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MODELLING OF UNSTEADY, INCOMPRESSIBLE SEPARATION ON AN AEROFOIL USING AN INVISCID FLOW ALGORITHM

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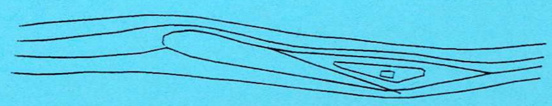

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ABSTRACT

Presented in this report is a new method for the prediction of unsteady, incompressible separated flow over a two-dimensional aerofoil. The algorithm was developed from an existing unsteady potential flow model[1] and makes use of an inviscid formulation for the flowfield. The aerofoil is represented by vortex panels of linearly varying strength which are piecewise continuous at the corners. Discrete vortices with finite cores are used to model the separating shear layers.

Following a brief summary of unsteady separation modelling, the theoretical framework is presented and the subsequent numerical implementation is discussed in detail.

Results are given for flows which tend asymptotically to the steady state and conclusions are drawn regarding the usefulness of the method.

SYMBOL GLOSSARY

A	total influence coefficient of γ_j
B	part influence coefficient of γ_j
C	part influence coefficient of γ_{j+1}
C_n	normal lift coefficient
$C_{m_{\frac{1}{4}}}$	quarter chord moment coefficient
C_p	pressure coefficient
c	aerofoil chord
D	part influence coefficient of γ_s
D_o, D_v	distance parameters associated with coalescence
d_1, d_2	distances associated with error estimation
e_v	error estimate of coalescence
G	discrete vortex coefficient
h, Δh	total head
I_1, I_2, I_3, I_4	integrals associated with vortex panel method
$\underline{i}$, $\underline{j}$	unit vectors
K	discrete vortex strength
L	aerofoil panel length
N	number of panels
N_v	number of discrete vortices
N_c	number of recently shed vortices not involved in coalescence
$\hat{n}$	unit normal vector
p	pressure
$\vec{q}$, $\vec{\Delta q}$	velocity vector
R_1, R_2	regions with different total head
$\vec{r}$	position vector
s, Δs	distance along panel
t, Δt	time
U	freestream velocity
x, y	co-ordinates of points on surface
z	complex number

GREEK SYMBOLS

Γ	circulation
γ	vorticity strength
Δ, λ	wake panel lengths
η	regulating function associated with vortex core
Θ, $\Delta\Theta$	wake panel angles
ρ	density
σ	radius of vortex core
Φ, $\Delta\Phi$, Φ_c	velocity potential
ω	vorticity

SUBSCRIPTS

a, a', b, b'	positions either side of vortex sheet
c_i	control point
i, j	index of aerofoil panels
m	time step counter
n	normal direction
p	index of wake panels
s	conditions at separation point
v	discrete vortex

1 INTRODUCTION

The study of phenomena associated with unsteady flow around aerofoils has been of consuming interest to aerodynamicists for many years. An understanding of such flows is important, for instance, in the design of helicopter rotors. In this case during forward flight transonic effects are important for the half cycle of advancing blade motion and dynamic stall is a predominant feature while the blade is retreating. The progress which has been made[1], both experimentally and computationally, in these areas is also of benefit to those considering the performance of turbomachinery and wind turbines etc.

The discrete vortex method has been applied to unsteady aerofoil problems for some time. Geising[3], Basu and Hancock[4] and the present authors[1], have used the method to predict unsteady, incompressible inviscid flows, whilst Ham[5], Baudu et al[6] and Ono et al[7] have had some success in modelling unsteady, incompressible, separated flows. Clements and Maull[8] provided an early history of the method, and made subsequent use of it to model vortex shedding from a square based body. Other more recent uses of the method have been the asymptotically steady analyses of Sarpkaya[9] and Katz[10], who considered a flat plate and a thin cambered aerofoil respectively. These latter efforts highlight the attempts that have been made to reproduce what are essentially viscous phenomena by the use of inviscid algorithms. All these incorporate the assumption that the flow is irrotational over the entire region except at the body and its wake elements. In such schemes, the vorticity shed from the body is usually derived from velocities sampled at the edges of the shear layer, an approach validated by the experiments of Fage and Johansen[11] and by the analysis of boundary layer separation on aerofoils by Sears[12,13].

Recently the detailed mathematical and numerical techniques associated with discrete vortex methods were reviewed by Leonard[14]. Application of the point vortex, vortex blob and newer contour dynamics methods to two-dimensional vortical flows were discussed as well as developments in three-dimensional vortex methods. Leonard was subsequently part of a team which incorporated the vortex blob, or core, method into a new numerical scheme for the prediction of separated flows[15].

Three versions of the original algorithm were developed; a pure vortex method, a method with added quasi-steady integral boundary layer calculations, and a method which incorporates a truly unsteady implicit finite difference boundary layer scheme. Some valuable and interesting results were presented for a range of bluff body, aerofoil and tilt rotor problems. Further developement is, however, needed, especially to improve on the drag predictions.

Presented in this report are the first results from a new method to predict the unsteady flow over an aerofoil undergoing upper surface separation. The method is of the inviscid type and uses vortices with finite cores. Reliance is not placed on the explicit evaluation of the shear velocities for the determination of the shed vorticity, which is, rather, one of the variables in a "Kutta" condition. The method was developed from an existing unsteady potential flow model[1], and the location of the separation point is a necessary input into the algorithm.

2 THEORETICAL DESCRIPTION OF MODEL

The model at time t_m, is set up as shown in figure 1. The aerofoil is represented by N panels from upper to lower trailing-edge over which there is placed a vortex sheet of linearly varying strength that is piecewise continuous at the panel corner points. With upper surface separation present, the distribution of vorticity within the separated zone is constrained to take starting and finishing values of zero. The circulation around the aerofoil is Γ_m, where $\Gamma_m = \int \gamma \, ds$, and the vorticity shed at previous times is represented by discrete vortices except in the region close to the upper surface separation point, where it takes the form of $N_p - 1$ constant strength vortex panels. Two additional constant strength vortex panels appear at time t_m, one at each separation point, to account for the latest change in aerofoil circulation, in accordance with Kelvin's theorem[16]. The strengths of the emanating sheets are determined by making use of Helmholtz's theorem[17] of continuity of vorticity which, when applied with the former theorem, results in the following condition;

$$\Delta_1 \gamma_s + \lambda \gamma_{N+1} = \Gamma_{m-1} - \Gamma_m \qquad (1)$$

where Δ_1 and λ are the lengths of the respective panels.

In order to obtain a solution for the unknown bound vortex sheet strengths, the boundary condition of zero flow normal to the surface is applied at the mid-points (control points) of the aerofoil panels resulting in the following system of equations;

$$\vec{U}.\hat{n}_i + \sum_{j=2}^{N} A_{ij}\gamma_j + A_{i1}\gamma_s + A_{iN+1}\gamma_{N+1} + \sum_{g=2}^{N_p} A_{ig}\gamma_{pg} + \sum_{g=1}^{N_v} G_{ig}K_g = 0\,, \quad i = 1,2,\ldots,N \qquad (2)$$

The second, third and fourth terms in equation (2) are the normal induced velocities at the i^{th} control point due to the bound vortex sheet and the two separating panels at time t_m respectively. These terms contain the unknown vortex stengths whereas the first, fifth and sixth terms can be completely evaluated and are the normal induced velocities at the i^{th} control point due to the freestream, the remaining wake panels and all wake vortices respectively. The theoretical details associated with equations (1) and (2) are considered in the appendix.

The expressions (1) and (2) amount to a system of N + 1 unknown γ values. However, as Δ_1 and λ are also unknown a solution can be obtained only by iteration from initial values assigned to both of these variables. It follows that the iterative scheme must incorporate some means of assigning new values to Δ_1 and λ and this is achieved by considered the Bernoulli equation as it applies to vortex sheets.

The dynamical boundary conditions for vortex sheets have been examined by Geising[18] and a similar approach is used here. If we assume that a separated wake, as illustrated in figure 2, gives rise to two isolated regions R_1 and R_2 with total heads h_1 and h_2 respectively, then the Bernoulli equation applied across each separation point yields the following results (see fig. 2);

<u>upper surface separation point</u>

$$\left(\frac{P_a}{\rho} + \frac{\gamma_a^2}{2} + \frac{\partial\phi_a}{\partial t}\right) - \left(\frac{P_a'}{\rho} + \frac{\gamma_a'^2}{2} + \frac{\partial\phi_a'}{\partial t}\right) = h_1 - h_2 = \Delta h$$

$$\Rightarrow \quad \frac{\gamma_a^2}{2} = \frac{\partial(\phi_a' - \phi_a)}{\partial t} + \Delta h = \frac{\partial\Delta\phi_a}{\partial t} + \Delta h \quad , \quad \text{with } P_a = P_a'$$

$$\Rightarrow \quad \frac{\gamma_s^2}{2} = \frac{\partial\Delta\phi_s}{\partial t} + \Delta h \qquad (3)$$

trailing-edge separation point

$$\left(\frac{p_b}{\rho} + \frac{\gamma_b^2}{2} + \frac{\partial \phi_b}{\partial t}\right) - \left(\frac{p_{b'}}{\rho} + \frac{\gamma_{b'}^2}{2} + \frac{\partial \phi_{b'}}{\partial t}\right) = h_1 - h_2 = \Delta h$$

$$\Rightarrow \quad \frac{\gamma_b^2}{2} = \frac{\partial(\phi_{b'} - \phi_b)}{\partial t} + \Delta h = \frac{\partial \Delta \phi_b}{\partial t} + \Delta h \quad , \text{ with } p_b = p_{b'}$$

$$\Rightarrow \quad \frac{\gamma_{N+1}^2}{2} = \frac{\partial \Delta \phi_{N+1}}{\partial t} + \Delta h \qquad (4)$$

subtracting (4) from (3) we get:

$$\frac{\gamma_s^2}{2} - \frac{\gamma_{N+1}^2}{2} = \frac{\partial}{\partial t}(\Delta \phi_s - \Delta \phi_{N+1}) \qquad (5)$$

In order to simplify the right hand side of equation (5) we acknowledge that:

$$\phi_{a'} - \phi_{b'} + \phi_a - \phi_{a'} + \phi_b - \phi_a + \phi_{b'} - \phi_b = 0$$

$$\Rightarrow \quad \phi_{a'} - \phi_{b'} + \phi_b - \phi_a = \Delta \phi_s - \Delta \phi_{N+1}$$

The left hand side represents the circulation around the aerofoil, Γ_m, and therefore;

$$\frac{\gamma_s^2}{2} - \frac{\gamma_{N+1}^2}{2} = \frac{\partial \Gamma_m}{\partial t} \simeq \frac{\Gamma_m - \Gamma_{m-1}}{\Delta t} \qquad (6)$$

Equation (6), which is the unsteady "Kutta" condition, can be derived by considering the boundary layer, which in this case is infinitely thin, at the separation points as was shown by Sears[12,13]. This is an example of the link between the viscous nature and inviscid dynamics of separation as the boundary layer thickness diminishes.

By examining equations (1), (3), (4) and (6) it will become apparent that the relevant iterative scheme for Δ_1 and λ is:

$$\Delta_1 = \left|\frac{\gamma_s}{2}\right| \Delta t$$

$$\lambda = \left|\frac{\gamma_{N+1}}{2}\right| \Delta t$$

Within the iterative cycle, the trailing edge panel is aligned with the local stream direction but, for numerical reasons which will be discussed later, this is not the case for the upper surface panels.

Once a converged solution has been obtained, the unsteady pressure co-efficient is determined from Bernoulli's equation. In region R_1 (see fig. 2). This is:

$$C_p = 1 - \frac{\gamma^2}{U^2} - \frac{2}{U^2}\frac{\partial \phi}{\partial t}$$

In region R_2 the equation becomes:

$$C_p = 1 - \frac{\gamma^2}{U^2} - \frac{2}{U^2}\frac{\partial \phi}{\partial t} - \frac{2}{U^2}\Delta h$$

$$= 1 - \frac{\gamma^2}{U^2} - \frac{2}{U^2}\left[\frac{\partial \phi_a}{\partial t} + \frac{\partial \Delta \phi_a}{\partial t} + \frac{\partial}{\partial t}(\phi - \phi_{a'}) + \Delta h\right]$$

$$= 1 - \frac{\gamma^2}{U^2} - \frac{2}{U^2}\frac{\partial \phi_c}{\partial t} - \frac{\gamma_s^2}{U^2}$$

$$\text{i.e.} \quad C_p = 1 - \frac{(\gamma^2 + \gamma_s^2)}{U^2} - \frac{2}{U^2}\frac{\partial \phi_c}{\partial t}$$

where Φ_c = continuous potential in region R_2.

The potential function is approximated by integrating the velocity field from upstream of the aerofoil to the leading edge and then around the surface, proceeding through the upper surface separation point in a continuous manner. The term $\frac{\partial\phi}{\partial t}$ is taken as $\frac{(\phi_m - \phi_{m-1})}{\Delta t}$, and the loads are determined by integrating the pressure distribution.

Once a complete solution has been obtained at time t_m, the model is then set up for time t_{m+1}. Existing vortices are convected to their new positions by calculating the velocities of their centres and using the first order Euler scheme:

$$\vec{r}_{v_{m+1}} = \vec{r}_{v_m} + \vec{q}_{v_m}\left(t_{m+1} - t_m\right)$$

The same scheme as above is used to convect the extra trailing-edge panel to its new position as a discrete vortex. The upper surface panels, however, are treated differently, as detailed in the next section.

3 COMPUTATIONAL DETAILS

(a) Upper Surface Separation

As illustrated in figure 1 the separation point is located on one of the aerofoil panels between two corner points as this positioning is essential if a solution is to be obtained. Restrictions which follow from this are:

(i) separation point must be kept away from the corner points, otherwise there is one less unknown and a solution cannot be obtained.

(ii) separation point must be kept away from the control points, otherwise infinite velocity components arise and the solution is meaningless.

Considering (i) and (ii) the best location for the separation point would be either at a distance of one quarter or three quarters of the panel length from one of the corner points, however numerical experiments have shown that the latter of these positions yields the most stable results. Care must also be taken with the distribution of aerofoil panels around the separation point to ensure that the distance to the control point is at least of the same order as Δ_1. If separation occurs on the first panel a fully attached potential flow solution is obtained via an existing model[1].

At the end of each time step the vorticity emanating from the upper surface does not immediately take the form of a discrete vortex but remains as a sheet for a number of time steps. The reason for this is illustrated in figure 3, where the velocity components of a constant stength vortex panel and an equivalent point vortex, placed at the centre of the panel are plotted at various stations. From this figure it may be seen, that the discrete vortex approximation to a vortex sheet is very poor close to the sheet which leads, in this case, to an erroneous solution in the wake immediately downstream of the separation point. In arriving at a method of convecting this vorticity various schemes were tried:

(i) calculate the velocity, $\overline{q}$, at each of the panel ends (taken as the mean of the control point velocities on either side) and hence compute the new length, $\Delta_{new} = \overline{q}\,\Delta t$. The vorticity was adjusted to maintain the overall panel circulation, ie,

$$\gamma_{new} = \frac{(\gamma\Delta)_{old}}{\Delta_{new}}$$

(ii) Panels were adjusted so that the vorticity strengths were the same as that at the separation point, ie, $\gamma_{P_2} = \gamma_{P_3} = \gamma_{P_4} = \ldots = \gamma_S$ The lengths were then computed from

$$\Delta_{new} = \frac{(\gamma_S\,\Delta)_{old}}{\gamma_{S\,new}}$$

(iii) Panels were convected as a whole, ie $\Delta_{new} = \Delta_{old}$, $\gamma_{new} = \gamma_{old}$

Scheme (i) proved to be too unstable when the velocity field around the separation point became erratic leading to massive fluctuations in length and vorticity. Scheme (ii) suffered from similar stability problems due to the fact that large fluctuations in γ_S were propagated immediately throughout the near wake. Greatest stability was achieved with scheme (iii) and this is due to the fact that any fluctuations in γ_S only propagate one panel at a time, thereby avoiding massive instantaneous changes in the local velocity field.

Unlike the trailing-edge panel, geometric restrictions have been introduced to control the separated upper surface panels. The angle between the first panel and the local surface tangent, Θ_p, is fixed and the angular deflection of each subsequent panel has an upper limit of $\Delta\Theta_p$.

Once the panels have been convected as described above, the outermost panel becomes a discrete vortex, except at the start when the wake contains less than N_p panels.

(b) Discrete Vortex Modelling

Initially point vortices were used to represent the shear layers. However, it was soon realised that stable solutions would not be obtained due to the singular nature of the flow in the vicinity of such vortices along with their proximity to the aerofoil surface. To overcome this problem, and obtain acceptable solutions, vortices with finite cores have been used. The resulting vorticity field can be written as follows;

$$\omega(\vec{r}) = \frac{1}{2\pi} \sum_{g=1}^{N_v} K_g \gamma_v (|\vec{r} - \vec{r}_g|) \qquad (7)$$

where the function γ_v describes the distribution of vorticity within the core and satisfies the normalising condition, $\int_0^\infty \gamma_v r\,dr = 1$
The velocity field is obtained by inserting equation (7) into the Biot-Savart equation to obtain[15]

$$\vec{q} = \frac{1}{2\pi} \sum_{g=1}^{N_v} K_g \eta(|\vec{r} - \vec{r}_g|) \begin{pmatrix} y_g - y \\ x - x_g \end{pmatrix}$$

where η is a function which makes the velocity regular throughout the core and is defined by the equation;

$$\frac{d(r^2\eta)}{dr} = r\gamma_v$$

Three types of core have been used (σ is the core radius):

(i) $\gamma_v = \frac{1}{\sigma r}$, $\vec{q} = \frac{1}{2\pi}\sum_{g=1}^{N_v} \frac{K_g \hat{r}_n}{\sigma}$ inside core ie constant velocity.

(ii) $\gamma_v = \frac{2}{\sigma^2} e^{-\frac{r^2}{\sigma^2}}$, $\vec{q} = \frac{1}{2\pi}\sum_{g=1}^{N_v} \frac{K_g}{r}(1-e^{-\frac{r^2}{\sigma^2}})\hat{r}_n$ throughout flowfield.

(iii) $\gamma_v = \frac{2}{\sigma^2}$, $\vec{q} = \frac{1}{2\pi}\sum_{g=1}^{N_v} \frac{K_g}{\sigma^2}\vec{r}_n$ inside core ie constant vorticity.

Although tests were carried out using all three of the above cores the best results have been obtained with core (iii).

Once the vortices have been released into the stream they convect according to the induced velocities at their centres. It has been found necessary, however, to impose restrictions whenever unacceptable motions occur. These motions are due to an inappropriate time step for vortices close to the surface of the aerofoil. If left unhindered these may cross over the aerofoil surface. Initially such vortices were eliminated from the computation, but this produced unacceptable peaks in circulation and lift and so a different scheme was developed whereby they were reflected from the surface. This was an improvement but did not stop the problem of some vortices settling very near to the surface, and hence not convecting downstream.

This problem has been resolved by further ensuring that all vortices are kept outwith a given distance from the surface. At present this distance has been taken to be equal to the core radius, σ, and any vortex found within this region is relocated at the limiting boundary along the normal to the surface. Vortices that are close to the separation point very often do not reach this boundary for a few time steps and in such cases the temporary limiting distance used is the maximum normal distance to the surface yet achieved. Figure 4 illustrates these restrictions.

The large amount of time expended when vortex methods are used in computations usually dictates that a limit be placed on the total number of vortices contained in the wake . This is achieved by suitable coalescence. Vortices may be coalesced for other computational reasons such as the prevention of wake disruption[9] caused by vortices of opposite sign. In the model described herein, two methods of coalescing vortices were used, one for each of two regions:

(i) within a distance, D_o, of the aerofoil surface, vortices of opposite sign which come closer than a certain distance, D_v, are coalesced into a single equivalent vortex. The total circulation is conserved but not the first moment of vorticity as this would result in the combined vortex being far removed from the immediate vicinity. Instead, the location is calculated as if both vortices were of the same sign, ie $z_3 = \frac{|K_1| z_1 + |K_2| z_2}{|K_1| + |K_2|}$ where z_3 is the new position and z_1 and z_2 are the respective positions of the two vortices.

(ii) outwith a distance, D_o, of the aerofoil surface any two vortices are coalesced if an error criterion is satisfied. The total circulation and the first moment of vorticity are conserved in the combination, which is carried out only if the error is less than a certain value, e_v. The expression used to calculate this error is similar to that used in reference 15:

$$\frac{|K_1 K_2|}{|K_1 + K_2|} \cdot \frac{|z_1 - z_2|^2}{\Delta t \, d_1^{1.5} d_2^{1.5}} < e_v$$

The two methods are needed for the following reasons: in the region close to the aerofoil it is desirable to coalesce vortices of opposite sign and this would not be a likely result of implementing method (ii) due to the error criterion; in this same region it is undesirable that vortices of the same sign be coalesced as this leads to stronger vortices and hence larger velocity gradients on the surface which can produce unstable results; in the region far from the surface the method should automatically coalesce vortices which are farther apart than those in the close-in region, and method (ii) does this. It should be noted that the most recent N_c vortices to be shed are not involved in coalescence so that the shear layer can initially remain relatively undisturbed.

(c) Miscellaneous Points

All of the results presented in the next section were obtained using a thirty panel representation of the aerofoil, as this number has been found to be satisfactory[1]. To calculate the velocity potential, a reference point is located three chord lengths upstream from the leading-edge and the change in potential calculated across each of thirty equal length panels which form a line between both points. The choice of time step was a balance between the cost of the computation, the flow resolution required and the sensitivity of the solution to the length $\Delta_1 \propto \Delta t$. For all cases here, $\Delta tU/c = 0.05$

Four iterations are carried out per time step as this number was found to be sufficient for acceptable convergence. The numerical parameters that were assigned the same value in all of the tests were;
$Np = 4$, $\Theta p = 10^o$, $\sigma = 0.05$, $D_o = 1$, $D_v = 0.1$, $e_v = 5 \times 10^{-4}$
Others are mentioned in the next section.

4 RESULTS

Figure 5 illustrates the results obtained following a step change in incidence from 0 - 18.25° for the NASA GA(W) - 1 aerofoil. For this test $\Delta\Theta_p = 0^o$ and $x_{s/c} = 0.575$. From figure 5(a) it may be seen that the wake at $tU/_c = 15$ consists of two well defined shear layers which come together a short distance downstream followed by a thin region which extends far downstream while gradually opening out. This representation compares well with other wake models [19,20], and there is no need to make initial assumptions concerning the wake shape. Figures 5(b) and 5(c) show the time dependant behaviour of the normal force and quarter chord moment. Whilst the initial response will not be physically accurate as the fixed separation point does not correctly model the true initial conditions, the approach to a steady value can be observed. The build up in pressure near the leading-edge to the steady state is particularly evident in figure 5(d) and the settled chordwise pressure distribution shown in figure 5(e), compares very favourably with the experimental data[21] ($Re = 6.3 \times 10^6$, $M = 0.15$). An isometric projection of the pressure time history is presented in figure 5 (f) and illustrates well the constant pressure region downstream of the separation point.

A step change from 0 - 20.05° was applied to the same aerofoil and figure 6 illustrates the results obtained with $\Delta\Theta_p = 0^o$ and $x_s/c = 0.475$. The shear layers in figure 6(a) enclose a larger near wake region than that which was formed in the previous case although the thin far wake is similar. The normal force and quarter chord moment approach a steady value in figures 6(b) and 6(c), and the build up in leading-edge pressure in figure 6(d) is more marked than that in figure 5(d). The good agreement of the settled solution with the measurements is evident in figure 6(e).

Figure 6(f) illustrates the pressure time history and provides a good view of the build-up to the steady state as well as the constant pressure region.

Figure 7 illustrates results obtained from a test where separation occurs near to the leading-edge after a step change in incidence from 0 - 21.14°, again using the same aerofoil. In this case $\Delta\Theta_p = 3^o$ and $X_s/c = 0.125$. From figure 7(a) it can be seen that the shear layer emanating from the upper surface starts to break up soon after it is shed and this is due to the more severe flowfield perturbations which accompany increasing amounts of separation. The result of this is that the near wake is wide and the far wake is no longer thin, exhibiting a periodic structure composed of alternately signed vortex clusters. The initial response of the normal force and quarter chord moment in figures 7(b) and 7(c) corresponds to the passage of the first vortex cluster, although the forward movement of the separation point has not been modelled. The moment exhibits more of the oscillatory nature of the flow whereas the normal force is not unduly perturbed in its approach to a steady value. Due to massive upper surface separation the behaviour of the leading-edge pressure, illustrated in figure 7(d), is markedly different from the previous cases, and the computed pressure distribution compares very favourably, figure 7(e), with the measured data. The wake pressure is not always constant due to the passage of vortices over the aerofoil, however for comparison purposes a computed pressure distribution has been chosen, near $tU/c = 20$, that exhibits the closest approximation to a uniform wake pressure. The pressure time history is shown in figure 7(f), which illustrates well the vortex shedding and subsequent passage over the aerofoil.

The final test case, illustrated in figure 8, is that of a NACA 23012 aerofoil which undergoes a step change in incidence form 0 - 18.60°, with $\Delta\Theta_p = 0^o$ and $X_{s/c} = 0.2$. Because of the lower angle of attack the wake in figure 8(a) is not as wide as that in figure 7(a), although the far wake broadens out more in this case than in either of the two cases of separation nearer the trailing edge. The shear layers break up soon after being shed and transform into alternately signed clusters which are convected downstream. The normal force, and quarter chord moment can be seen to approach a steady value in figures 8(b) and 8(c) although the moment exhibits more of the unsteady fluctuations, a feature which can be discerned from all of the results presented herein. The leading-edge pressure in figure 8(d) builds up to a final value in a fairly short time and the good correlation between the two pressure distributions can be seen from figure 8(e). For comparison purposes in this case, a "steady" calculation[20] was performed at the same angle of incidence and separation point position as those used for the "unsteady" calculation. From the three-dimensional projection in figure 8(f) it can be seen that after the passing of the initial vortex the wake pressure remains relatively constant close to the separation point but exhibits increasing amounts of unsteadiness nearer to the trailing-edge. This would suggest the presence of vortex clusters close to the aerofoil in this region.

5 CONCLUSIONS

A new method for the prediction of unsteady, imcompressible, separated flow around an arbitrary aerofoil has been developed. An inviscid formulation is used for the flowfield and the shear layers are represented by discrete vortices with finite cores. The first results of asymptotically steady separated flow about an aerofoil with a fixed separation point are most encouraging. The algorithm is thus regarded as being very useful and future work will be concerned with the incorporation of a moving separation point into the model to enable a proper investigation of aerofoil dynamic stall during ramp and oscillatory motions to be carried out.

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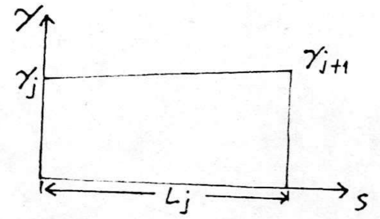
APPENDIX

(a) Vortex panel method

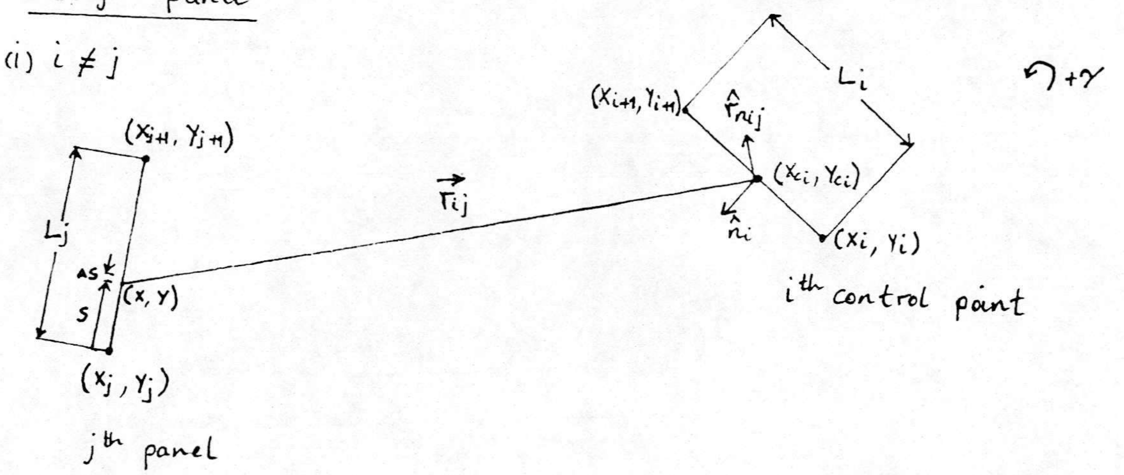


The vorticity at any point along the panel is:

$$\gamma = \gamma_j + \left(\frac{\gamma_{j+1} - \gamma_j}{L_j}\right) s$$

Derivation of the influence coefficient at the i^{th} control point due to the j^{th} panel

(i) $i \neq j$



velocity component normal to i^{th} panel induced by element of vorticity across Δs

$$|\overrightarrow{\Delta q}_{nij}| = \frac{\gamma \Delta s}{2\pi |\vec{r}_{ij}|} (\hat{r}_{nij} \cdot \hat{n}_i)$$

∴ total normal velocity component can be written:

$$|\vec{q}_{nij}| = \frac{1}{2\pi} \int_0^{L_j} \frac{\gamma (\vec{r}_{nij} \cdot \hat{n}_i)}{|\vec{r}_{ij}|^2} \, ds$$

where

$$\hat{n}_i = \frac{(y_{i+1} - y_i)\,\underline{i} + (x_i - x_{i+1})\,\underline{j}}{L_i}$$

$$\vec{r}_{nij} = (y - y_{ci})\,\underline{i} + (x_{ci} - x)\,\underline{j}$$

and $$|\vec{r_{ij}}|^2 = s^2 + bs + c$$

where $$b = -\frac{2}{L_j}\left\{(x_{ci} - x_j)(x_{j+1} - x_j) + (y_{ci} - y_j)(y_{j+1} - y_j)\right\}$$

$$c = (x_{ci} - x_j)^2 + (y_{ci} - y_j)^2$$

And so:

$$|\vec{q_{n_{ij}}}| = \frac{1}{2\pi}\int_0^{L_j} \frac{\left[\gamma_j + \frac{(\gamma_{j+1} - \gamma_j)s}{L_j}\right]\left\{\left[y_j - y_{ci} + \frac{(y_{j+1} - y_j)}{L_j}s\right]\underline{i} + \left[x_{ci} - x_j - \frac{(x_{j+1} - x_j)s}{L_j}\right]\underline{j}\right\}.\hat{n}_i}{s^2 + bs + c}\,ds$$

$$= \frac{1}{2\pi L_j}\left[(I_1\gamma_j + I_2\gamma_{j+1})\underline{i} + (I_3\gamma_j + I_4\gamma_{j+1})\underline{j}\right].\hat{n}_i \qquad (A1)$$

where

$$I_1 = \int_0^{L_j} \frac{\left[L_j(y_j - y_{ci}) + (y_{j+1} - 2y_j + y_{ci})s - \frac{(y_{j+1} - y_j)}{L_j}s^2\right]}{s^2 + bs + c}\,ds$$

$$I_2 = \int_0^{L_j} \frac{\left[(y_j - y_{ci})s + \frac{(y_{j+1} - y_j)}{L_j}s^2\right]}{s^2 + bs + c}\,ds$$

$$I_3 = \int_0^{L_j} \frac{\left[(x_{ci} - x_j)L_j - (x_{j+1} - 2x_j + x_{ci})s + \frac{(x_{j+1} - x_j)}{L_j}s^2\right]}{s^2 + bs + c}$$

$$I_4 = \int_0^{L_j} \frac{\left[(x_{ci} - x_j)s - \frac{(x_{j+1} - x_j)}{L_j}s^2\right]}{s^2 + bs + c}\,ds$$

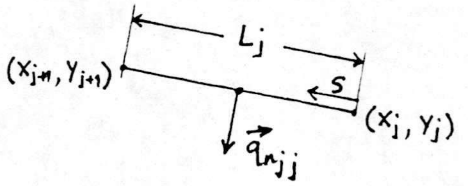
From equation A1 we can obtain the coefficients of γ_j and γ_{j+1}:

$$|\vec{q}_{n_{ij}}| = B_{ij}\gamma_j + C_{ij}\gamma_{j+1}$$

where $$B_{ij} = \frac{1}{2\pi L_j}(I_1 \underline{i} + I_3 \underline{j}).\hat{n}_i$$

$$C_{ij} = \frac{1}{2\pi L_j}(I_2 \underline{i} + I_4 \underline{j}).\hat{n}_i$$

(ii) $i = j$



$$|\vec{q}_{n_{jj}}| = \int_0^{L_j} \frac{\gamma\, ds}{2\pi(\frac{L_j}{2} - s)}$$

$$= \frac{1}{2\pi}\int_0^{L_j} \frac{\left[\gamma_j + \frac{(\gamma_{j+1} - \gamma_j)s}{L_j}\right]}{\frac{L_j}{2} - s}\, ds$$

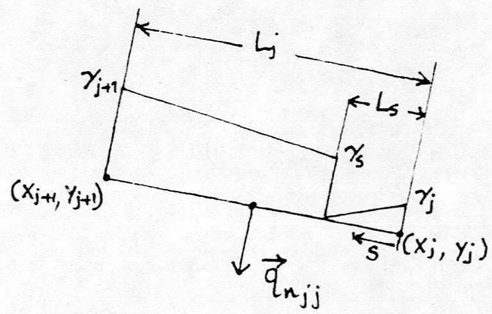
$$= \frac{1}{2\pi}\left\{\left[-\gamma_j \ln\left|\frac{L_j}{2} - s\right|\right]_0^{L_j} - (\gamma_{j+1} - \gamma_j)\int_0^{L_j}\left(\frac{1}{L_j} - \frac{1}{L_j - 2s}\right) ds\right\}$$

$$= \frac{1}{2\pi}(\gamma_j - \gamma_{j+1})$$

$$= B_{jj}\gamma_j + C_{jj}\gamma_{j+1}$$

where $B_{jj} = \frac{1}{2\pi}$, $C_{jj} = -\frac{1}{2\pi}$

(iii) $i = j$ and separation occurs on panel.



$$|\vec{q}_{n_{jj}}| = \int_0^{L_s} \frac{\gamma\, ds}{2\pi\left(\frac{L_j}{2} - s\right)} + \int_{L_s}^{L_j} \frac{\gamma\, ds}{2\pi\left(\frac{L_j}{2} - s\right)}$$

$$= \frac{\gamma_j}{2\pi} \int_0^{L_s} \frac{\left(1 - \frac{s}{L_s}\right)}{\left(\frac{L_j}{2} - s\right)} ds + \frac{1}{2\pi} \int_{L_s}^{L_j} \frac{\left[\gamma_s + \left(\frac{s - L_s}{L_j - L_s}\right)(\gamma_{j+1} - \gamma_s)\right]}{\frac{L_j}{2} - s} ds$$

$$= \frac{1}{2\pi} \left\{ \gamma_j \left[1 + \left(\frac{L_j}{2L_s} - 1\right) \ln\left|\frac{L_j - 2L_s}{L_j}\right|\right] - \gamma_s \ln\left|\frac{L_j}{L_j - 2L_s}\right| - \left(\frac{\gamma_{j+1} - \gamma_s}{L_j - L_s}\right)\left[L_j - L_s + \left(\frac{L_j}{2} - L_s\right) \ln\left|\frac{L_j}{L_j - 2L_s}\right|\right.\right.$$

$$= B_{jj}\gamma_j + C_{jj}\gamma_{j+1} + D_{jj}\gamma_s$$

where

$$B_{jj} = 1 + \left(\frac{L_j}{2L_s} - 1\right) \ln\left|1 - \frac{2L_s}{L_j}\right|$$

$$C_{jj} = -\left[1 + \left(\frac{\frac{L_j}{2} - L_s}{L_j - L_s}\right) \ln\left|\frac{L_j}{L_j - 2L_s}\right|\right]$$

$$D_{jj} = 1 - \left(\frac{\frac{L_j}{2}}{L_j - L_s}\right) \ln\left|\frac{L_j}{L_j - 2L_s}\right|$$

(b) Matrix of coefficients

After the boundary conditions have been applied at each of the control points there are N equations in N+1 unknowns:

$$A_{11}\gamma_s + A_{12}\gamma_2 + A_{13}\gamma_3 + \dots\dots + A_{1N-1}\gamma_{N-1} + A_{1N}\gamma_N + A_{1N+1}\gamma_{N+1} = a_1$$

$$\vdots \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \vdots$$

$$A_{N1}\gamma_s + A_{N2}\gamma_2 + A_{N3}\gamma_3 + \dots\dots + A_{NN-1}\gamma_{N-1} + A_{NN}\gamma_N + A_{NN+1}\gamma_{N+1} = a_N$$

(Note that coefficients of γ_s are placed in column 1 as $\gamma_1 = 0$)

The necessary extra equation comes from specifying the shed circulation:

$$\Delta_1\gamma_s + \lambda\gamma_{N+1} = \Gamma_{m-1} - \Gamma_m$$

$$\Rightarrow \Delta_1\gamma_s + \lambda_1\gamma_{N+1} + \frac{1}{2}\sum_{\substack{j=1 \\ j\neq N_s}}^{N}(\gamma_j + \gamma_{j+1})L_j + \frac{\gamma_{N_s}}{2}L_s + \frac{(\gamma_s + \gamma_{N_s+1})(L_{N_s} - L_s)}{2} = \Gamma_{m-1}$$

$$\Rightarrow \left[\Delta_1 + \frac{(L_{N_s} - L_s)}{2}\right]\gamma_s + \frac{(L_1 + L_2)}{2}\gamma_2 + \dots\dots + \frac{(L_{N_s-1} + L_s)}{2}\gamma_{N_s} + \frac{(L_{N_s} - L_s + L_{N_s+1})}{2}\gamma_{N_s+1}$$

$$+ \dots\dots + \frac{(L_{N-1} + L_N)}{2}\gamma_N + \left(\frac{L_N}{2} + \lambda\right)\gamma_{N+1} = \Gamma_{m-1}$$

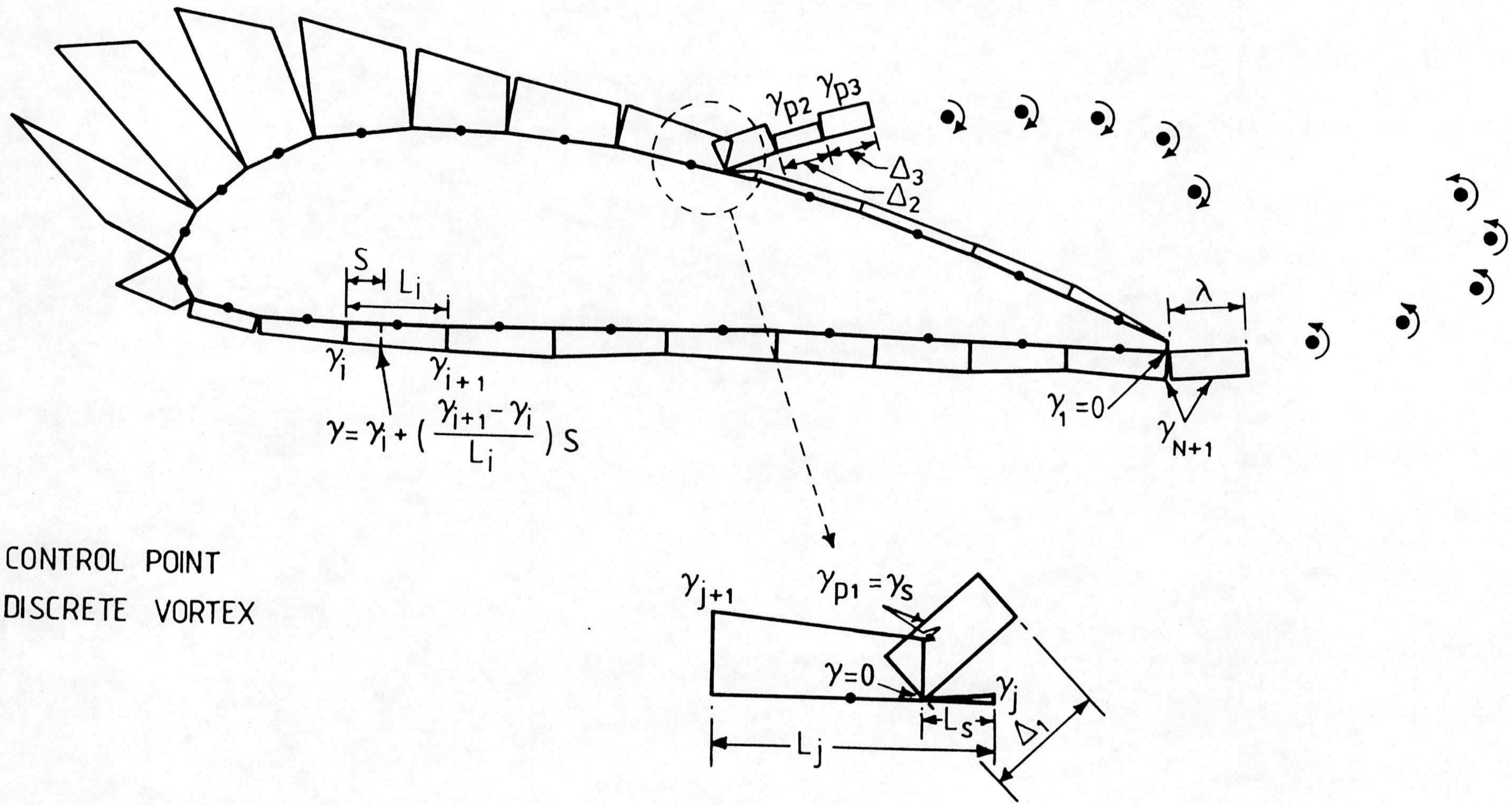


Fig.1 UNSTEADY SEPARATION MODEL AT TIME t_m

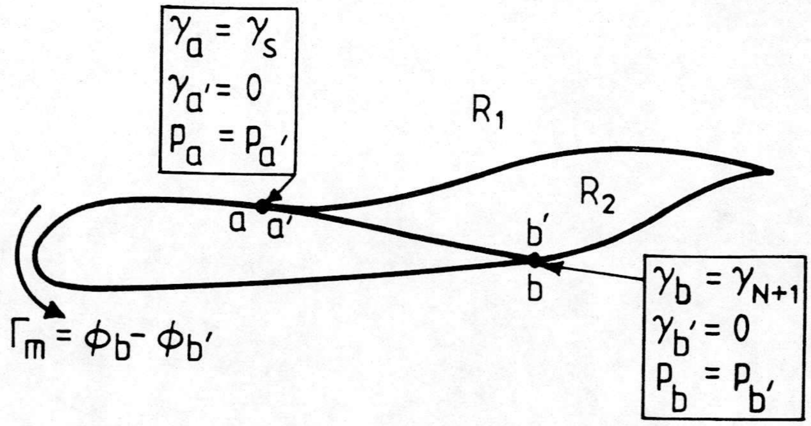


Fig. 2. INVISCID FORMULATION

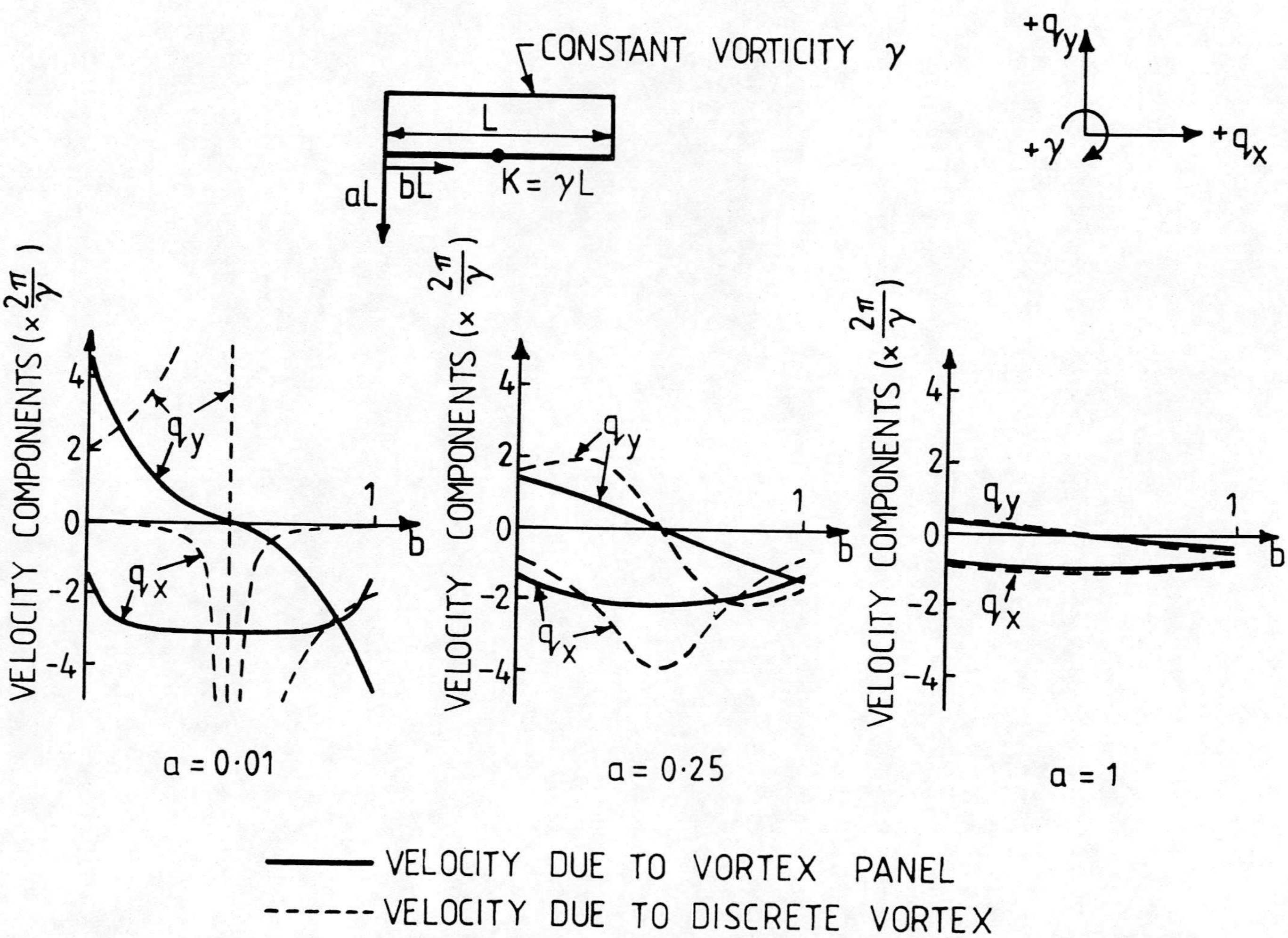
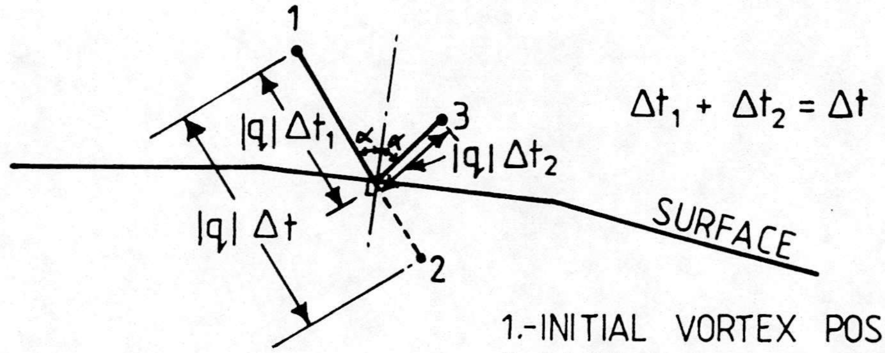
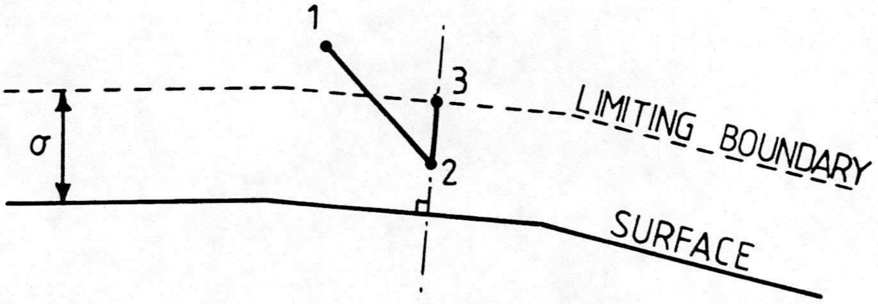


Fig. 3. COMPARISON BETWEEN THE LOCAL VELOCITY FIELDS INDUCED BY A VORTEX PANEL AND AN EQUIVALENT DISCRETE VORTEX



1.-INITIAL VORTEX POSITION
2.-ILLEGAL VORTEX POSITION ACROSS SURFACE
3.-FINAL VORTEX POSITION

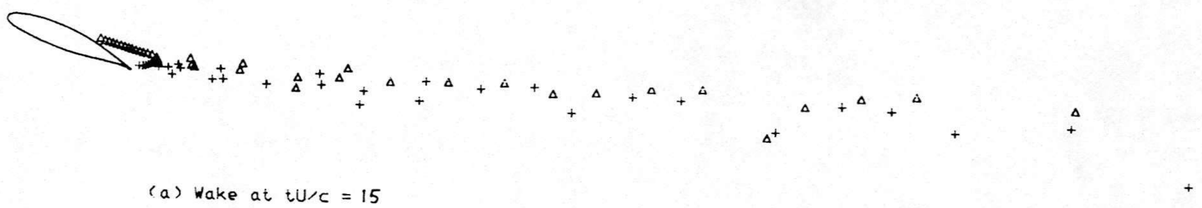
(a). REFLECTION FROM SURFACE



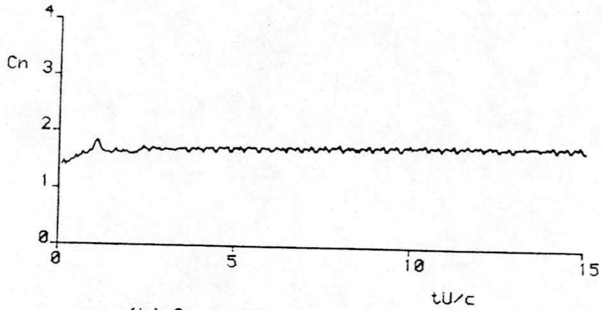
1.-INITIAL VORTEX POSITION
2.-ILLEGAL VORTEX POSITION WITHIN BOUNDARY
3.-FINAL VORTEX POSITION

(b.) RELOCATION AT BOUNDARY

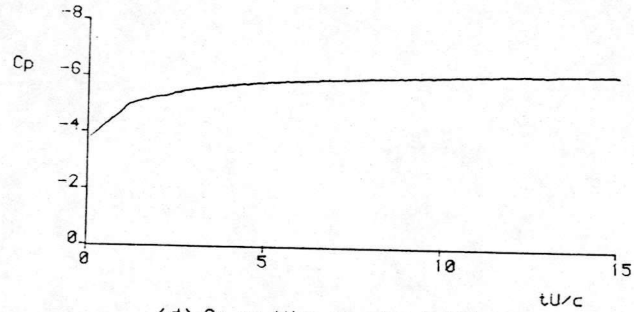
Fig. 4. RESTRICTIONS ON VORTEX MOTION



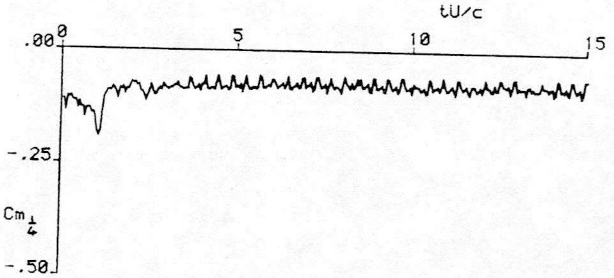
(a) Wake at tU/c = 15
△ clockwise circulation + anticlockwise circulation



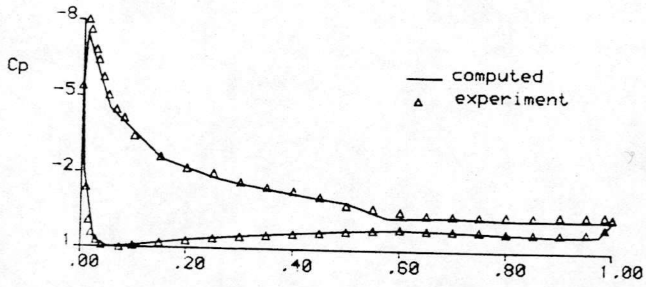
(b) Cn vs tU/c



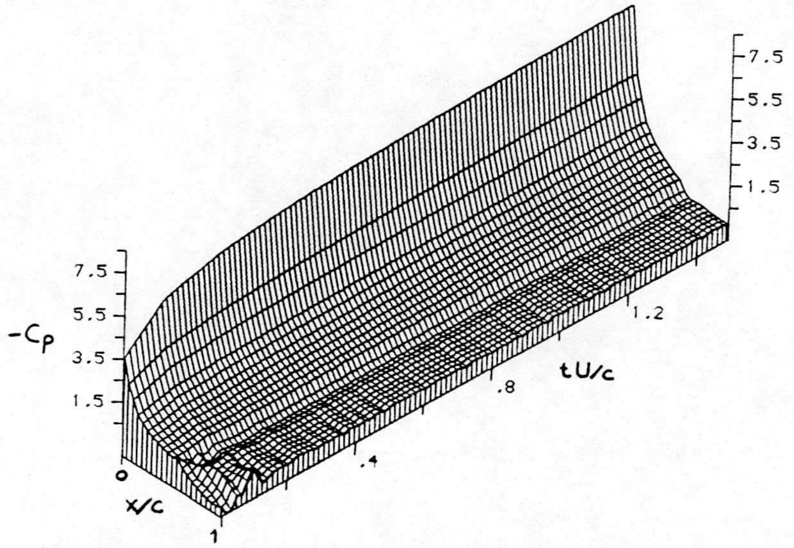
(d) Cp vs tU/c at x/c =0.025



(c) $Cm_{\frac{1}{4}}$ vs tU/c



(e) Comparison between computed and steady experimental Cp



(f) Pressure-Time history.

FIG 5. RESULTS OBTAINED FOLLOWING A STEP CHANGE IN INCIDENCE FROM 0-18.25 DEG. USING THE GA(W)-1 AEROFOIL

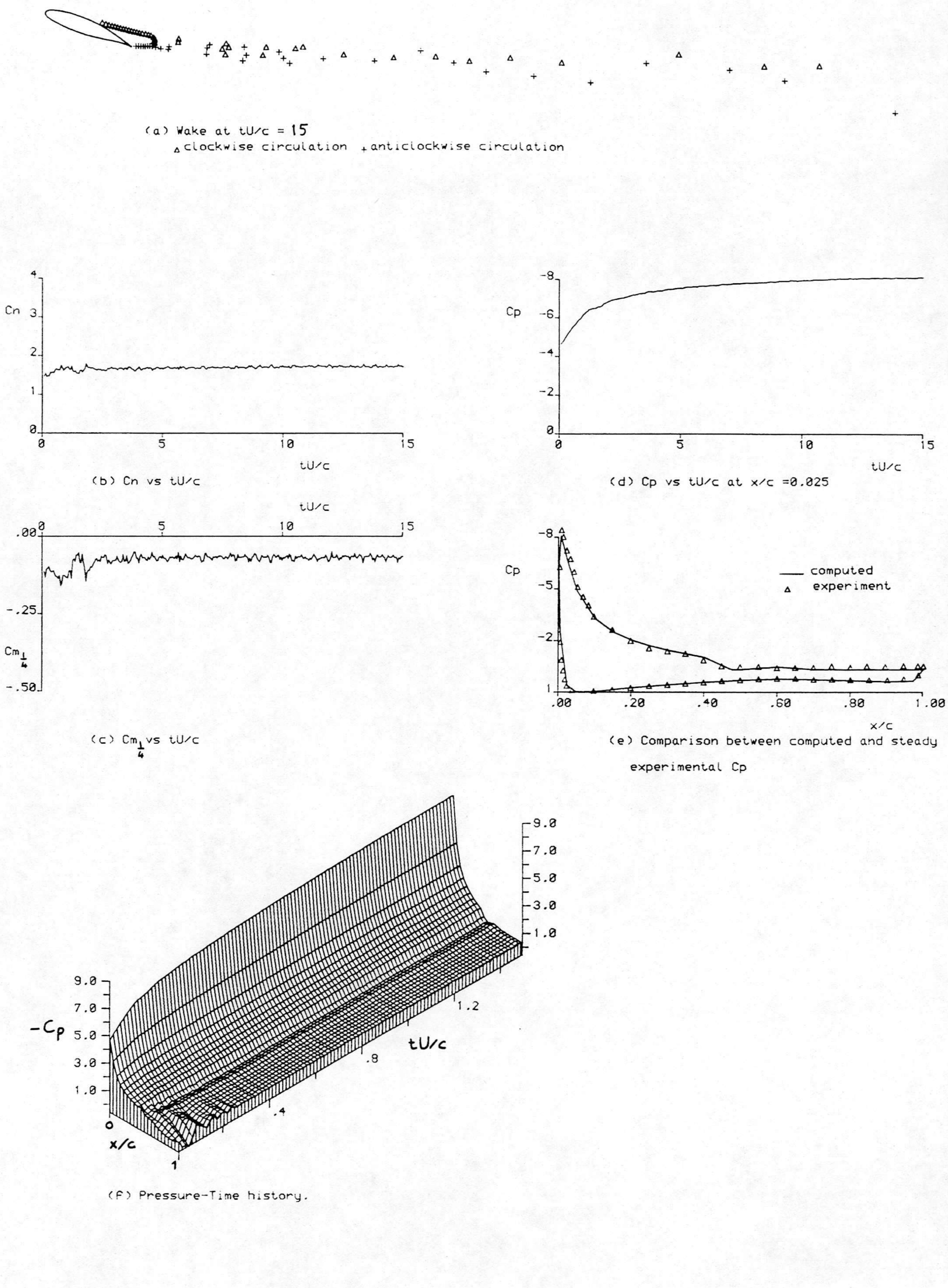
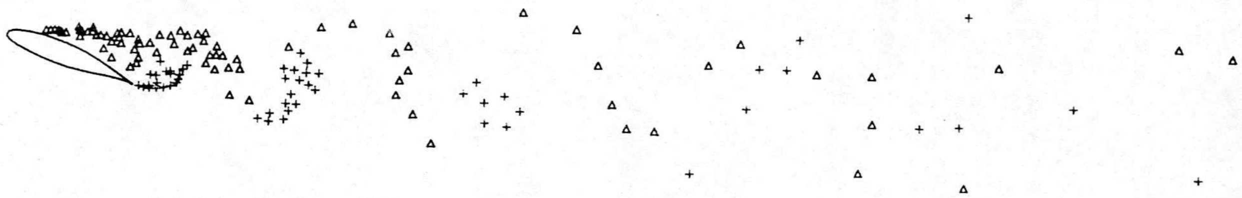
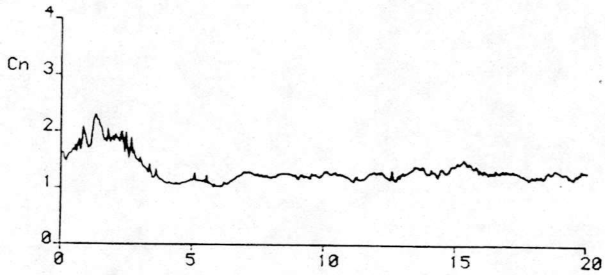


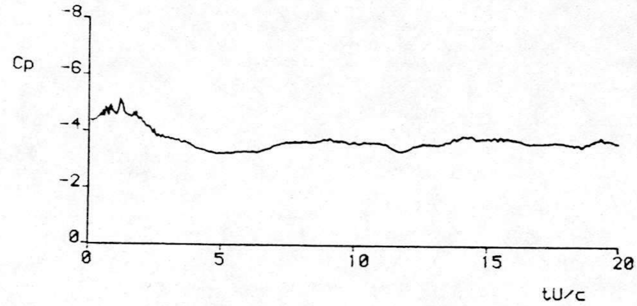
FIG 6. RESULTS OBTAINED FOLLOWING A STEP CHANGE IN INCIDENCE FROM 0-20.05 DEG. USING THE GA(W)-1 AEROFOIL



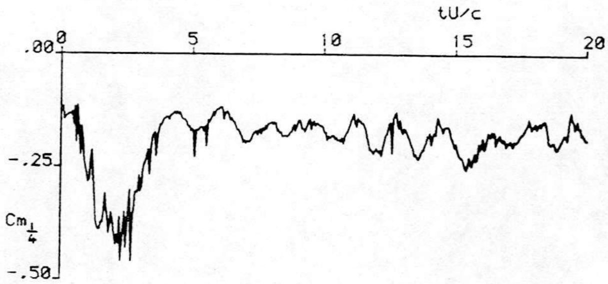
(a) Wake at tU/c = 20
△ clockwise circulation + anticlockwise circulation



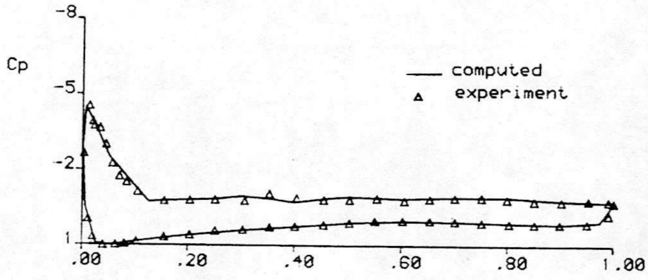
(b) Cn vs tU/c



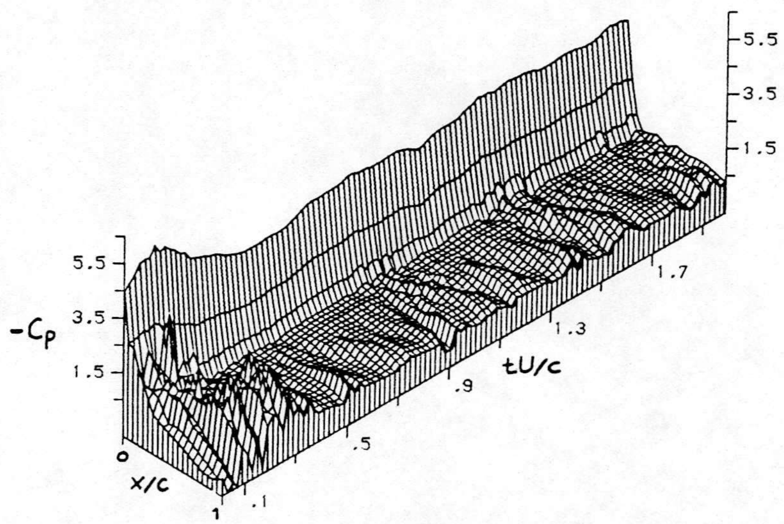
(d) Cp vs tU/c at x/c =0.025



(c) $Cm_{\frac{1}{4}}$ vs tU/c

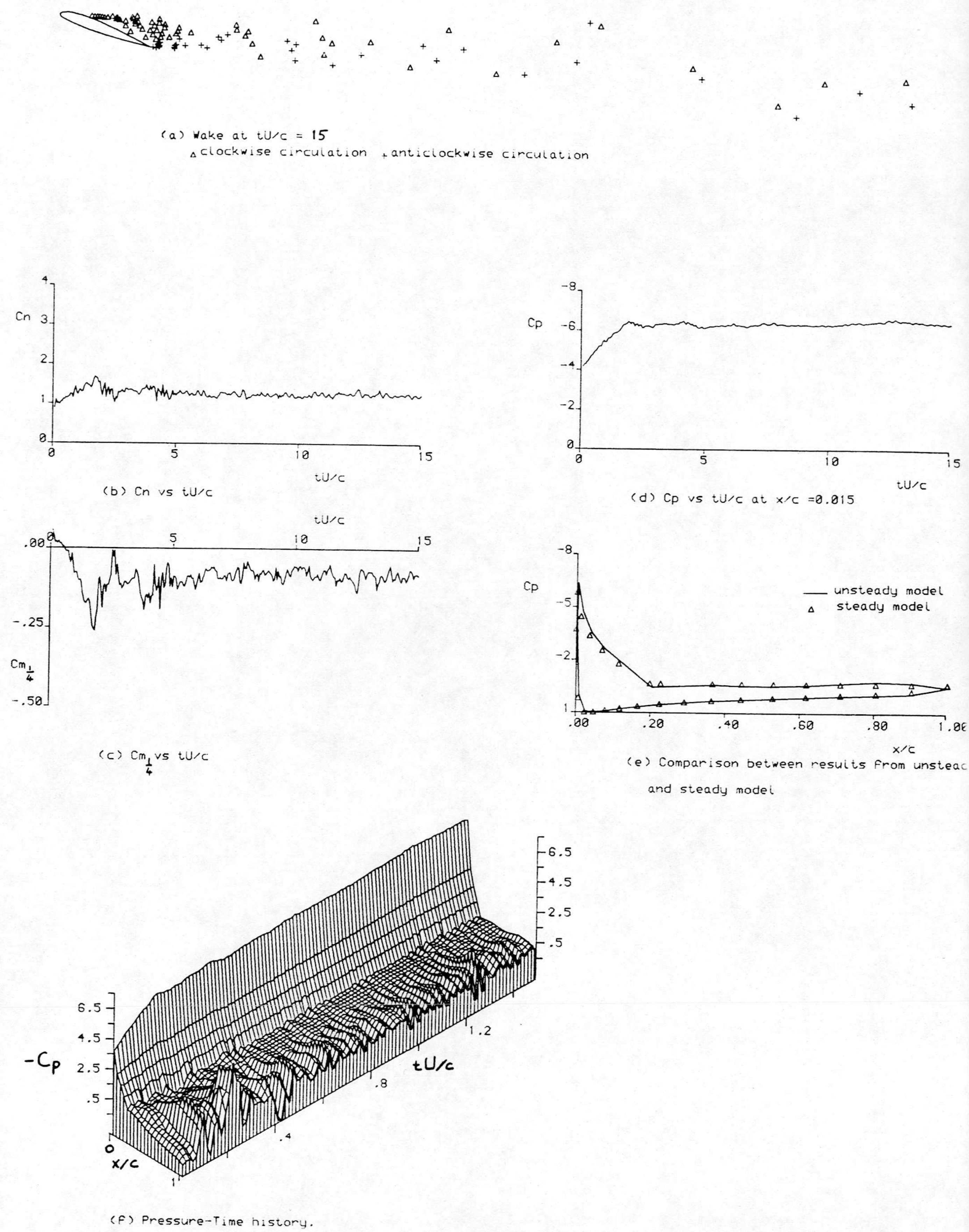


(e) Comparison between computed and steady experimental Cp



(f) Pressure-Time history.

FIG 7. RESULTS OBTAINED FOLLOWING A STEP CHANGE IN INCIDENCE FROM 0-21.14 DEG. USING THE GA(W)-1 AEROFOIL



(a) Wake at tU/c = 15
△ clockwise circulation + anticlockwise circulation

(b) Cn vs tU/c

(c) $Cm_{\frac{1}{4}}$ vs tU/c

(d) Cp vs tU/c at x/c =0.015

(e) Comparison between results from unsteac
and steady model

(f) Pressure-Time history.

FIG 8. RESULTS OBTAINED FOLLOWING A STEP CHANGE IN INCIDENCE FROM 0-18.60 DEG. USING THE NACA 23012 AEROFOIL