UNIVERSITY OF GLASGOW

DEPARTMENT OF AERONAUTICS AND FLUID MECHANICS

THE PREDICTION OF TURBULENT BOUNDARY LAYERS

by

R.A.McD. GALBRAITH, B.Sc., Ph.D.

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SUMMARY

In the past there has been some argument about how the problem of predicting turbulent boundary layers could best be handled. The present paper attempts to alleviate some of the disagreement and separately treats the model and the solution procedure. It is shown that the choice of dependent variable for the closure hypothesis is of minor importance when compared to the final form of the model. Consideration is also given to the way in which simple models, such as the constant eddy-viscosity assumption, may be improved and also the possible limitation of such procedures. This, it is suggested, is when, of necessity, the specification of the modelled quantity is by a complicated rate equation. Here the use of the turbulent transport equations as a basis for the model is accepted. However, it is concluded that until the complex and hopefully more general turbulence models can predict relatively uncomplicated flows, with at least as good an accuracy as the simpler models discussed, then their use in such flows is superfluous./...

/superfluous. Finally, a brief discussion on integral methods suggests, that where a suitable velocity profile family exists, such procedures can form the basis of a fast, simple and accurate method of solution.

1) Introduction

The prediction of turbulent boundary layer development has from its inception been bedevilled by an inability to formulate a concise and accurate model for the apparently random nature of the flow. Many models and prediction procedures have been proposed, with various degrees of success. Most of the earlier workable methods were of an integral form, incorporating some simple auxiliary equation derived from scant empirical data. In general, such methods rarely gave satisfactory predictions for flows, other than those from which the models were developed.[1] This situation remained until the early 1960's.

About this time two major developments occurred. First, a new breed of integral procedures was developed.[2,3,4]. Second, as computers became more widely available, much attention was focussed on solving the governing equations in their basic form using simple closure hypothesis, such as a constant eddy-viscosity model and a suitable numerical analysis.[5,6,7].

The/...

The net result of the above was a minor split in attitudes towards methods of predicting turbulent boundary layer development. Those who favoured integral methods, continued to do so, and not only improved their accuracy, but extended their applicability to a wide variety of flow situations.[8,9,10,11] In contrast to this, there was a generally accepted obsolescence of integral procedures, by the exponents of finite difference methods. This was apparently a result of integral methods lack of generality, incurred through their implicit need of at least a suitable skin friction law, and more recently, an accurate velocity profile family.

In due course, when finite difference procedures had themselves been successfully developed, a split occurred among the exponents of "Differential Methods". The simple eddy-viscosity and mixing-length models, received much criticism by those who favoured the development of more complex and ideally completely general models, based on some turbulent transport equation(s). It had also been suggested that the simple eddy-viscosity concept should be discarded on the basis of it not being a local property of the flow. However, as with the integral methods, those who favoured the simpler forms of eddy-viscosity and mixing length, continued to develop new procedures and model formulations for handling a wide variety of flow situations.[12]

The present report attempts to reconcile some of the above conflicts./...

/conflicts. It is suggested that, at present, no model or numerical procedure is supreme, and it is not clear in which direction future efforts, in developing new models, should be channelled. One thing is clear, however, for "production" programs, the simplest and most efficient procedure, capable of satisfying the designers requirements, should at all times be used. To make the correct choice, one must, therefore, be aware of the available techniques and be able to recognise that a particular flow may be satisfactorily predicted via a particular model and a particular solution procedure. One cannot help making a poor but illustrative analogy between the above and the analytic solution of differential equations.

In this report it is demonstrated that, for relatively simple flow situations, simple models can give better predictions of flow development than some of the more sophisticated proposals. Further, the choice of modelled quantity is of minor importance when compared to the precise form of the model itself. This, of course, implies an indifference, for a particular model, between eddy-viscosity, mixing length, turbulent kinetic energy and entrainment, etc., but sensitivity to the succinctness and plausibility of the numerous models that have been proposed.

The way in which the constant eddy-viscosity model may be intuitively improved, whilst remaining relatively uncomplicated,/...

/uncomplicated, was investigated and the limitations of such models are discussed. It is suggested that much of the constant eddy-viscosity model's success can be attributed to its apparently reasonable predictions of two particular separating flows, where the averaging effect of the constant value, conceals the true poor quality of the predictions.

As regards the limitations of intuitive improvements to simple models, it is concluded that when the adequate prediction of a flow, requires the specification of the independent variable to be modelled via a rate equation, then it would seem reasonable, where enough data is available, to use the turbulent transport equations as a basis for the model. This does not preclude the use of intuitive arguments in the specification of the individual terms appearing in such equations. Nevertheless, a wide and useful range of flows may still be very satisfactorily predicted, using very simple procedures.

Finally, it is concluded that where a suitable velocity profile family exists, then the incorporation of the model into an integral procedure will yield a simple solution without any severe loss of accuracy.

The current investigation took the following form. First, an uncomplicated and flexible integral solution procedure for/...

/for incompressible two-dimensional boundary layers was programed and tested using an arbitrary closure hypothesis. Then, without alteration to the main program, three closure hypotheses of similar formulation but differing dependent variables were used in turn, to predict the development of a variety of flow situations: they were found to give very similar results. Second, various flow developments were predicted using a constant eddy-viscosity model, which was subsequently improved to satisfactorily predict flows proceeding to separation.

2) Notation

x,y	distance along and normal to the surface.
u,v	velocities in the boundary layer in the x,y directions
U	velocity outside the boundary layer
p	static pressure
δ	boundary layer thickness
δ^*	displacement thickness $= \int_0^\infty (1 - \frac{u}{U})\, dy$
θ	momentum thickness $= \int_0^\infty \frac{u}{U}(1 - \frac{u}{U})\, dy$
H	form parameter $= \frac{\delta^*}{\theta}$
H*	form parameter $= (\frac{\delta - \delta^*}{\theta})$
v_e	entrainment $= \frac{d\{U(\delta - \delta^*)\}}{dx}$
C_e	entrainment coefficient $= v_e/U$
ρ	density
ν	kinematic viscosity

ν_e effective viscosity

ν_τ kinematic eddy-viscosity

Re_θ $= U\theta/\nu$

τ shear stress

τ_w wall shear stress

u_τ friction velocity $= \tau_\omega/\rho$

cf skin friction coefficient $= \tau_\omega/\frac{1}{2}\rho U^2$

G velocity defect parameter $= \dfrac{\int_0^\infty \{(U - u)/u_\tau\}^2\, dy}{\int_0^\infty \{(U - u)/u_\tau\}\, dy} = \sqrt{(\frac{2}{cf})}\ \dfrac{H - 1}{H}$

π pressure gradient parameter $= \dfrac{\delta}{\tau_\omega}\ \dfrac{dp}{dx}$

Subscripts

2D pertaining to two-dimensionality

eq pertaining to equilibrium conditions

exp experiment

max maximum value of

0.4 value at $y/\delta = 0.4$

3) The Integral Solution Procedure

The basic procedure follows that of Patel & Head[13], which may be briefly described as follows.

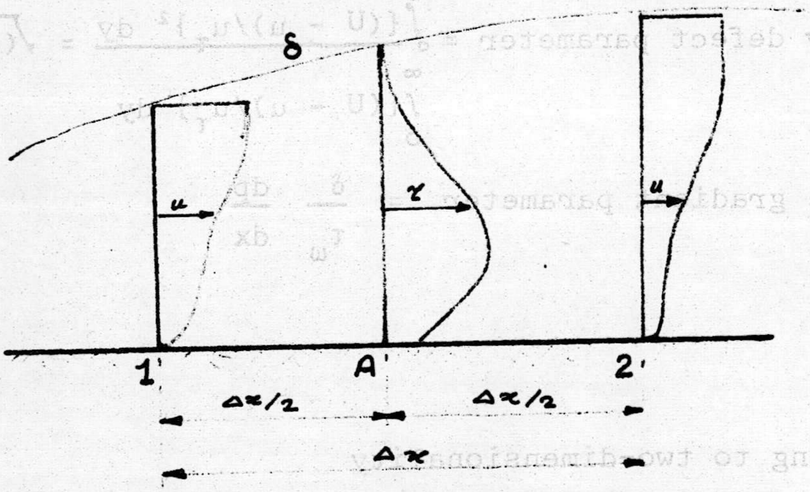
First we assume that the velocities are adequately represented throughout/...

throughout by the Thompson profile family (as described in ref. 14) which provides relationships of the form.

$$\frac{u}{U} = f_1 (H, Re_\theta, y/\delta) \qquad (1)$$

$$cf = f_2 (H, Re_\theta) \qquad (2)$$

We further assume that at some initial station (1) the values of H and Re_θ are known



The object is to determine the values of H and Re_θ at the second station (2) a short distance downstream.

If we assume a value of H at (2) then we can determine the value of Re_θ which satisfies the momentum integral equation.

$$\frac{d\theta}{dx} = \frac{cf}{2} - (H + 2) \frac{\theta}{U} \frac{dU}{dx} \qquad (3)$$

using relations (1) and (2) and so define the velocity profile there./...

there. It is then possible to determine the shear-stress profile, at the mid station (A) of the interval, from the continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (4)$$

and the conservation of momentum equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \frac{1}{\rho}\frac{\partial \tau}{\partial y} \qquad (5)$$

written in the integral form

$$\frac{\tau_{y_1}}{\rho} = \frac{\tau\omega}{\rho} - U\frac{dU}{dx}y_1 + 2\int_0^{y_1} u\frac{\partial u}{\partial x}\,dy - u\int_0^{y_1}\frac{\partial u}{\partial x}\,dy \qquad (6)$$

and then applied in finite difference form over the interval.

At this stage in the procedure, the mean flow has been completely described and a check can be made as to whether or not that particular choice of H yields a flow field, which satisfies any criterion we may choose to apply. If the check is unsatisfactory, repeat the procedure with a different value of H at (2). In fact, rather than proceeding by trial, it has been found more convenient to choose three different values of downstream H, and interpolate to find that value of H, for which the criterion is satisfied.

It will be recognised that the procedure just described is rather clumsy, expensive in computing time, and if (for example) an entrainment model is used, computes unnecessary information/...

information. However, it is extremely flexible and transparent and cannot in itself introduce any form of instability into the computations. It is therefore well suited for the present work.

4) Flow Predictions Using Similar Shear-Stress, Eddy-Viscosity and Entrainment Models.

4.1 Background

In 1970 Head & Patel[8] described an improved and very successful entrainment model, which was based on heuristic and qualitative physical arguments. The model consisted of prescribing the entrainment coefficient (Ce) for equilibrium layers, and modifying this value to take account of deviations from equivalent equilibrium conditions.

Specifically the model is given by,

$$Ce = Ce_{eq} \, F(r_1) \tag{7}$$

and

$$Ce_{eq} = H^* \left\{\frac{1}{U}\frac{dU\theta}{dx}\right\}_{eq}, \tag{8}$$

where

$$r_1 = \left.\frac{1}{U}\frac{dU\theta}{dx}\right|_{2D} \Bigg/ \left.\frac{1}{U}\frac{dU\theta}{dx}\right|_{eq}, \tag{9}$$

$$F(r_1) = \frac{1}{2r_1 - 1} \text{ for } r_1 > 1$$

(see fig.1)(10)

$$F(r_1) = \frac{5 - 4r_1}{3 - 2r_1} \text{ for } r_1 < 1,$$

and $\left\{\frac{1}{U}\frac{dU\theta}{dx}\right\}_{eq}$ follows from the momentum integral equation, using the value of π that acrues from the empirical π -G relation given by Nash for equilibrium layers, i.e.,

$$G = 6.1\ (\pi - 1.81)^{\frac{1}{2}} - 1.7 \qquad (11)$$

The predictions of flow development obtained by this method were as good, and often better, than those of more complicated procedures (figs. 2,3).

Later analysis of measured boundary layer development showed that the maximum value of eddy viscosity in the outer part of the layer varied between wide limits and exhibited distinct trends similar to those assumed by Head & Patel for the entrainment coefficient. That is, for layers developing faster than the corresponding equilibrium layer (i.e. $r_1 > 1.0$), the maximum value of the eddy viscosity is less than would be expected under self preserving conditions, and vice versa. In fact, Head & Galbraith[16] showed that, for equilibrium layers, the ratio $v_e\delta/\nu_\tau\big]_{max}$ remains substantially constant over a wide range of pressure gradients. Galbraith[15] also demonstrated that the eddy viscosity and the entrainment were very closely related, even in non-equilibrium layers (figs. 3,4,5) and, although unable to substantiate the validity of Head & Patel's arbitrary correction to Ce_{eq} (i.e. eqn. 10), or indeed suggest an improved formulation, he clearly showed that the above mentioned trends did exist (fig. 6).

It thus appeared that similar model formulations for the entrainment and eddy viscosity would yield similar predictions of flow development. Indeed, as mentioned in sect.1,/...

sect. 1, the choice between entrainment and eddy viscosity, as the dependent variable of the model, was not nearly as important as its precise formulation. This led Galbraith[15] to state,

> "The behavior of the entrainment and eddy viscosity in the outer part of layer are not just vaguely similar but very closely related so that any correlation proposed for $(\nu_\tau/U\delta^*)_{max.}$ should be equally valid for Ce δ/δ^*. There is also every reason to expect that similar correlations might apply to say the dissipation integral and other such hypothesis".

The present section tests this proposal by predicting experimentally obtained flow developments, using similar model formulations for the entrainment, eddy viscosity and the value of the shear stress at position $y/\delta = 0.4$.

The reason for modelling the shear stress directly, and not, for example, the dissipation integral, is given in the following section.

4.2) <u>The model formulations</u>

All three models tested were of similar form in that they followed Head & Patel's method of specifying the equivalent equilibrium value of the dependent variable, and then modifying that, to account for non-equilibrium conditions. Although the specification of the equilibrium value varied slightly between models, the modifying function described/...

described by eqn.10 was always used, since, as discussed above, no obvious improvement was evident.

Using ϕ to represent the quantity being modelled, then all three hypothesis take the following form.

$$\phi = \phi_{eq} \, F(r_1) \qquad (12)$$

with $F(r_1)$ given by eqn. 10 and

$$\phi = F_2(\pi_{eq}) \qquad (13)$$

where π_{eq} follows from Nash's π - G relation for equilibrium layers and is a function of the local values of H and Re_θ.

In the case of the entrainment and eddy viscosity, the choice of function for ϕ_{eq} was easily obtained by fitting an appropriate curve through the results obtained from the direct analysis of measured boundary layer developments as described in Ref. 14. This gave the following.

$$Ce \left.\frac{\delta}{\delta^*}\right|_{eq} = 0.18 - e^{-(0.3\pi_{eq} + 2.52)} \qquad (14)$$

and $$\left.\frac{\nu_T}{U\delta^*}\right|_{0.4,eq} = 0.024 - e^{-(0.525\pi_{eq} + 4.95)} \qquad (15)$$

The comparisons of these functions, with the above mentioned results, are given in figs. 7,8.

Turning now to the shear-stress model, (i.e. the variation of/...

of τ/τ_ω at $y/\delta = 0.4$), it should be noted that it had originally been intended to model the dissipation integral. However, during the programing and testing of the basic solution procedure, an initially temporary and simple closure hypothesis was used. This took the now unusual form of eqns. 12, 13 with eqn. 13 being specified by

$$\left.\frac{\tau}{\tau_\omega}\right]_{0.4} = 0.65 + \pi_{eq} \quad . \qquad (16)$$

The linearity was not speculative, for it had been noticed, during the preparation of ref. 16, that the contours of (τ/τ_ω)max were virtually coincident with those of π (see fig. 9): a rather interesting result. The maximum value of τ/τ_ω was a little inconvenient for the purpose required and so the value at $y/\delta = 0.4$ was chosen quite arbitrarily. Again the results of ref. 14 were used to obtain eqn. 16 and these are compared in fig. 10.

Due to the success and usefulness of this model, it was retained as part of the investigation.

4.3) Results

Figures 11-19 illustrate predictions of flow developments from the above three models for the following flows.

1.	Wieghardt	zero pressure gradient	1400	**
2.	Bradshaw	a = -0.15	2500	
3.	Bradshaw	a = -0.255	2600	
4.	Schubauer & Spangenberg	Flow B	4500	
5.	Schubauer & Spangenberg	Flow E	4800	
6.	Ludwieg & Tillmann	dp/dx>>0.0	1200	
7.	Perry	adverse pressure gradient	2900	
8.	Tillmann	re-attaching flow	1500	
9.	Bradshaw & Ferriss	relaxing flow	2400	

When all three predictions from the models appear as a solid line, as in the case of Wieghardt's flow, then they lie within the thickness of the given line.

The results clearly show that the three models considered gave nearly equivalent predictions. Any small deviations from each other were more a function of the deficiencies inherent in the velocity profiles, rather than of the models themselves.

It will be seen that, in general, the accuracy obtained from these models is good, except for the flows of Perry, Ludweig & Tillmann and Tillmann. Even here, however, the results are as good, or as bad, as some of the better known models.

** The numeric index is that used at the Conference, held at Stanford University, on the prediction of two-dimensional turbulent boundary layers (ref.17).

4.4) Discussion

It will be recalled that the aim of the present section was to provide some evidence in support of an indifference towards the choice of dependent variable for a particular model. That which has been presented, it may be argued, is not an entirely independent check, since the use of the models closely follows their derivation. However, in each case, the model formulation is similar and the only "tuning" carried out was for the dependent variable's equivalent equilibrium value. Of course, a different profile family may yield a slightly different specification. In fact, in a finite-difference procedure, where one is not restricted by the profile family, this problem does not arise and so the specification of the equivalent equilibrium value (i.e. eqn. 13) could take the form,

$$\phi = A + B\pi_{eq} \qquad (17)$$

as is the case with the shear-stress model (eqn. 16). Whether this would give similar results, to those already presented, remains to be seen.

If one accepts the results with the above qualification, then their significance is very clear. It is, that one need no longer argue, if one ever did, whether it is the eddy viscosity, entrainment, dissipation integral or turbulent kinetic energy, etc., that should or should not be modelled. Each/...

Each one, it would appear, gives just as good a prediction as the rest, provided it takes the same form and the required empiricisms are of similar quality. The choice between them then becomes one of ease of programing, the availability of reliable empirical data and, if an integral procedure is to be used, the deficiences of the appropriate profile family.

A further implication of the results stems from the accuracy of the predictions being just as good as some of the more complicated procedures,which suggests that, for many flows, a simple but realistic procedure will suffice. This is reassuring for those who require fast and simple methods, which may be included as a small part of a large inviscid - viscous flow calculation. A fuller discussion of this point is given in section 6 of this report.

5) Improved Eddy-Viscosity Models

5.1 Introduction

Here the eddy viscosity concept was employed to investigate the dificiencies of simple models and the way in which they may be improved, whilst remaining relatively uncomplicated, to satisfactorily handle a wider range of flows than had hitherto been possible.

To avoid unnecessary discussion of eddy viscosity variation in the wall region and the associated controversy,[18] which is not the concern of this paper, the value of $\nu_T/U\delta^*$ at $y/\delta = 0.4$ has been modelled throughout.

Initially the eddy viscosity was simply taken as a universal constant, which only gave satisfactory predictions for zero pressure gradient flow, and then subsequently improved to adequately handle equilibrium and separating flows.

Recalling (from the previous section) that the modelled quantity is of minor importance when compared to its specification and that the eddy viscosity and entrainment coefficient show similar variations throughout the entire development of the layer, then it is not unrealistic to expect that the results, of the above outlined investigation, will be applicable to the entrainment concept. Indeed, there is every reason to expect that their applicability is relatively independent of the modelled quantity.

5.2 The Model $\left.\frac{\nu_T}{U\delta^*}\right]_{0.4}$ = constant.

This is the simplest specification of eddy viscosity we can have. Originally, the possibility of the ratio $\nu_T/U\delta^*$ being treated as a constant was considered by Clauser[19], who had suggested that, in the outer region of the flow, the ratio $\nu_T/U\delta^*$ could, as a rough approximation, take/...

take the universal value of 0.018. Although the magnitude of the constant varies from author to author, the constant eddy viscosity model has been employed in the analysis of quilibrium layers[20,21], and in numerous prediction procedures,[6,7] with varying degrees of success and criticism.

In the present model the eddy viscosity is described by the relation

$$\left.\frac{\nu_\tau}{U\delta^*}\right]_{0.4} = 0.017 \quad . \qquad (18)$$

This value, which is a little higher than that normally used (e.g., 0.016), is taken from Galbraith & Head's analysis for zero pressure gradient flow, the results of which are presented in Fig. 8.

The predictive capability of the above model for the flows 1 to 5 and 9 of sect. 4.3 and also Goldberg's flow[22] (pressure distribution No. 3), is illustrated in figs. 20 to 26.

It can be seen that for Wieghardt's flow the predictions are very satisfactory and every bit as good as more recent complex models (see fig. 16). However, in the case of Bradshaw's two equilibrium flows, figs. 21, 22, the model over predicts the development of H and under predicts that of/...

of cf. The more severe the pressure gradient the poorer the prediction. It is salutary to note that this is also the case with the similar model used by Cebeci & Smith[6], in their finite-difference procedure (figs. 17, 18).

Galbraith[15] has shown that where $\frac{dG}{d(\log Re_\theta)}$ is positive there is a corresponding reduction in $\nu_T / U\delta^*\rfloor_{max}$ relative to the equivalent equilibrium value (fig. 28). It is, therefore, not unreasonable to expect that any model which under estimates the true variation of eddy viscosity will result in an over prediction of G and hence H. Thus the increasing disagreement, between prediction and experiment, is a consequence of the above model under predicting the value of $\nu_T / U\delta^*\rfloor_{0.4}$ in the equilibrium adverse pressure gradients (see fig. 27).

In the two separating flows of Schubauer & Spangenberg, the predictions (figs. 23, 24) are apparently quite satisfactory, at least over the first part of the flow. Similar results from other authors, for these two flows, have probably accounted for much of the constant eddy viscosity model's success and long life. However, a closer inspection reveals a quite unrealistic prediction of flow development.

Consider the initial stages of flow E's development, where H and hence G are increasing only slowly. The visible over/...

over prediction of H and under prediction of cf is a consequence of the model giving too small a value of $\nu_T/U\delta^*]_{0.4}$; see fig. 27. However, as the layer approaches separation, the true value of $\nu_T/U\delta^*]_{max}$ will gradually decrease to around 0.2 of the equivalent equilibrium value, (see fig. 28) and so the model's value of $\nu_T/U\delta^*]_{0.4} = 0.017$, increasingly over predicts the true value, with the consequent under prediction of H and corresponding over prediction of cf. The model's unfavourable predictive capability for separating flows is, therefore, somewhat concealed by its constant value falling around the mid range of the true variation of $\nu_T/U\delta^*]_{0.4}$, and thus over the flow tends to average out the discrepances of prediction.

In Schubauer & Spangenberg Flow B (fig. 25) where, from the outset, the experimental variation of H increases more rapidly than that of Flow E, with an implicit reduction in the value of $\nu_T/U\delta^*]_{0.4}$, the model is initially more satisfactory than it was in the case of Flow E. This is reflected in a good prediction for this part of the flow. This quality of prediction, however, only exists until the rate of increase of H or G is such that, even with the increase in $\nu_T/U\delta^*]_{0.4}$ due to the severity of the equivalent equilibrium layer, the true value of $\nu_T/U\delta^*]_{0.4}$ is less than that given by the model. After this stage has been reached, the model increasingly under predicts the development of H.

It/...

It can therefore be seen that the constant eddy viscosity model is only applicable to zero pressure gradient flows. Even here, as with the case of Weighardt, only over the fully developed and well behaved region, with the starting or non-equilibrium part of the flow being handled with care.

5.3) <u>The model $\nu_T/U\delta^* = F(\pi_{eq})$</u>

Head & Galbraith[16] have shown that even in equilibrium layers, of which zero pressure gradient is a special case, there is substantial variation with pressure gradient in the value of $\nu_T/U\delta^*\rfloor_{max.}$ This section takes account of this variation by considering $\nu_T/U\delta^*\rfloor_{0.4}$ to be a function of π_{eq} given by eqn. 15, i.e.,

$$\left.\frac{\nu_T}{U\delta^*}\right\rfloor_{0.4} = 0.024 - e^{-(0.525\pi_{eq} + 4.95)} \qquad (19)$$

As figs. 21,22 show, the predictive capability of the simple eddy viscosity model has been extended to satisfactorily handle the two equilibrium layers investigated by Bradshaw. In contrast to this improvement, however, the predictions for the two separating layers of Schubauer & Spangenberg (figs. 23,24) have been impaired and are quite unsatisfactory.

The poor prediction of the two separating flows is not entirely/...

entirely unexpected for, as can be seen in fig. 27, the new model seriously over estimates the true variation of the eddy viscosity, resulting in an under estimation of the shape factor development. These two flows, however, exhibit substantial deviations from equilibrium conditions. Any model used to predict such flow situations must satisfactorily account for their manifestations in the modelled quantity.

5.4) Eddy Viscosity in Non-Equilibrium Layers

Various authors, e.g., 3,4,23 have considered ways of improving simple models to account for non-equilibrium flow conditions. In sect. 4,Head & Patel's method[8] was discussed and applied to the eddy viscosity concept as well as the modelling of the shear stress. Their method, which was originally based on qualitative and heuristic arguments, did not have the modifying function (eqn. 10) checked against experiment until the analysis of Galbraith & Head[14] provided some consistent data. Even then, however, Galbraith[15] was only able to substantiate the general trend of the proposed function,due to a large amount of scatter which was not entirely unexpected.

The justification for Head & Patel's model, therefore, lies in the quality of the predictions obtained which has been clearly demonstrated in sect. 4. Head & Patel also suggested that further improvements in predictive accuracy could be obtained in two ways. First, all experimentally investigated flows contain some degree of three-dimensionality. This/...

This will manifest itself through an imbalance of the momentum integral equation, from which a suitable modification to the model may be made by assuming that the three-dimensionality of the layer is of a plane convergent/divergent nature and that the increased/decreased rate of growth may be treated as if it arose from purely two-dimensional causes; Head[24] has recently provided additional evidence and thoughts on this.

Second, and more important, they limited the rate at which the entrainment coefficient was allowed to change. This action appears to have been mainly intuitive and originated from the poor predictions obtained for Golberg's flow (fig. 2c). Again, justification for this course of action came from the improved predictions and, more recently, from the work of Galbraith[15].

As before, he substantiated the correctness of introducing such a lag term but could not verify the appropriетness of the one used. Here, however, Galbraith's results are significant in the development and improvement of simple models.

Figures 28, 29 recast Galbraith's results such that $(\nu_T/U\delta^*)_{max}/(\nu_T/U\delta^*)_{eq}$ is correlated against $\frac{dG}{d(\log Re_\theta)}$, instead of $\theta \frac{dH}{dx}$ as originally used, and it can be seen from/...

from fig. 28, that,at least for separating flows, there can be little doubt that there is a substantial reduction in the relative value of $\nu_T/U\delta^*]_{max}$. Also, from the analysis of Perry's flow, in which there is a significant imbalance of the momentum integral equation, the assumed three-dimensional causes appear to modify the value of the eddy viscosity such that it parallels the variation arising from purely two-dimensional causes. Results presented by Head[23] indicate that, in contrast to the reduced value of $\nu\ /U\delta^*]_{max}$ in Perry's possible convergent flow, there is a marked increase for divergent flows.

For two-dimensional non-equilibrium flows, other than those where G is increasing monotonically, notably relaxing layers, the correlation presented in fig. 28 is no longer valid, as is clearly demonstrated in fig. 29.

Bradshaw & Ferriss's flow[25], it will be recalled, consisted of a layer, the first part of which developed in an adverse pressure gradient conducive to the development of a particular equilibrium layer. At some position downstream this pressure gradient was removed and the subsequent development in zero pressure gradient was intensively investigated.

Golberg's[22] flow was in many respects similar to that of Bradshaw/...

Bradshaw & Ferriss's, but the initial development was in the presence of a severe adverse pressure gradient causing the layer to proceed towards separation. Just before the onset of separation the pressure gradient was removed and the layer allowed to relax in zero pressure gradient. In this flow, however, the entire development of the layer was thoroughly investigated.

Now it can be seen from fig. 29 that, at the point of relaxation, the respective eddy viscosities are where they now would be expected. In the case of Bradshaw & Ferriss's flow, close to equilibrium conditions, whilst for Golberg's flow, close to separation. The subsequent trajectories of the eddy viscosity are those of a slow recovery. In fact, a lagging behind the rate at which the mean velocity profile adjusts to the new situation: it exhibits a damped response. Head & Patel's[8] inclusion of a limiter, for the rate at which the entrainment coefficient is allowed to change, is thus seen to be reasonable. Any model purporting to account for such deviations from equilibrium conditions must exhibit, albeit crudely, this damped response.

The author has found that the parameters $\theta\frac{dH}{dx}$ and $dG/d(\log Re_\theta)$ are too closely coupled to the solution and do not give as stable predictions, using the current procedure, as the parameter r_1 ($\equiv \frac{1}{U}\frac{dU\theta}{dx}\Big]_{2D} \Big/ \frac{1}{U}\frac{dU\theta}{dx}\Big]_{eq}$).

Hence/...

Hence, the improved model, considered in the present section, which can satisfactorily predict nominally two-dimensional separating flows, is that used in sect. 4, and is based on the parameter r_1 and the modifying function given by eqn. 10. For the sake of comparison, the results of sect. 4 are here repeated in figs. 20 to 24 where the very satisfactory agreement between experiment and the predictions, for both the equilibrium and separating layers, may be seen.

For the two relaxing layers (i.e. figs. 25, 26) it can be seen that Bradshaw & Ferriss's flow is very satisfactorily predicted, whilst the result for the more severe case considered by Golberg is very poor (except over the first part of the flow), both absolutely and relative to the prediction of Bradshaw et al[26]. However, Head and Patel[8] (see fig. 2c) obtained very satisfactory results after the inclusion of a simple lag term and taking due account of three-dimensionality.

Returning to Bradshaw & Ferriss's flow (figs. 19, 25), the present predictions are in better agreement with the measurements than the more complicated models, especially for the skin friction. The present model may, therefore, be considered adequate for predicting moderately relaxing layers, although the extent to which this can be applied is unknown/...

unknown and its acceptance does not imply validity. The major defect of the model, i.e., the exclusion of the damped response, is ever present, but in this case the incurred error appears to be small.

6) Concluding Discussion

6.1 Closure Hypotheses

The results of sect. 4 showed that the predictive capability of a particular model is relatively insensitive to the choise of modelled quantity, but very sensitive to its formulation. In sect. 5, a crude eddy-viscosity model was improved in an intuitive and pragmatic way until it could satisfactorily predict separating and, possibly, mildly relaxing layers. It was further pointed out that in relaxing layers the response of the eddy viscosity, once the pressure gradient had been removed, visibly lagged behind the rate at which the mean velocity profile adjusted to the new situation. In these more complicated flows it would, therefore, seem reasonable to specify the eddy viscosity by a rate equation.

If one prescribed the eddy viscosity by some differential equation (e.g., ref. 23), the entrainment could take a similar form and so too the shear stress. Here, however, the simple models based on much empirical data become less distinguishable from the more complex** turbulence models

** Here "complex" refers to the amount of detailed turbulence modelled.

which also take the form of a differential equation(s). It could indeed be suggested that, in recognising the need to specify the modelled quantity by a differential equation in the more complex flows, the exponents of simple models have come round to the stance held by those who favoured more complicated procedures. On the face of it, this would appear to be the case.

Consider first the more complex turbulence models where the underlying theme appears to be one of a search for generality. The assumption being, that the more complex the model the more general it will be, and thus the more desirable. Eventually, one model would suffice for a whole range of flow situations. Currently, however, such models are in need of improvement, even in the simple flows which have been considered here. Nevertheless, they do possess the distinct advantage that there is no requirement to conjure up the basic form of the model, since this is generally dictated by the turbulent transport equation(s) chosen.

The exponents of simpler models appear to develop them pragmatically for an increasing range of flow situations, where it is assumed that the modelling of the mean flow quantities is adequate, but in this they severely limit the number of flows that they can handle. They may argue, of course, that this limited range is not only much/...

much larger than would be expected but also is very useful and, further, that the predictive accuracy is relatively very good. So why use a cumbersome complicated model when a simple one will suffice? It will be easier to program and cheaper to run.

However, when the flow under consideration is such that the model equation(s) can no longer be of a simple form then it would seem reasonable to use the exact turbulence transport equations as a basis for the model. As already stated,the use of these equations does not preclude the use of intuitive speculations and qualitative arguments about the flow structure,and does not imply the retention of all the terms appearing in the chosen equation(s).

Similarly, their use neither restricts the choice of quantity to be modelled nor does it imply superiority of one over the other. It is perfectly in order for Ng & Spalding[27] to model the eddy viscosity using a Reynolds-stress model whilst Bradshaw et al[26] model the shear stress, McDonald & Camarata[28] use an extended mixing length, Green et al [29] the entrainment.

All the last three works base their model on the Turbulent Kinetic Energy Equation.

6.2) Integral Solution Procedures

So far the choice of solution procedure has not been discussed,/...

discussed. It is the author's opinion that this should be treated separately from the model formulation. In general, once the model has been chosen then consider the solution procedure to be employed, and this should not exclude the possible use of an integral technique. After all, the very satisfactory results of sect. 4 were obtained by just such a method. Also, Patel & Head[13] developed a simple and fast integral solution procedure for use with the Bradshaw et al[26] model, and this generally yielded improved accuracy of prediction over the original method (see fig. 30).

It may, therefore, be said that where a suitable family of velocity profiles exist there can be little reason why it should not form the basis of a solution procedure which will, in general, be simple, fast and economic. Such procedures will, therefore, appeal to those who wish a fast algorithm to form part of an inviscid-viscous flow program.

Such analysis not only requires the accurate prediction of the boundary layer flow parameters like δ^*, but also the skin friction, sometimes the heat transfer and the effects of transpiration, etc. Here again, integral methods have distinct advantages. First, most velocity profile families accurately model the flow in the vicinity of the wall[1,18], and this results in good predictions of skin friction. Second, it has been demonstrated[30,31] that,

for/...

for particular flows, good calculations of heat transfer may be obtained by first solving the momentum layer by a fast integral procedure giving profiles of shear stress, eddy viscosity and, via a Prandtl No. assumption, eddy conductivity from which the thermal layer may be solved using a finite difference procedure. Finally, much work has also been carried out for boundary layers with distributed injection, and a suitably three parameter family of two-dimensional velocity profiles has been developed[32].

7) Conclusions

1) In any closure hypothesis, the modelled quantity is of minor importance when compared to its specification.

2) Similar model formulations, using different dependent variables with an equivalent quality of empiricism, will yield very similar predictions.

3) Simple models may be easily improved to satisfactorily predict a wide variety of flow situations, often with better accuracy than current more complex turbulence models.

4) When the flow is of such complexity that to obtain satisfactory predictions the model is specified by a rate equation, it would seem reasonable to base it on one or more of the turbulent transport equations.

5) Conslusion 4 does not imply the obsolescence of simple models.

6) The method of solution is relatively independent of the closure hypothesis.

7) Integral procedures can provide very simple and fast solution algorithms. Where a suitable velocity profile family exists, such procedures should always be seriously considered.

ACKNOWLEDGMENTS

The author wishes to express his thanks to Dr. Andrew Grant for useful discussions during the preparation of this paper.

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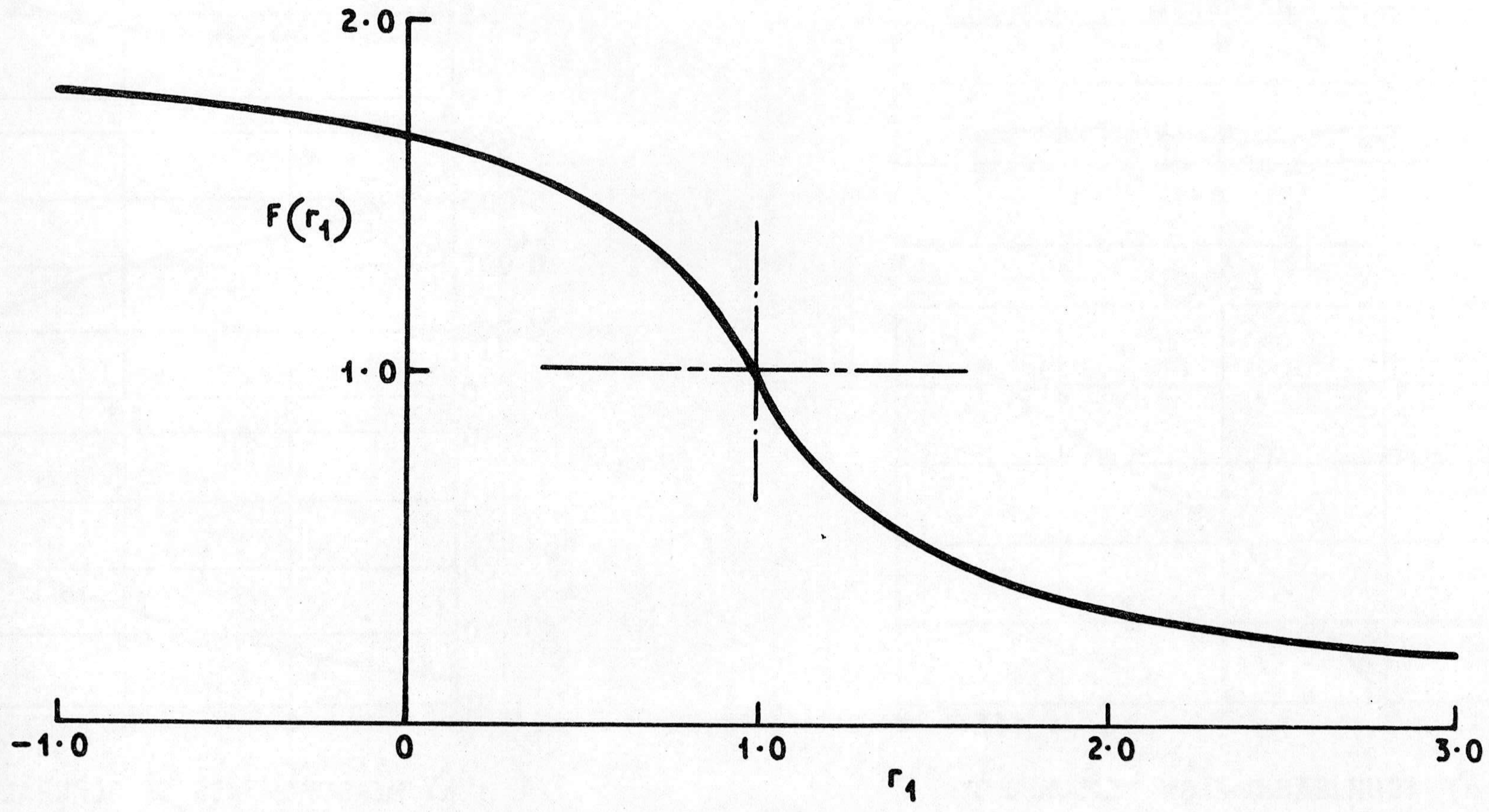
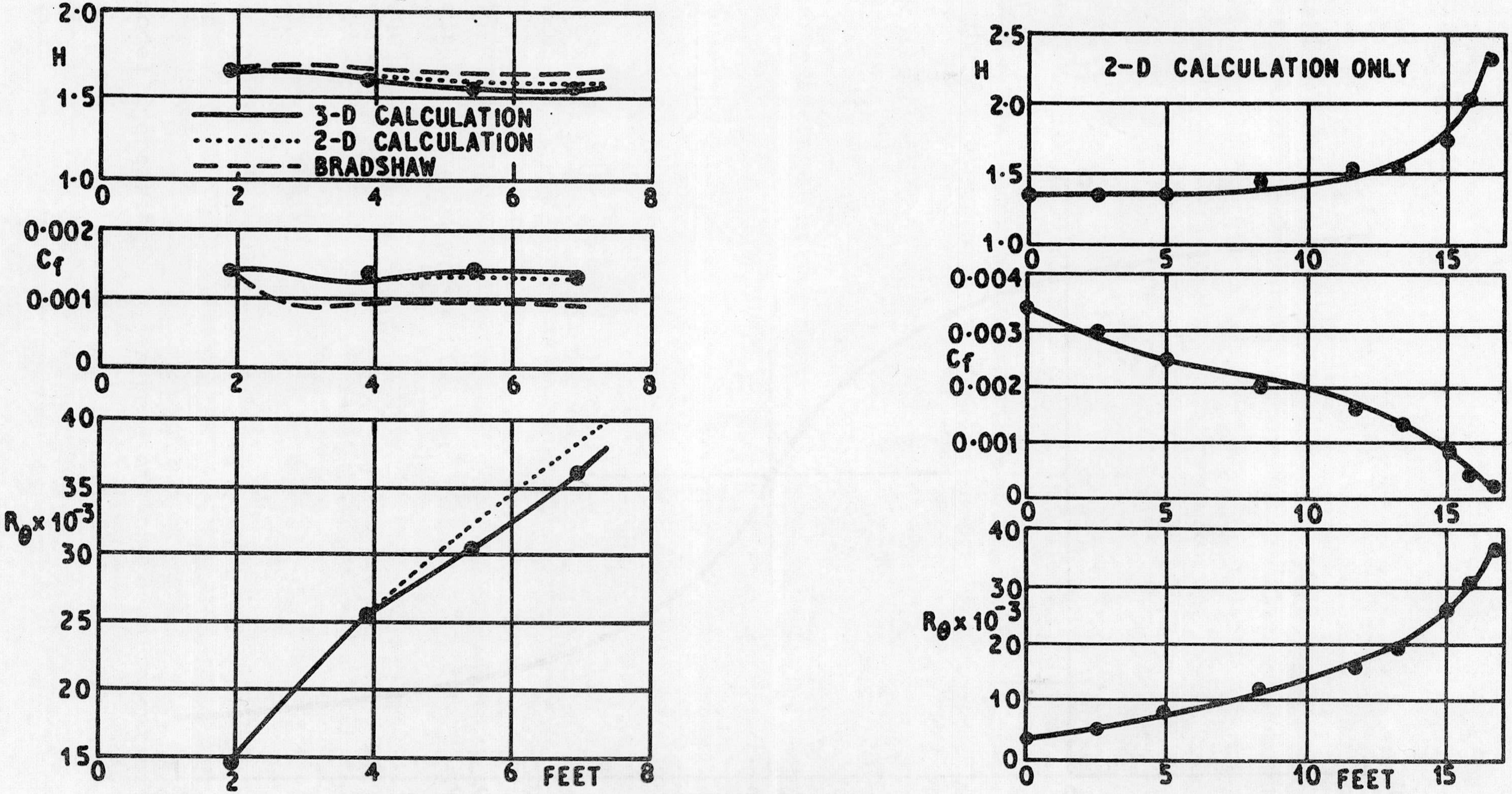


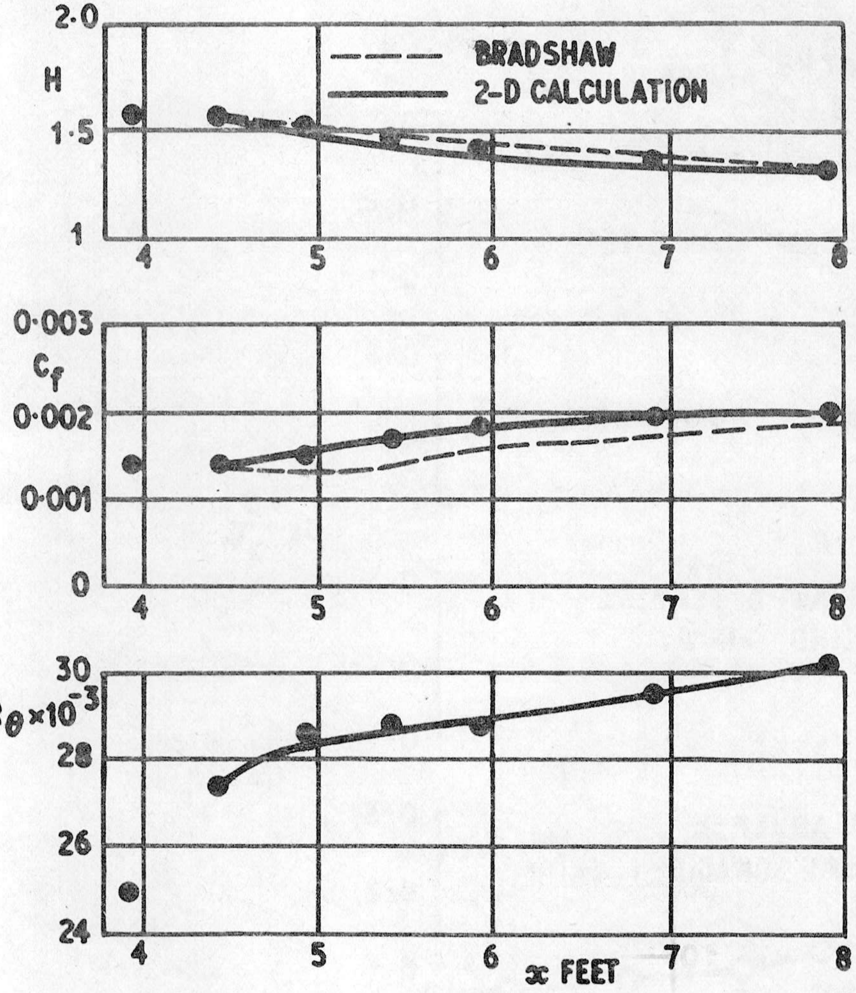
FIG. 1. THE FUNCTION $F(r_1)$



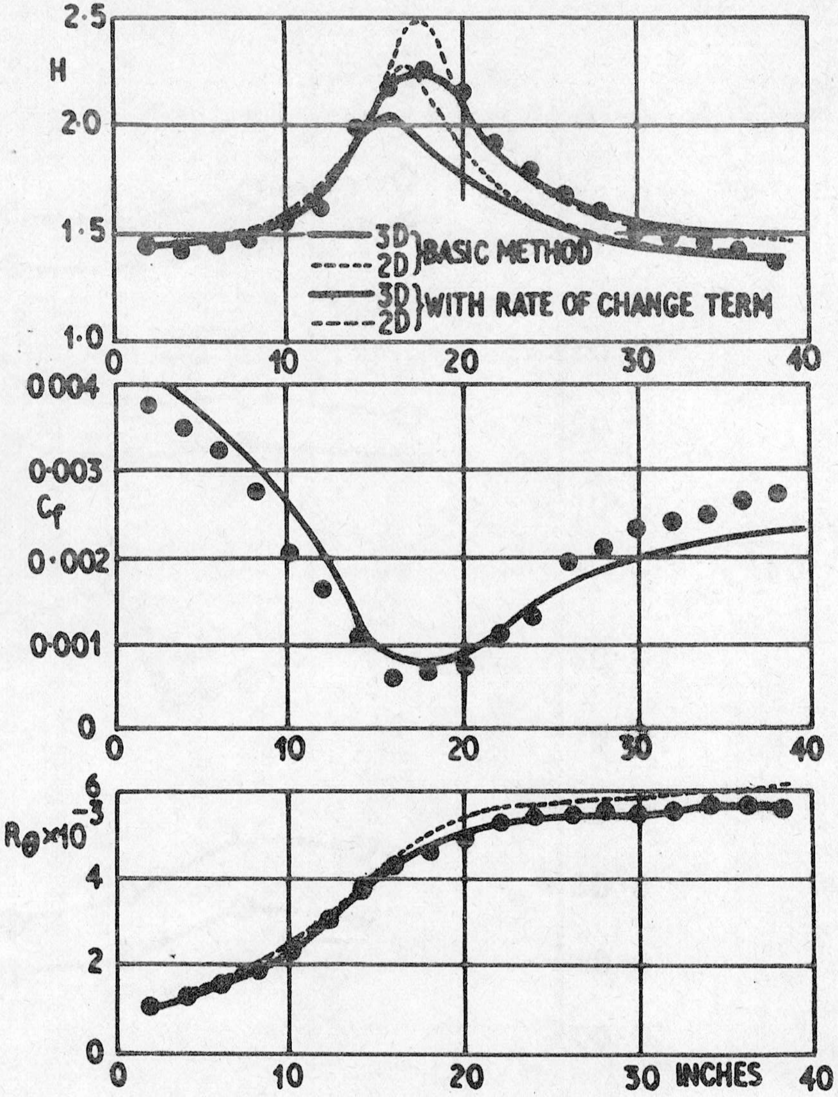
a) EQUILIBRIUM FLOW MEASURED BY BRADSHAW (a = −0·255)

b) MEASUREMENTS OF SCHUBAUER AND SPANGENBERG (FLOW E)

FIG. 2. IMPROVED ENTRAINMENT METHOD RESULTS (REPRODUCED FROM REF. 8)



2c. RELAXING FLOW MEASURED BY BRADSHAW & FERRISS.



2d. MEASUREMENTS BY GOLDBERG. (PRESSURE DISTRIBUTION 3)

FIG 2. CONTINUED.

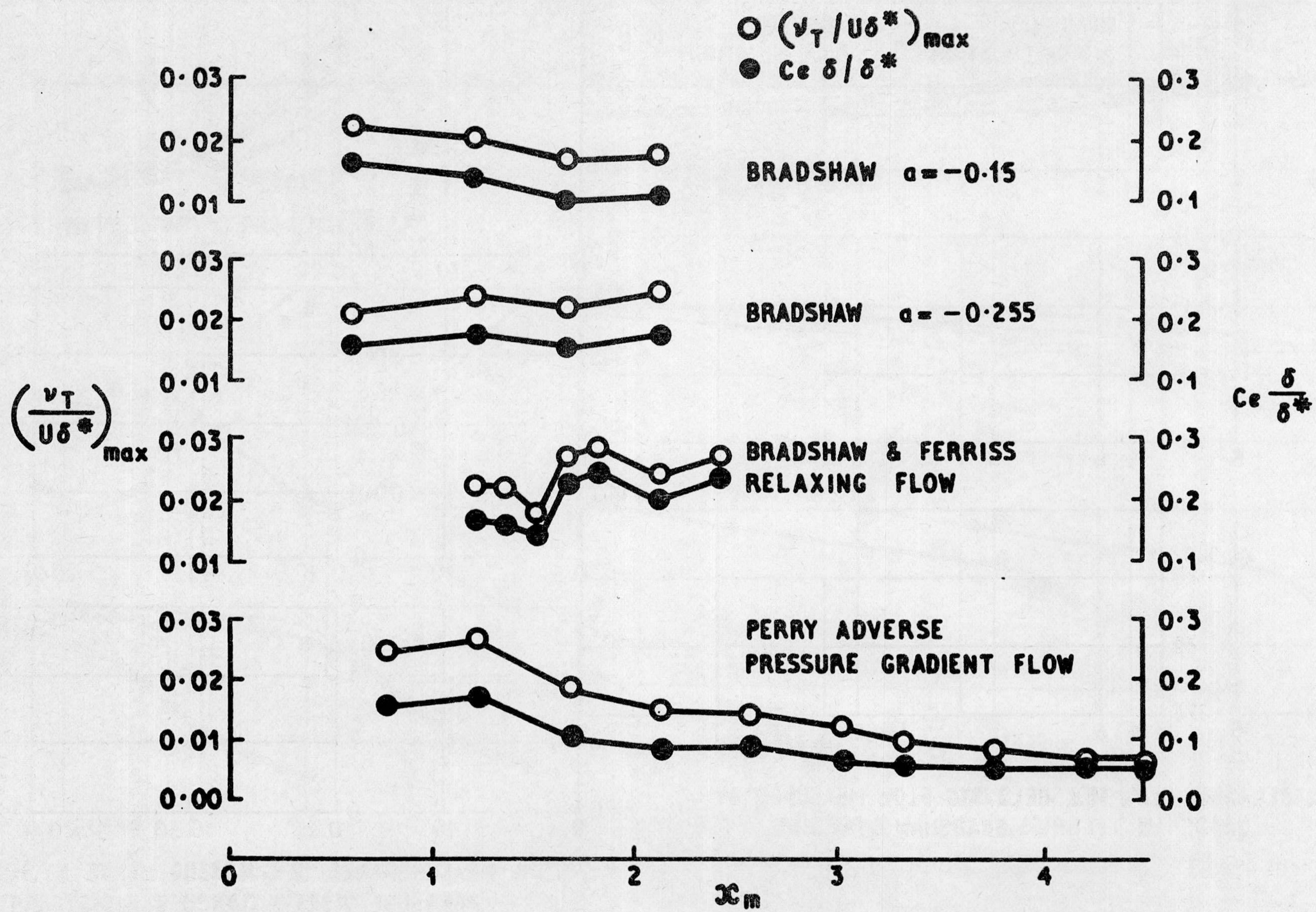


FIG. 3. COMPARISON OF $(\nu_T/U\delta^*)_{max}$ AND $Ce\,\delta/\delta^*$ DEVELOPMENTS

(TAKEN FROM REF. 15)

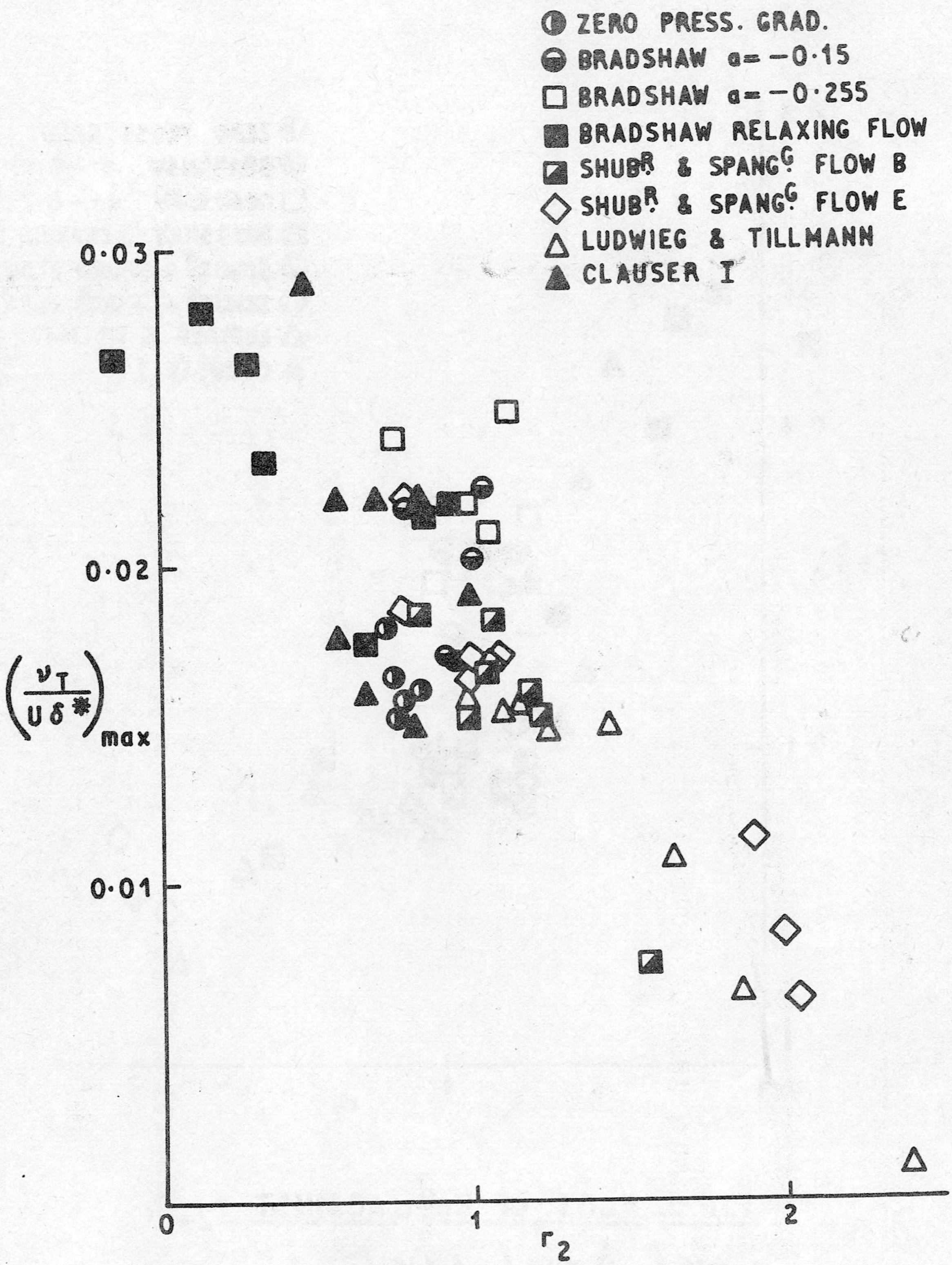


FIG. 4. PLOT OF $(\nu_T/U\delta^*)_{max}$ AGAINST r_2.

NOTE:- $r_2 = \frac{1}{U}\frac{dU\theta}{dx}\Big]_{exp.} \Big/ \frac{1}{U}\frac{dU\theta}{dx}\Big]_{eq}$, $\frac{1}{U}\frac{dU\theta}{dx}\Big]_{eq}$ OBTAINED FROM NASH'S (1965)[3] π – G RELATION

(REPRODUCED FROM REF. 15)

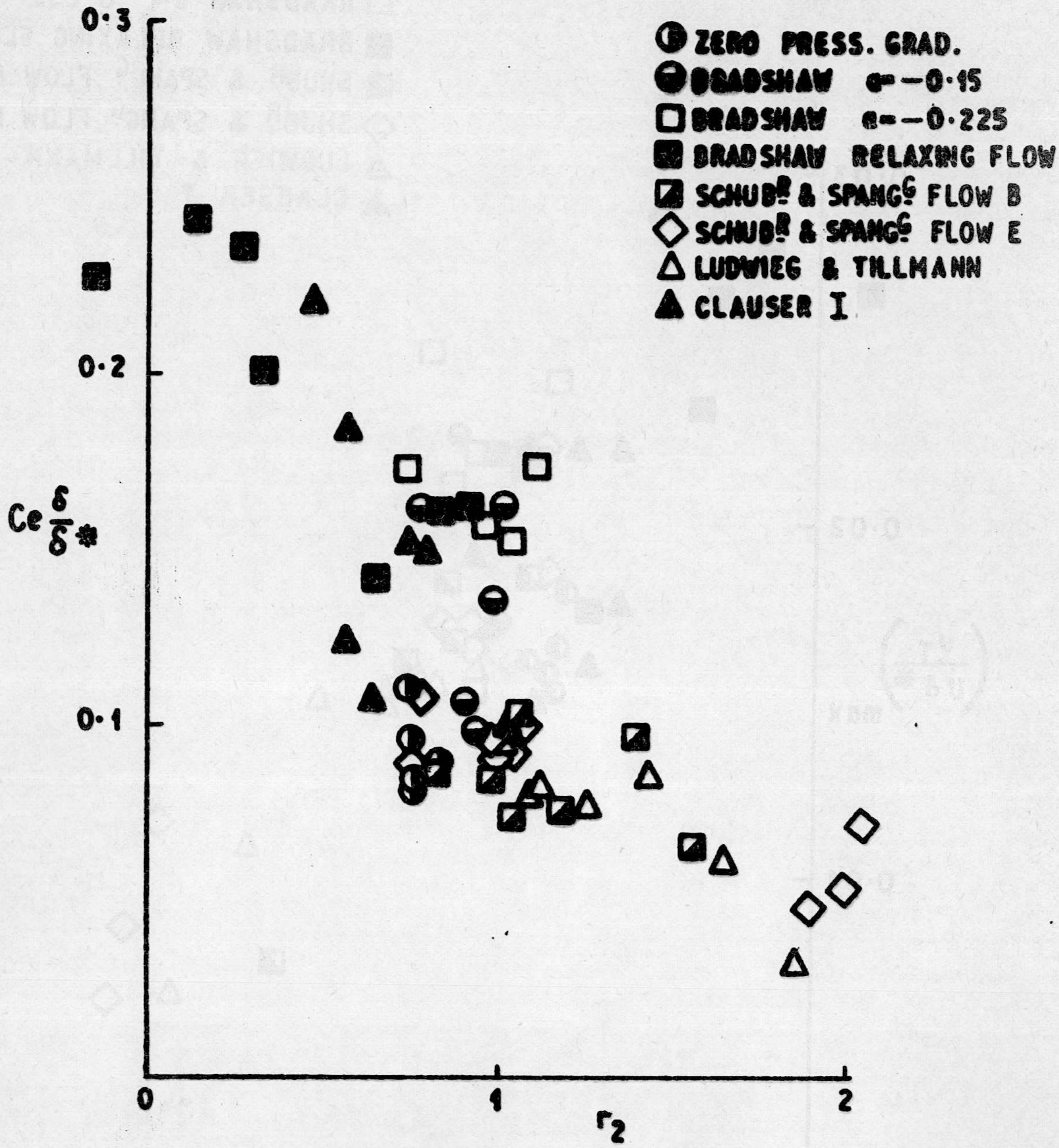


FIG. 5. PLOT OF $Ce\frac{\delta}{\delta^*}$ AGAINST r_2

NOTE:- $r_2 = \left.\frac{1}{U}\frac{dU\theta}{dx}\right)_{exp} \Big/ \left.\frac{1}{U}\frac{dU\theta}{dx}\right)_{eq}$, $\left.\frac{1}{U}\frac{dU\theta}{dx}\right)_{eq}$ OBTAINED FROM NASH'S (1965)[3] π–G RELATION

(REPRODUCED FROM REF. 15)

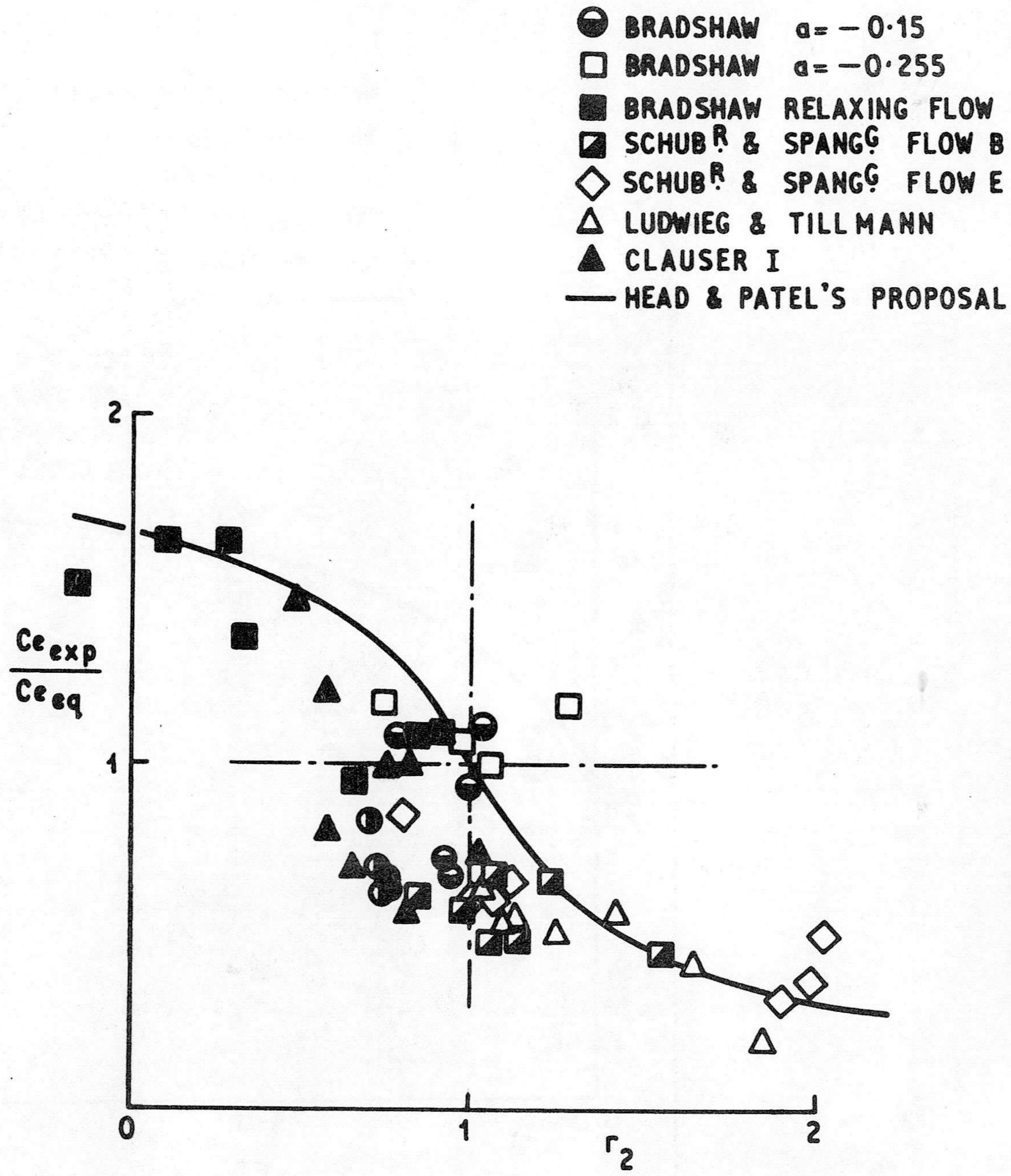


FIG. 6. PLOT OF Ce_{exp}/Ce_{eq} AGAINST r_2.

NOTE:- $r_2 = \left.\frac{1}{U}\frac{dU\theta}{dx}\right/_{exp.} \Big/ \left.\frac{1}{U}\frac{dU\theta}{dx}\right/_{eq}$, $\left.\frac{1}{U}\frac{dU\theta}{dx}\right/_{eq}$ OBTAINED FROM NASH'S (1965) $\pi - G$ RELATION

(REPRODUCED FROM REF. 15)

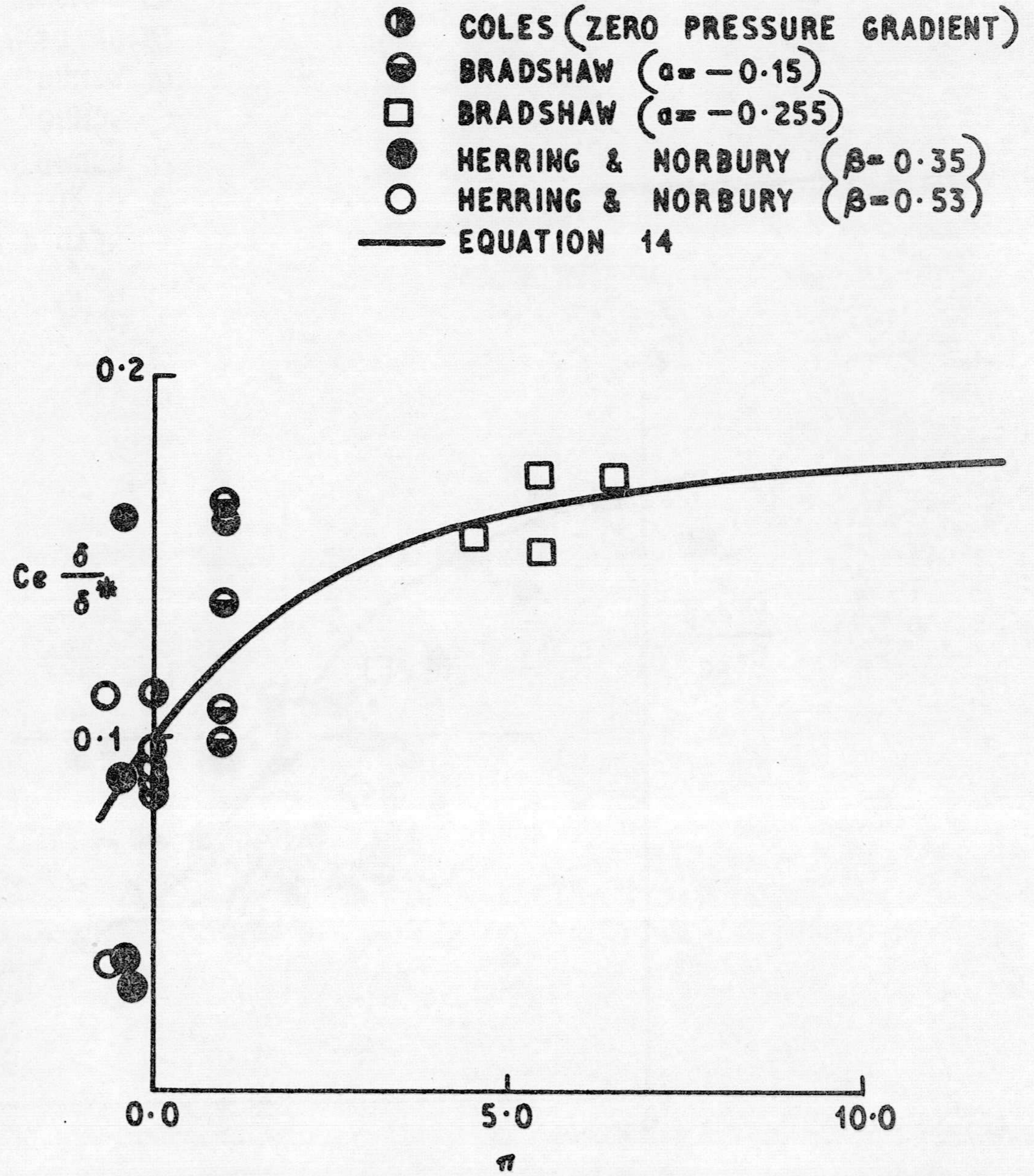


FIG. 7. VARIATION OF $Ce \frac{\delta}{\delta^*}$ FOR EQUILIBRIUM LAYERS

(DATA TAKEN FROM THE ANALYSIS OF REF. 14)

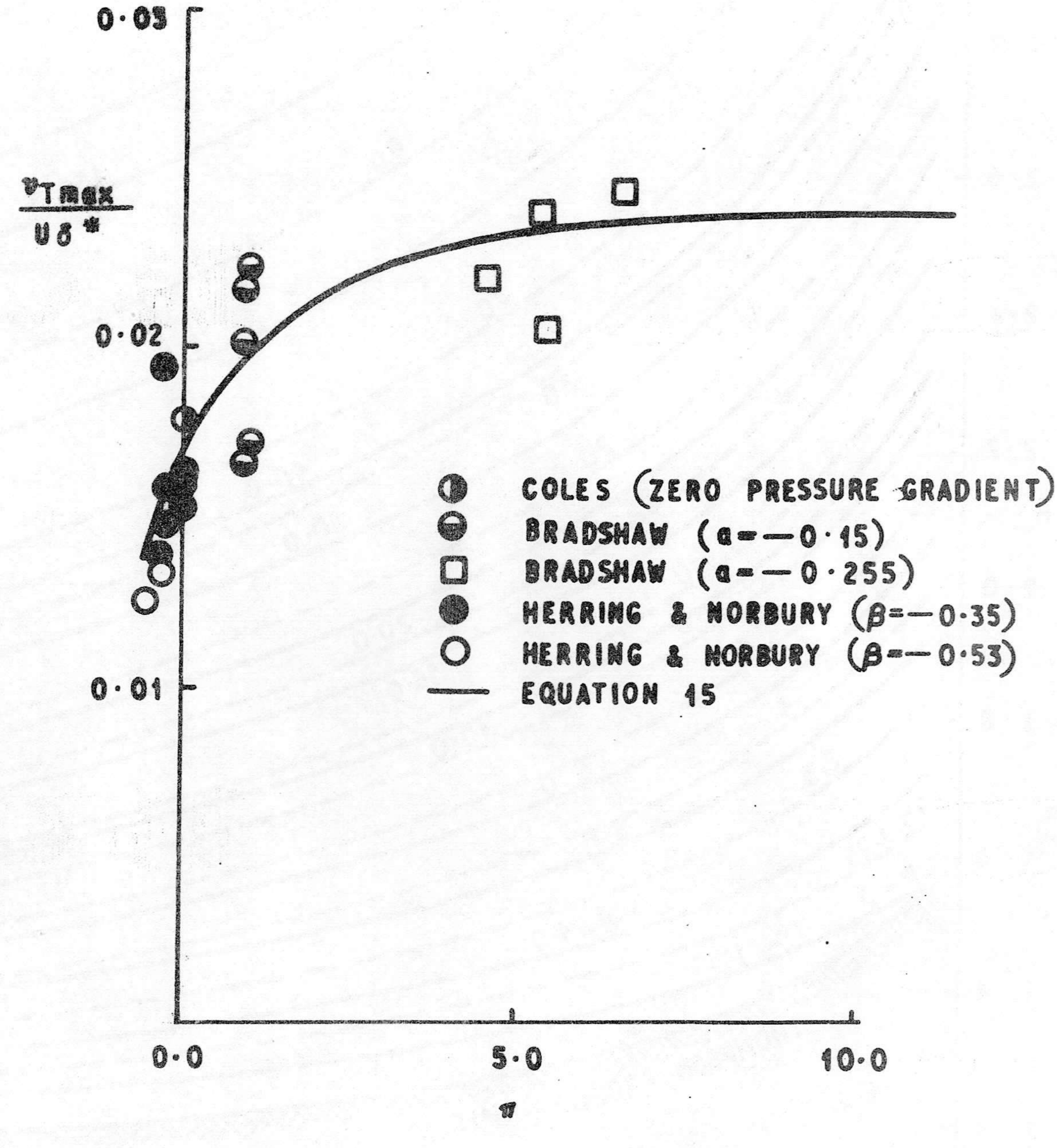


FIG. 8. VARIATION OF $\nu_{T max}/U\delta^*$ FOR EQUILIBRIUM LAYERS.

(DATA TAKEN FROM THE ANALYSIS OF REF. 14)

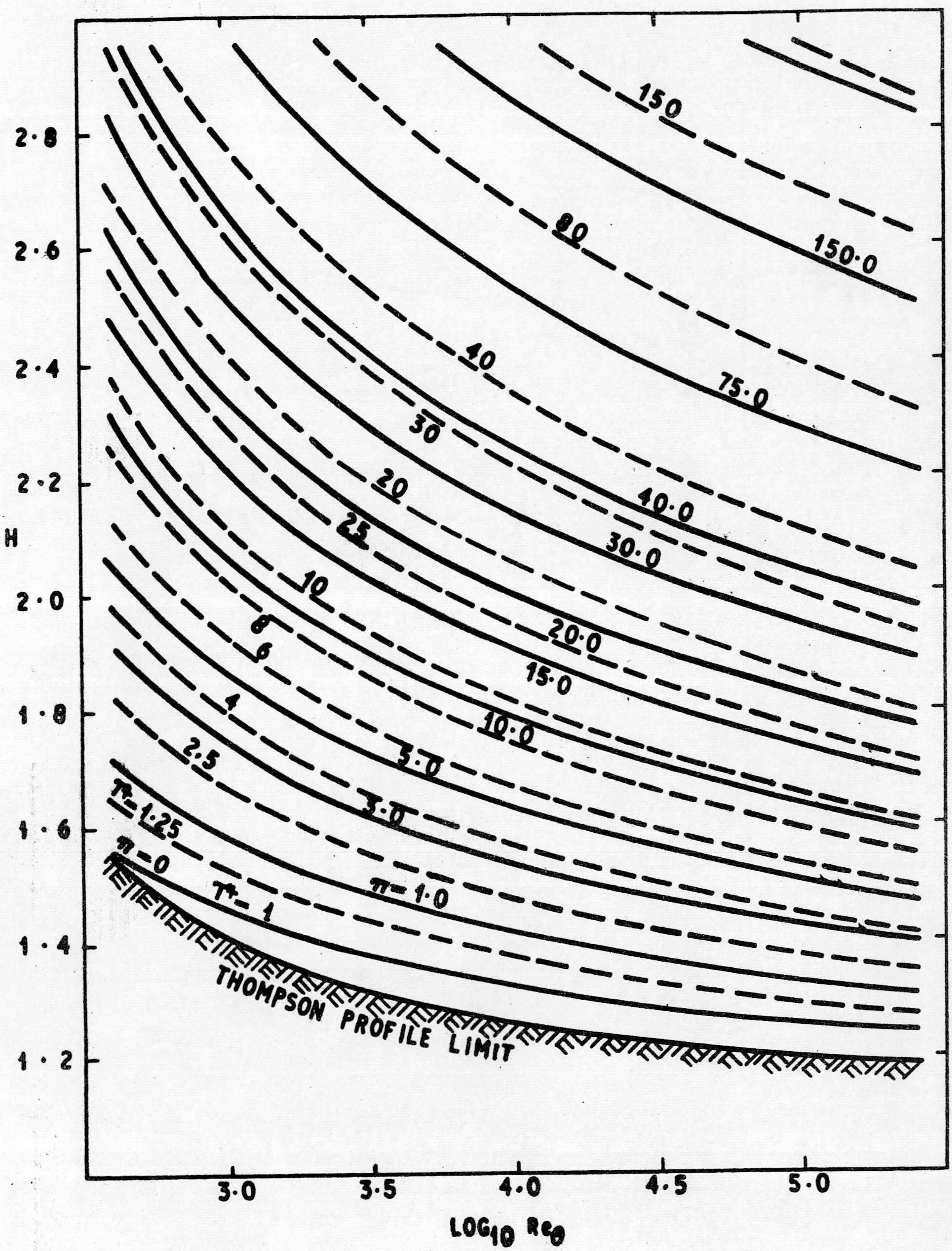


FIG. 9. COMPARISON OF π AND $\tau^{+} = \tau/\tau_{\omega}|_{max}$ CONTOURS FOR EQUILIBRIUM LAYER.

(FROM ANALYSIS OF REF. 16.)

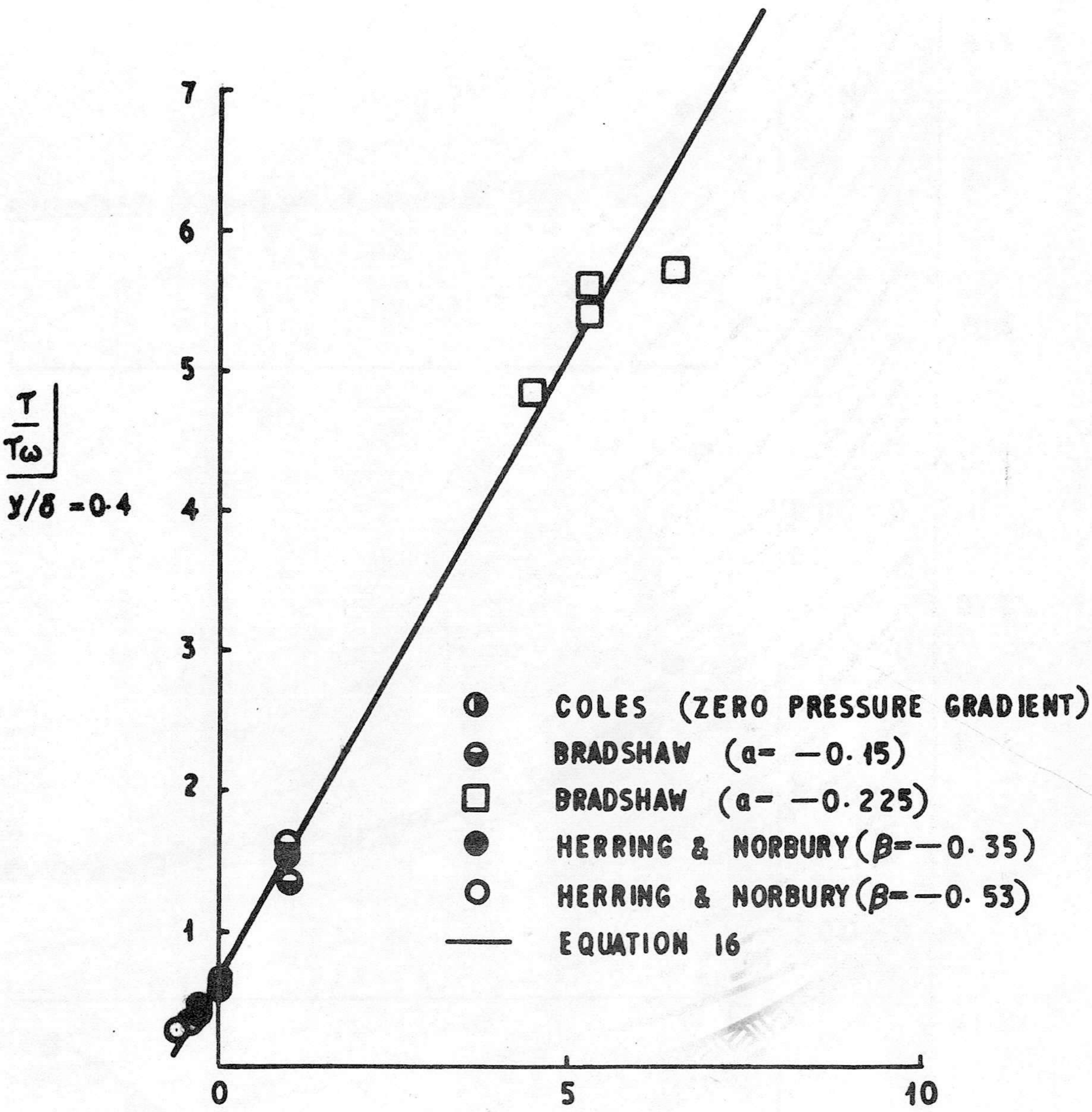


FIG. 10. VARIATION OF $(T/T_\omega)_{y/\delta = 0.4}$ FOR EQUILIBRIUM LAYERS.

(DATA TAKEN FROM THE ANALYSIS OF REF. 14)

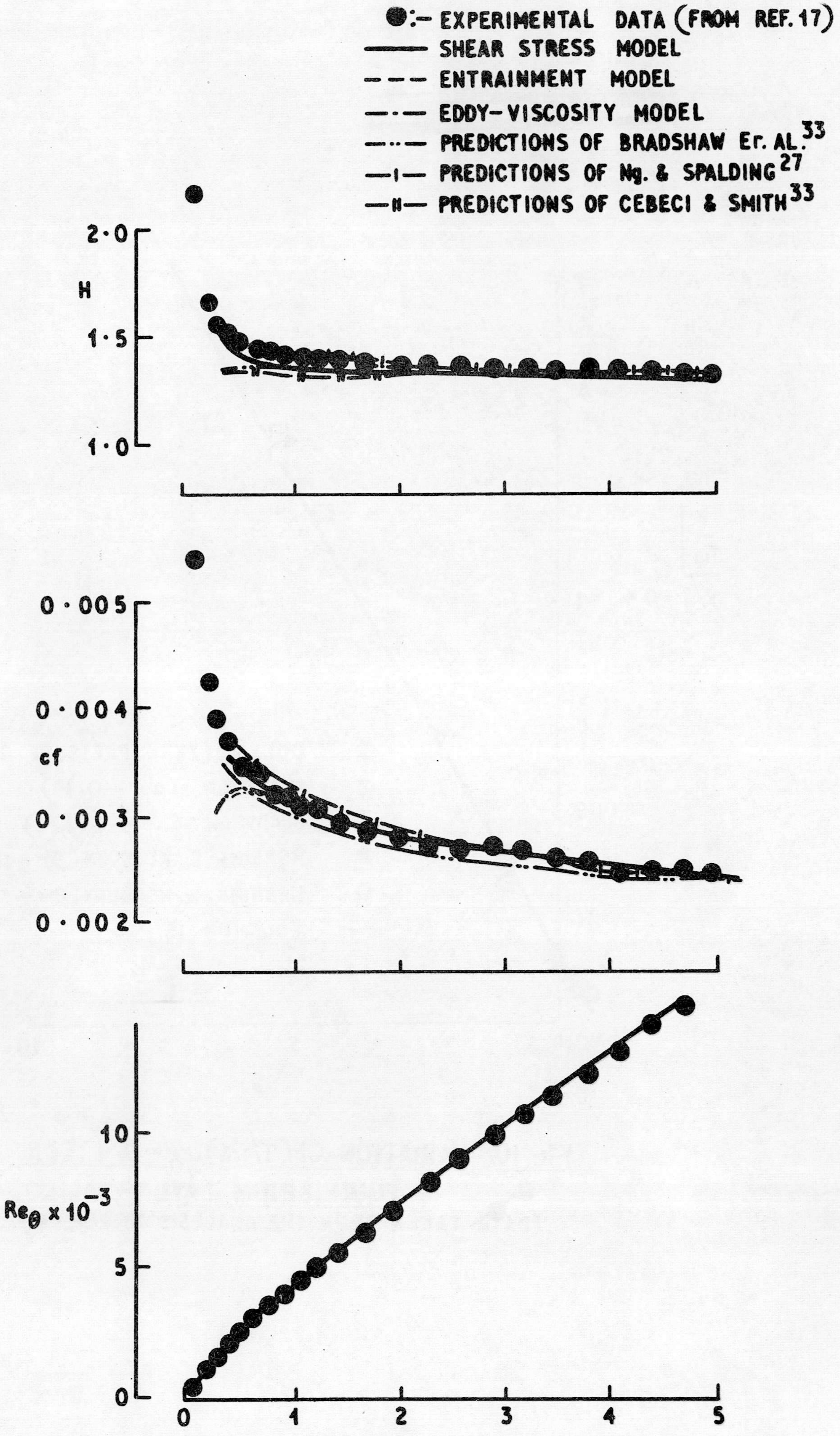


FIG. 11. WIEGHARDT, ZERO PRESSURE GRADIENT

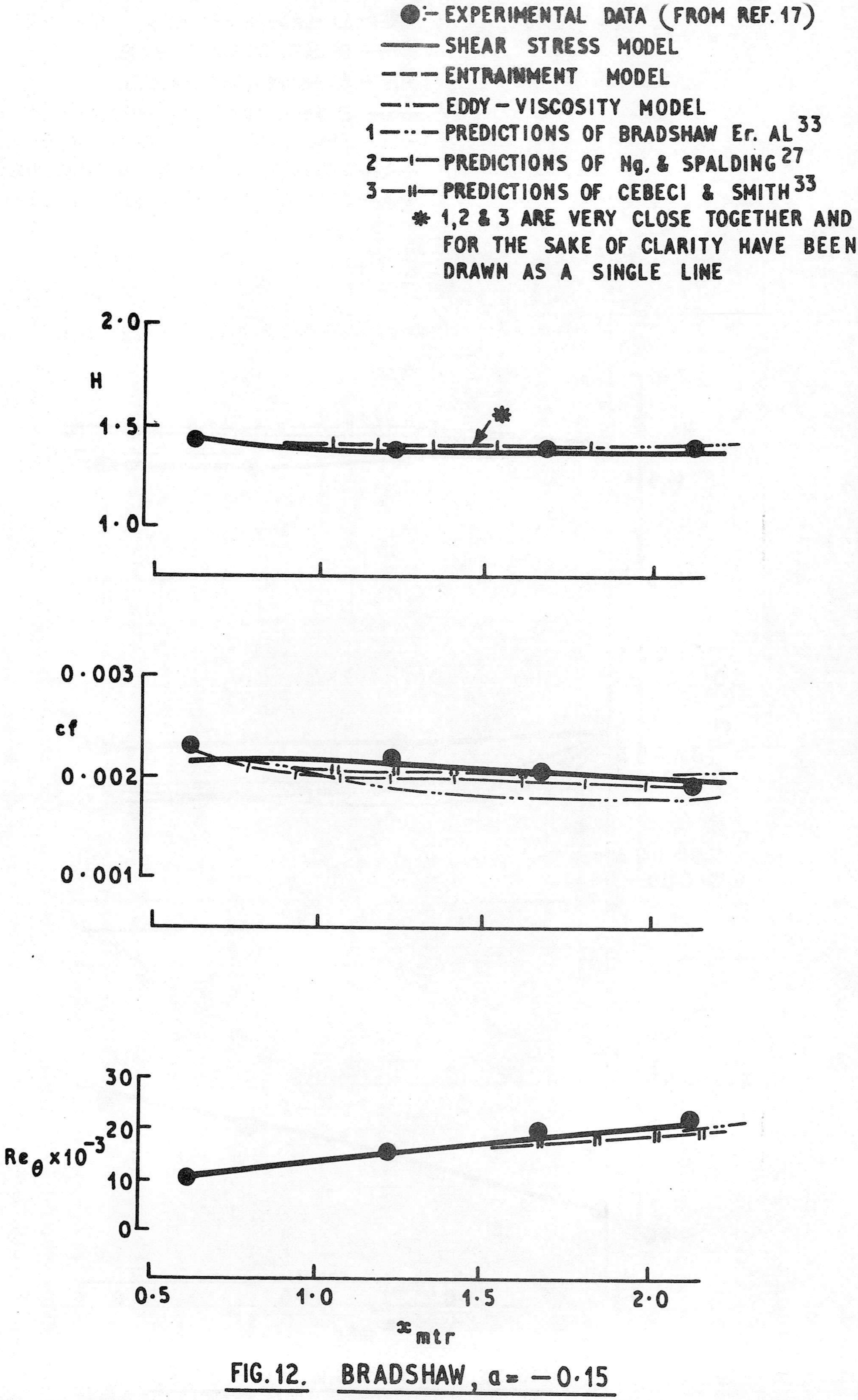


FIG. 12. BRADSHAW, $a = -0{\cdot}15$

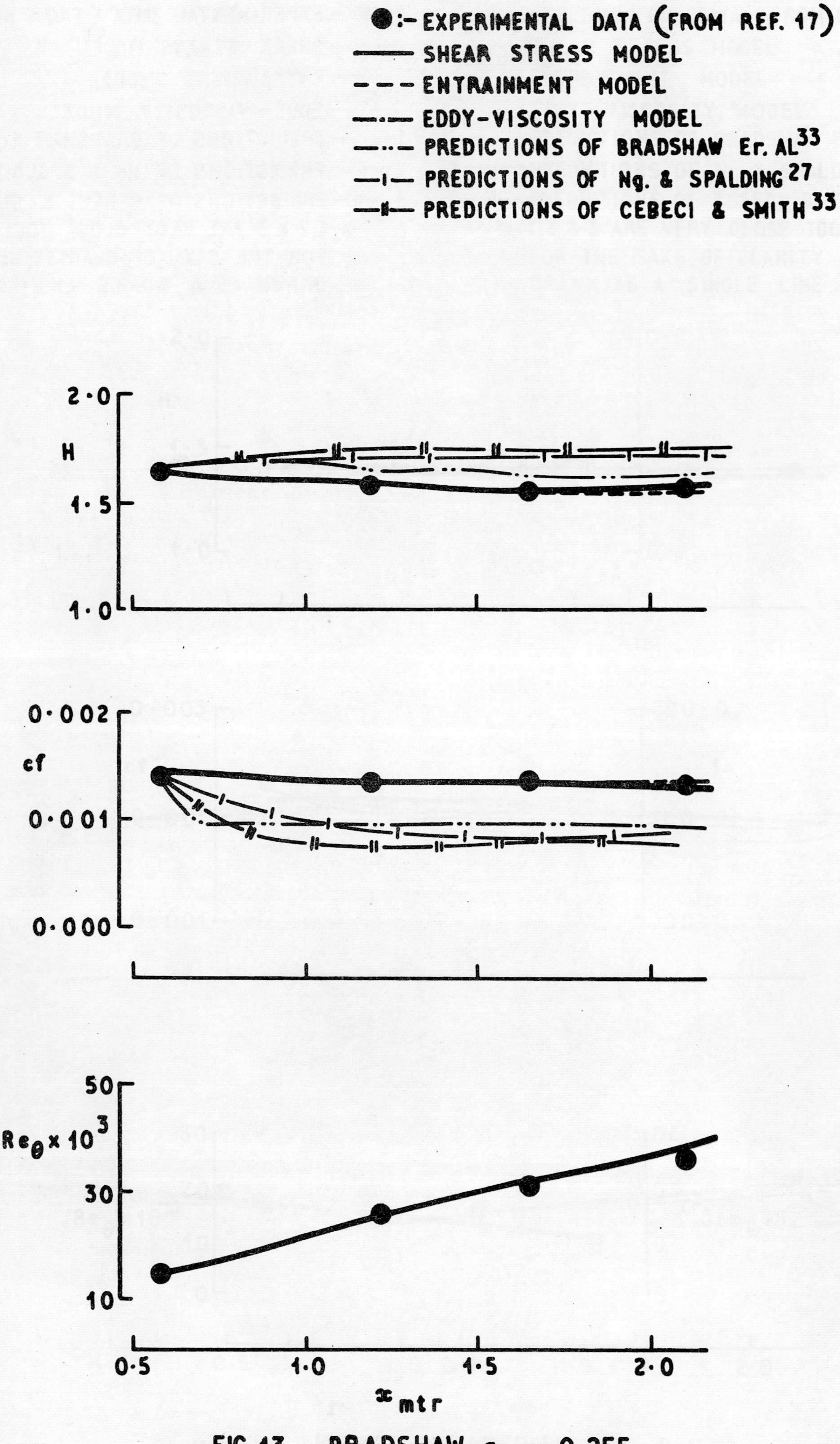


FIG. 13. BRADSHAW, a = — 0·255

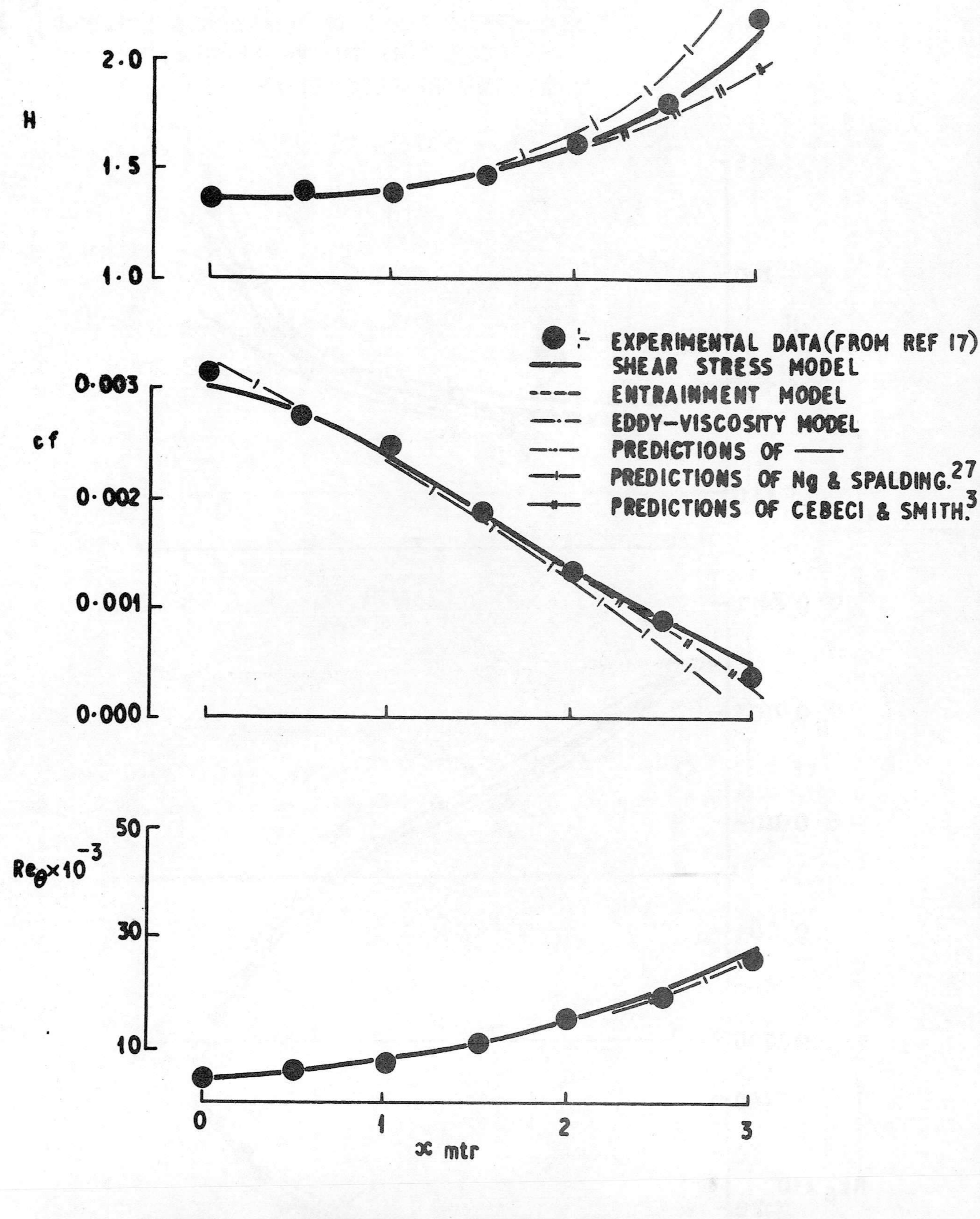


FIG. 14. SCHUBAUER & SPANGENBERG, FLOW B

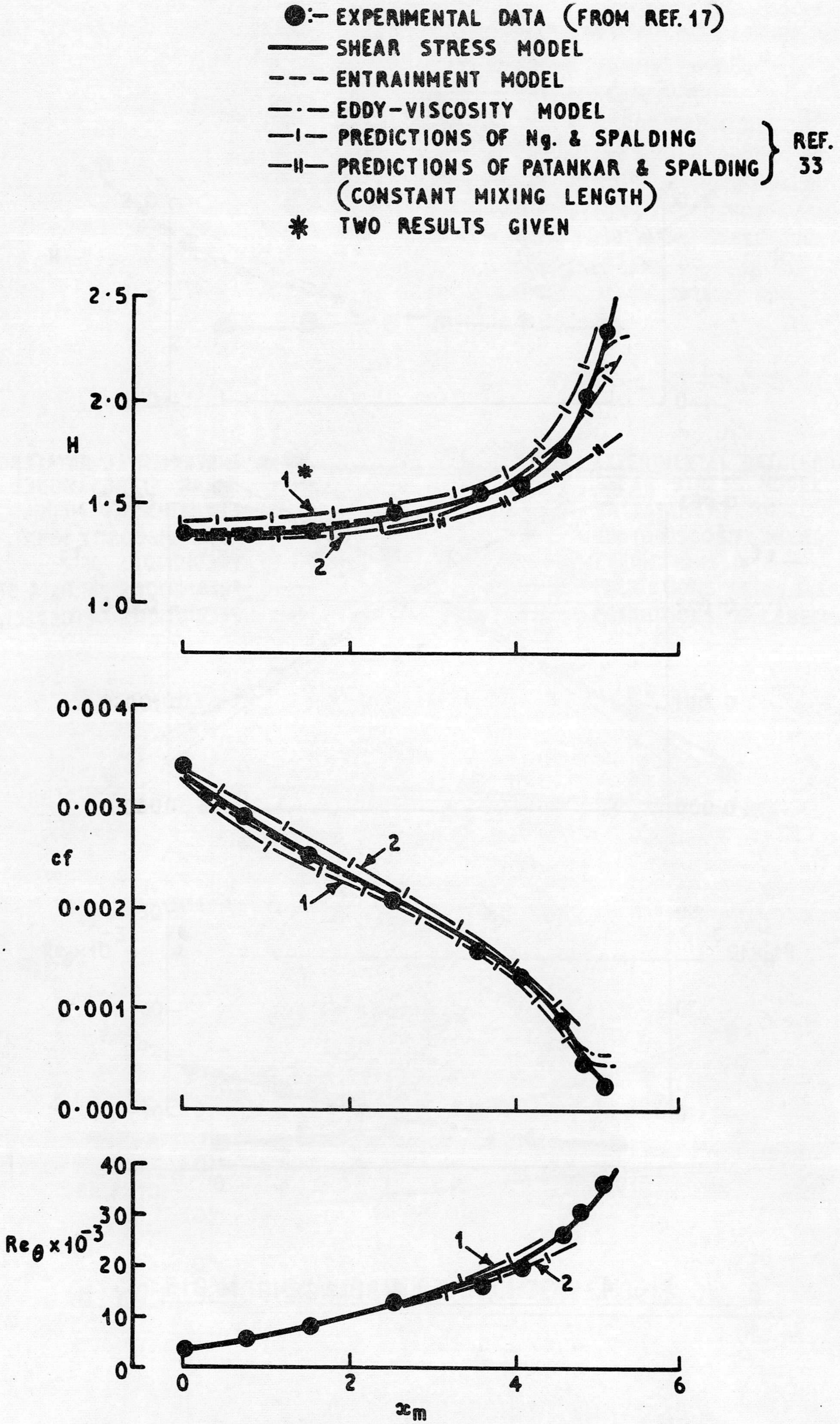


FIG. 15. SCHUBAUER & SPANGENBERG (FLOW E)

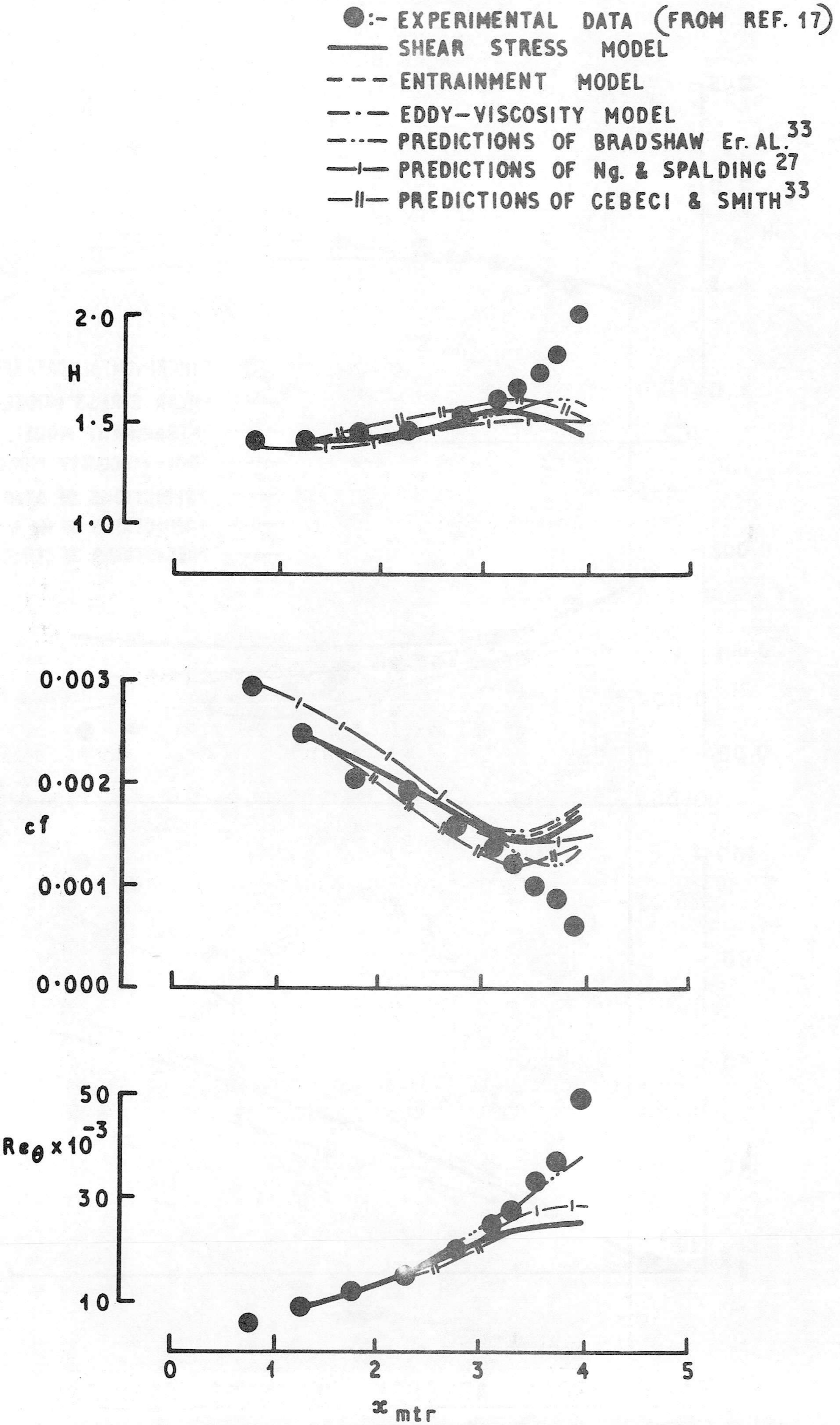


FIG. 16. LUDWIEG & TILLMANN, dp/dx >> 0·0

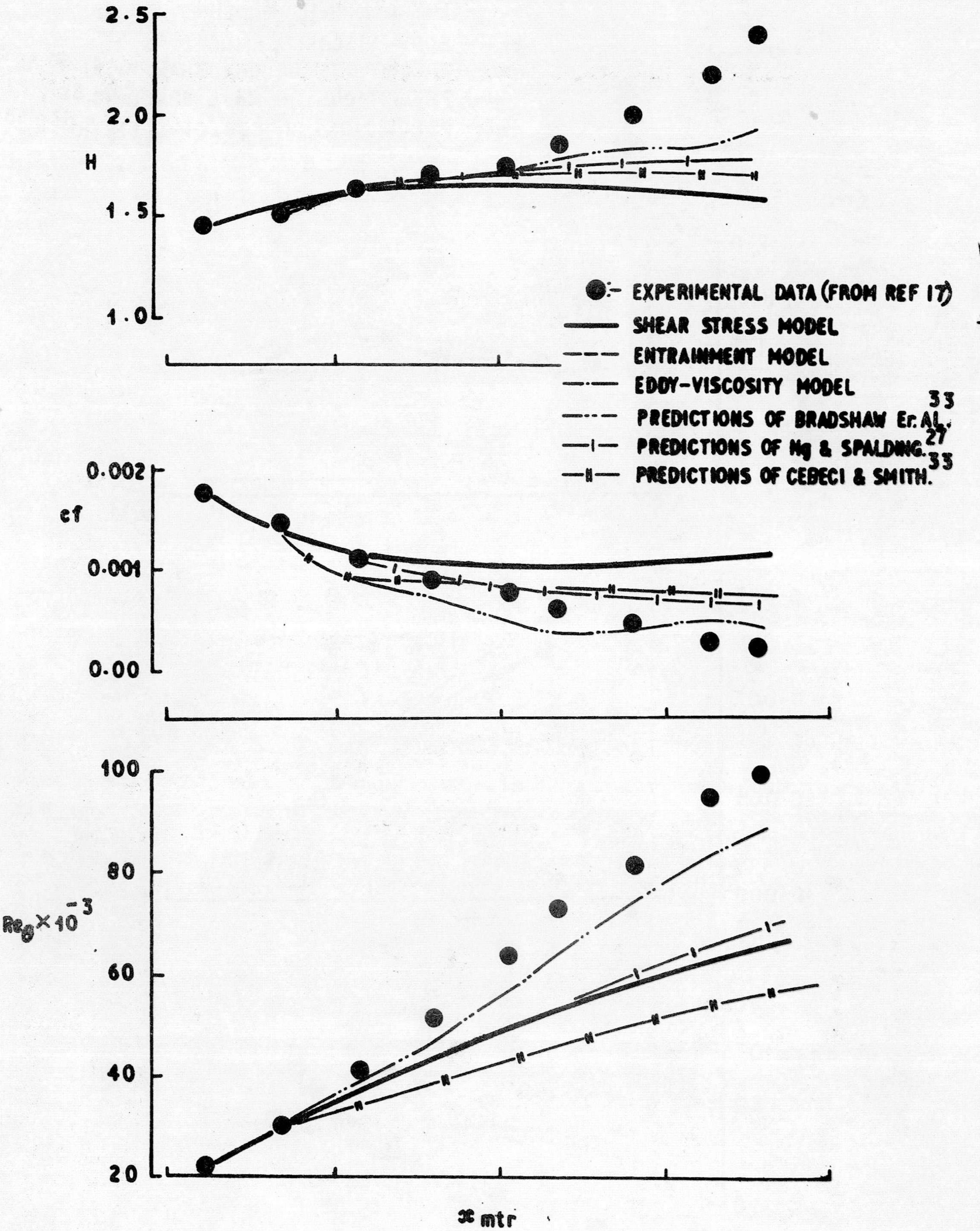


FIG. 17. PERRY, ADVERSE PRESSURE GRADIENT.

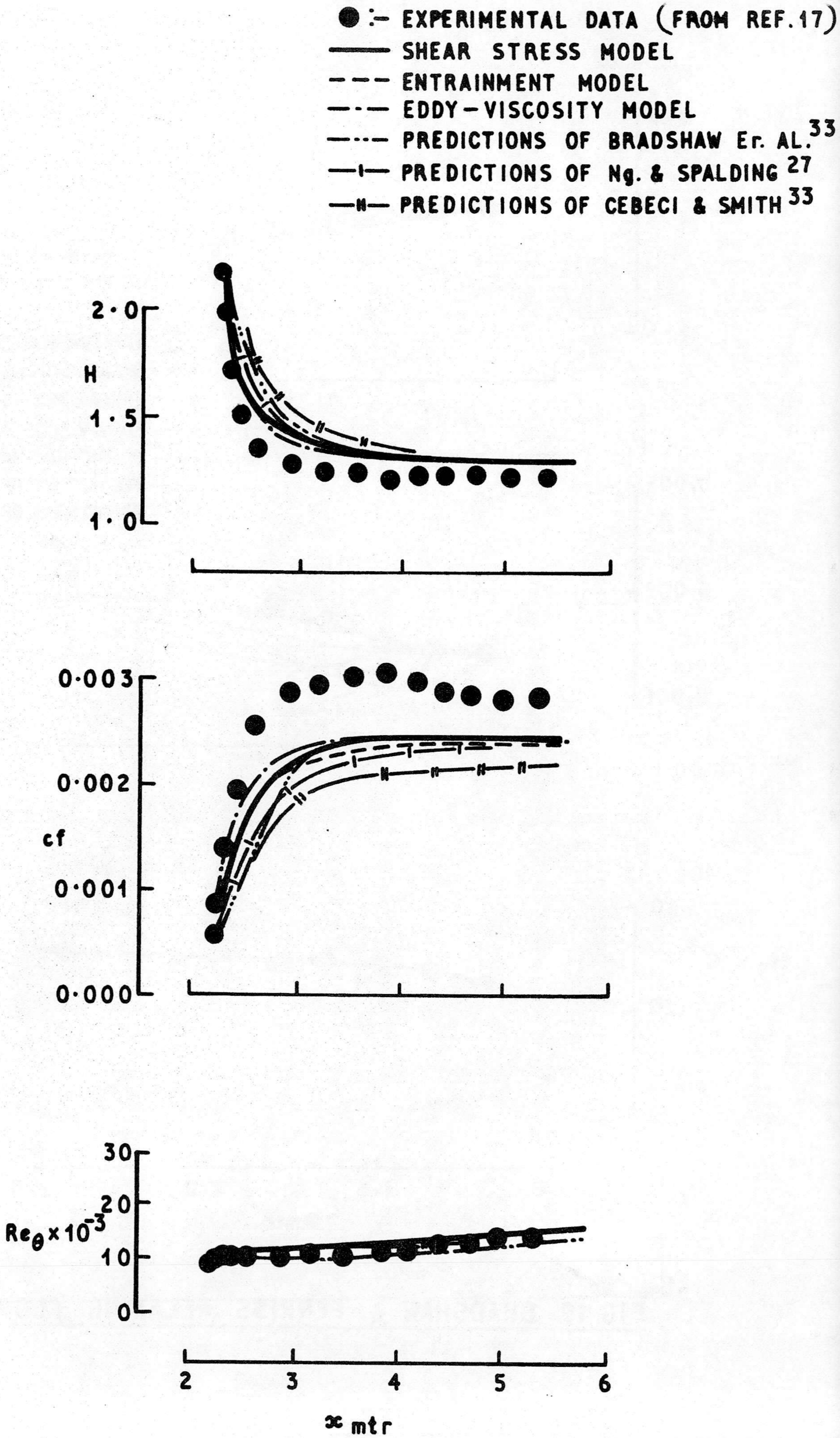


FIG. 18. TILLMANN LEDGE FLOW

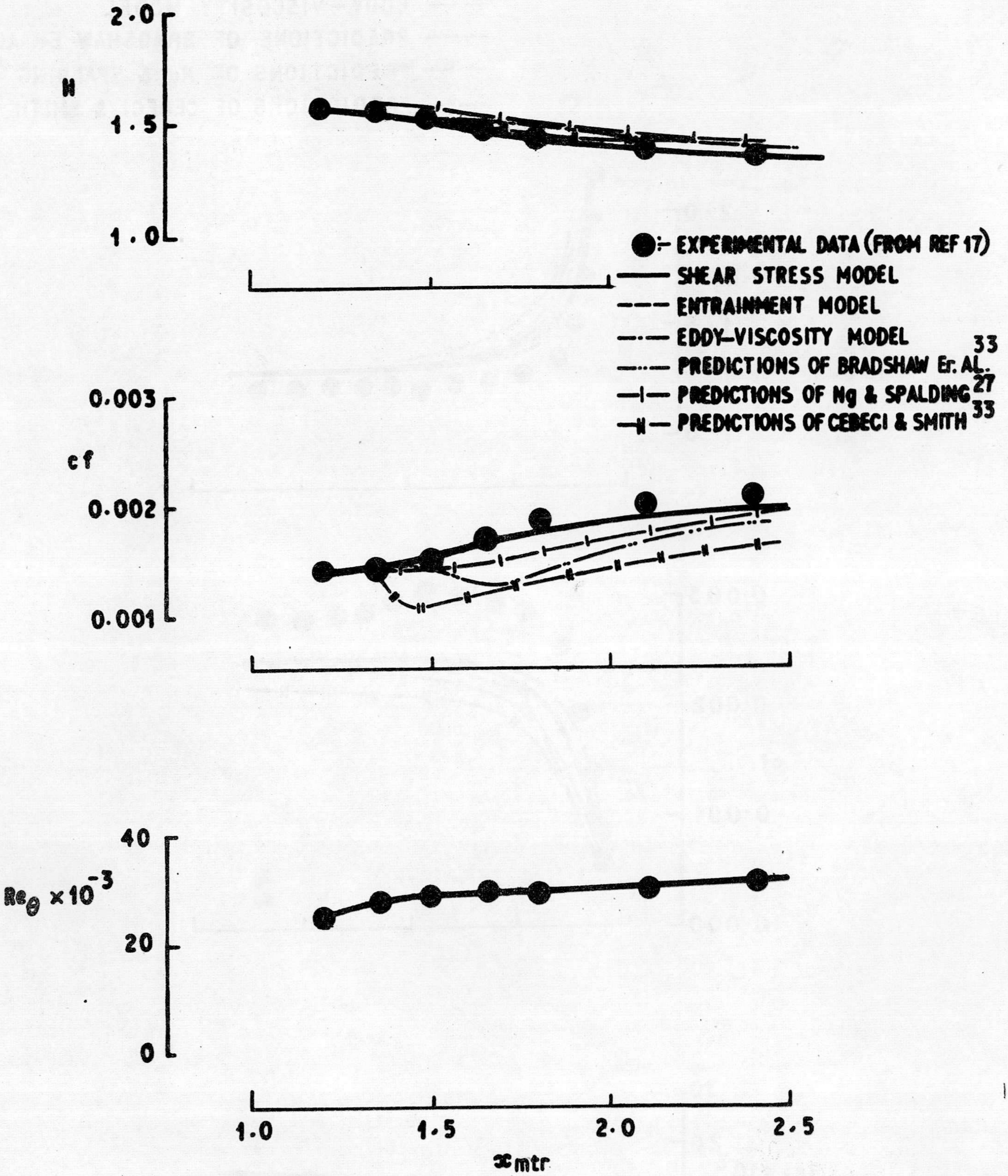


FIG. 19. BRADSHAW & FERRISS, RELAXING FLOW.

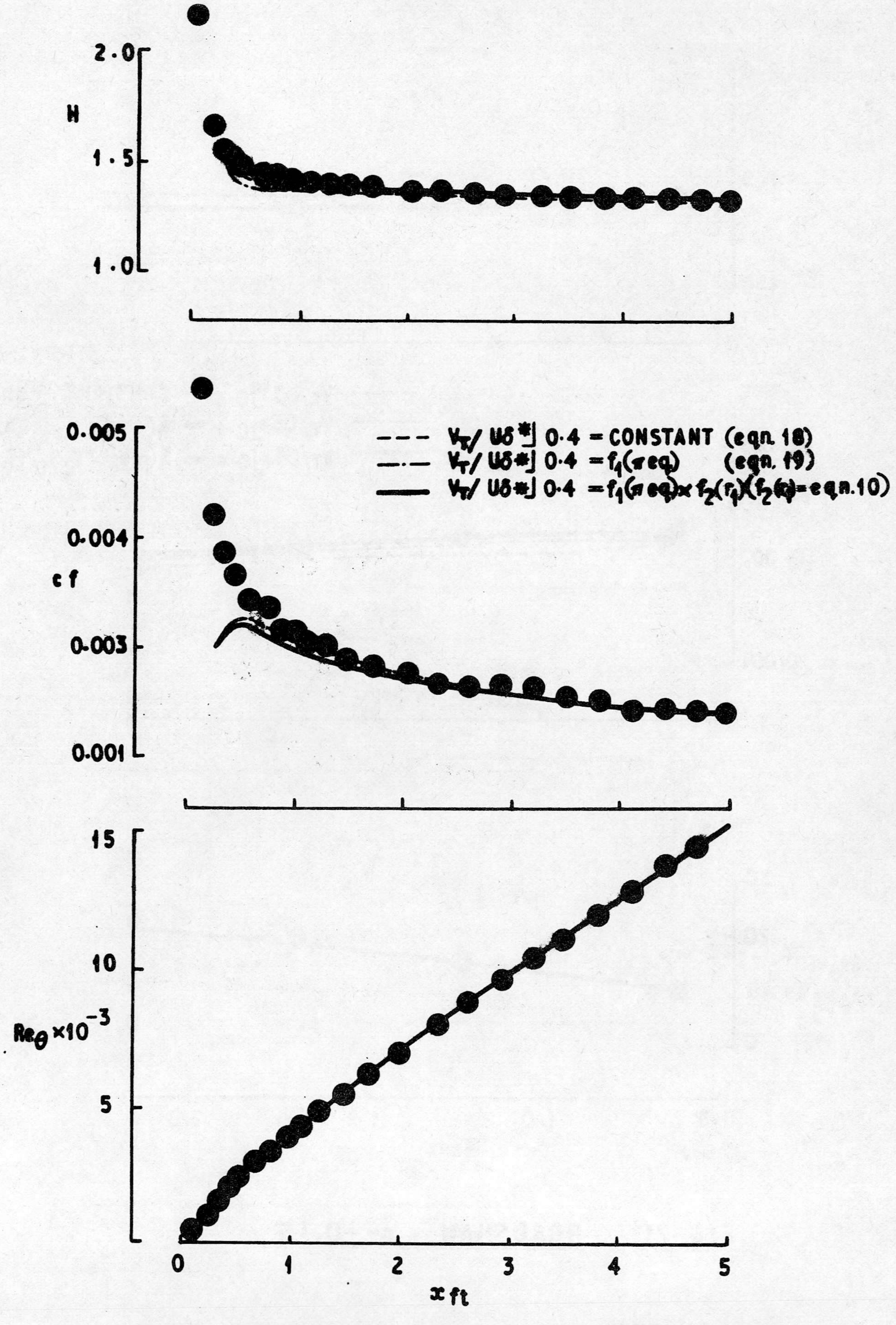


FIG. 20. WIEGHARDT, ZERO PRESSURE GRADIENT.

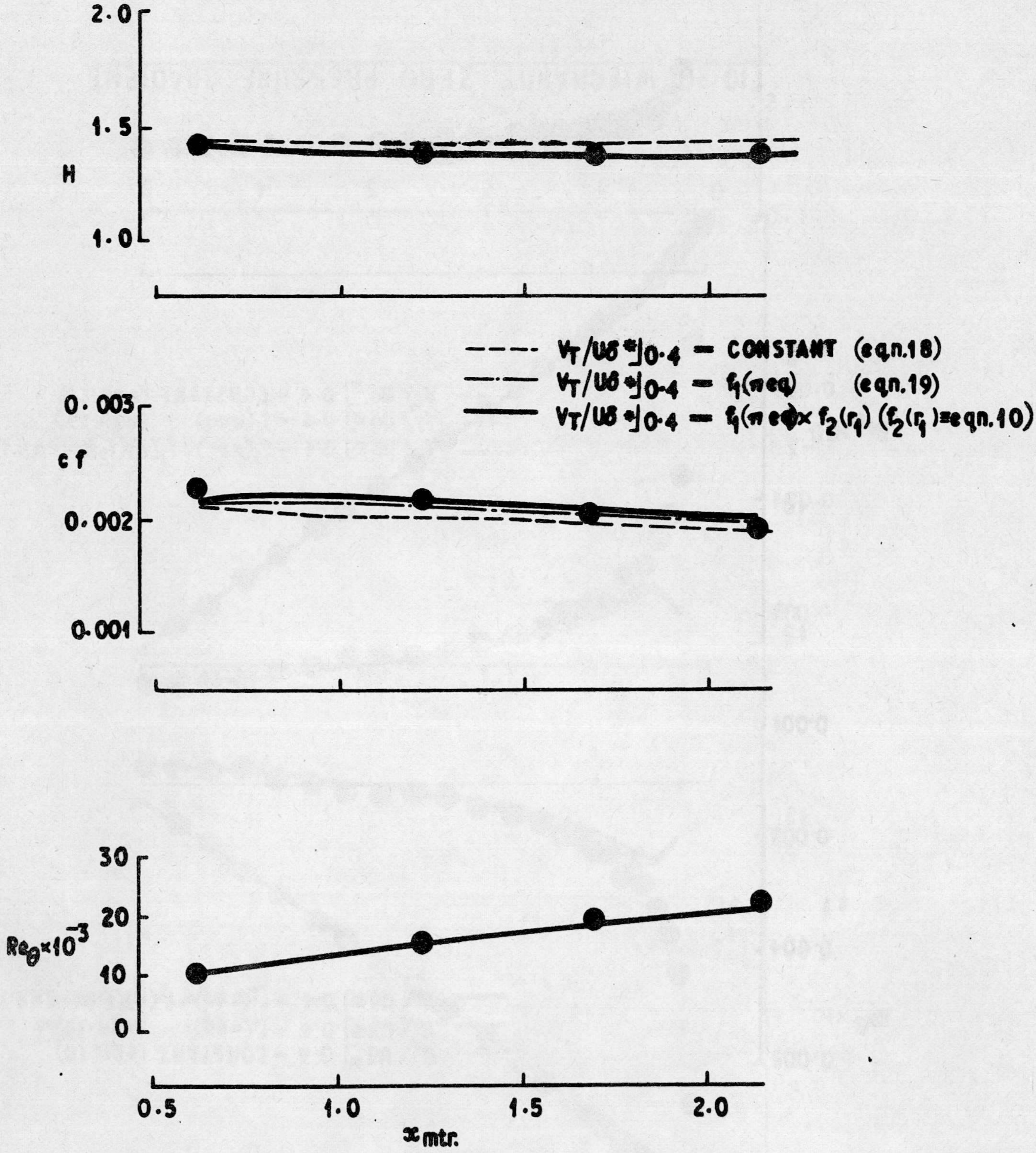


FIG. 21. BRADSHAW, a=−0.15

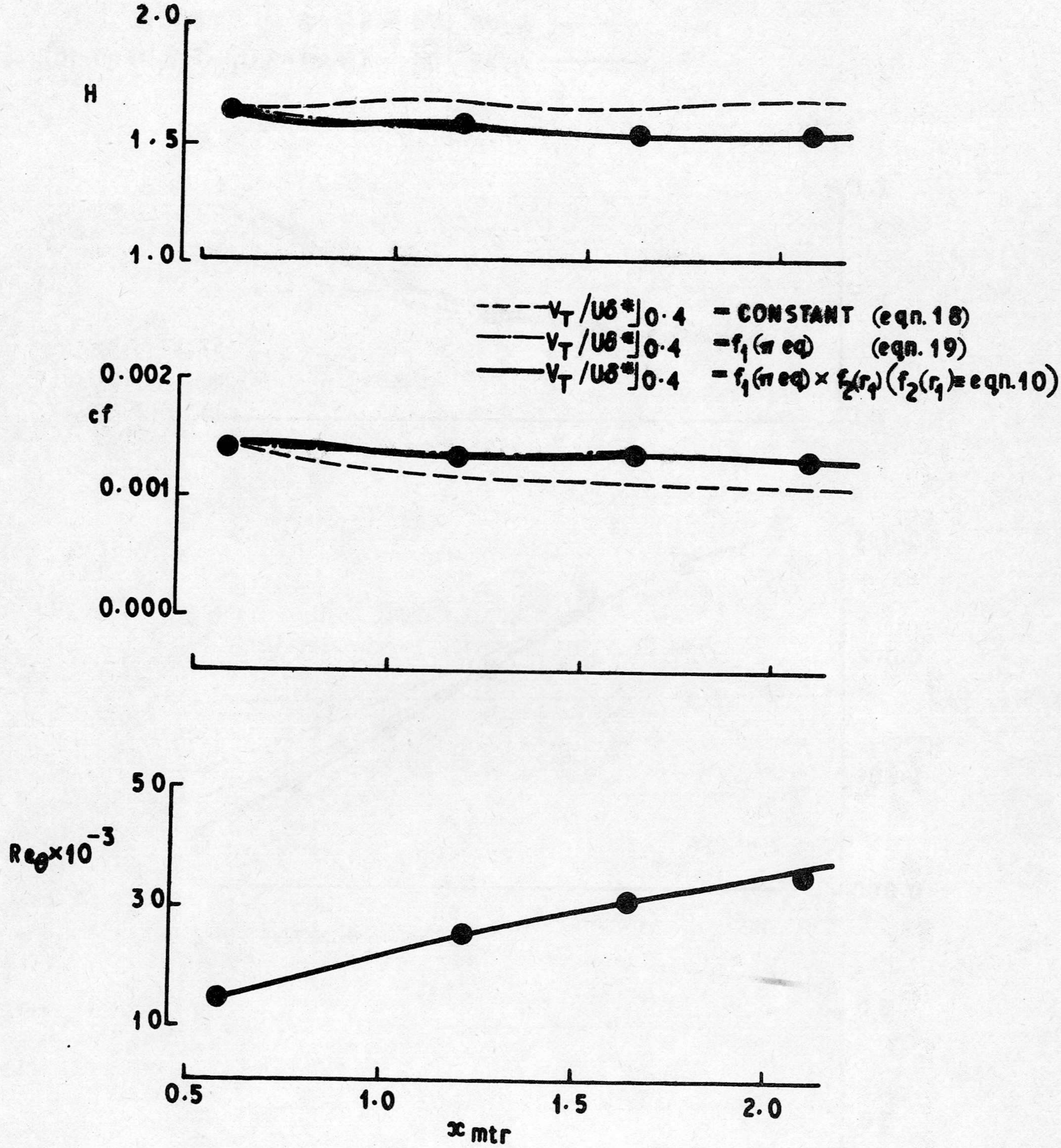


FIG. 22. BRADSHAW, $a = -0.255$.

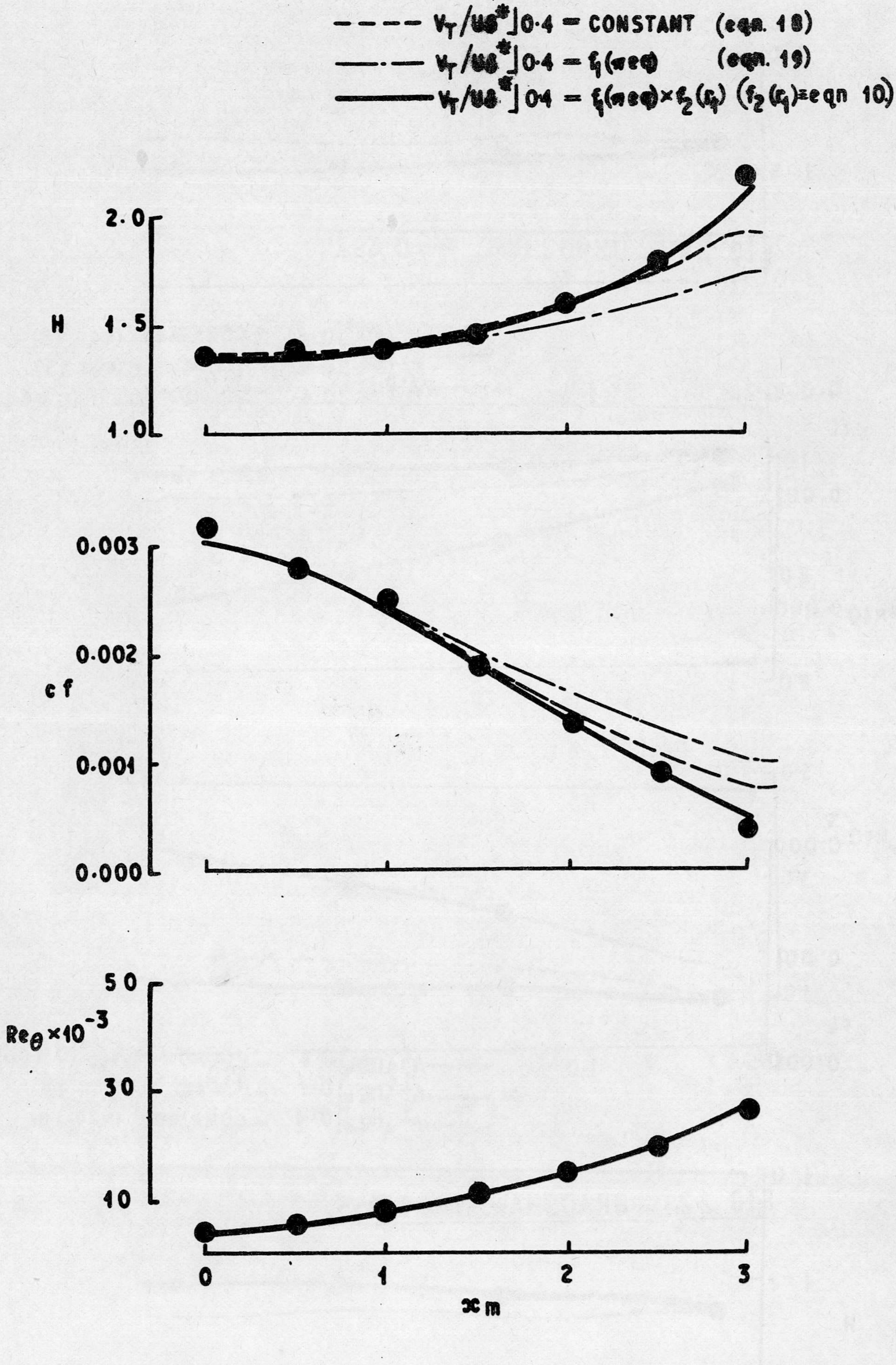


FIG. 23. SHUBAUER & SPANGENBERG, FLOW B.

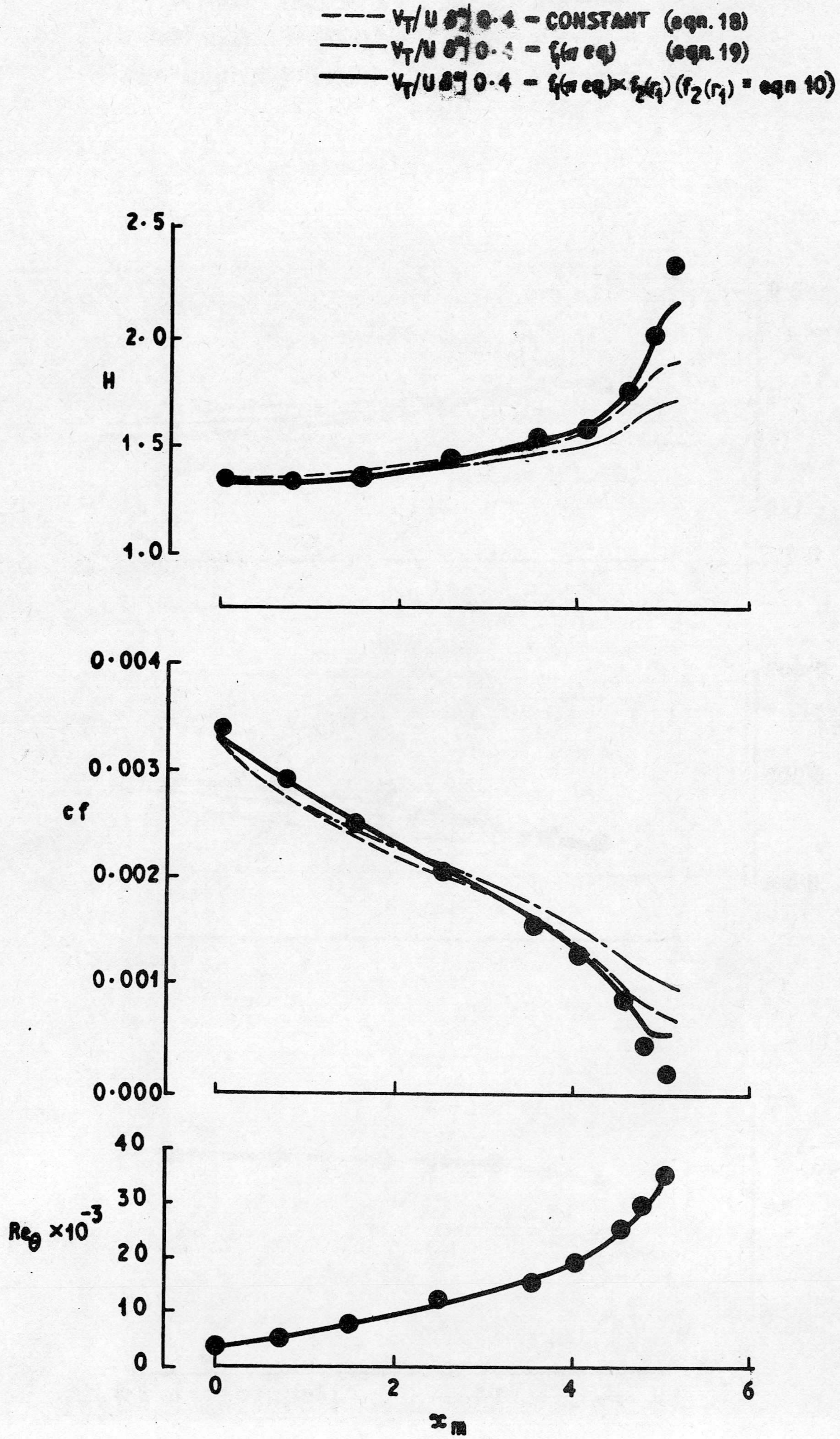


FIG. 24. SHUBAUER & SPANGENBERG, FLOW E.

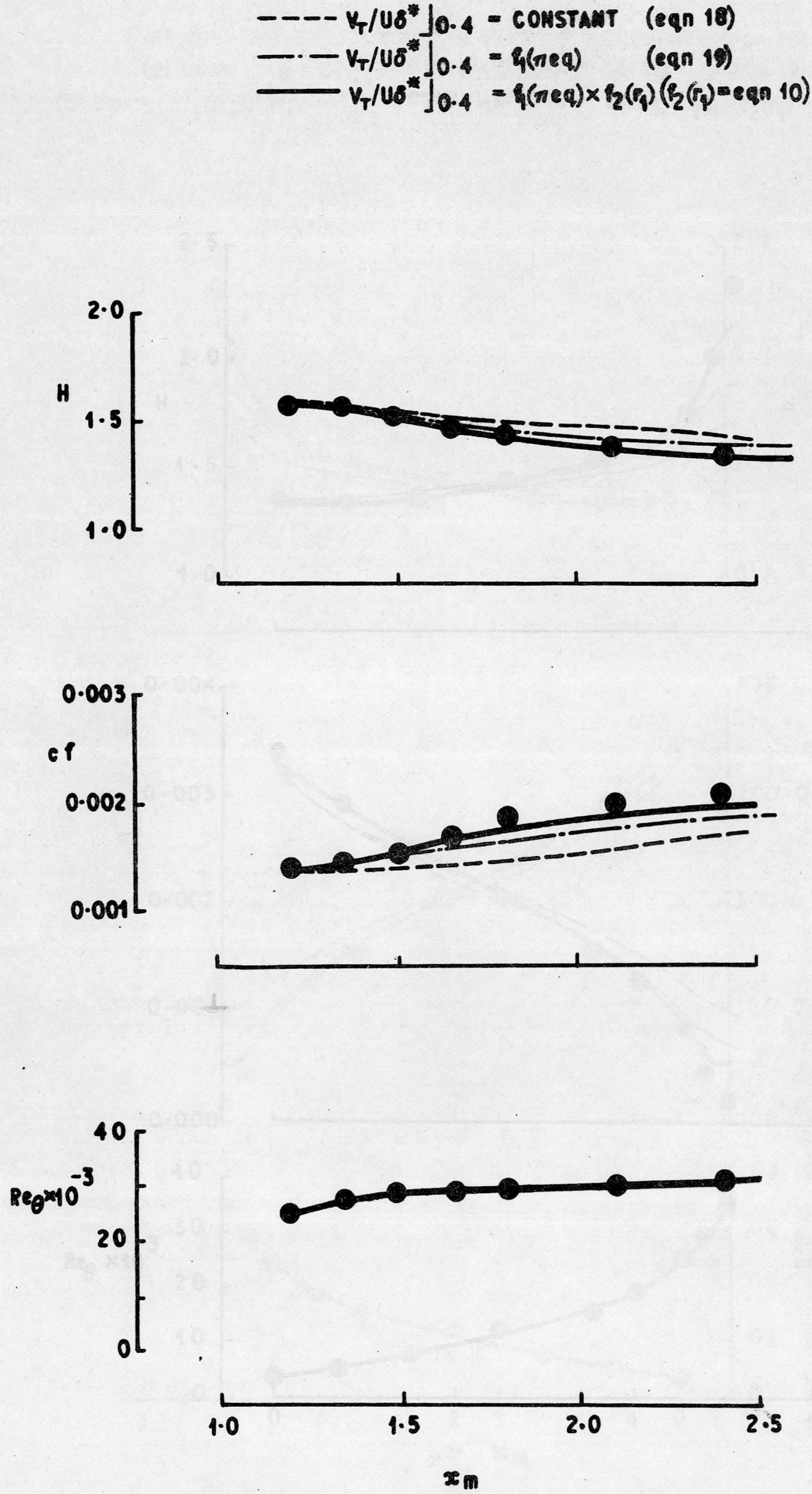


FIG. 25. BRADSHAW & FERRISS'S RELAXING FLOW

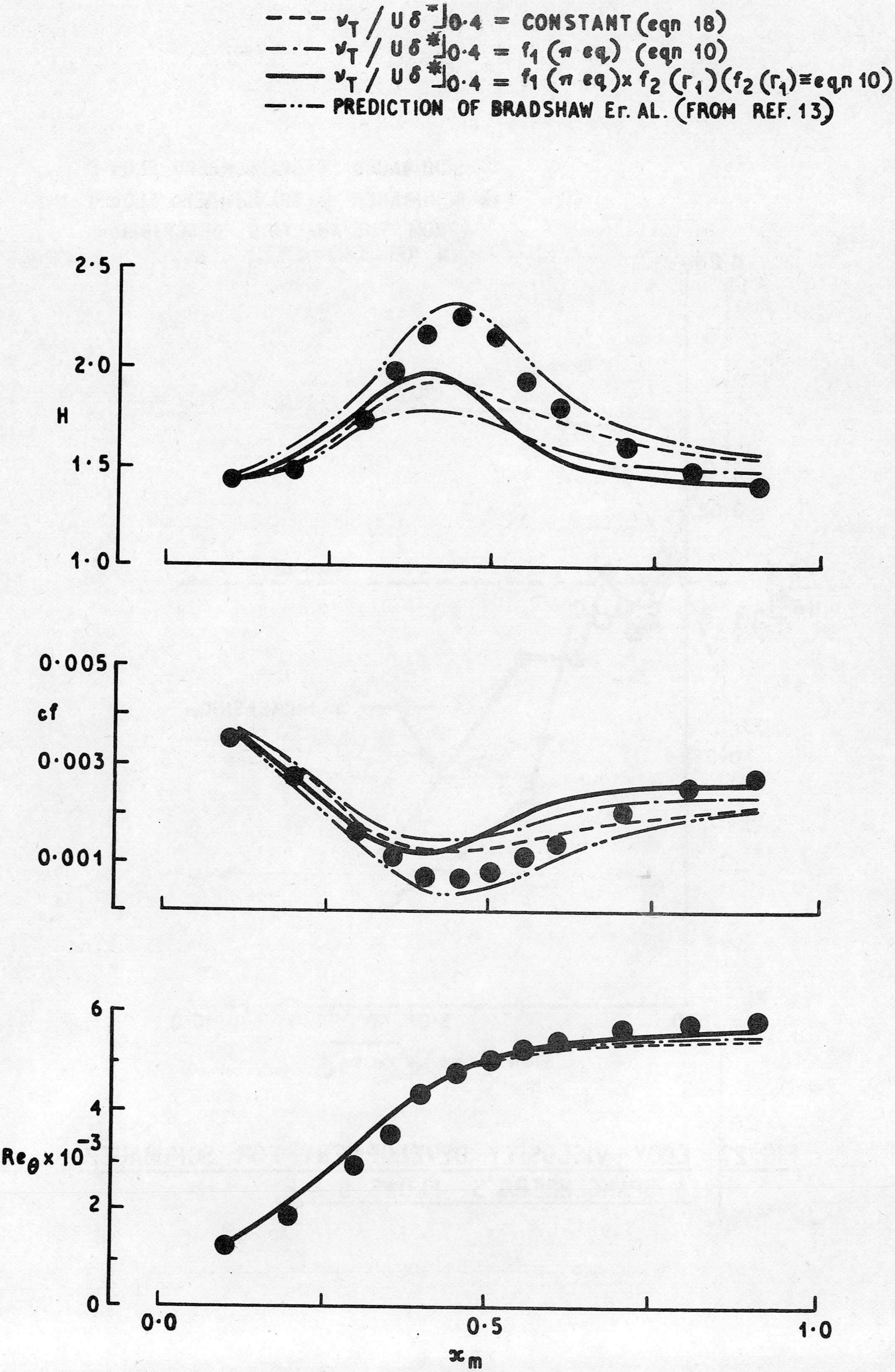


FIG. 26. GOLDBERG. PRESSURE DISTRIBUTION No.3.

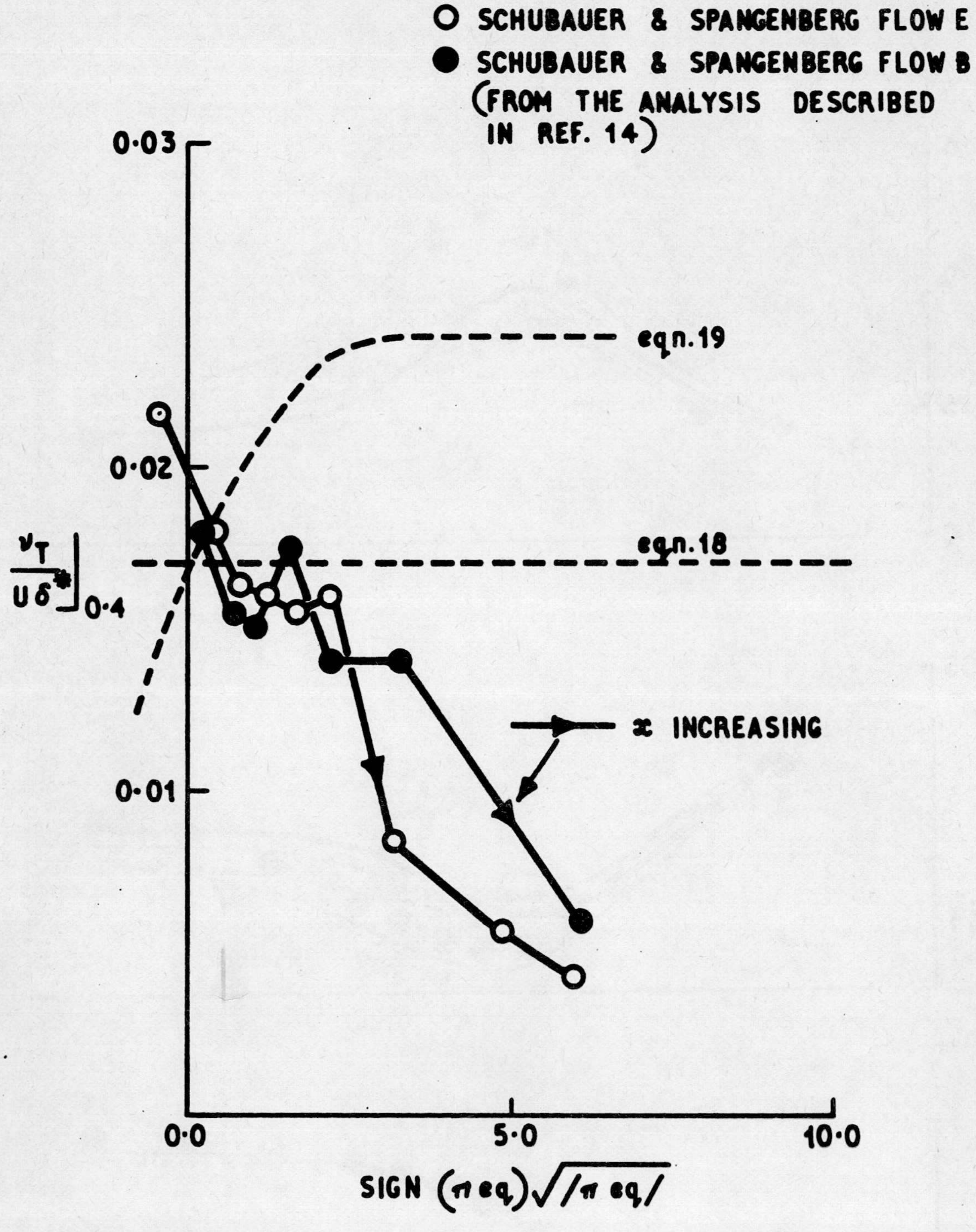


FIG. 27. EDDY VISCOSITY DEVELOPMENT FOR SCHUBAUER & SPANGENBERG'S FLOWS B & E

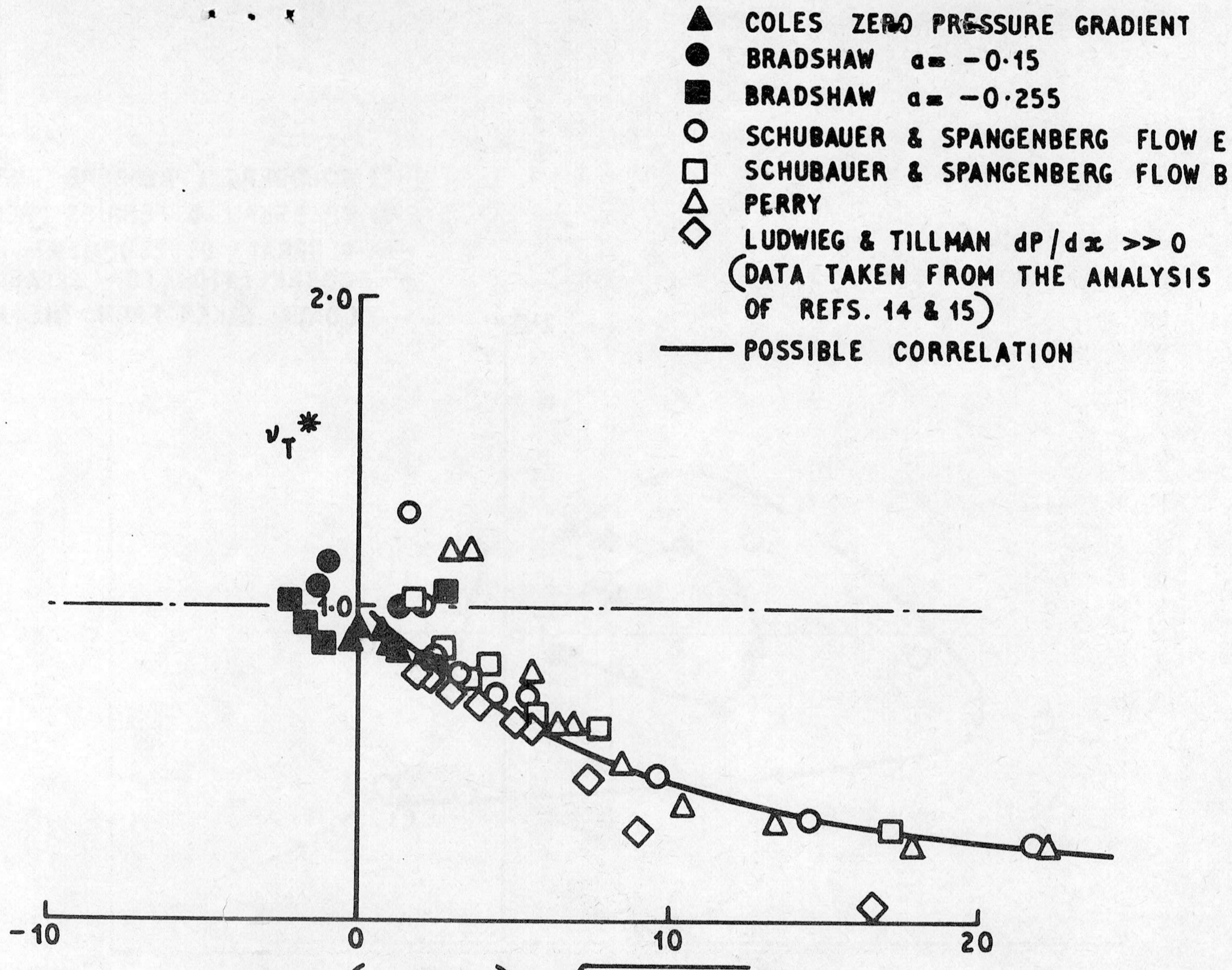


FIG. 28. EDDY VISCOSITY IN SEPARATING & EQUILIBRIUM LAYERS

$$\left(\nu_T^* = \left[\nu_{T\,max}/U\delta^*\right]_{exp} \Big/ \left[\nu_{T\,max}/U\delta^*\right]_{eq}\right)$$

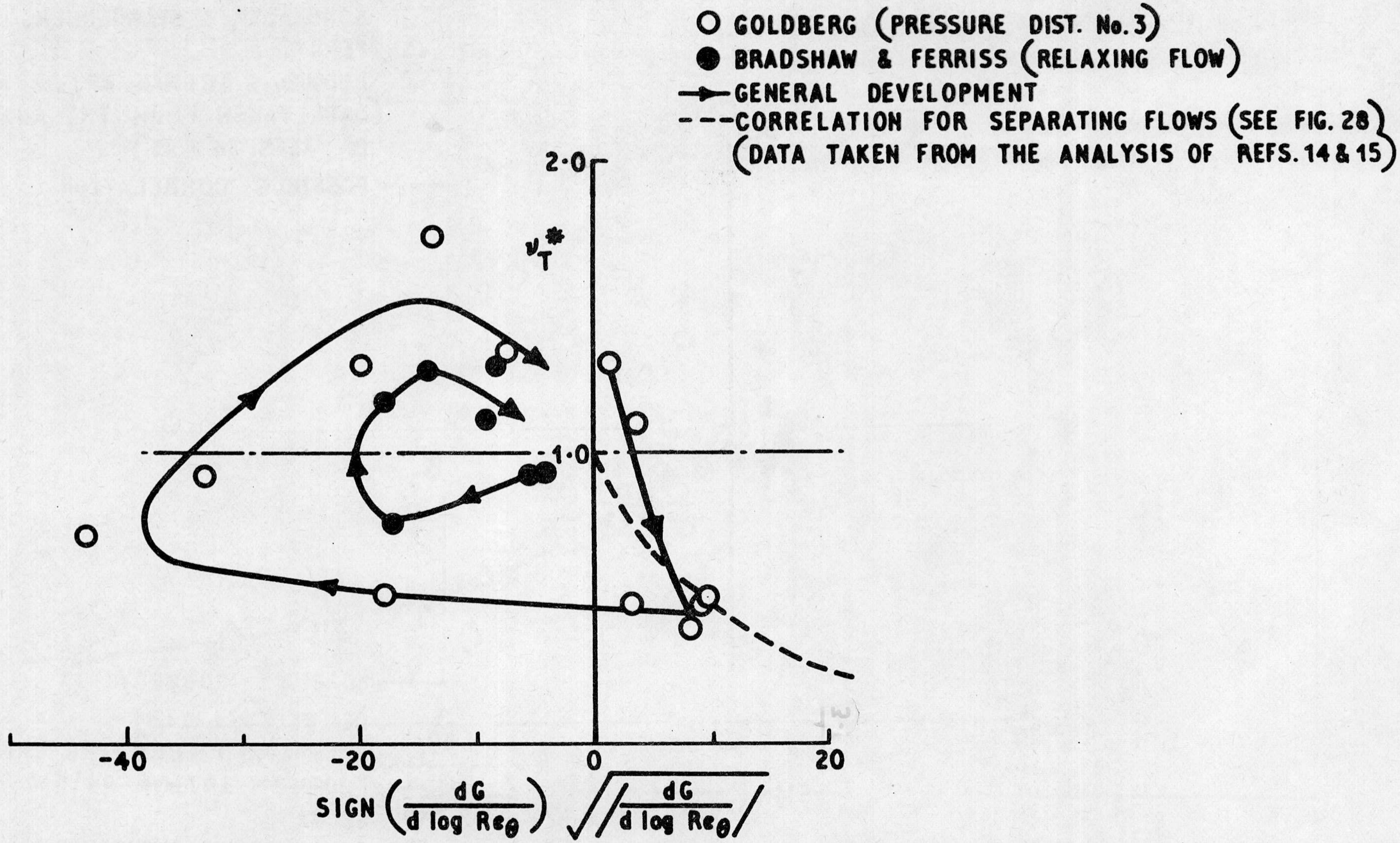
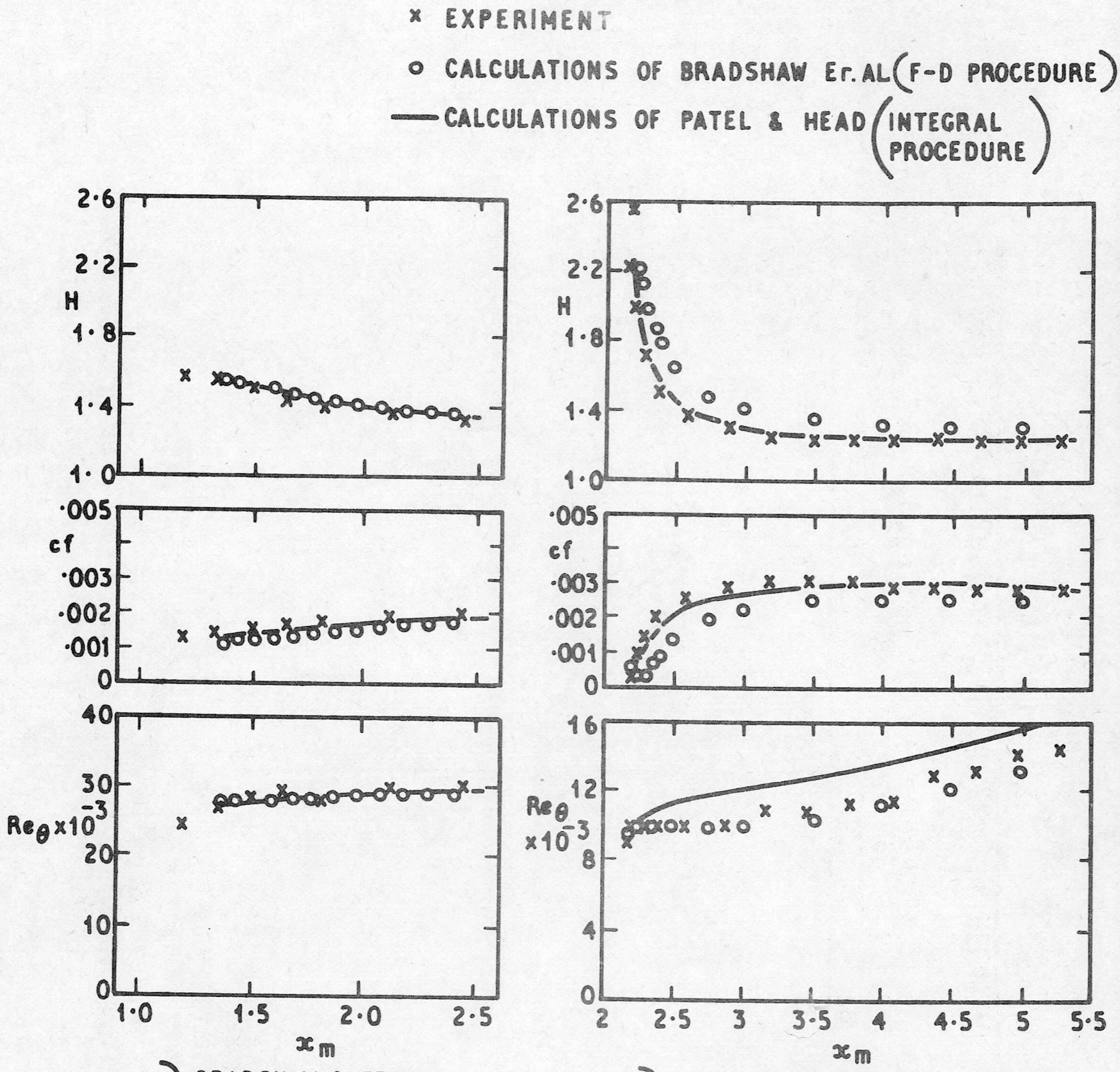


FIG. 29. EDDY VISCOSITY DEVELOPMENT IN RELAXING LAYERS

$\left(\nu_T^* = \left[\nu_{T\,max}/U\delta^*\right]_{exp} \Big/ \left[\nu_{T\,max}/U\delta^*\right]_{eq}\right)$



a) BRADSHAW & FERRIS, RELAXING FLOW

b) TILLMAN RE-ATTACHING FLOW

FIG. 30. PREDICTIONS FROM A FINITE-DIFFERENCE AND AN INTEGRAL PROCEDURE USING THE SAME CLOSURE HYPOTHESIS

(TAKEN FROM PATEL & HEAD, REF. 13)