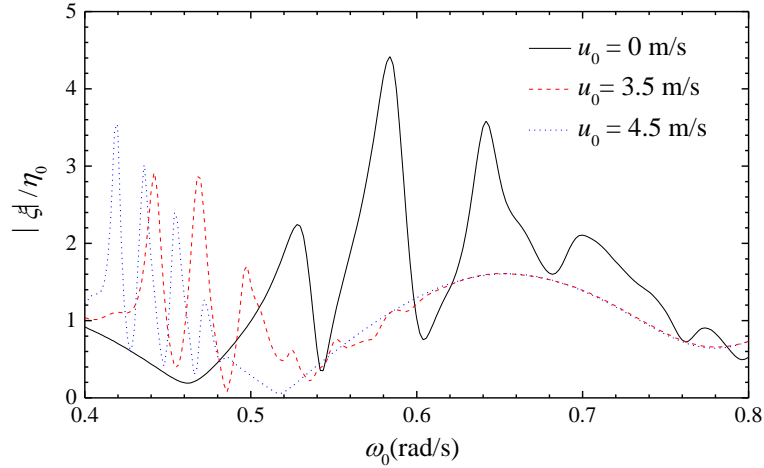
Item	Value
Length (L)	300 m
Breath (B)	75 m
Draft (h)	22 m
Displacement (V)	495,000 m ³
Gap width (b)	14.8 m

211

212 *4.1. Forward speed effect*

213 When two side-by-side arranged rectangular barges are travelling together with
 214 forward speed in head sea, the wave resonance in the gap is expected to exhibit unique
 215 characteristics compared with stationary condition. To figure out the forward speed
 216 effects on gap resonance, investigations will be conducted under different forward speed
 217 conditions.

218 Forward speed has been shown to modify the frequencies of resonant waves. Fig. 6
 219 demonstrates the variation of resonant modes with forward speed. Table 2 summarizes
 220 the resonant frequencies at various resonant modes. When the two barges are stationary
 221 in wave, resonant wave elevation is observed around 0.532 rad/s, 0.584 rad/s and 0.64
 222 rad/s. In the presence of forward speed, the resonant frequencies shift to lower values. In
 223 the body-fixed coordinate, the oscillating frequency of wave flow seen by the gap is larger
 224 than ω_0 if the two barges are travelling in head sea. It partly explains the shift of resonant
 225 frequencies in the presence of forward speed. It is also shown that the resonant
 226 frequencies are compressed within a narrow frequency range. Moreover, such effect
 227 becomes more prominent as the forward speed increases.



228

229

Fig. 6. Resonant frequencies of wave elevation at different forward speeds.

230

Table 2

231

Resonant frequencies of wave elevation at different forward speeds

	1 st Mode	2 nd Mode	3 rd Mode
$u = 0$ m/s	0.532 rad/s	0.584 rad/s	0.64 rad/s
$u = 3.5$ m/s	0.442 rad/s	0.468 rad/s	0.496 rad/s
$u = 4.5$ m/s	0.418 rad/s	0.436 rad/s	0.454 rad/s

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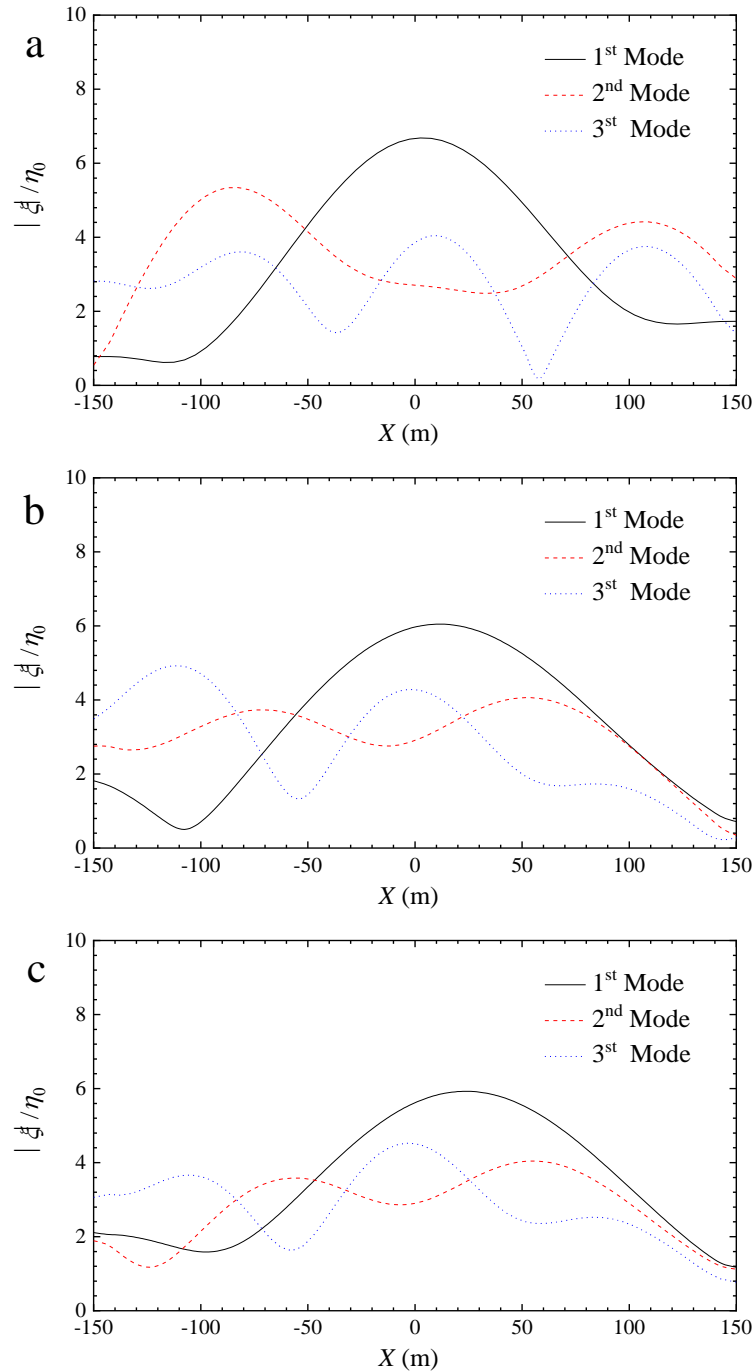
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Fig. 7 demonstrates the amplitudes of resonant wave elevation along the central line of the gap ($-150 \text{ m} < X < 150 \text{ m}$, $Y = 0 \text{ m}$) at different forward speeds. It is shown that wave patterns in the gap are closely associated with the resonant modes. In Fig. 7 (a), one single crest is observed around $X = 0 \text{ m}$. The wave amplitude then decays gradually as it moves to the gap ends. At the 2nd mode, two crests appear at $X = -90 \text{ m}$ and $X = 110 \text{ m}$, respectively. Nevertheless, a trough is observed around $X = 0 \text{ m}$. At the 3rd mode, three crests are observed at $X = -80 \text{ m}$, $X = 0 \text{ m}$ and $X = 110 \text{ m}$ whereas two troughs appear at $X = -50 \text{ m}$ and $X = 60 \text{ m}$. Additionally, the maximum wave elevation drops at higher order of resonant modes. Fig. 8 presents a clearer demonstration of the wave surface oscillation at various resonant modes. The wave patterns of odd number modes are symmetric with respect to midship, while even number modes are antisymmetric. Analogous mode shapes can be expected occur at even higher modes. It appears that the water surface within the gap oscillates as a flexible plate. Similar phenomenon of waves between two barges in beam sea has been reported by Feng and Bai [26] as well. It can be also seen that forward speed effect reduces the amplitude of resonant waves. Although the nature of free surface

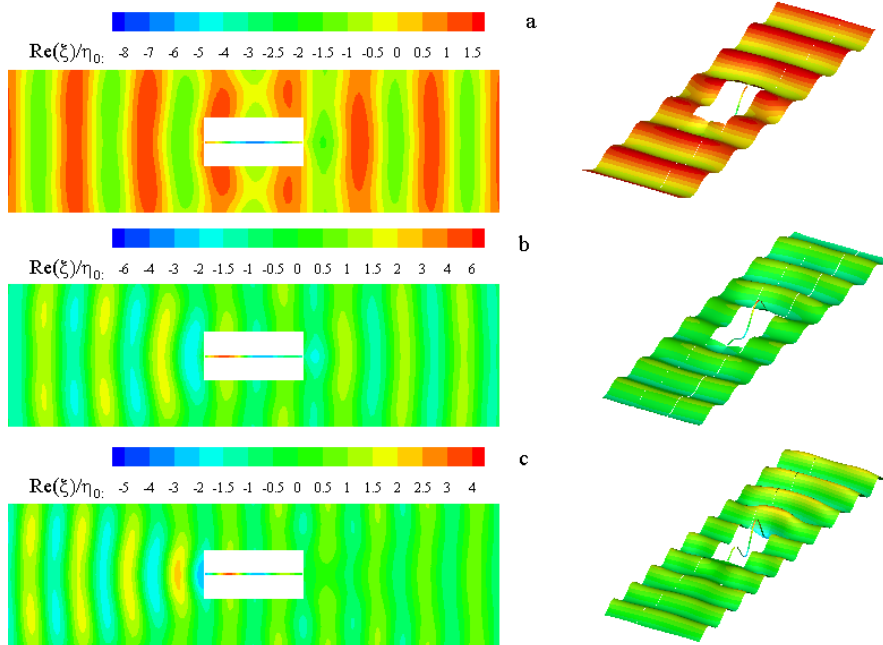
248 oscillation seems not to be changed by the presence of forward speed according to Fig. 7
 249 (b) and Fig. 7 (c), the wave patterns at resonant modes are reshaped. Fig. 9 compares the
 250 wave pattern of 3rd resonant mode when the two barges are travelling at $u = 0$ m/s and u_0
 251 $= 3.5$ m/s, respectively. Compared the upper half part, no wave crest can be found at the
 252 upstream region of gap in the lower half part. It seems that the trapped free surface no
 253 longer oscillates like a flexible plate in the presence of forward speed.



254
 255 **Fig. 7.** Dimensionless wave elevation along the central line of gap at different forward

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speeds. (a) $u_0 = 0$ m/s; (b) $u_0 = 3.5$ m/s; (c) $u_0 = 4.5$ m/s.

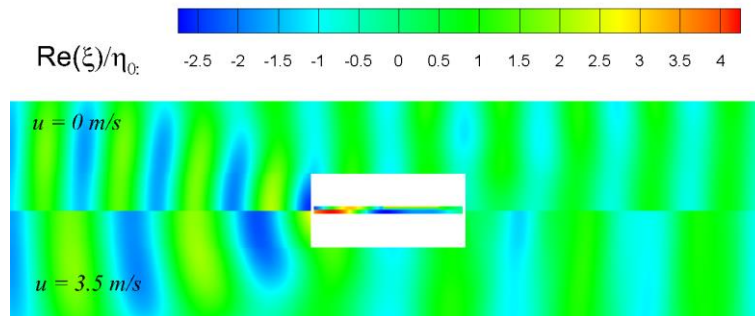


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Fig. 8. Real part of wave patterns $\text{Re}(\xi)/\eta_0$ at various resonant modes, $u_0 = 0$ m/s (body-fixed coordinate system). (a) 1st mode; (b) 2nd mode; (c) 3rd mode.



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Fig. 9. Real part of wave patterns $\text{Re}(\xi)/\eta_0$ at 3rd resonant mode (body-fixed coordinate system).

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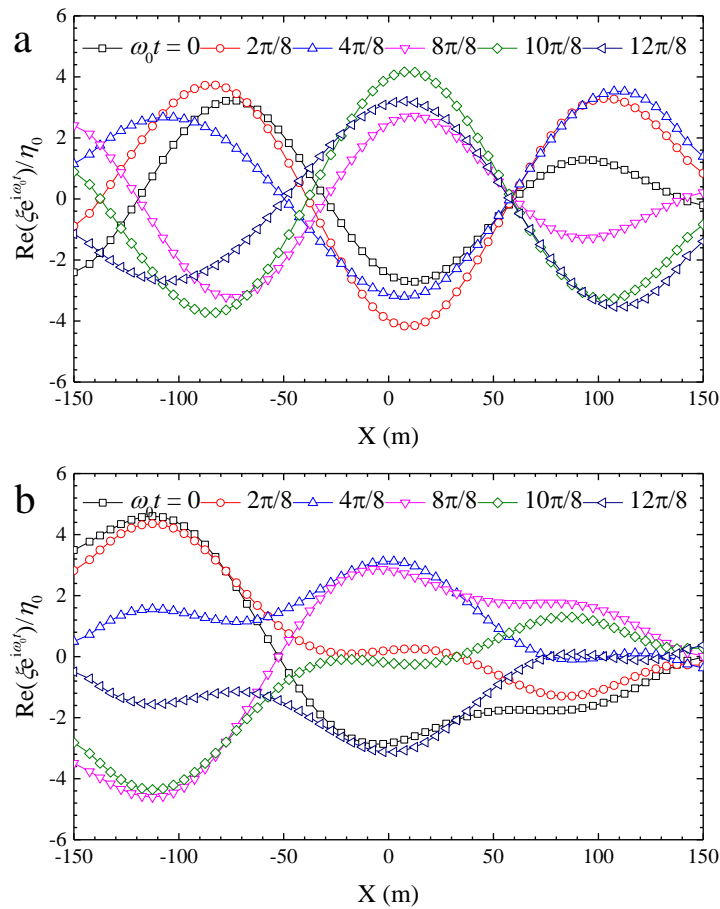
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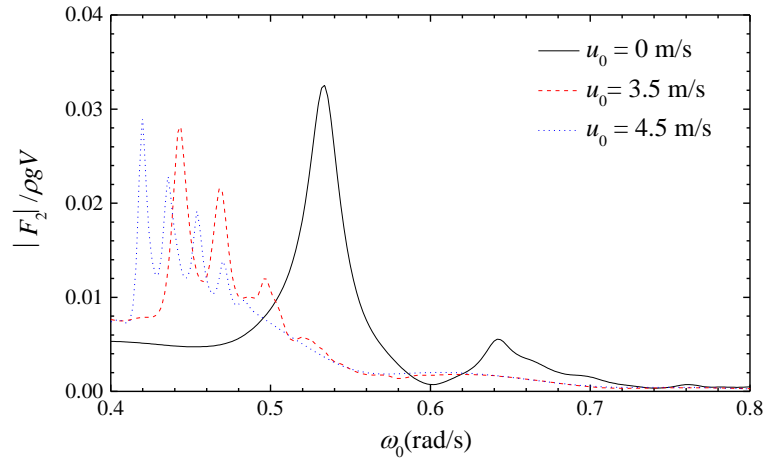
To further investigate the gap water surface oscillation, Fig. 10 displays the instantaneous wave elevation along the central line at 3rd resonant mode. As shown in Fig. 10 (a), the wave flow within the gap behaves very like a standing wave, especially within the mid gap zone ($-50 \text{ m} < X < 50 \text{ m}$). Combing the wave surface profile at various time steps, it can be concluded that the wave surface indeed oscillates like a flexible plate. The standing wave pattern becomes less evident near the gap ends. This is because the physical boundary condition at gap ends is a Dirilect condition rather than a Neumann condition. It means that the fluid flow is free to move in and out of the envelop through the gap ends, resulting in fluid exchange and loss of wave energy trapped in the gap.

272 Consequently, the standing wave patterns around the gap ends are less obvious. When the
 273 two barges are travelling ahead, the standing wave behaviour becomes nearly invisible
 274 (see Fig. 10 (b)) and the oscillations of water surface are different from a flexible plate
 275 oscillation mode. As the barges are travelling ahead, the fluid exchange between the gap
 276 and outer field become more prominent. In this circumstance, the majority of wave energy
 277 used to be trapped within the envelope in stationary cases is now carried to the
 278 downstream. Please note that the wave energy also escapes from the bottom of the gap.
 279 Based on the CFD technology, Feng et al. [27] conducted a detailed investigation on the
 280 wave flow at the gap and they clearly showed the wave flow escaped from the bottom.
 281 However, the wave dissipation at the bottom is mainly due to the flow separation and
 282 shedding occurring at the square edge [28]. Since the linear potential flow theory is used
 283 in our model, it is out of the scope of the present study.



284
 285 **Fig. 10.** Instantaneous wave elevation along the central line of gap at 3rd resonant mode.
 286 (a) $u_0 = 0$ m/s; (b) $u_0 = 3.5$ m/s.

287 In addition to wave elevation, the hydrodynamic wave force represents another
 288 aspect of gap resonances. The wave-induced transverse exciting forces on single barge
 289 are estimated at different forward speeds and the simulation results are presented in Fig.
 290 11.



291
 292 **Fig. 11.** Transverse wave exciting force on single barge at different forward speeds.

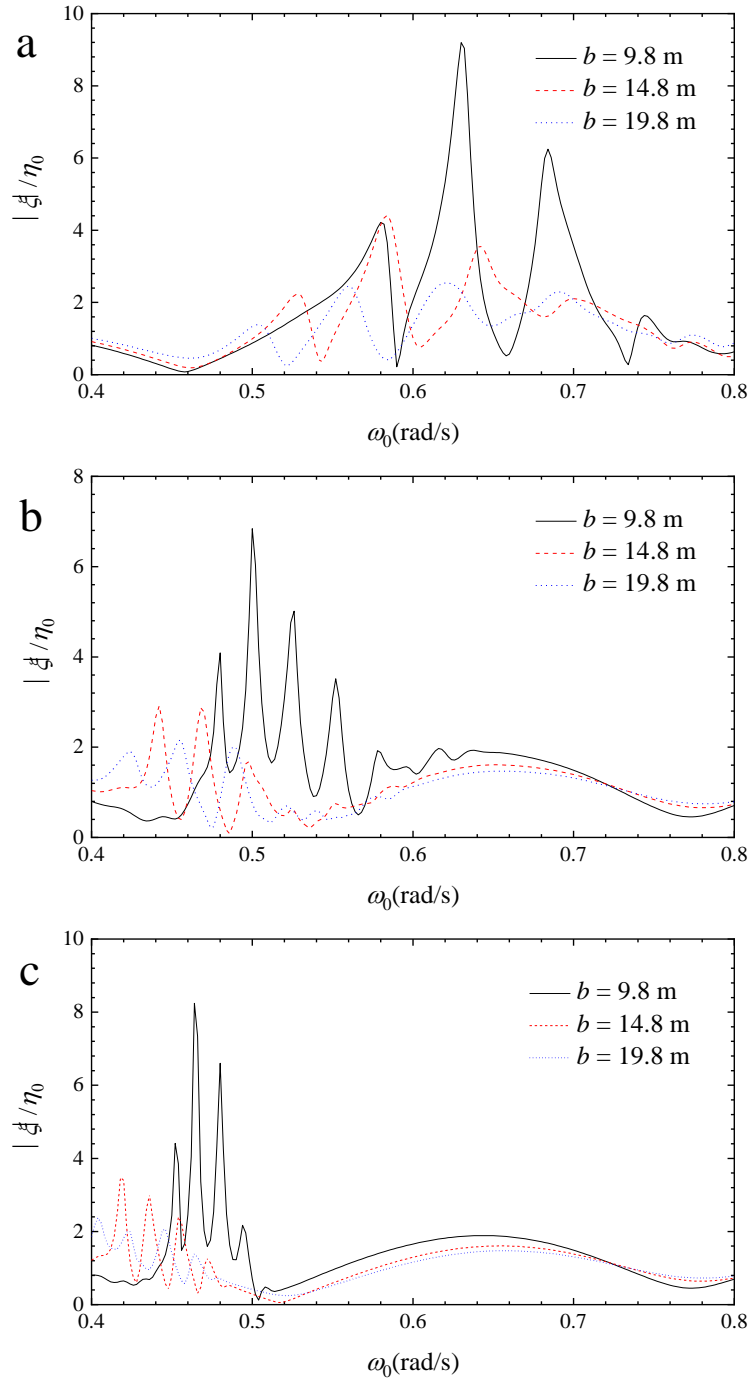
293 The resonant modes of transverse wave force vary with forward speed. When the two
 294 barges are stationary in waves, resonant wave exciting force is observed at around 0.534
 295 rad/s and 0.642 rad/s, corresponding to the 1st and the 3rd resonant modes of wave
 296 elevation in Table 2. As the water wave pattern of even number resonant modes are
 297 antisymmetric with respect to midship (see Fig. 8 (b)), the transverse wave forces acting
 298 on the upstream half ship and downstream half ship just offset each other, resulting in a
 299 limited resultant transverse force. Therefore, the wave force is not excited at 2nd mode. In
 300 the presence of forward speed effect, extra resonant wave forces are excited and the
 301 resonant frequencies shift to lower value within a narrow frequency range. When the two
 302 barges are travelling ahead at $u = 3.5$ m/s, the resonant wave forces are excited at 0.444
 303 rad/s, 0.468 rad/s and 0.496 rad/s, corresponding to the 1st, 2nd and 3rd resonant modes of
 304 wave elevation. Transverse wave force is excited at 2nd resonant mode because the wave
 305 patterns of even number order resonant modes are on longer antisymmetric with respect
 306 to midship. As a result, transverse wave force acting on the upstream half ship and
 307 downstream half ship would not offset each other. Such forward speed effect becomes
 308 more pronounced with the increase of forward speed. It can also be seen from Fig. 11 that

309 the peak value of the 1st mode of resonance force decreases in the presence of forward
310 speed. The decreased transverse force amplitude could reduce the risk of breaking
311 mooring lines or hawsers during lighting operations.

312 According to the above discussions about gap resonance, forward speed plays a
313 positive role in lightering operations. Forward speed contributes to the reduction of wave
314 elevation and attracting or separating force by carrying wave energy to the downstream.
315 Additionally, forward speed is able to modify the resonant frequencies of gap wave,
316 indicating that the resonant responses can be avoided by adjusting the forward speed on
317 the basis of local sea states. It will substantially reduce the chance of ships being subject
318 to large amplitude impact loads and hence avoid potential accidents.

319 4.2. *Gap width effect*

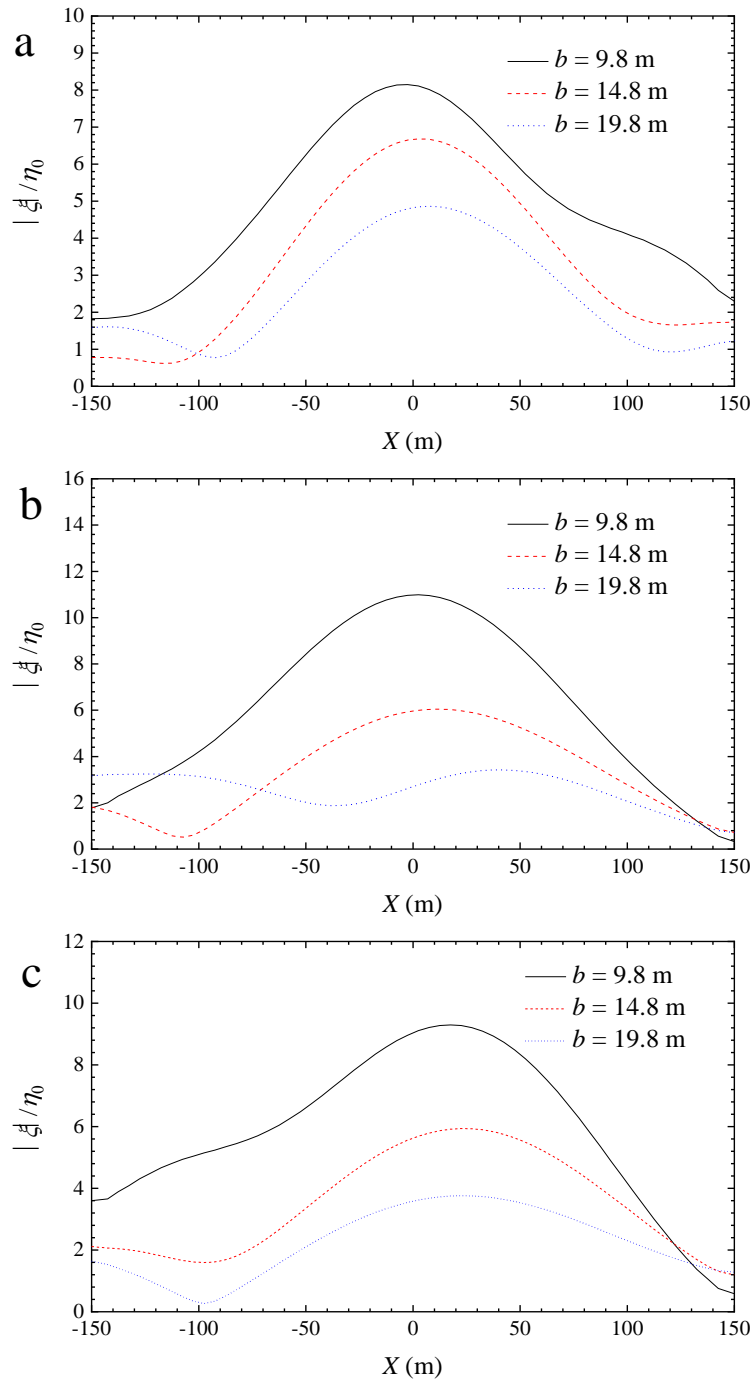
320 The gap width between the two barges is varied from 9.8 m to 19.8 m with an
321 increment 5 m to investigate the gap resonant responses at different gap widths.
322 Additionally, gap resonances will be also investigated at zero and nonzero forward speed
323 conditions to figure out whether the gap width effect would alter when coupled with
324 forward speed effect.



325
 326 **Fig. 12.** Resonant frequencies of wave elevation at different gap widths. (a) $u_0 = 0$ m/s;
 327 (b) $u_0 = 3.5$ m/s; (c) $u_0 = 4.5$ m/s.

328 Fig. 12 compares the resonant frequencies at different gap widths. It is shown clearly
 329 that the resonant frequencies shift to lower values within a narrow range with the increase
 330 of gap width. This trend remains the same no matter the barges are stationary or moving
 331 ahead, manifesting that the gap width effect on the resonant frequencies is independent
 332 from forward speed effect. It also indicates if the sea state is known, the resonant

333 frequencies can be avoided by adjusting the gap width between two barges.



334

335

336

Fig. 13. Wave elevation along the central line of gap with different gap widths at 1st resonant mode; (a) $u_0 = 0$ m/s; (b) $u_0 = 3.5$ m/s; (c) $u_0 = 4.5$ m/s.

337

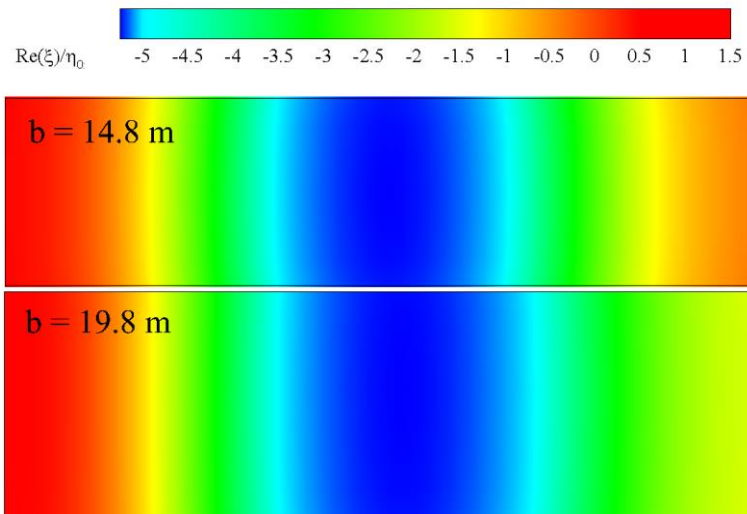
338

339

340

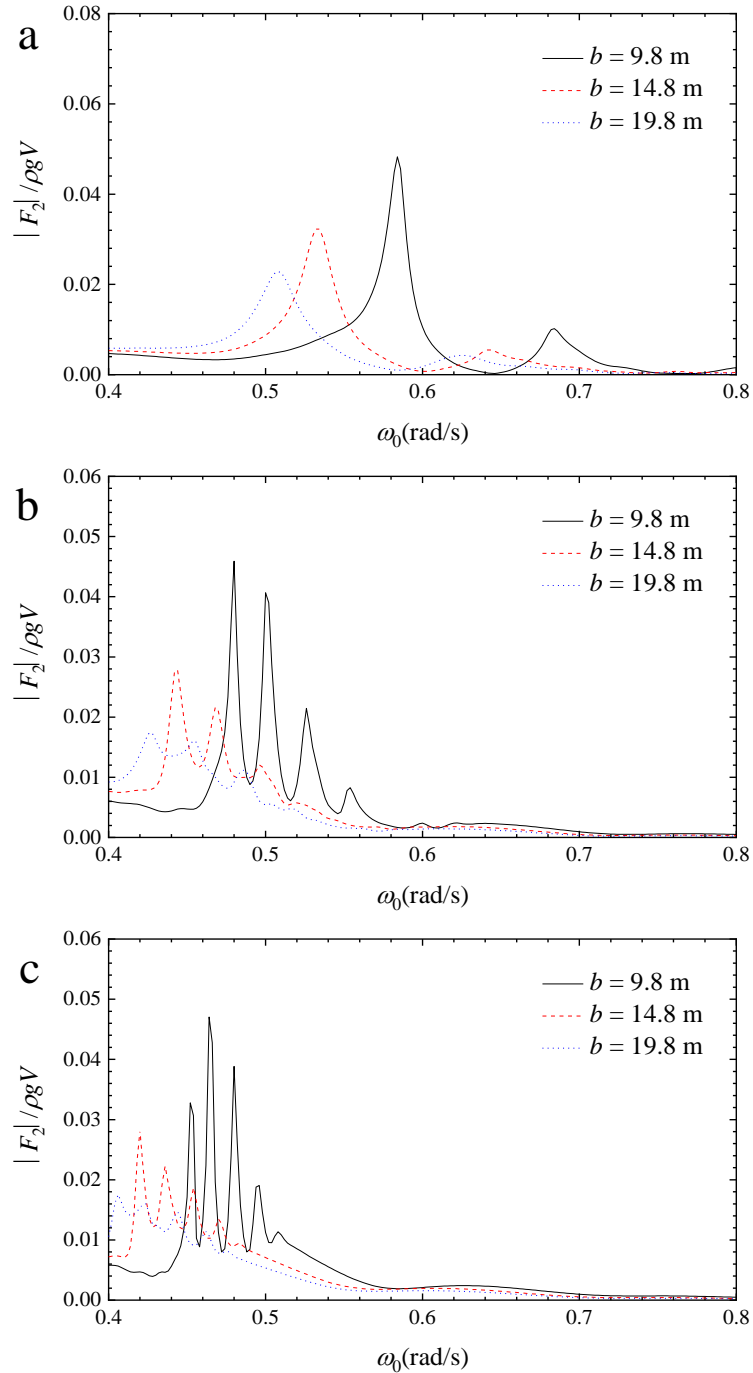
The wave elevation along the central line of gap at different gap widths is examined and the simulation results at 1st resonant mode are presented in Fig. 13. In general, increase of gap width contributes to the alleviation of gap resonance by reducing wave amplitudes within the gap, regardless of forward speed. The hydrodynamic interaction

341 between the two barges becomes less pronounced with the increase of gap width and
 342 hence the gap resonance will reduce. Meanwhile, the change of gap width does not
 343 reshape the wave pattern at resonant mode. Fig. 14 compares the wave patterns within the
 344 gap with different widths ($b = 14.8$ m and $b = 19.8$ m, respectively) when the two barges
 345 are stationary in head sea. As shown, the wave patterns within the gap are very similar. A
 346 wave trough appears in the middle of gap and the wave elevation increases gradually as
 347 it moves to the gap ends.



348 **Fig. 14.** Real part of wave patterns $\text{Re}(\xi)/\eta_0$ with different gap widths at 1st resonant
 349 mode, $u_0 = 0$ m/s (body-fixed coordinate system). Upper part represents case $b = 14.8$
 350 m; lower part represents case $b = 19.8$ m.
 351

352 The variation of transverse exciting forces with incident wave frequencies at different
 353 gap widths are shown in Fig. 15. The variation trend of wave forces follows a similar way
 354 as that of wave elevations. Both in stationary and travelling conditions, the resonant
 355 frequencies of transverse wave force shift to lower value within a narrow frequency range.
 356 Meanwhile, the amplitudes of wave force also drop as a result of less prominent
 357 hydrodynamic interaction.



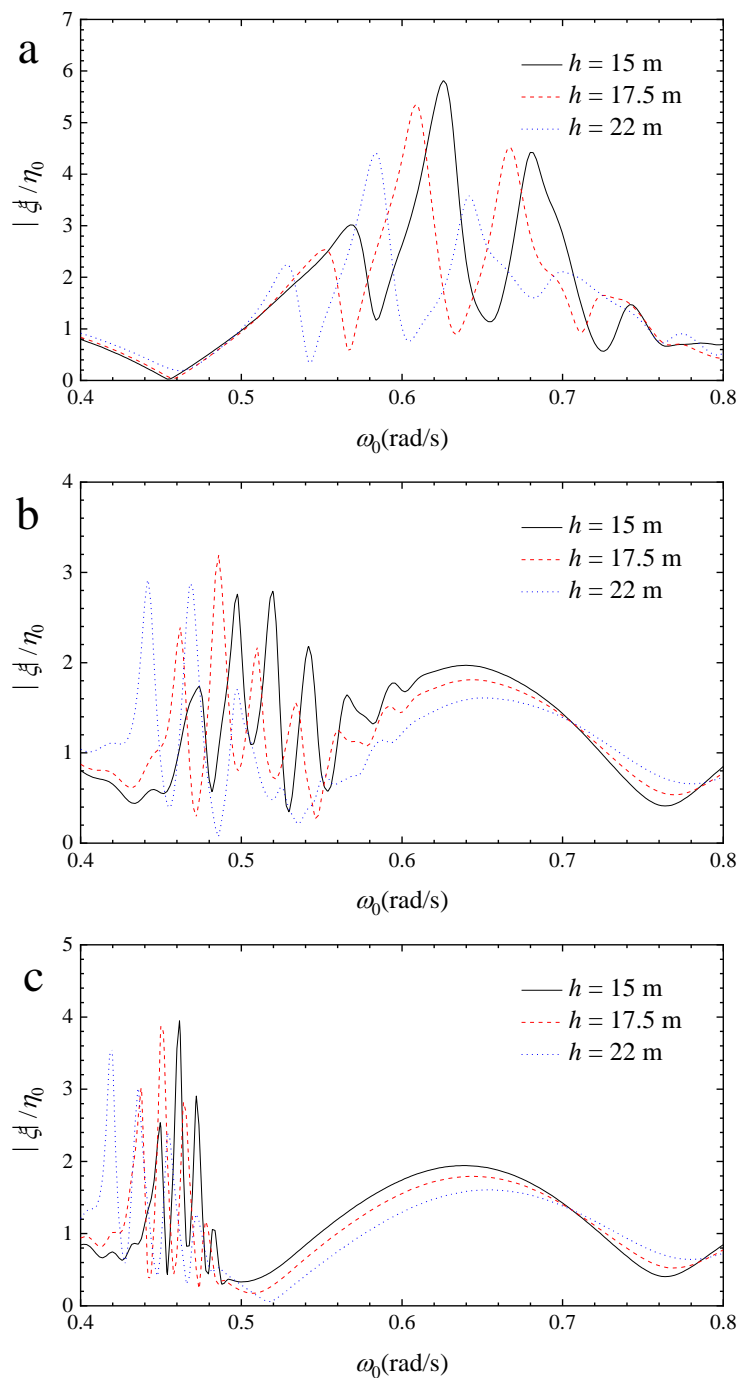
358
 359 **Fig. 15.** Transverse wave exciting forces at different gap widths. (a) $u_0 = 0$ m/s; (b) $u_0 =$
 360 3.5 m/s; (c) $u_0 = 4.5$ m/s.

361 4.3. Draft effect

362 Draft effect on the wave resonance is examined by adjusting the draft of the barges.
 363 The forward speed is also adjusted to investigate whether the draft effect will vary in the
 364 case of different forward speeds.

365 Fig. 16 presents the variation of wave elevation resonant frequencies with barge draft.

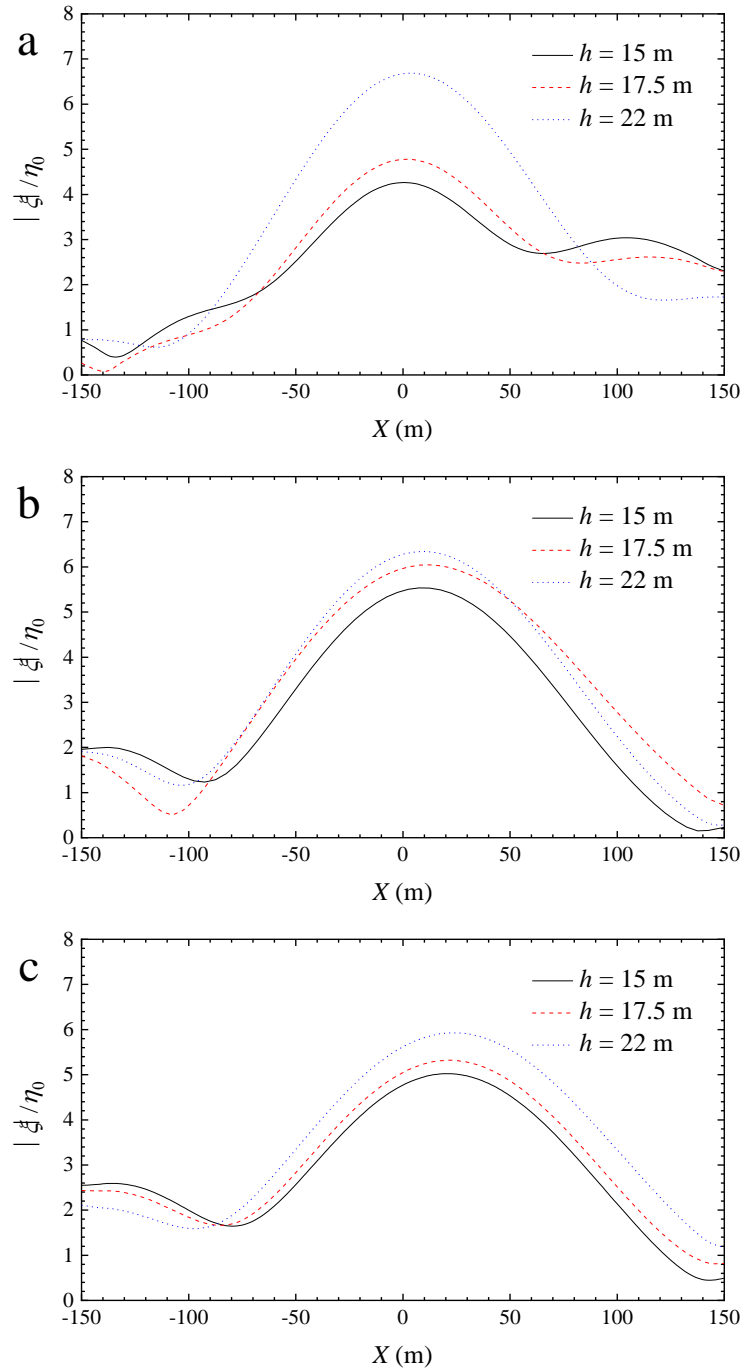
366 When the draft increases, the resonant frequencies shift to lower level. Moreover, this
 367 variation trend remains the same regardless of forward speed.



368
 369 **Fig. 16.** Resonant frequencies of wave elevation at different barge drafts. (a) $u_0 = 0$ m/s;
 370 (b) $u_0 = 3.5$ m/s; (c) $u_0 = 4.5$ m/s.

371 The dependency of resonant wave elevation on barge draft is displayed in Fig. 17. In
 372 general, resonant wave elevation increases with barge draft reflecting that the gap
 373 resonance becomes more prominent. A deeper draft makes could trap more wave energy

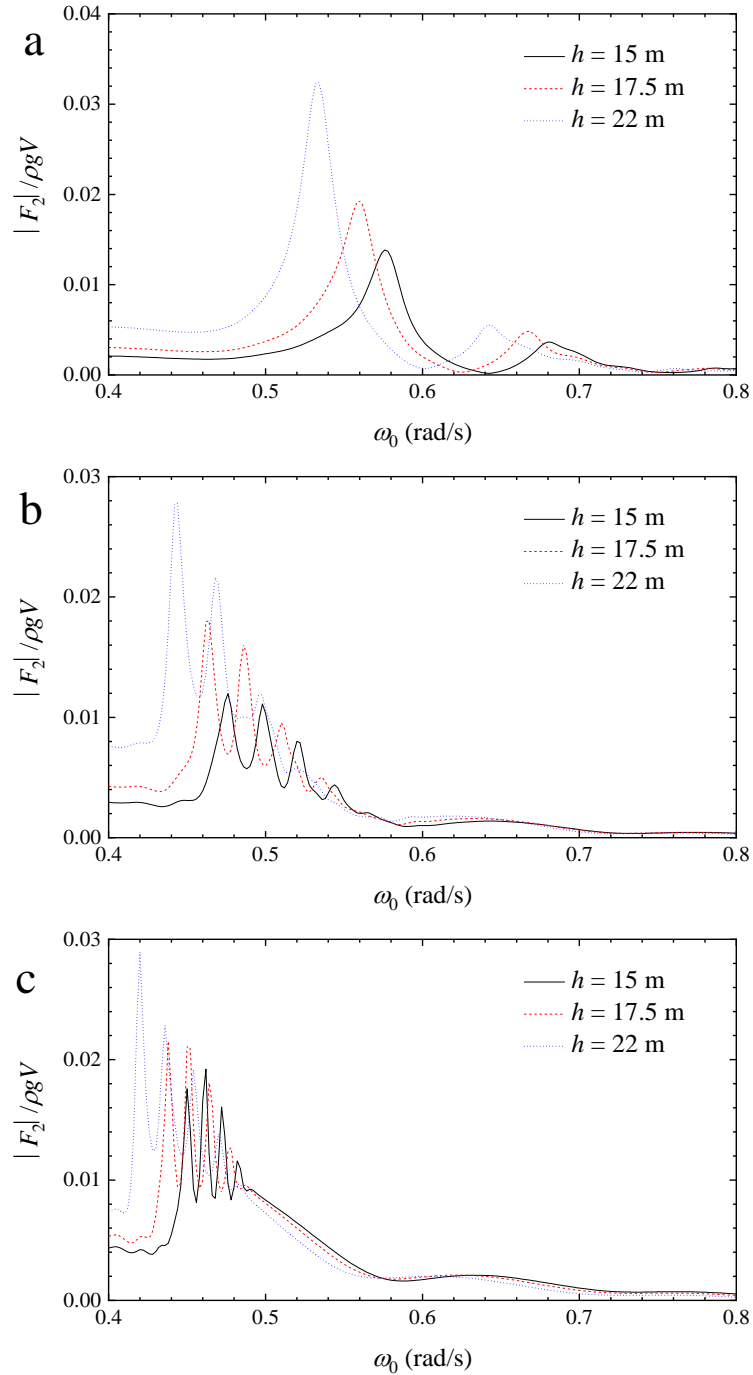
374 within the envelope. Consequently, the gap resonance becomes increasingly pronounced
 375 as a reflection of increasing amount of wave energy trapped. Once again, the variation of
 376 wave elevation with draft is independent from forward speed.



377
 378 **Fig. 17.** Dimensionless wave elevation at different drafts. (a) 1st mode, $u_0 = 0$ m/s; (b)
 379 1st mode, $u_0 = 3.5$ m/s; (c) $u_0 = 4.5$ m/s.

380 Fig. 18 displays the variation of transverse wave force as a function of incident wave
 381 frequency at different barge drafts. The transverse wave force varies with draft in a similar

382 way like resonant wave elevation. The resonant frequencies of transverse wave force shift
 383 to lower value with the increase of draft. Meanwhile, there is also a positive correlation
 384 between transverse wave force and barge draft.



385
 386 **Fig. 18.** Dimensionless wave force at different drafts. (a) $u_0 = 0$ m/s; (a) $u_0 = 3.5$ m/s; (a)
 387 $u_0 = 4.5$ m/s;

388 5. Conclusions

389 In the present study, a 3-D Rankine source method is applied to investigate the gap
390 resonance between two travelling rectangular barges arranged side-by-side with
391 concentration on the forward speed effects. The gap width effect and barge draft effect
392 are also studied as well as their coupling effects with forward speed. A modified
393 Sommerfeld radiation condition, accounting into the Doppler shift effect, is applied in
394 this paper to manage the low forward speed problem ($\tau < 0.25$). The following conclusions
395 have been drawn from the present study.

- 396 1. Forward speed effect reshaped the wave pattern within the gap and modified the
397 resonant frequencies of gap resonance. Simulation results showed that the gap
398 water surface no longer oscillated like a flexible plate in the presence of forward
399 speed and the standing wave pattern became nearly invisible. In addition, the
400 wave elevation was also reduced accounting into forward speed effect. Analysis
401 manifested that forward speed accelerated the fluid exchange between the gap
402 and outer fluid field, leading to the reduction of wave energy trapped within the
403 envelop. Consequently, the gap resonance was reduced, in terms of wave
404 amplitudes and wave exciting forces. In addition to the reduction of gap
405 resonances, forward speed also shifted the resonant frequencies to lower values
406 within a narrow frequency range. It indicated that the resonant responses can be
407 avoided by adjusting forward speed in a lightering operation.
- 408 2. Augment of gap width contributed to the reduction of gap resonance as the
409 hydrodynamic interaction between the two barges became less prominent.
410 Meanwhile, the resonant frequencies shifted to lower values.
- 411 3. More wave energy gest trapped within the gap with a deeper draft, leading to the
412 amplification of gap resonance responses. In addition, resonant frequencies
413 shifted to lower values with the augment of draft.
- 414 4. The variation trend of gap resonance with respect to gap width and draft seems
415 to be independent from the forward speed.

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