Wireless Sensor with Data and Energy Packets

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***Abstract*—We study an energy harvesting wireless sensor nods which harvests energy and senses and transmits data. Both data and energy are represented as discrete quantitities using the previously introduced in "Energy Packet Network" paradigm. For each ata packet, the sensor requires and consumes K_e energy packets for sensing and storage and K_t energy packets for transmission. Assuming random processes for sensing and energy harvesting, we obtain a two-dimensional random walk model and reduce its complexity using companion matrices and matrix algebra techniques. The resulting solution allows us to obtain, in steady-state, all the metrics of interest such as the backlog of energy and data in the sensor. We also consider the case when M sensors operate in proximity and create some interference for each other.**

***Index Terms*—Wireless Sensors; Energy Harvesting; Energy Packets; Data Packets; Random Walk; Markov Chains; Companion Matrices.**

I. Introduction and Previous Work

A wireless sensor network consists of several sensor nodes, typically a small device that may have magnetic, thermal, optical, chemical and mechanical sensors to get measurement. They includes three basic components: *a sensing subsystem* for data gathering from the physical surrounding environment, *a processing subsystem* for local data processing and storage, and *a wireless communication subsystem* for data receiving and transmission [1]. A WSN typically consists of a number of sensor nodes few tens to thousands. WSNs can be used in many different areas such as [2]: military [3], [4], natural disasters [5], biomedical health monitoring [6], [7], and hazardous environment exploration and seismic sensing [8]. WSNs show some differences from traditional network with respect to its source and design needs. Source needs can be considered as communication range, amount of energy, limited storages or buffers and limited processing mechanism for each node. Design needs basically depend on the application and the environment for which WSN is used. The environment is also one of the most important parameter, in order to determine the size of WSNs and the network topology. For example, while a few nodes could be enough for indoor environments, much more nodes could be needed for outdoor environments to provide reliable operation with sensed data.

Finite battery capacity is one of the major limitations of wireless sensor nodes, since nodes only be able to operate as long as there is an unfinished battery. This finite lifetime causes interruption of the applications and regularly change of batteries. Use of large batteries for longer lifetimes could be a solution, but we will need to concern about increased size, weight and cost. An alternative technique addressing the problem of finite operation time of a sensor node is energy harvesting.

Energy can be harvested from the environment from solar and other forms of light and electromagnetic sources, thermal, vibrational, piezoelectric, and used as electrical energy in sensors and computing or communication systems powered in this manner [9], [10], [11], [12], especially in remote sensing and security applications [12], [13]. When energy is harvested, the manner in which it is exploited and scheduled for consumption must take into account both the harvesting process and the usage needs [14], [15].

Earlier work [16] introduced the idea that energy storage and usage can be represented through discrete mathematical models, where an "energy packet" is an abstraction that exploits the analogy between the random arrival of harvested energy and the random arrival of packets in data networks. Similarly it exploits the notion of an energy packet buffer (i.e. a battery or capacitor), similar to a data buffer or queue of work in a computer system. Recent papers [17], [18], [19] have exploited this paradigm to study energy harvesting wireless sensors with energy losses and leakage. In [20], the case where the energy required for one data packet transmission is exactly $K > 1$ energy packets was considered, assuming that energy is used only for packet transmission but also for not for packet processing and other sensor node electronics, while [21] has studied the model where a data packet transmission requires exactly two energy packets: one for processing and one for packet transmission, or $K_e = K_t = 1$, in the notation of the present paper.

Here we generalise the approach to arbitraryK_e and K_t values. The motivation is the that the node electronics and the transmitter may have to vary the power levels they use to deal with the speed of processing or the transmission power to overcome errors. This generalisation then leads us to a two-dimensional random walk model which is harder to solve in terms of closed form formulas.

II. Mathematical Model

In the mathematical model we use, data (from sensing) and energy (from energy harvesting) packets arrive from the environment at random according to two distinct and independent Poisson processes, at average rates λ and Λ, respectively. Data packets are stored in a finite capacity buffer of size B, while the energy store (battery or capacitor) has a limited capacity of E energy packets. Since the processing and transmission of a packet occurs very fast, we can assume that the time taken for packet processing and transmission is negligible compared to the rates of data collection and energy harvesting which depend on external physical processes and transducers.

In a sensor node, the harvested energy is consumed for node electronics (sensing-processing-storing processes) and packet transmission. We assume that $K_e > 1$ and $K_t > 1$ energy packets required for node electronics and data transmission, respectively. Therefore, whenever a sensor node has less than K_e amount of energy packet, data can not be sensed and stored, and whenever there is more than K_e amount of energy, data is sensed and stored and also it could be transmitted immediately if the remaining energy packet number is greater or equal to K_t.

$N(t)$ and $M(t)$ are respectively, the number of data and energy packets in the sensor node at time $t \geq 0$, so that state of the system can be represented by pair of $(N(t), M(t))$. Let us write $p(n, m, t) = Prob[N(t) = n, M(t) = m]$. From the above remark, we need only consider $p(n, m, t)$ for the state space S of pairs of integers $(n, m) \in S$ such that:

$$S = \{(0,0), (n,0), (0,m), (l,k) : \ 1 \leq n \leq B, \ 1 \leq m \leq E, \\ 1 \leq l < B, 1 \leq k < K\}, \ where\ K = K_e + K_t.$$

In [21], the energy expended per packet for node electronics and data transmission are equal and are provided by a single energy packet, resulting in a one-dimensional Markov chain. However, when we consider the general case for K_e and K_t where they can take arbitrary values, system is no longer modeled as 1D Markov chain but 2D Markov chain in Figure 1, which makes the analysis harder. Since the energy consumption for many sensor node applications is mainly dominated by data transmission subsystem [22], we assume $K_t > K_e$ for the current system model. According to defined system model we can write following global balance equations:

$$\begin{aligned}
p(0,0)[\Lambda] &= \Lambda p(1, K-1) + \lambda p(0, K), \\
p(0,m)[\Lambda] &= \Lambda p(0, m-1) + \\
&\lambda p(0, m+K) 1[E \geq m+K], \\
&\mathbf{1 \leq m < K_e} \\
p(0,m)[\Lambda+\lambda] &= \Lambda p(0, m-1) + \\
&\lambda p(0, m+K) 1[E \geq m+K], \\
&\mathbf{K_e \leq m < E} \\
p(0,E)[\lambda] &= \Lambda p(0, E-1), \\
p(n,0)[\Lambda] &= \Lambda p(n+1, K-1) + \lambda p(n-1, K_e), \\
&\mathbf{1 \leq n < B} \\
p(n,m))[\Lambda] &= \Lambda p(n, m-1) + \lambda p(n-1, m+K_e), \\
&\mathbf{1 \leq n < B,\ 1 \leq m < K_e} \\
p(n,m)[\Lambda+\lambda] &= \Lambda p(n, m-1) + \lambda p(n-1, m+K_e), \\
&\mathbf{1 \leq n < B,\ K_e \leq m < K - K_e} \\
p(n,m)[\Lambda+\lambda] &= \Lambda p(n, m-1), \\
&\mathbf{1 \leq n < B,\ K - K_e \leq m < K} \\
p(B,m)[\Lambda] &= \Lambda p(B, m-1) + \lambda p(B-1, m+K_e), \\
&\mathbf{K_e \leq m < K - K_e} \\
p(B,m)[\Lambda] &= \Lambda\ p(B, m-1), \\
&\mathbf{K - K_e \leq m < K} \\
p(B,0)[\Lambda] &= \lambda\ p(B-1, K_e).
\end{aligned}$$

Finding a closed-form solutions for stationary probability distributions and other quantities by using these balance equations is elusive, so that we need to use different approaches for this system model.

We can use the traditional approach where we define generator matrix Q to find the stationary probabilities. Q is an nxn matrix of n states Markov chain. In our system model, it can be easily observed that $n = E + BK + 1$. Expressing the stationary probability of each state π_i as a row vector π we can write this as a matrix equation $\pi Q = 0$

The π_i are unknown and are the values we wish to find. Since π_i is a probability distribution we also know that the normalisation condition holds: $\sum_{x_i \in S} \pi_i = 1$. Thus, with these $n+1$ equations (global balance equations and normalisation condition) we can solve to find the n unknowns.

In our system, in order to find quantities of interests, we need to deal with $E+BK+2$ equations, so that the complexity of the solution increases dramatically with increasing data and energy buffer sizes. To deal with the further complexity of the solution, we require spending more time and energy. Thus, apart from the traditional approach, we use a different solution method decreasing the solution complexity.

A. Solution with Companion Matrices

For the sake of solution simplicity, we start representing sensor states as S_j, such that:

$$S_j = p(n, m) : j = nK - m + E.$$

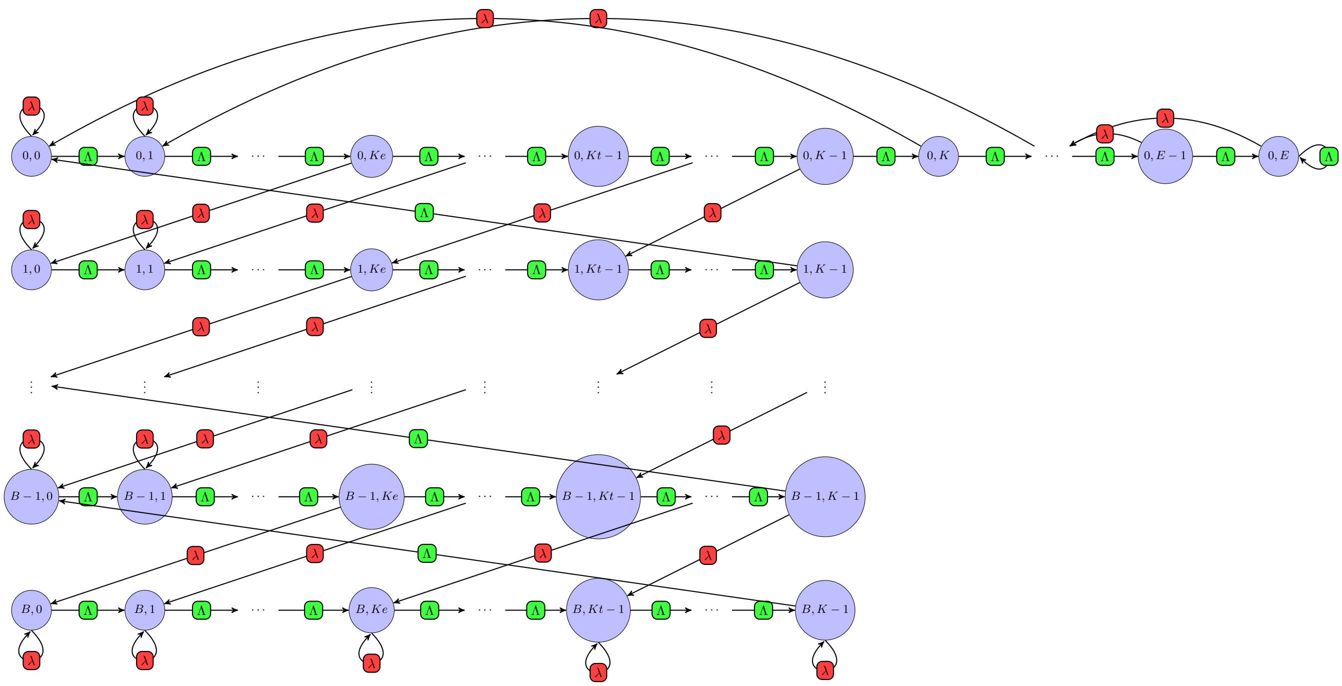

Fig. 1: Two-dimensional Markov chain state diagram representation of the system

We also consider each row of the two-dimensional Markov chain in Figure 1 as a vector V_i where $0 \leq i \leq B$, so that we will have:

$$\begin{aligned} V_0 &= [S_E, S_{E-1}, \cdots, S_1, S_0], \\ V_1 &= [S_{E+K}, S_{E+K-1}, \cdots, S_{E+2}, S_{E+1}], \\ V_2 &= [S_{E+2K}, S_{E+2K-1}, \cdots, S_{E+2+K}, S_{E+1+K}], \\ &\vdots \\ V_B &= [S_{E+BK}, \cdots, S_{E+2+BK-K}, S_{E+1+BK-K}]. \end{aligned}$$

Besides the fact that complicated state transition behaviors among the states, once we carefully observe the diagram in Figure 1, it could be realized that every row, except the first and the last one, has exact same state transition behaviors. Therefore, we might have some recurrence relations which might reduce the number of total equations and the system complexity. Figure 2 shows the state representation of i^{th} row of the two-dimensional state diagram or vector V_i where $0 < i < B$. We observe in Figure 2 that for vector V_i, there are 3 different transition behaviors among the states so that we can subdivide the vector into 3 separate regions by which we can write following equations:

- *For Region1,* $0 \leq m < K_e$*:*

$$S_{N+1} = S_N - (\frac{\lambda}{\Lambda})\ S_{N-K-K_e}, \tag{1}$$

- *For Region2,* $K_e \leq m < K_t$*:*

$$S_{N+1} = S_N + (\frac{\lambda}{\Lambda})\ (S_N - S_{N-K-K_e}), \tag{2}$$

- *For Region3,* $K_t \leq m < K$ *:*

$$S_{N+1} = (1 + \frac{\lambda}{\Lambda})\ S_N. \tag{3}$$

Note that the (1) and (2) are linearly recursive sequence of order $K + K_e + 1$ whose minimum value is 8. We know that there is no solution in radicals to the general polynomial equations of degree 5 and more by Abel&Ruffini theorem [23]. Thus, it is not easy to solve these equations and express a closed-form solution for stationary probability distributions. However, we may use companion matrices of each equation to express transitions among states. To provide consistency among the companion matrices, we will consider each one as a square matrix with dimension $K + K_e + 1$. Once we consider the vector V_1, we can write state transitions by using companion matrices as follows:

$$\begin{bmatrix} S_{E+2} \\ S_{E+1} \\ \vdots \\ S_{E+2-K-K_e} \end{bmatrix} = \begin{bmatrix} (1+\frac{\lambda}{\Lambda}) & 0 & \dots & 0 & 0 \\ 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & 0 \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix} \begin{bmatrix} S_{E+1} \\ S_E \\ \vdots \\ S_{E+1-K-K_e} \end{bmatrix}$$

or equivalently:

$$\overrightarrow{S_{E+2}} = C_3\ \overrightarrow{S_{E+1}}.$$

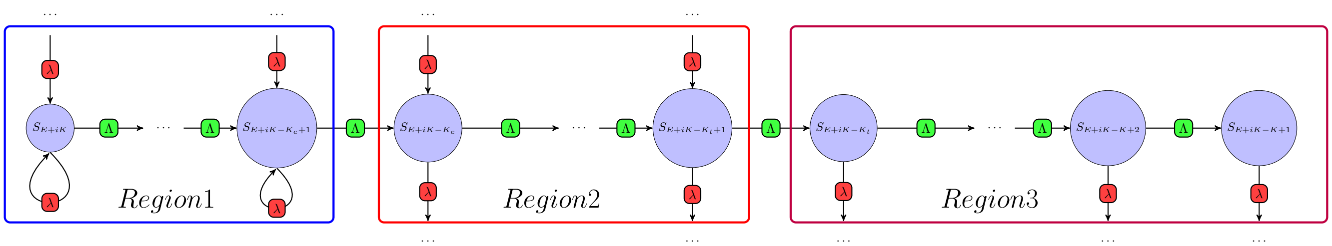

Fig. 2: State diagram representation of the vector V_i

Other state vectors in *Region3* can also be expressed iteratively as following:

$$\begin{aligned}\overrightarrow{S_{E+3}} &= C_3\ \overrightarrow{S_{E+2}} = C_3^2\ \overrightarrow{S_{E+1}},\\ \overrightarrow{S_{E+4}} &= C_3\ \overrightarrow{S_{E+3}} = C_3^3\ \overrightarrow{S_{E+1}},\\ &\vdots\\ \overrightarrow{S_{E+K_e+1}} &= C_3\ \overrightarrow{S_{E+K_e}} = C_3^{K_e}\ \overrightarrow{S_{E+1}}.\end{aligned}$$

Similarly, for *Region2*:

$$\begin{aligned}\overrightarrow{S_{E+K_e+2}} &= C_2\ \overrightarrow{S_{E+K_e+1}} = C_2\ C_3^{K_e}\ \overrightarrow{S_{E+1}},\\ \overrightarrow{S_{E+K_e+3}} &= C_2\ \overrightarrow{S_{E+K_e+2}} = C_2^2\ C_3^{K_e}\ \overrightarrow{S_{E+1}},\\ &\vdots\\ \overrightarrow{S_{E+K_t+1}} &= C_2\ \overrightarrow{S_{E+K_t-1}} = C_2^{K_t-K_e}\ C_3^{K_e}\ \overrightarrow{S_{E+1}},\end{aligned}$$

and for *Region1*:

$$\begin{aligned}\overrightarrow{S_{E+K_t+2}} &= C_1\ \overrightarrow{S_{E+K_t+1}} = C_1\ C_2\ C_3^{K_e}\ \overrightarrow{S_{E+1}},\\ \overrightarrow{S_{E+K_t+3}} &= C_1\ \overrightarrow{S_{E+K_t+2}} = C_1^2\ C_2^2\ C_3^{K_e}\ \overrightarrow{S_{E+1}},\\ &\vdots\\ \overrightarrow{S_{E+K}} &= C_1\ \overrightarrow{S_{E+K-1}} = C_1^{K_e-1}\ C_2^{K_t-K_e}\ C_3^{K_e}\ \overrightarrow{S_{E+1}},\\ \overrightarrow{S_{E+K+1}} &= C_1\ \overrightarrow{S_{E+K}} = C_1^{K_e}\ C_2^{K_t-K_e}\ C_3^{K_e}\ \overrightarrow{S_{E+1}},\end{aligned}$$

where

$$C_1 = \begin{bmatrix} 1 & 0 & \dots & 0 & -\frac{\lambda}{\Lambda} \\ 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix}, C_2 = \begin{bmatrix} (1+\frac{\lambda}{\Lambda}) & 0 & \dots & 0 & -\frac{\lambda}{\Lambda} \\ 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix}$$

We showed that we can express all the state vectors in V_1 in terms of $\overrightarrow{S_{E+1}}$ and companion matrices. In fact, the same procedure can be followed for the other row vectors and any state vector $\overrightarrow{S_N} : S_N \in V_i, 0 < i < B$ can be expressed as:

$$\overrightarrow{S_N} = \begin{cases} {C_3}^{\alpha}\ C^{\lfloor \frac{N-E-1}{K} \rfloor}\ \overrightarrow{S_{E+1}} & 0 \le \alpha \le K_e \\ {C_2}^{\alpha-K_e}\ {C_3}^{K_e}\ C^{\lfloor \frac{N-E-1}{K} \rfloor}\ \overrightarrow{S_{E+1}} & K_e < \alpha \le K_t \\ {C_1}^{\alpha-K_t}\ {C_2}^{K_t-K_e}\ {C_3}^{K_e}\ C^{\lfloor \frac{N-E-1}{K} \rfloor}\ \overrightarrow{S_{E+1}} & K_t < \alpha < K \end{cases} \tag{4}$$

where $\lfloor \cdot \rfloor$ is a function that returns the largest integer less than or equal to its argument, C is the multiplication of companion matrices, i.e., $C = C_1^{K_e}\ C_2^{K_t-K_e}\ C_3^{K_e}$, and the parameter $\alpha = (N-E+K-1) \pmod K$. Thus, we are able to express the state vectors $\overrightarrow{S_N} : S_N \in V_i, 0 < i < B$ with respect to companion matrices and the state vector $\overrightarrow{S_{E+1}}$.

After studying on the majority of the vectors, we can now consider V_0 and write following characteristic equations:

- *For* $0 < N \le E - K + 1$*:*

$$S_N = (\frac{\lambda}{\Lambda})(1 + \frac{\lambda}{\Lambda})^{N-1} S_0 \tag{5}$$

- *For* $E - K + 1 < N \le E - K_e$*:*

$$S_{N+1} = (1 + \frac{\lambda}{\Lambda}) S_N - \frac{\lambda}{\Lambda} S_{N-K} \tag{6}$$

- *For* $E - K_e < N \le E$*:*

$$S_{N+1} = S_N - \frac{\lambda}{\Lambda} S_{N-K} \tag{7}$$

Thus, we can express $\overrightarrow{S_{K+1}}$ by companion matrix from equation (6):

$$\begin{bmatrix} S_{K+1} \\ S_K \\ \vdots \\ S_{1-K_e} \end{bmatrix} = \begin{bmatrix} (1+\frac{\lambda}{\Lambda}) & 0 & \dots & 0 & -\frac{\lambda}{\Lambda} \\ 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix} \begin{bmatrix} S_K \\ S_{K-1} \\ \vdots \\ S_{-K_e} \end{bmatrix}$$

where the states $\{S_j : j < 0\}$ are redundant, i.e., probability of these states is zero, and they are used to provide consistency among dimensions of companion matrices.

Thus, we may keep the iteration and express other state vectors as follows:

$$\begin{aligned}\overrightarrow{S_{K+1}} &= C_2\ \overrightarrow{S_K},\\ \overrightarrow{S_{K+2}} &= C_2\ \overrightarrow{S_{K+1}} = C_2^2\ \overrightarrow{S_K},\\ &\vdots\\ \overrightarrow{S_{E-K_e+1}} &= C_2\ \overrightarrow{S_{E-K_e}} = C_2^{E+1-(K+K_e)}\ \overrightarrow{S_K}.\end{aligned}$$

and

$$\begin{aligned}\overrightarrow{S_{E-K_e+2}} &= C_1\ \overrightarrow{S_{E-K_e+1}} = C_1\ C_2^{E+1-(K+K_e)}\ \overrightarrow{S_K},\\ \overrightarrow{S_{E-K_e+3}} &= C_1\ \overrightarrow{S_{E-K_e+2}} = C_1^2\ C_2^{E+1-(K+K_e)}\ \overrightarrow{S_K},\\ &\vdots\\ \overrightarrow{S_{E+1}} &= C_1\ \overrightarrow{S_E} = C_1^{K_e}\ C_2^{E+1-(K+K_e)}\ \overrightarrow{S_K}.\end{aligned}$$

We may also express $\overrightarrow{S_K}$ by using (5) as:

$$\overrightarrow{S_K} = \begin{bmatrix} S_K \\ S_{K-1} \\ \vdots \\ S_2 \\ S_1 \\ S_0 \\ S_{-1} \\ \vdots \\ S_{-K_e} \end{bmatrix} = \frac{\lambda}{\Lambda} S_0 \begin{bmatrix} (1+\frac{\lambda}{\Lambda})^{K-1} \\ (1+\frac{\lambda}{\Lambda})^{K-2} \\ \vdots \\ (1+\frac{\lambda}{\Lambda}) \\ 1 \\ \frac{\Lambda}{\lambda} \\ 0 \\ \vdots \\ 0 \end{bmatrix} = (\frac{\lambda}{\Lambda}) S_0 \overrightarrow{\gamma}$$

Thus, we may write:

$$\overrightarrow{S_N} = \begin{cases} {C_2}^{N-K} \ (\frac{\lambda}{\Lambda}) S_0 \overrightarrow{\gamma} & K \leq N \leq E - K_e + 1 \\ {C_1}^{N-\varsigma_1} \ {C_2}^{\varsigma_2} \ (\frac{\lambda}{\Lambda}) S_0 \overrightarrow{\gamma} & E - K_e + 1 < N \leq E + 1 \end{cases} \quad (8)$$

where $\varsigma_1 = E - K_e + 1$ and $\varsigma_2 = \varsigma_1 - K$.

We also may replace $\overrightarrow{S_{E+1}}$ in (4) and rewrite it as:

$$\overrightarrow{S_N} = \begin{cases} {C_3}^{\alpha} \ C^{\lfloor \frac{N-E-1}{K} \rfloor} C' & 0 \leq \alpha \leq K_e \\ {C_2}^{\alpha - K_e} \ {C_3}^{K_e} \ C^{\lfloor \frac{N-E-1}{K} \rfloor} C' & K_e < \alpha \leq K_t \\ {C_1}^{\alpha - K_t} \ {C_2}^{K_t - K_e} \ {C_3}^{K_e} \ C^{\lfloor \frac{N-E-1}{K} \rfloor} C' & K_t < \alpha < K \end{cases} \quad (9)$$

where $C' = C_1^{K_e} C_2^{E+1-(K+K_e)} (\frac{\lambda}{\Lambda}) S_0 \overrightarrow{\gamma}$.

Also, we may write following characteristic equations for the states in V_B:

- *For $BK + E - K + 1 \leq N \leq BK + E - K_t$:*

$$S_{N+1} = S_N \quad (10)$$

- *For $BK + E - K_t < N < BK + E$:*

$$S_{N+1} = S_N - \frac{\lambda}{\Lambda} S_{N-K-K_e} \quad (11)$$

Therefore, we may write followings:

$$\overrightarrow{S_{BK+E-K_t+1}} = \cdots = \overrightarrow{S_{BK+E+1-K}}$$

and

$$\begin{aligned} \overrightarrow{S_{BK+E-K_t+2}} &= C_1 \ \overrightarrow{S_{BK+E+1-K}}, \\ \overrightarrow{S_{BK+E-K_t+3}} &= C_1^2 \ \overrightarrow{S_{BK+E+1-K}}, \\ &\vdots \\ \overrightarrow{S_{BK+E}} &= C_1^{K_t-1} \ \overrightarrow{S_{BK+E+1-K}}. \end{aligned}$$

We can express the state vector $\overrightarrow{S_{BK+E+1-K}}$ from (9) as,

$$\overrightarrow{S_{BK+E+1-K}} = C^{B-1} C'$$

Therefore, we may write $\overrightarrow{S_N} : S_N \in V_B$:

$$\overrightarrow{S_N} = \begin{cases} C^{B-1} C' & BK + E1 - K \leq N \leq BK + E + 1 - K_t \\ {C_1}^{N-(BK+E-K_t+1)} C^{B-1} C' & BK + E - K_t + 1 < N \leq BK + E \end{cases} \quad (12)$$

Thus, we can express any state vector $\overrightarrow{S_N} : \{S_N : 0 \leq N \leq BK + E\}$ as a function of companion matrices C_1, C_2, C_3, required energy packet amounts K_t and K_e, the vector $\overrightarrow{\gamma}$, and the state S_0 by combining equations (8), (9), (12). We also know that the normalisation condition holds $\sum_{i=0}^{BK+E} S_i = 1$, so that we can find stationary probability distribution of S_0 and of all the other states in the system.

B. A Numerical Example

Since the rate of energy is in power units, the average total power consumed by the sensor node is:

$$\xi = (1 - S_0)\Lambda, \quad (13)$$

where the reduction $S_0 \Lambda$ is due to the lost energy packets when the battery or capacitor is full. On the other hand, the average radiated power is:

$$\phi = \kappa \xi, \quad (14)$$

where $\kappa = \frac{K_t}{K}$.

When we assume that there are M identical sensors operating at the same power level [21] and using BPSK transmission, the probability that a bit is received correctly is given by:

$$Q(\sqrt{\frac{\eta K_t}{\eta \kappa \xi \zeta \ (M-1) + \eta \xi \zeta \ (\frac{M-M'}{M}) 1[M > M'] + N}}), \quad (15)$$

where the denominator stands for interference, plus the noise power level denoted by N. Also, η is the reduction of transmitted power that is received at the receiver, ζ is a factor representing the effect of side-band frequency channels among the M' separate frequency channels and it is typiaccly much smaller than 1, and $Q(x) = \frac{1}{2}[1 - erf(\frac{x}{\sqrt{2}})]$.

When we assume a simple system with parameters $K_t = 3, K_e = 2, E = 10, B = 3, \Lambda = 10, \lambda = 2$, we have 8×8 companion matrices. Once we follow the solution procedure, we can calculate $S_0 = 0.1592$. We can observe the effect of the number of sensor nodes on the receiver error probability in Figure 3 with the hypothetical parameters $\eta = 0.5, \ \zeta = 0.02, \ M' = 50, N = 1$.

III. Conclusions

This paper analyses wireless sensor nodes that harvest energy and sense data from the environment, and store and transmit data in the form of discrete packets, and also use a discrete representation of energy. It is assumed that a data packet can be sensed, processed and stored by the node only when there are at least K_e energy packets available in the node. Also, these K_e energy packets will be effectively expended each time a data packet is successfully received by the node. Otherwise, the data will not be received and the sensed data will go unnoticed and it will be lost.

On the other hand, in order to transmit a data packet the node requires an additional number of K_t energy packets, where we assume that $K_t > K_e$. Again all of the K_t energy packets will be consumed for one transmission. Thus

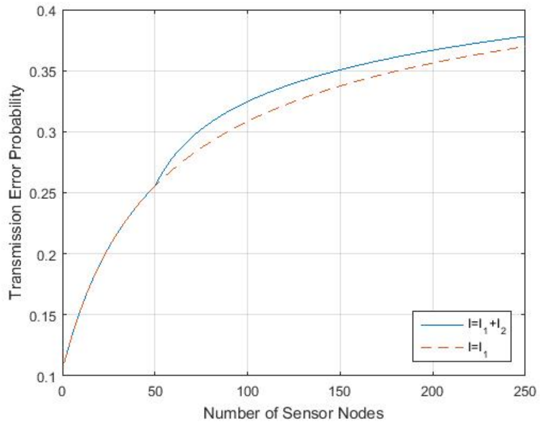

Fig. 3: Transmission error probability vs number of sensor nodes

the successful sensing and transmission of one data packet requires the consumption of a total of $K = K_E + K_t$ energy packets. In our model, both the processing and transmission of a packet are assumed to occur very rapidly, if enough energy is available so that an arriving data packet is instantaneously stored if the amount of energy available is more than K_e but less than K, while it will be both stored and transmitted when the amount of energy available is at least K. Under these assumptions, we construct a two-dimensional continuous time Markov chain to represent the behaviour of the system. We then propose a solution method for this model that uses companion matrices and linear algebra to reduce its computational complexity. We also exploit certain regularity properties of the matrix structure resulting in efficient numerical computation of all the metrics of interest. In particular we can obtain the steady-state distribution of the backlog of data and energy packets, the system throughput in terms of successfully transmitted packets and the possible loss of energy when the energy storage device is full and energy is harvested. The analysis also allows us to include the effect of a communication environment where, in addition to noise, multiple wireless sensor nodes may interfere with each other resulting in increased data errors and reduced effective throughput. Future work will address more practical system where non-transmitted data may be lost due to time-outs, and energy may be lost through leakage. We also plan to consider related multi-hop and networked systems.

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