AN ASPECT OF AEROFOIL LIFT AND PRESSURE DISTRIBUTION

Lecture given to the Glasgow University
Maclaurin Society

on 17th November, 1982

by

Dr. R.A.McD. Galbraith

G.U.Aero.Rept. 8207. Nov., 1982.

R 8207.

AN ASPECT OF AEROFOIL LIFT AND PRESSURE DISTRIBUTION

Lecture given to the Glasgow University
Maclaurin Society

on 17th November, 1982

by

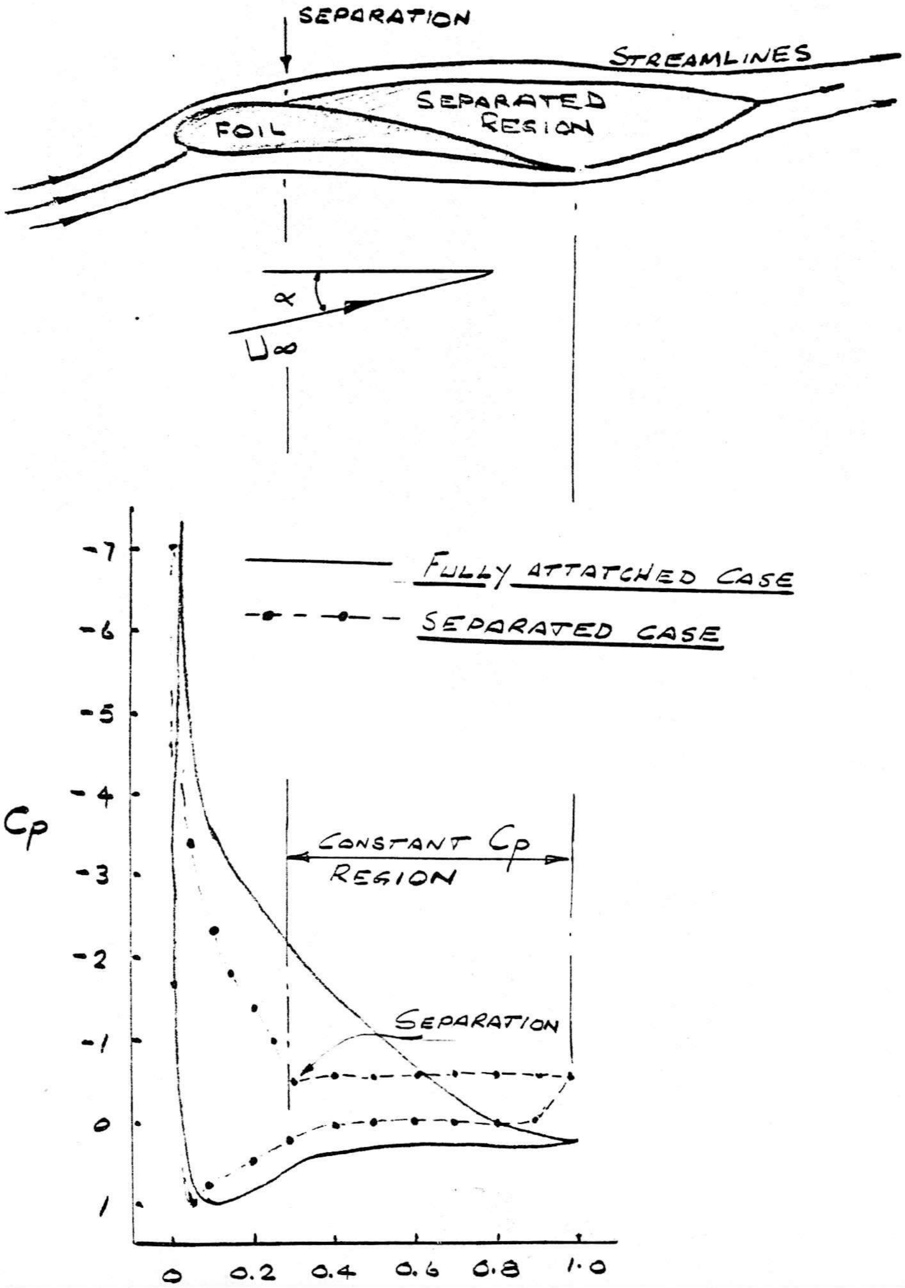
Dr. R.A.McD. Galbraith

G.U.Aero.Rept. 8207. Nov., 1982.

INTRODUCTION

First, may I thank you all for your invitation and kind introduction. I have never lectured to mathematicians before and, not being of that discipline, it is both a little worrying and interesting. As you know, engineers are not mathematicians, but in general, they must possess a working knowledge of particular aspects of mathematics. Indeed, history has shown that in the field of aeronautics, many mathematicians have become engineers and many engineers mathematicians.

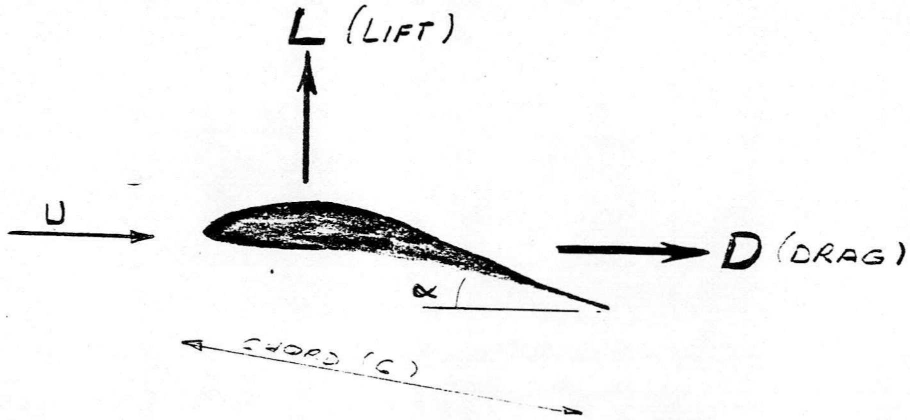
In the next half hour or so I hope to describe one of my department's current interests, that of modelling the flow past an aerofoil at angles of attack where upper surface trailing edge separation is present. The salient features of such a flow are as shown.



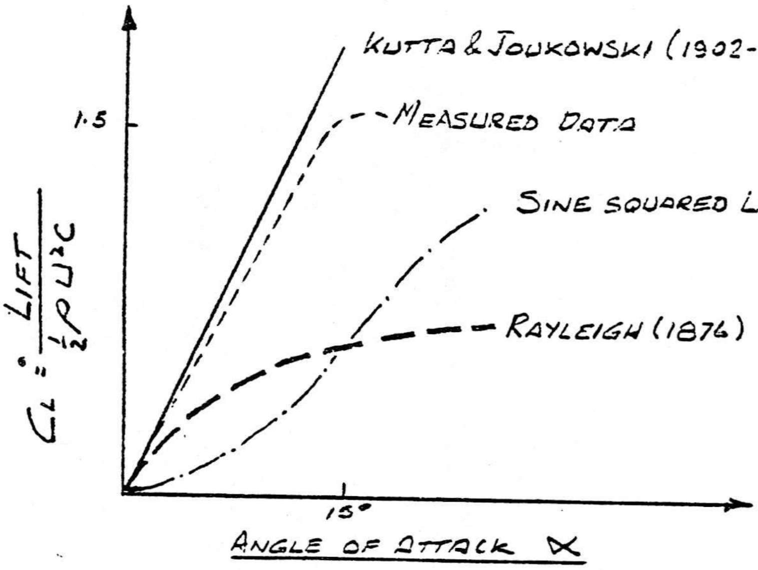
The important features to note in the diagram are that on the top surface the flow, after separation, does not follow the aerofoil contour and associated with this is a highly turbulent region, termed a wake, in which the surface pressure is observed to be a constant.

Before considering the mathematical details of method of modelling used at Glasgow, I would like to give you a very brief and incomplete history of the initial developments in aerofoil theory. These, of course, were uniquely related to the quest for powered flight by the early aviators. It was at the beginning of the nineteenth century that the first of many gifted men (Sir George Cayley) succinctly articulated the problem of mechanical flight in a statement still valid today. It was, he said, "that of making a surface support a weight by the application of power to counterbalance the resistance of air".

That surface, of which he talked, was, of course, the aerofoil. He further recognised that, when an aerofoil was immersed in a moving fluid, the resultant force on it could be decomposed into a force transverse to the direction of flow called the LIFT and an in-line force called the DRAG.



The drag force is particularly important for in the field of classical hydrodynamics where the viscous nature of the fluid is ignored, no such force exists. This was not as observed and the rather awkward conflict is termed D'alemberts paradox. Moreover, the early theories of lift were also in very poor agreement with the observations, as can be seen from the diagram.

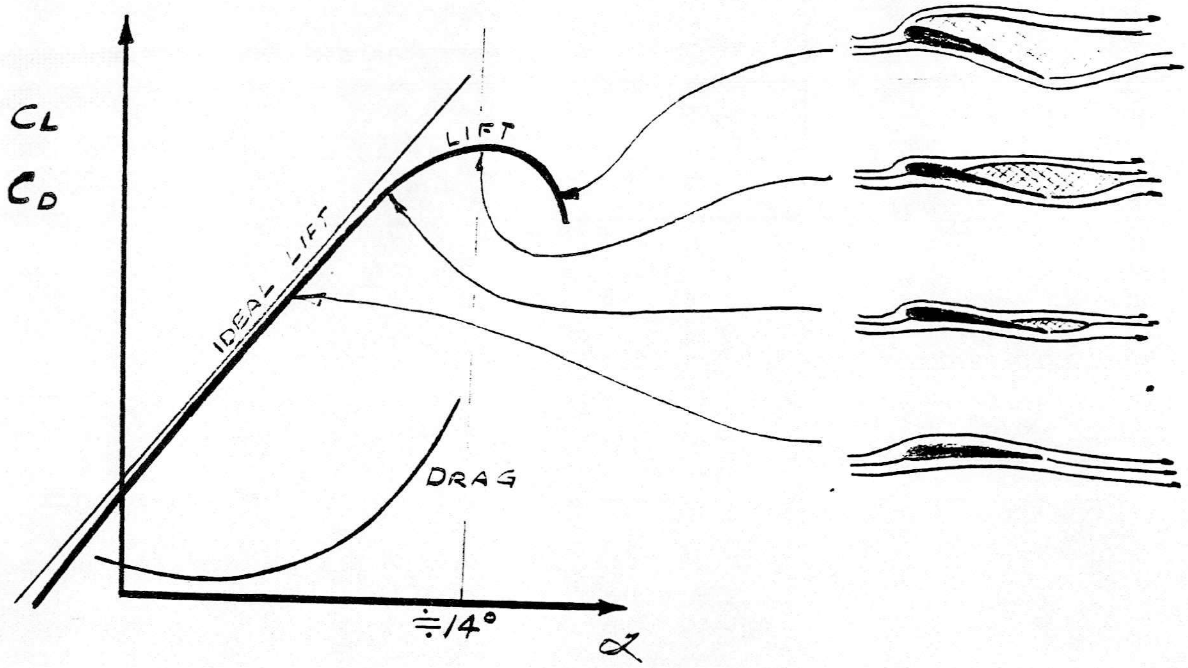


It is therefore hardly surprising that the early aviators relied and developed an empirically based discipline (hydraulics) to design fluid dynamic machines and aircraft. Indeed, the first powered flight of an aircraft was made in December 1903 by the Wright brothers at Kill Devil Hills, two years before the modern theory of lift (based on classical hydrodynamics) was first published. It was also three years before the conflict between classical hydrodynamics and hydraulics was resolved by introducing the concept of boundary layers which were used to explain the drag force and the phenomenon of flow separation.

Since those early days of powered flight, a mere eighty years or so ago, there has been a staggering pace of development.

The Kitty Hawk's (the Wright's plane) first flight was at around 30 mph at a height of 10 feet and a duration of 54 seconds. Today we have exceeded the speed of sound and reached altitudes far beyond the bounds of aerodynamic flight. All of this development has been based on an exquisite blend of speculation, empiricism and theory.

Even although such advances have been made we are still unable to accurately predict the characteristics of an arbitary aerofoil. High speed digital processors have, however, made possible the treatment of some interesting cases. Typical of these flows is that of an aerofoil prone to trailing edge separation and the statically measured characteristics of these foils are, in general, as indicated.



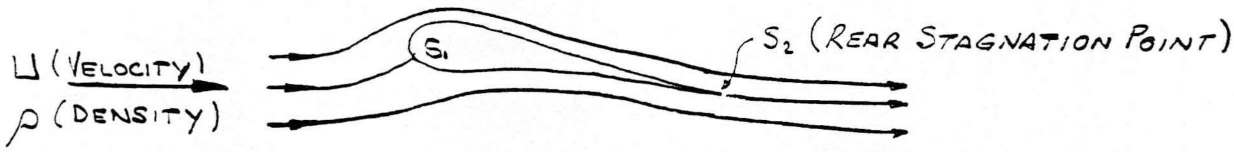
It may be seen that as the angle of attack (α) increases, the measured data deviates from the classical theory and although these departures are initially small there is an obvious breakdown of agreement for angles in excess of around 14°. Corresponding to the obvious limit of the lift there is a steep rise in the amount of drag. These limits are normally referred to as stall and the curves drawn are typical of turbulent stall. The generally accepted view on the flow development during such a stall is that, as shown above, the flow on the upper surface breaks away progressively from the trailing edge towards the leading edge. A fully stalled condition arises when the entire upper surface lies within the separated region.

This, then, is the flow for which the model I now wish to describe has been developed.

DESCRIPTION OF THE MODEL

In the fully attached case of aerofoil flow the breakthrough in predicting the lift arrived when Kutta suggested that the flow should leave the trailing edge smoothly. This is now termed the Kutta condition and is generally stated, that when an aerofoil is placed in a moving stream, it will generate around itself sufficient circulation to place the near stagnation point on the trailing edge.

Pictorially this is as shown :



This then sets a unique value on the circulation (K) and, independent of the aerofoil shape, the lift is simply given as

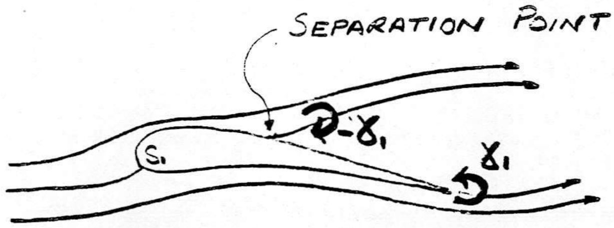
$$L = \rho U K$$

with the circulation being defined as

$$K = \oint V ds \quad \text{(V is the tangential velocity)}$$

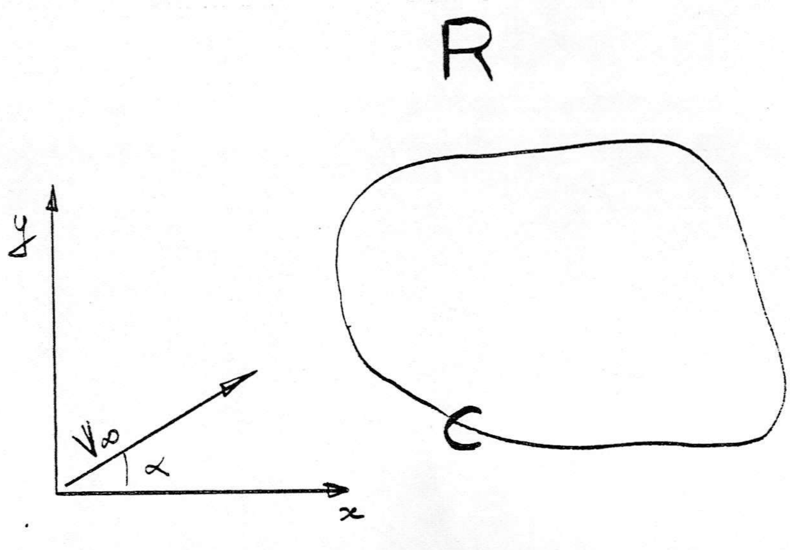
It is the viscous nature of the flow which generates the circulation but when the value is known then the classical hydrodynamic theory may be used.

For the aerofoil with trailing edge separation, this condition no longer applies and an alternative method of determining the circulation must be used. It is now common to accept that the vorticity shed at the trailing edge and the separation point be equal in magnitude but opposite in sign.



This is very similar to the classical Kutta condition and is often referred to as the modified Kutta condition. The difference is simply that the classical Kutta condition refers to the limit when the separation point and the trailing edge co-incide. It may therefore be considered that it is the location of the separation point that determines the circulation and hence lift. The main difficulty here is that separation is exceedingly difficult to predict and so, for the purposes of describing the model, it will be treated as an empirical input.

For the fully attached case the mathematical description of problem is to calculate the potential flow in a region R exterior to a simple impermeable contour C, i.e.,



The fluid velocity at any point is given by

$$\bar{V} = \bar{V}_\infty + \bar{v}$$

where $\bar{V}_\infty$ is the incident uniform flow and $\bar{v}$ the perturbation velocity due to the presence of C.

For such a flow

$$\bar{V} = \text{grad}\ \Phi$$

and $$\nabla^2\Phi = 0$$

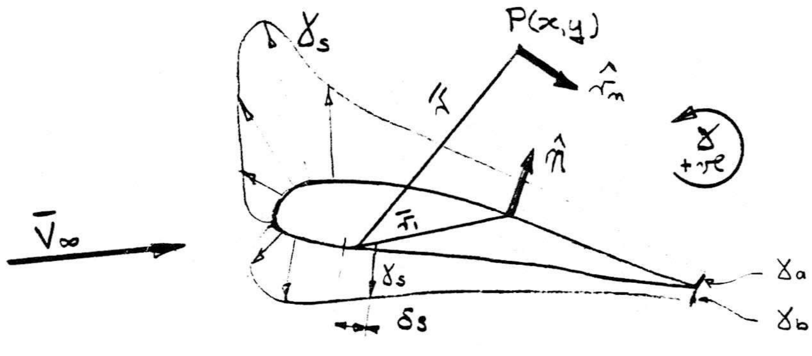
where Φ is some potential function,

and finally on the contour

$$\frac{\partial\Phi}{\partial n} = \text{grad}\ \Phi \cdot \hat{n} = \bar{V}\cdot\hat{n} = 0$$

(Neumann boundary condition)

One method of satisfying the above is to replace the contour C by a vortex sheet of variable unknown strength (γ_s) as shown.



It is in the nature of vortex sheets that the induced velocity $\overline{\delta v}$ at point P due to an element δ_s is given by

$$\overline{\delta v} = \frac{\gamma_s \delta_s}{2\pi |r|} \hat{r}_n$$

and hence $$\bar{v} = \oint_c \frac{\gamma_s}{2\pi |r|} \hat{r}_n \, ds$$

and so the total velocity at P is

$$\bar{V} = V_\infty + \bar{v}$$

hence the normal velocity to the contour at 1 may be obtained from

$$\bar{V}_n = \bar{V} \cdot \hat{n}$$

and since the contour is impermeable

$$\bar{V}_n = 0$$

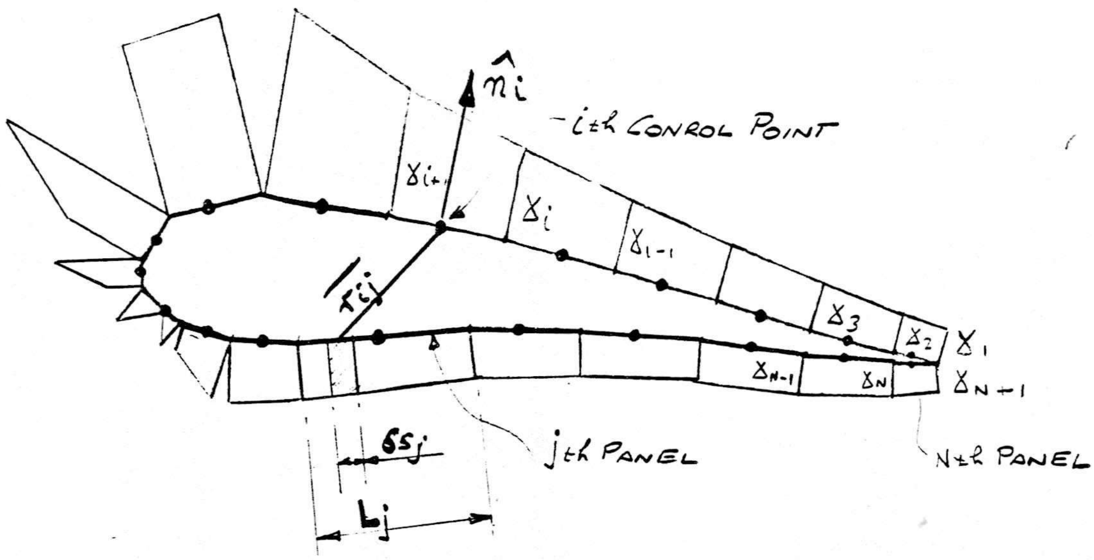
For an exact solution, the above equation must be satisfied at all points on the contour and as described there is no unique solution unless an additional condition is imposed. That condition is the Kutta condition and simply means that the net vorticity at the trailing edge is zero, i.e.,

$$\gamma_a = \gamma_b$$

If the function γ_s is obtainable then the surface velocity may be obtained from

$$V = |\gamma_s|$$

Analytically this is very difficult to solve and a numerical procedure is normally adopted in which the continuous vortex sheet is represented by a finite number of straight segments (called panels) with a linear distribution of sheet strength as shown.



At the centre of each panel is a control point at which the Neumann boundary condition is to be satisfied.

Now the jth panel will induce a velocity at the i th control point given by

$$\bar{v}_{i_j} = \int_0^{L_i} \frac{\gamma_s \, ds_j}{2\pi |r_{i_j}|} \, \hat{r}_{ni_j}$$

and so the induced normal velocity is simply

$$\overline{v_{r_{i_j}}} = \overline{v_{i_j}} \cdot \hat{n}_i$$

Each panel contributes to normal velocity at the i th control point and so

$$\overline{v_{n_i}} = \sum_{j=1}^{N} \overline{v_{n_{i_j}}}$$

which may be written in the form

$$\overline{v_{n_i}} = \frac{1}{2\pi} \sum_{j=1}^{N} C_{i_j} \gamma_j$$

where C_{i_j} are termed the influence coefficients.

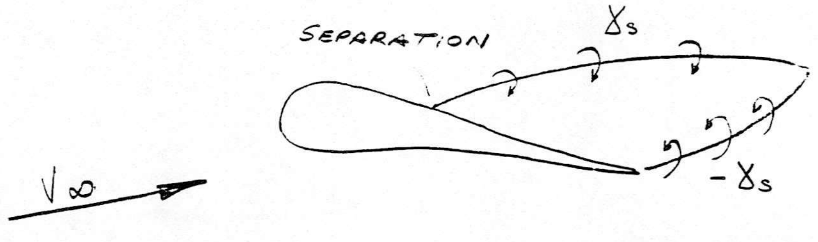
Thus for each control point

$$\frac{1}{2\pi} \sum_{j=1}^{N} C_{i_j} \gamma_j + \overline{V_\infty} \cdot \widehat{n_i} = 0$$

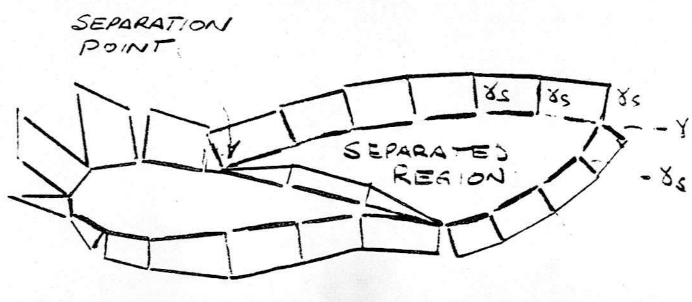
This represents N linear equations with N + 1 unknowns and the final equation follows from the Jutta condition expressed as

$$\gamma_1 + \gamma_{N+1} = 0$$

The extension of the method to that of partially separated flow is to assume that the separated region is bounded by two vortex sheets of equal but opposite strength, i.e.,



the condition of zero normal velocity on the aerofoil contour is preserved so that the sheet strength distribution is as follows :



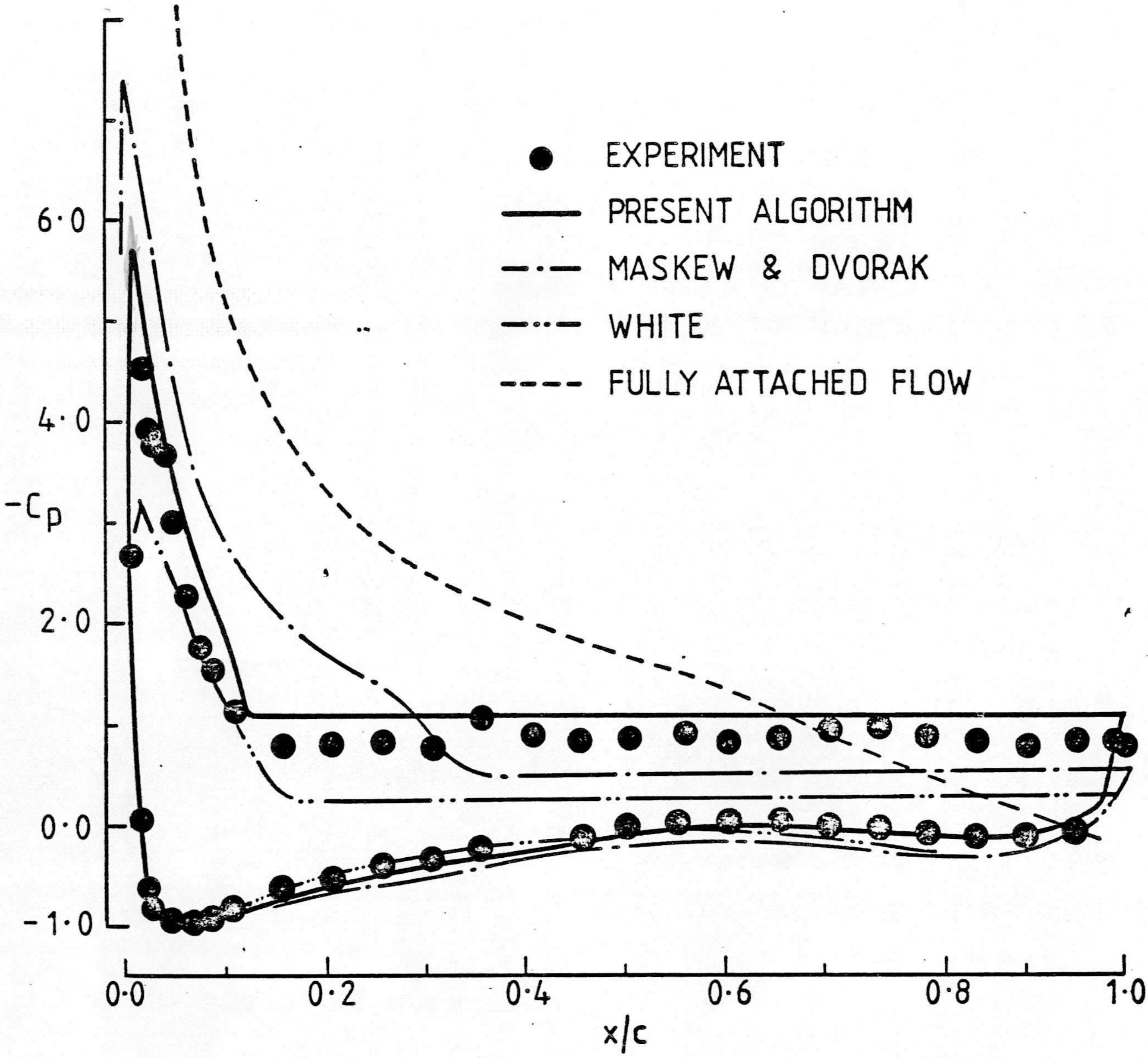
In the separated region the pressure is assumed constant and its boundary is initially unknown. This necessitates some starting profile which, of course, will not necessarily lie along streamlines since the boundary condition is not explicitly satisfied of the profile. To fulfil the requirement that the bounding sheets lie on streamlines, it is necessary to adjust the region in an itterative manner until a suitable boundary is obtained.

On the face of it, this seems a minor extension to the fully attached case but numerical and programming difficulties were much greater than expected. In particular, the handling of the panel on which separation occurs and in the itterative method for the separated region.

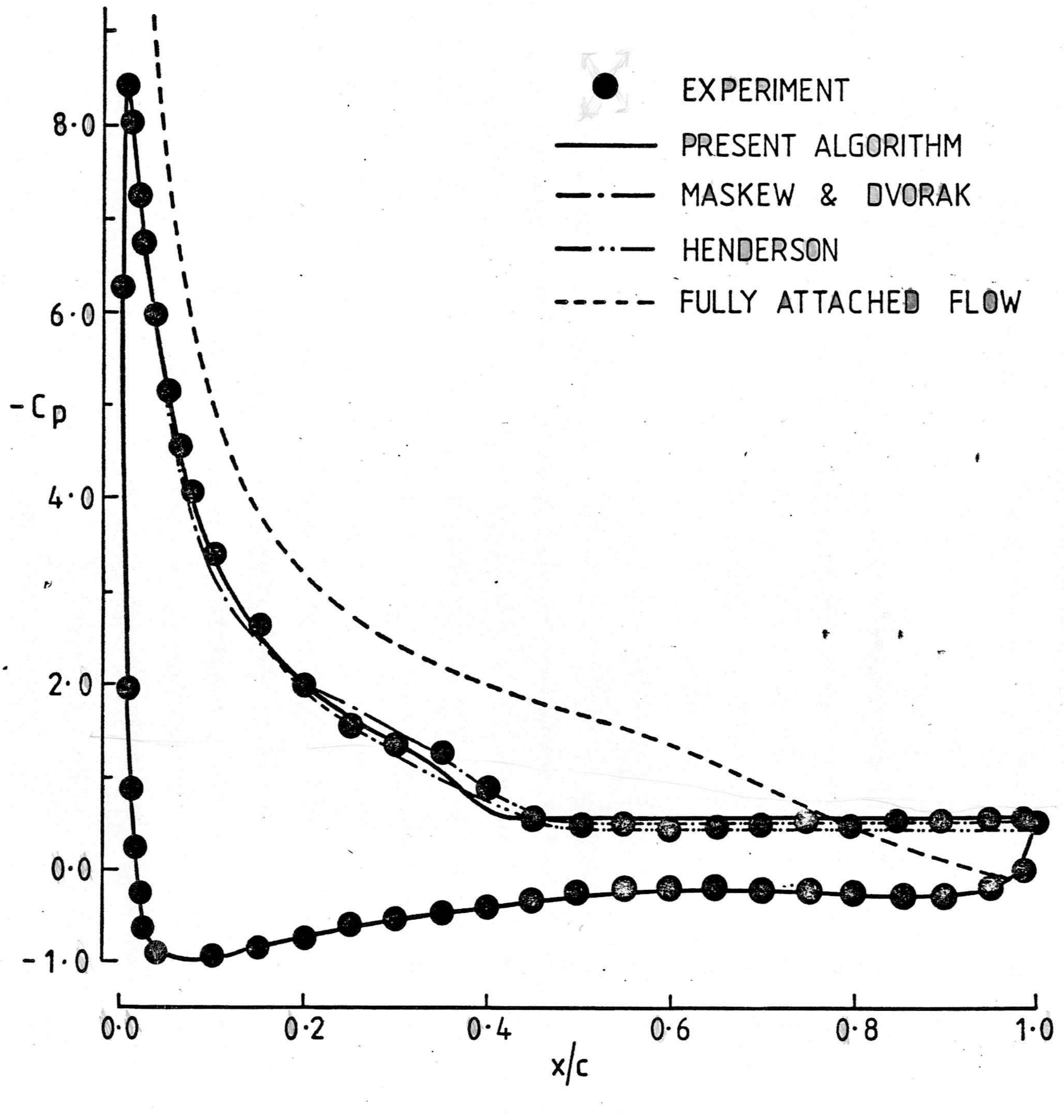
The final test of any predictive procedure, however, is, of course, how well it predicts an accepted test case. As may be seen from the following figures, it is very satisfactory. The agreement with the measured data is good and very much better than the method based on fully attached flow. The overall predicted lift curve slope shows the same quality of agreement with the data and it is a most gratifying result.

It would be quite wrong, however, to assume that all aerofoil data may be predicted to the same accuracy. The results presented were simply checks on the method, given that the location of the separation point is known. Separation is a most difficult phenomenon to predict and even the best methods may be out by as much as twenty to thirty percent.

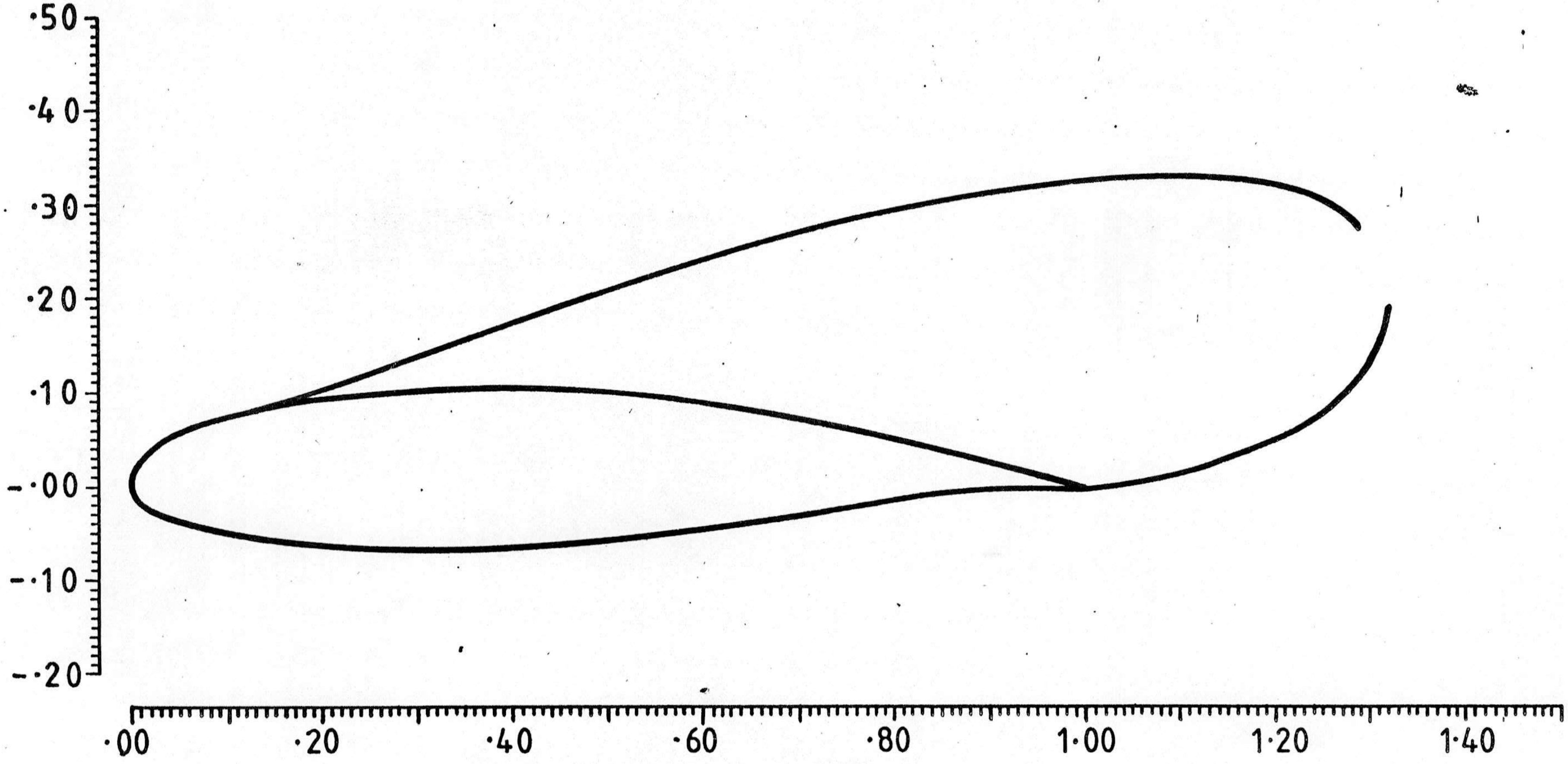
I trust that this rather hurried talk in which I have covered a lot of ground has satisfactorily illustrated some of the current methods for predicting the flow past aerofoils and I have not abused your discipline too much.



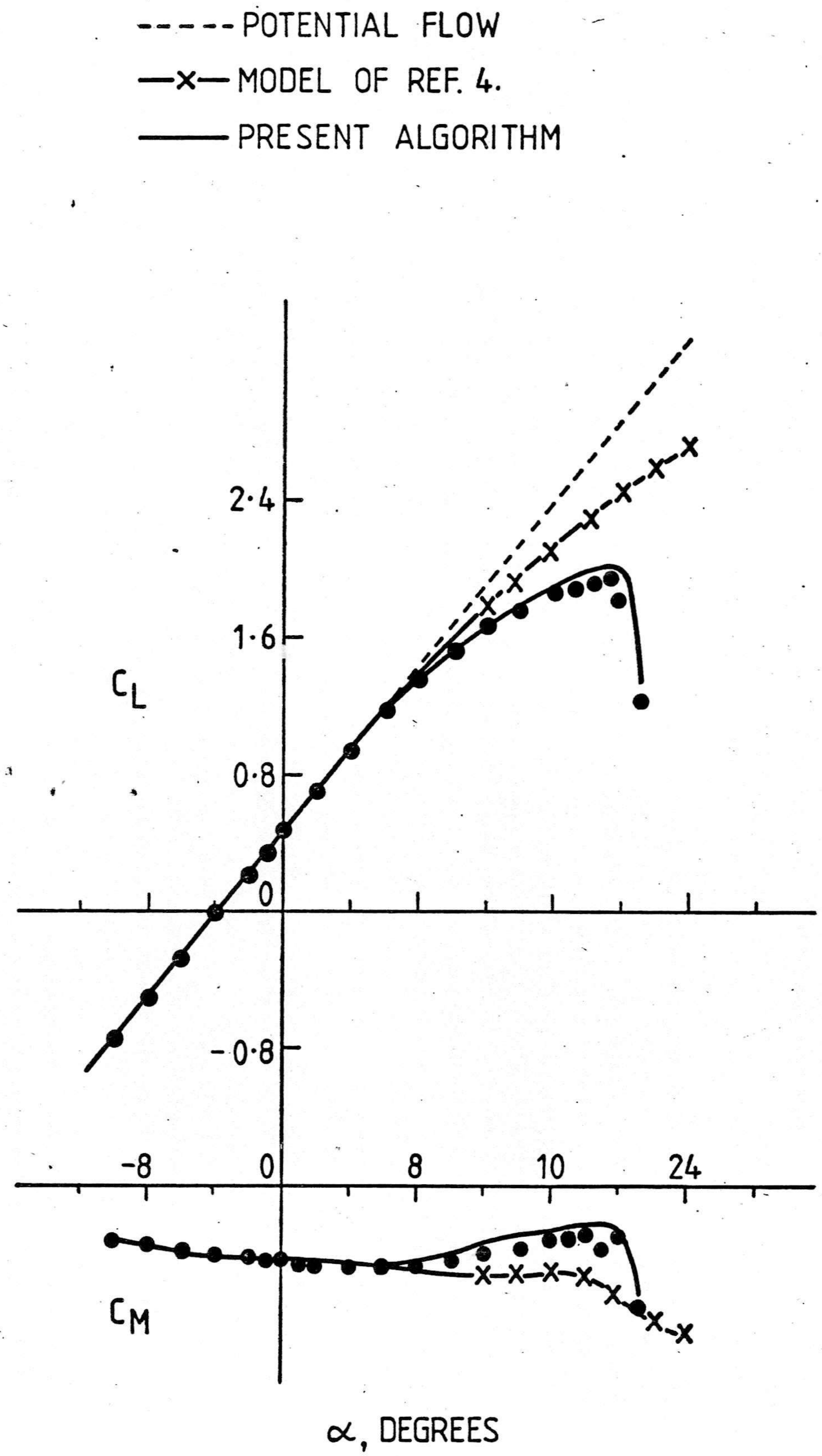
PRESSURE PROFILES FOR THE GA(W)-1 AEROFOIL AT 21·14° ANGLE OF ATTACK



PRESSURE PROFILES FOR THE GA(W)-1 AEROFOIL AT 20° ANGLE OF ATTACK



WAKE SHAPE FOR THE GA(W)-1 AEROFOIL
AT 21·14° ANGLE OF ATTACK



COMPARISON OF EXPERIMENTAL AND THEORETICAL AERODYNAMIC COEFFICIENTS

$$\left(g_i = x_i \frac{c_\infty}{c_i} + \sum \frac{\gamma_i}{R_i} / c_\infty \right)$$