Mathematical Play: Games, problem solving, investigations and discussions.

Kartar Singh Uppal

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Abstract

This dissertation looks at the learning and teaching of mathematics through the issue of play and generalisation. Here, play is defined as providing a medium for learning. Generalisation is seen as being central to what mathematics is. The study initially examines the connections between play and mathematics. It goes on to investigate the effect of five significant shifts in my teaching, which have been driven by this initial study: Use of whiteboards, Starting with a problem, Generalisation, Room layout and Discussion.

The study is carried out within a reflective practitioner qualitative research framework and is presented in a diary format in chronological order.

In the learning of mathematics I have realised that mathematics is about problem solving and generalisation used correctly could greatly aid students' understanding of mathematics. My teaching of mathematics has been greatly enhanced by using 'circle time' with small whiteboards by starting with a problem. I feel I have developed for students an experience of doing mathematics in a playful manner.

In memory of Maureen Hickman, who passed away whilst writing this dissertation

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1. Introduction

I thought first about teaching mathematics when I was seventeen. At the time I believed algebra was the core skill if taught well would enable students to think logically and solve problems in a wide variety of disciplines. I have been mainly self taught through the School Mathematics Project 11-16, with teacher assistance when I got stuck. I felt then that a didactic style with someone who understood the mathematics would create the optimum learning environment. I was convinced that this style allowed an authority figure to explain the concepts and allowed questions to be asked immediately dispelling any misconceptions. In hindsight I felt traditional teaching was a better medium for learning than reading it for yourself. University mathematics confronted me for the first time with traditional teaching. The Pure lectures were difficult to grasp because the concepts were so abstract and the style of thinking and presenting was so different to school mathematics. Applied lectures were better because the concepts seemed more real, but I could not always follow the pace of the lectures. At the time I felt I was not as able as other mathematicians, but I rarely worked with other students.

My teaching career began in 1996 in a selective school and I was traditionally didactic. Lessons would generally begin with some sort of algebraic proof of a general result, this would be followed by a number of examples to illustrate the generality. I noticed over the next seven years having worked in a variety of selective schools that many students struggled to follow the 'lecture' and they would become disruptive. When questions were set many students struggled to answer them, which would result in me bringing the whole class together to explain again. I would at this point start giving hints so that they could get a start.

I have come to realise recently in my current selective school that standing at the front of a classroom and lecturing students in a didactic way has really been ineffective as a method to teach mathematics, which in hindsight should have been clear to me from university lectures.

One of my major insights has come from reading about enactivism in *'Teaching mathematics: toward a sound alternative'* by Brent Davis (1996), as part of the first part of the course. It offers me a new way of looking at knowing, which fits my postmodern beliefs, began a re-examination of what mathematics is and centrally offers the idea of 'play' as a vehicle to learn. I am interested to see what connections there are between mathematics and play. I hope to exploit these to see if the notion of play will change my practice for the better. This interest in the notion of play has been sparked by the fact that I have noticed recently that young children are exposed to a more playful environment at primary school, but this rapidly becomes more formal in secondary school. Play to me has always been very enjoyable, but it has struck me that at primary schools they manage to connect it to the curriculum. Whilst at secondary school it seems to me a distraction from learning. I am curious to see what connections there are between play and mathematics and if I can exploit these connections to change my practice for the better.

2. Literature

2.0 Introduction

This chapter surveys some of the literature on play, the nature of mathematics and in particular the connections between play and a variety of mathematical activities (problem solving, games, investigations and discussions), which I term as 'mathematical play'. The aim is to influence my practice because I feel the notion of play will change my practice for the better.

2.1 Concept of Play

Play is a difficult concept to define, which I see as providing a medium for learning (Bergen, 1988). I feel it achieves this because during an activity which is playful there is an opportunity to change parts of a problem, for example when solving equations one of the coefficients can be changed so that a new question is devised. It can be rule governed especially when playing a game of any sort. There is always activity within play, which normally makes it pleasurable and motivating for example in a game of football. The goals of play tend to be shifting, as one problem is solved another is created because play seems to have goal flexibility. This, I suspect, is caused by students having a desire to see what happens when something is changed. My interest in play is rooted in my stance as an enactivist, which I will now explore.

2.2 Enactivism

'Mathematics is truly about us and our world' (Davis, 1996:81). Davis here is attacking what has been for so long a central tenet of mathematics and its teaching; that there is a separate platonic world where mathematical concepts exist and it is the job of a teacher to help a student to access this world. Some teachers respond by using a 'transmission mode' of teaching.

It is important to see how Davis (1996) has come to reach this position to appreciate the role of play in all its guises. Davis writes as an Enactivist, which I will explore by first considering constructivism and social constructivism. Constructivism is based on two principles (von Glasersfeld, 1989:162):

(a) knowledge is not passively received but actively built up by the cognising subject;

(b) the function of cognition is adaptive and serves the organization of the experiential world, not the discovery of ontological reality

which are immediately problematic because it is unable to account for cultural knowledge (Davis, 1996). Its focus on how the isolated individual comes to know but ignores the fact that individuals do not exist by themselves, but by a social bond through language games (Lyotard, 1979).

Social constructivism by its very nature tackles some of the problems of constructivism because it assumes that *'human beings are not self-contained, self-sufficient subjects contingently and externally related to one another, but beings who are formed, from the very beginning, in and through their social interactions'* (Levin, 1989:150). Hence the cultural situation and social interaction are seen to be important for knowledge development (Vygotsky, 1978). It seems we have a solution but there are subtle differences between the two frameworks, which encompass a major assumption. Firstly, the position of events in learning. Constructivists put the subjective construction of knowledge before social mediation, whilst the social constructivists see the events occurring the other way round (Davis, 1996). Secondly, constructivists focus on an individual's learning, whilst social constructivists concentrate on the social context in which learning occurs. It becomes apparent that both frameworks are founded on the modernist belief that the *'self'* is separated from each other and the world. Davis offers an alternative postmodernist notion of *'self'* as *'defined as a network of relationships, and so, as histories, contexts, and participants vary, identities change'* (Davis, 1996:192) which only reiterates Lyotard's (1979) view of the self. This notion of the self is not recent. The Buddhist concept of *'anatta'* (Rahula, 1990) which literally means *'not-self'* is used to describe the fact this permanent *'self'* is a mental construct we project on the flow of consciousness. In meditation when experience is analysed and broken down into its constituent parts no *'self'* is found.

Enactivists have taken on board the *'no permanent self'* idea of Buddhism because they see the transformational process which occurs to the self as *'not something that happens* *to the self, it is the self'* (Davis, 1996: 191). Davis also sees the language games of Wittgenstein (1953) in cognition, which *'does not occur in minds and brains, but in the possibility for (shared) action'* or *'play'* (Davis, 1996:192). Davis' ideas are enactivist because he offers models of cognition and learning which are historical, situational, dynamic, intersubjective and consensual (Varela et al, 1991). They focus on the phenomenal and experiential. Play is crucial for enactivism because all phenomenona exists only in playing. Playing is seen as a movement which *'has no goal that brings it to an end; rather it renews itself in constant repetition'* (Gadamer, 1990: 103), which encompasses everything like conversation, games and so on. Hence all knowing is doing. All doing arises from our interaction with our complex and active environment.

Mathematics, itself, can be put under the spotlight of enactivism to reveal two possibilities; *mathematics* and *mathematical* (Heidegger, 1977). *Mathematics* is the widely accepted static body of knowledge that has emerged through inquiry; whilst *mathematical* is the orientation to inquiry, which has allowed *mathematics* to evolve. It involves comparing, ordering, creating and naming, which mathematicians would describe as noticing of sameness, pattern and regularity. The *mathematical* possibility takes the familiar definition of learning, which is exactly what the Greeks intended it to mean.

2.3 Greeks

The Greeks offer a good starting point to see connections between play and mathematics. Play, the English word, is derived from the Greek word *paidiá* meaning childish play or harmless amusement (Liddell & Scott, 1925/1968). The word is derived from *pais* meaning 'child'. Education is also derived from *pais* and in Greek is the word *paideiá*, which was used to mean both the process of a child's rearing and training and the result of that process: its mental culture, learning and education (Liddell & Scott, 1925/1968).

Plato saw *paidiá* offering a natural setting for *paideiá* and one in which the player's natural underlying dispositions are revealed. The word 'mathematics' comes from a Greek word meaning 'learning', so it is possible to see that the Greeks saw a connection between play and mathematics. Unfortunately, Plato lost the connection between play and education as he moved to the higher forms of knowledge (Davis, 1996). Huizinga's (1955) work on the role of human play as essentially that of contest and that the contests have a civilising function is important to explore to see the link between play and learning mathematics.

2.4 Huizinga: Nature of Higher Play

Modern research in Play and Mathematics has been limited probably by the fact that play is seen as the domain of early years education.

Huizinga (1955) offers an early definition of the nature of higher play, which illuminates Gadamer's stance (2.2 Enactivism). Huizinga (1955) sees play as voluntary activity, which makes it free and pleasurable. Forced play is no longer play. It steps outside of ordinary or 'real' life, so it does not have the immediate satisfaction of needs and wants because it occurs in a temporary world with its own arrangement. It can at any time run away with the participants, so they no longer realise where they are. Pretend play is a good example of this stepping out. A third characteristic linked to the second is the fact that play is bounded in time and space. It contains its own meaning. The temporary world is bounded by the space it is played in be it the arena, the card table or the stage. Play does play itself to an end, but it does restart itself either when the participants decide to have another game or when they decide to change the rules and play again. Whilst in progress there is movement, change, alternation, repetition, succession, association and separation.

There is always in play the feature *'it creates order, is order'* (Huizinga, 1955:10), which is apparent from the fact that any change from the order ruins the games. Its order creates its beauty, but it could be just that in creating something beautiful we create an orderly form. All play is governed by rules; it determines what 'holds' in the temporary world. If the rules are broken the whole play world collapses and the game is over. There is always tension in play created by the fact the player is trying to achieve something. Finally play inevitably creates social groupings.

Huizinga's (1955) characteristics of play are very abstract. Fromberg (1992) offers a model of play rooted in children's play which has important links with Huizinga's (1955) work.

2.5 Fromberg: Nature of Children's Play

Research on play in early childhood states that young children's play is *symbolic, meaningful, active, pleasurable, voluntary and intrinsically motivated, rule-governed and episodic* (Fromberg, 1992). Firstly the *symbolic* is where reality is represented by 'as-if' or 'what-if' attitudes. For a child pretending to be a dog, the 'what-if' behaviour is the imagining of the roles and feelings of a dog, whilst the 'as-if' behaviour is the play itself that allows the child to experience particular feelings and attributes. Play is *meaningful*, in that it connects or relates to experience. For children this occurs because it is integrally connected to their everyday experiences. Children are always *active* when they play. It is *pleasurable* because children focus on their play intently and enjoy that intensity of involvement. The *voluntary* and *intrinsically motivated* aspects of children's play relates to the motivation inspired by curiosity, mastery or financial. It is rule-governed implicitly or explicitly, which is not always apparent to the observer because the rules are usually child imposed. Fromberg (1992) sees the *episodic* nature of play as the emerging and shifting goals that develop throughout the play. In fact as children play their goals develop in various ways. Their play has goal flexibility.

It seems there is only a tentative link between the work of Fromberg (1992) and Huizinga (1955), but I propose to show that Fromberg's (1992) definitions in play are similar to Huizinga's characteristics of higher forms of play. The *symbolic* in early child's play with its pretending quality seems to relate to the stepping out of real life in Huizinga's (1955) features.

Play is *meaningful*, in that it connects or relates to experience. Huizinga (1955) also sees the meaning of play, when it is played within limits of time and space, which naturally connects it to experience. Children are always *active* when they play, which only reiterates Huizinga's (1955) view that there is change, movement and hence activity in play. The *pleasurable* and *voluntary* and *intrinsically motivated* aspects are similar to Huizinga's (1955) notion of voluntary. Both see the rule-governed nature of all play. Fromberg (1992) sees the *episodic* nature of play as the emerging and shifting goals that develop throughout the play, which Huizinga (1955) sees as the *tension* where the player wants to 'succeed' and will inevitably involve him changing his goals as the activity progresses.

It now seems appropriate to look at recent research into the role of play in promoting mathematical thinking

2.6 Play: Development of Mathematical Thinking of Children

The role of play in developing mathematical thinking of young children is well documented (Jarrell, 1998). Mathematical thinking develops because children work with objects, put them into relationships and think about those relationships during play (Piaget, 1973). In experiencing mathematics, children can better develop mathematical concepts (Copeland, 1984). Rogers and Miller (1984) have shown this by carefully designing a game on factoring which raised achievement for under performers if played frequently. Playing with *'specifically designed toys can also lead to greater understanding of the rules involved in mathematical concepts…than can be provided by observation of the same stimuli but without manipulation'* (Zammarelli & Bolton, 1977:160). Both research studies highlight the importance of grounding early mathematical work in concrete objects (Kamii, 1985), which hopefully allows them to see how mathematical symbols connect to and represent objects and actions, which are familiar. They can then make new connections within mathematics, rather than adults telling them what holds in the world of mathematics (Featherstone, 2000).

The advantages of play are not just limited to young children. Allen and Ross (1977) showed with eighth-grade students that playing a program called *'equations'* and with a mathematics play kit enabled students to apply mathematical ideas better.

In middle childhood the use of play in early childhood will be apparent because they have a much better understanding of the relationships between objects (Kamii, 1993) and they can use the knowledge to solve quickly more complex problems (Baroody, 1987). Their symbolic understanding will be at the point where they can make the connection between these relationships and the abstract symbols used to represent the relationships (Piaget, 1952).

Vygotsky (1978) offers a good model on how adults can influence play and hence help children develop their mathematical understanding. For Vygotsky (1977) development occurs between two levels. The lower level is what the child can do independently, without any assistance. The upper level of the child's ability is what the child can do with help from another person directly or indirectly. The zone of proximal development (ZPD) is the distance between these levels. It defines where the child's learning can happen. The zone is a changing structure because as a child masters one skill, new ones emerge at the upper level. Play is seen to provide support at the upper level of the ZPD and assist in making children: *renounce reactive behaviour*; it *promotes symbolic thinking* and *provides a context to practice planning and self-regulation* (Bodrova & Leong, 1998:278).

In mathematics, this means adults can support children's mathematical problem solving efforts by supporting them at the higher level. Jarrell (1998:63) offers three strategies to achieve this: *arranging the physical environment to make high-level mathematical thinking readily possible; asking questions that encourage children to think mathematically during play* and *playing with children games that have mathematical thinking embedded in them.* It is important to stress that teachers must work hard to facilitate play (Smilansky, 1968) and not to intervene to quieten or terminate children's activities (Trawick-Smith, 1994).

2.7 Play: Mathematics

It seems important to now show the link between play and mathematics at a higher level and hence its link with games, problem solving and investigations at school level. Huizinga (1955), once again, offers a link between play and mathematical invention. He sees the nature of play: it steps outside of ordinary or 'real' life; it is bounded in time and space; it is orderly and in consequence beautiful; it is governed by rules; play involves tension; play is voluntary; play creates social groupings. The mathematical world, with its invented mathematical concepts (Wittgenstein, 1956) could be this stepping out of 'real' life (Huizinga, 1955), but crucially it is far more limiting than play because it seeks to answer if this and this is true, what can be argued logically and deal with studying the hypothetical states of things (Peirce, 1902). All 'truths' are conventional and last longer than in play (Ernest, 2004). Play certainly has far more freedom with any 'truths' being imaginary. In mathematics the play has to hold up, be true within that structure. In the world of play both the constraints and criteria for acceptability can be imaginative. Both live in a realm where there is a *temporary world within the ordinary world, dedicated to the performance of an act apart* (Huizinga, 1955:10). The world exists when you're working on some mathematical problem, say the solution of an algebraic equation, but ceases to exist when you move onto something unrelated. Hence it is bounded in time and space (Huizinga, 1955).

Poincaré (1946) sees the importance of order and hence beauty in mathematics, because the most useful combinations of ideas tend to be the most beautiful and these are not invented by using strict logical connections, but by allowing room for disorder and play. This is similar to the voluntary aspect of Huizinga's (1955) work.

The fact that play involves rules is mirrored in mathematics by the fact that mathematicians explore, and ultimately use, the rules that govern the behaviour of mathematical entities, which inevitably creates tension because of the uncertainties of success (Featherstone, 2000). Mathematics creates social groupings because mathematicians do work together as well as alone, which only mirrors the nature of the social bond, where *each* (self) *exists in a fabric of relations* (Lyotard,1979:15).

If we assume that mathematics is a collection of language games (Wittgenstein,1956), with three characteristics: *the rules are the object of a contract, explicit or not, between players; if there are no rules there is no games; every utterance should be thought of as a 'move' in a game* (Lyotard, 1979:10). Then in both mathematics and play the rules are always open to the possibility of change, but they remain fixed for a lot longer in mathematics (Ernest, 2004).

As a postmodernist the assumption that play and mathematics are rule-structured activities is problematic because they assume there is an underlying structure to the activity: play with its rules and mathematics with its language games. Postmodernist would deny there is any structuralist claims to be made about any activity. In consequence truth is far more fragile being relative, localised and temporal. Order is a misnomer for the position when any order that is claimed is highly sensitive to initial conditions. Hence chaos is embraced to counter any claims made about order within mathematics. In conclusion I find it difficult to define mathematics, because like all human knowledge it has a crisis in its legitimatisation (Lyotard, 1979). Different metanarratives produce different narratives, but I have chosen to work with a rule-structured metanarrative to enable me pragmatically to produce some research.

2.8 Play: Mathematically Thinking

I propose in this section to show a similarity between mathematical thinking and play. Mathematical thinking can be seen about mathematising situations and applying mathematical powers in order to model situations. It will mean setting and solving problems by following logical deductions, it could also mean conjecturing and proving theorems. These models and theorems are continually evolving.

Lakatos (1976) created a model of mathematical thinking similar to Popper's (1972) falsification principle for scientific thinking, where multiple theories are acceptable for the same phenomenon until they are falsified. Truth in science is no longer possible for scientific theories. The process consists of starting with a problem or conjecture, there is a simultaneous search for proofs and counterexamples. New proofs explain old counter examples, new counterexamples undermine old proofs. Proof, here, is seen as a search for explanations, justifications and elaborations, which make the conjecture more plausible.

The conjecture is being made more accurate and detailed because of counterexamples. This is an ongoing process, which never stops. This process is very similar to mathematical modelling (Wheeler, 1982) where we specify a problem, set up a mathematical model, find a solution, interpret the solution and compare with the original problem and repeat this cycle if necessary. The similarities arise from seeing conjecturing like suggesting a mathematical model. Proving is like finding a solution. The counterexamples are like finding the discrepancies or errors in the model. In reality both frameworks deal with two quite distinct areas of mathematics. Lakatos is more relevant to Pure mathematics, whilst the modelling is clearly more in tune with traditional Applied mathematics.

I am not going to try and argue that play is equivalent to mathematics, but to the fact that doing mathematics can be playful (Featherstone, 2000). Historically Felix Klein (1849-1928) was one of the modern mathematician's to identify imagination as a major factor in a mathematicians work: *the mathematician...does not work in a rigorous, deductive manner, but rather uses fantasy* (Steen, 1989: 83). Thus mathematical thinking be it pure or applied is achieved frequently through playful means, with potentially several false starts, beginning in the middle or even working backwards and sometimes the whole idea is born out of fantasy. Mathematical conjectures or models are a stepping outside of the real world or a fantasy, which is a key element for Huizinga. The mathematical world be it a conjecture or model is bounded by time and space in terms of the arena where it is played, which Huizinga also sees as a characteristic.

Lakatos' (1976) model and the mathematical model of Wheeler (1982) are very structuralist, when in reality like mathematics, mathematical thinking is difficult to define because it depends on your metanarrative. To see mathematical thinking as play-like requires a very narrow view of play. Most students of mathematics will work within a predefined culture of mathematics, where any play is limited to within the language game being discussed. A mathematician has the true scope of entering a fantasy world to create new language games. I see very tentatively that moving from a conjecture or mathematical model to a proof or solution and then through counterexamples or errors back to a new conjecture or model is similar to the fact that for Huizinga (1955) there is always movement and change whilst play is progressing.

The continuous cycling through both the Lakatosian framework and the mathematical model framework could be seen as tentatively similar to Huizinga's (1955) idea that play is attempting to create order. The order is what is being attempted when the conjecture or mathematical model has to be reformulated. Every reformulation is refining the conjecture or model to the point where it seems to create order. I cannot see it ever creating a permanent order, but it is possible to create a temporary order.

As a postmodernist I feel a pluralistic approach to mathematical thinking is required, which means there are many other ways to think about mathematical thinking for example computer proofs and foundational set theory. Lyotard (1979) spoke of the absence of an overriding grand narrative, which for mathematical thinking would mean there is no universal explanatory theory to explain it, but rather a plurality of little narratives. These would contain for example Lakatos, Wheeler, foundational set theory and computer proofs. Each little narrative is seeking to achieve limited objectives. These little narratives offer a multitude of ways to proceed, but I am essentially involved at looking at mathematical thinking within school mathematics, so I feel Lakatos and Wheeler offer the best little narratives to work with. There could be others, which fit my objectives better, but I have not found or invented them. I need to make a pragmatic decision given all the tensions that exist to ensure some research can occur.

I can now turn my attention to mathematical play, where I am looking at games, problem solving and investigations.

2.9 Play: Games

One of the most prominent areas that mathematical play expresses itself is in games. This is natural because mathematical concepts are themselves games (Wittgenstein, 1956). More generally they are part of *language games,* these are linguistic practices, governed by certain rules and conventions. Everything can be seen as a language. The language games, when played out in a social context, reveal meanings and understandings as aspects of the games themselves (Wittgenstein, 1956).

Meaning, for Wittgenstein, is not something outside of language, but comes from language in use (Wittgenstein, 1956: 54). Sfard (2000, 2001) uses Wittgenstein's ideas to see communication occurring through language games in a social context and meaning emerging through communication. Concentrating on communication rather than cognition *'entails viewing learning mathematics as an initiation to a certain well defined* *discourse'*, so seeing *'learning-as-participation'* rather than *'learning-as-acquisition'* (Sfard, 2001:13-14). Hence *language is a presentational act*: presentation of meaning (Fleener et al., 2004: 447). In conclusion, by meaningfully playing the games of mathematics pupils are allowed to participate in language games, so giving them the opportunity to experience their world as mathematical (Fleener et al..2004) rather than just experiencing the fixed body of *mathematics* (Heidegger, 1977), which is not part of the pupils' experience.

These language games according to Genova (1995) have three senses of 'play'. In the first sense of play it is 'playing-with' language games. This is where the child is pretending, imagining and using language with a 'as if' quality. Fromberg (1992) refers to this as the *symbolic* nature of play, which clearly relates to the stepping out for Huizinga's (1955) features. The second sense of language games involves 'playing-at' games. Here we make appropriate moves in the language game. It does not imply we understand what we are doing. This clearly highlights the fact that all play is rule governed (Huizinga,1955). In the third sense of language games involves 'playing-in'. This is similar to acting in a play, so it assumes a script. Huizinga (1955) would see this as the fact that play: is bounded in time and space; creates social groupings ; creates order, is order and the fact that there is tension in play.

Games are activities similar to language games except they are not primarily involved with language. They involve one or more players in which actions are governed by rules (moves) and result in the game changing from one state to another. Games are played by making acceptable moves. In both learning and doing mathematics we play the games of mathematics (Kanes, 1991). The advantages of using mathematical games are well documented as: maintaining and consolidating skills; improving problem solving ability and helping to develop and construct mathematical concepts (Larouche et al., 1984). Ernest (1986) also sees games providing motivation. It is important to stress the difference between games and mathematical games. The latter is based on mathematical ideas and where winning the game is mathematical (Ainley, 1988). Playing games, exclusively, does not teach mathematics (Ernest, 1986) but helps children to learn mathematics or do mathematics (Ainley, 1988).

The typology of mathematical games highlights their 'play' characteristics. For example the mathematical game described below (Bright et al., 1985: 161) helps to highlight this issue:

The teacher gives the following instructions. 'I'll throw 2 dice, you put the numbers into the triangles in the expression $\Delta x + \Delta = 5$ *in order to form an equation. Solve the equation, then swap work and check the solution by substitution. If your solution was correct, add it to your progressive tally. If not, throw it away. After 10 throws of the dice the person with the highest tally wins'*

Now when the class plays this game, there will be a number of other interactive games being played. Kanes (1991) has classified these games; firstly as a initiating or *root* game, which, from above, is the game of trying to solve the equation you have devised. The companion games or *metagames,* will ghost the root game. For example the devising of admissible moves in the root game; the creation of a strategy or optimal strategy for playing the root game. It is a game of games. Hence the *metagame* will determine the progress, the play of other games, root or meta.

Kanes (1991:235) offers the following classification of the *metagames:*

Category of metagames	*Objective of characteristic metagames*	*Characteristic metagames*
PARTICIPATION & AFFECT	*Player establishes psycho-social relationship with root game*	*avoiding, negating, varying, postponing, completing, perfecting*
PROCEDURE	*Construction of admissible moves in root game*	*describing, denoting, interrogating, performing, recommending, copying, metaphor and metonymy*
STRATEGY	*Construction of strategy for playing root game*	*simplifying, analysing and synthesising, patterning, hypothesising, translating, concretising and abstracting, modelling, comparing, inferring*
HIGHER-ORDER	*Determining optimality of strategies*	*examining, considering, managing, editing, surveying, prescribing, rationalising*

Table 1: Classification of metagames

Again, these categories can be linked into Huizinga's (1955) characteristics of play. In playing any root game; the player decides the extent and duration of his or her participation, which mirrors the fact that all play is voluntary and free (Huizinga, 1955). Play is always rule governed (Huizinga, 1955), which Kanes (1991) highlights as the fact that a player can decide to create moves which are acceptable. A strategy is a decision procedure, which imposes a structure on the successive moves of the game. In the example, above, it could be to put the smaller of the two numbers next to the x, to make the question easier for myself, which Huizinga (1955) sees as the fact that play is governed by rules. Rules which are imposed from the original setting up of the game and self imposed when trying to stick to a strategy. The higher-order metagame is effectively the game which exerts overall control over the whole gaming period. Its purpose is to seek the optimal strategy. Huizinga's (1955) refers to tension as being created when a player wants to succeed in the game. He sees the elements of tension and skill as crucial in all games of skill. I feel in finding the optimal strategy you will inevitably create tension within yourself as you try and find the optimal strategy.

I am not trying to say 'tension' and the higher-order metagame is identical, but that tension is present when the optimum strategy is sort.

Clearly, within mathematical games not all categories of metagames will be activated (Mayer, 1983), as much as in play not all characteristics are seen in each episode.

2.10 Play: Problem Solving and Investigations

Problem solving and investigations can be related, because any problem solved can be extended by varying one or more of the fixed quantities. Hence by introducing a variable, a problem is turned into an investigation.

Cognitive science (Marshall, 1995) offers the notion of schema to provide a theoretical basis for the development of students' ability to think about problems. Marshall, a cognitive scientist, defines a schema as a mechanism in human memory that allows for storage, synthesis, generalization and retrieval of similar experiences. It allows similar experiences to be organised. Schemas are not a new idea. Piaget (1971, 1977) used the idea of schema on his research on the development of scientific reasoning. His use of action: an activity of the mind linked to the experience of individuals with the physical world (Thompson, 1994) was important in formulating a definition for abstraction: the process by which the mind selects, organises and combines actions so that they can finally store them in memory (von Glasersfeld, 1995).

Schema are developed by reflecting on action (Sfard, 1991). Piaget (1985) originally termed this process of forming cognitive structures or schemas as reflective abstraction. For Piaget, reflective abstraction, is just one of three components of his theory of *functional invariants*. Accommodation is the fact that individuals have to adapt their functioning to the specific qualities of the things with which they are dealing. For example children realise they cannot change the properties of water and fire and so the original schema, which assumed you could change the properties of water and fire must be reconceptualised and then it can accommodate the new experience.

Assimilation, on the other hand, refers to the process by which objects or their attributes are incorporated into the individual's existing cognitive structures or schemas, often altering and developing these structures somewhat in the process. Individuals develop schema by having repetitive experiences in the organization of situations or events, which inevitably involves assimilation, accommodation and reflective abstraction (Greeno et al., 1996). Hence meaningful schema will only develop if individuals understand concepts to be able to recognize and construct patterns (Steele & Johanning, 2004). For individuals to use a schema they must make connections with prior knowledge. Sfard (1991) highlights the need for *structured cognitive schemata* which are vertically constructred. Each new layer forms an ever deeper connected hierarchy of knowledge. Horizontal schema are created when individuals rely on memorization (Sfard, 1991), which creates *unstructured, sequential cognitive schemata.* This is typified by being shallow with unconnected information.

Generalisation is at the heart of mathematics and to the connection between schemas and mathematical problem solving and algebraic thinking. Tall (1991) sees assimilation as an expansive generalization: a generalization where existing schema (cognitive structures) are extended without a change in current ideas. Accommodation is seen as a reconstructive generalisation. It reconstructs the existing schema. Dubinsky (1991) offers a simpler formulation where schema are generalised once an individual learns to apply an existing schema to a wider range of experiences.

Mason (1996) has offered in mathematics the importance of students' experiencing *'seeing a generality through the particular and seeing the particular in the general'* (p.65), which has parallels with the work of Krutetskii (1976), who sees two different aspects of generalisability like Mason. The first aspect is *'ability to see something general and still unknown to him in what is isolated and particular'* (Krutetskii, 1976:237), which is similar to Mason's seeing the general in the particular. The second aspect picks up seeing the particular in the general; *'a person's ability to see something particular and known to him in what is general and concrete'* (Krutetskii, 1976:237).

Understanding can be built simultaneously from particular to general and from general to particular (Sfard, 2003). In summary then schema are built by reflecting on actions (Sfard, 1991), but we must use schema to develop and refine them. Research in problem solving (Silver, 1981) has shown good problem solvers use the mathematical structure of problems to generalise solution strategies, which should not be a surprise to mathematicians who would see the structure of the problem as being key to its solution, so in solving related problems students are transferring what they learn to similarly structured problems. Reasoning in mathematics requires logical thought, whilst in play it requires less rigidity as long as the game can proceed. Confrey (1997) suggests using a context-based approach to help students tackle related problems and develop algebraic thinking. I think context problems work because what is seen to be 'similar' is actually influenced by context as well as by mathematical structure. The Algebra Working Group (1997) see growth and change, size and shape and number patterns as appropriate contexts. Lee and Wheeler (1987) have classified problems involving these contexts as *generalizing problems*. In their research, with linear and quadratic generalising problems, they found students did not usually check their generalisation to see if it worked for particular cases.

Mason et al (1985) and Balacheff (1988), independently, discovered that students had four stages of expressing generality in problems of this type. These stages where similar to Bell's (1976) work on the stages of generalising and proving. The first stage involved making conjectures about generality by looking at just a few cases. In the next stage students tested their generalisation against particular examples. This is followed by an awareness of the need to consider all possible cases. The fourth stage is about making explicit generalisations. Balacheff (1988) and Mason et al (1985) both found students used only the first two stages of generalizing and proving. These stages of Bell (1976) are typical mathematical behaviours which encompass thinking, figuring things out and reasoning. All children exhibit these behaviours when engaged in meaningful play (Whitton, 1998).

Steele and Johanning (2004) have worked with 7^{th}-grade pre-algebra students and explored how a teaching experiment based on creating effective schemas for solving algebraic problem situations involving contexts of growth and change and size and shape. Their findings showed the formation of both well-connected and partially formed schema, in roughly equal proportions. The sample size was only eight. In contrast to previous research (Cooper & Sakane, 1986; Balacheff, 1988; Lee & Wheeler, 1987; Stacey, 1989) students with well-connected schemas checked their generalisations. They tended to check them by drawing diagrams, which seems key to their success with generalization. Tables do tend to hide the structure of the problem, so it was no surprise to see students who relied on tables struggling to find a relationship between the quantities (Steele & Johanning, 2004). Problem solving, also, helps to develop *symbol sense* (Arcavi, 1994) because it is similar to algebraic thinking. Steele and Johanning (2004) recommend teachers creating problems that encourage students to draw and analyse diagrams, which promote algebraic thinking. This involves *seeing the general in the particular* and *to see the particular in the general* (Mason, 1996). They see a direct link between type of generalisation students construct and the schemas they are forming:

'Students extend their schemas to assimilate the new particular case into an existing general schema or accommodate their existing general schema to incorporate the new particular case' (Steele & Johanning, 2004: 88).

2.11 Play: Discussion

Discussion at first sight does not seem to have play characteristics but Legrand (1995) sees discussion as a scientific debate where conjectures are formulated, proposed, challenged, tested and justified. The scientific debate is similar to Lakatos' (1976) and Wheeler's (1982) model for mathematical thinking. The formulated or proposed aspect is similar to putting the conjecture or model forward. It is challenged and tested by counterexamples or errors found and justified through the search for proofs or better solutions. Discussion and mathematical thinking match up because Lakatos' and Wheeler's models share the same structure as Popper's (1972) 'falsification principle' for science. It is possible in the same way that Lakatos (1976) and Wheeler(1982) have been compared to Huizinga (1955) to see similar parallels with Legrand (1995).

Discussion is vital for mathematics because the subject is essentially linguistic, textual and semiotic, but embedded in social interaction and manifests itself as conversation (Harding, 1986). Discussion allows conversation to be playful because the conjecturing atmosphere gives students the confidence to try out ideas without feeling embarrassed about making mistakes. Huizinga (1955) would see this a stepping outside of ordinary life. Everything is offered as a conjecture those who are unsure often speak up to seek clarification and those who understand choose to listen and offer modifications through counter-examples, images, questions and suggestions. This I see as helping to form order, in Huizinga's (1955) schema. The conjecturing atmosphere within a mathematics discussion is different from a truly play conjecturing atmosphere. In the former the discussion is occurring within established rules of acceptance, whilst in the latter these criteria for acceptance can be totally imaginary and made as the discussion evolves. Truth is very brief in the play world, whilst in mathematics it has a certain permanence. In the Lakatosian model eventually the truth is found to be not as a general as it once seemed. The generality is re-formulated to take account of the counter-example. The truth now seems very situated to a very specific mathematical world. As a postmodernist I can already see that both my frameworks for play and mathematical thinking lack in-depth pluralism (Feyerabend, 1993), which a postmodern position expects, but to work within the criteria of a research dissertation requires some form of allegiance to some 'model' for both mathematics and play. I feel then that the scientific debate format, which has been taken up by Michaels and Sohmer (1999) with their concept of a position-driven discussion as most fruitful for my research, where a teacher leads a group of students in considering one central question with a fairly limited number of possible answers. Students are expected to take a position on the answer and required to support that position with evidence. Other students are encouraged to challenge and the teacher's role is not to say if an answer or hypothesis is correct, but to support and clarify the contributions of students.

2.12 Research Question

The main issue for my research is that I want to change my practice from being a traditional didactic teacher to a teacher who encourages mathematical thinking within pupils, so they are no longer passive receivers of knowledge but actively creating knowledge for themselves in a play like environment. I am keen to explore how the notion of play will change my practice.

As an enactivist it would be important to play the language games of mathematics (Wittgenstein, 1956) with the pupils, so giving them the opportunity to experience their world as mathematical (Fleener et al., 2004). I am going to start by introducing small whiteboards and see if this encourages them to participate in the language games of mathematics as Fleener, Carter and Reeder (2004) claim. I could start these episodes of play with a problem to gauge where the students are. The problem set should allow students to display their thinking about the problem, which should reveal their understanding depending on how explicit they are on the boards. I see that learning should start from their ideas or more specifically as a postmodernist the collective classes ideas, then it is possible to see the lower levels of the zone of proximal developments (Vygotsky, 1977). I could then see if this encourages children to think mathematically as Jarrell (1998) reports. Lakatos' (1976) model of mathematical thinking would be interesting to try and develop within the classroom. It naturally leads to looking at developing generalisation in the classroom, as a way of connecting problem solving to schema development. I feel that generalisation is the key objective that I am trying to encourage, through the medium of play because it is at the heart of what mathematics is. Finally, I feel it would be interesting to see what happens if I change the physical environment (Jarrell, 1998) to encourage discussion. It would be interesting to see how I could develop Michael and Sohmer's (1999) positive-driven discussion to one in which pupils are empowered to set the questions. Hopefully this will encourage more play like behaviour from my students.

My main research question is then "As I change some factors in the way I go about teaching, what behaviours change in my students and what issues are raised for me in trying to develop a classroom where generalisation and play are more prevalent?"

2.13 Summary

This chapter has dealt with many aspects of play starting from my philosophy of the concept and delving into early work by the Greeks and the connections between play in all its aspects, mathematics as an academic subject and educational research on play. I have settled on a research question, where I will focus on certain features of play involved in a mathematics classroom, which involve me being more playful in terms of starting with a problem, focusing on getting students to notice patterns, playing with room layout and encouraging discussion which in the next chapter I will discuss how I will research my question.

3. Methodology

Research Question

"As I change some factors in the way I go about teaching, what behaviours change in my students and what issues are raised for me in trying to develop a classroom where generalisation and play are more prevalent?"

3.0 Introduction

This methodology chapter begins with my personal beliefs: starting from my postmodernist beliefs I show how it is linked to enactivism and my research area play. I go on to discuss the relevance of four frameworks: activity theory, action research, reflective practice and the discipline of noticing and look at a variety of individual methods of research in terms of my postmodernist stance and then go on to discuss what I actually did for my research question. I finally outline my research timetable and how I analysed my data.

3.1 Personal Beliefs

I would describe myself as a Postmodernist (Lyotard, 1979), which encompasses deconstructionism (Derrida, 1977) and is ultimately a rejection of all structuralist claims.

A consequence of this is a belief that reality cannot be represented (Rorty, 1980) and whatever representation we try constitutes the *'text'* (Derrida, 1977). The text is problematic because the author does not have the definite guide to the meaning of the text (Barthes, 1968). Individuals do not exist by themselves, but by a social bond through language games (Lyotard, 1979).

As a Postmodernist then; the world is constituted by text and speech acts (Sfard, 1998); so in a classroom we focus on the language games (Wittengstein, 1953) occurring between the people present. To know the teacher or pupil is problematic, because we exist as a social bond, where *each* (self) *exists in a fabric of relations* (Lyotard, 1979:15).

Enactivism (Davis, 1996), for me, sums up my position in learning and knowing. Davis's (1996) notion of self as *'defined as a network of relationships, and so, as histories, contexts, and participants vary, identities change'* (p.192) only reiterates Lyotard's (1979) view of the self. He also sees the language games of Wittengstein (1953) in cognition, which *'does not occur in minds and brains, but in the possibility for (shared) action' (p.192)* or *play*. Enactivism is postmodernist in outlook because it rejects the ability to establish *'a new and irrefutable foundation'* (Davis, 1996: 7)

If we assume mathematics is a collection of language games (Wittgenstein, 1956), with three characteristics: *the rules are the object of a contract, explicit or not, between players; if there are no rules there is no game; every utterance should be thought of as a 'move' in a game* (Lyotard, 1979:10). These characteristics implicitly sum up exactly everything Huizinga (1955) points out as the nature of play. It is governed by rules; which determine what 'holds' in the temporary world. If the rules are broken the whole play world collapses and the game is over.

In learning a game it is important to play it. Play is important for enactivism because the phenomenon be it a conversation, a game exists only in playing. Playing is seen as a movement which *'has no goal that brings it to an end; rather it renews itself in constant repetition'*, (Gadamer, 1990: 103).

I am researching how the notion of play is changing my practice in the learning and teaching of mathematics because I feel doing mathematics can be playful for both teacher and students.

I am consciously trying this year (2005/06) to stop standing at the front of the classroom and telling the students about some aspect of Mathematics, with some effective questioning. Being passionate about enactivism and postmodernism has driven me to see the activity as essential in determining the quality of learning which occurs. Play is such a difficult concept to pin down because for some work is used to describe the same activity. It is difficult to decide whether play is an activity or a quality, which we can only vaguely describe.

Personally I am interested in developing my own practice so that I use the notion of play more in lessons and try to get myself and possibly the students to play and simultaneously develop my own personal use of activities which use games, problem solving and investigations.

As a postmodernist I would struggle to define either a teacher or a student and learning and teaching can be difficult to distinguish. I would rather speak of everyone as the potential to take a teacher's role or a pupil's role. Enactivism (Davis, Sumara & Kieren 1996) sees learning and action as one and the same.

3.2 Research Frameworks

In holding this enactivist philosophy I could see Activity theory as an appropriate framework (Leont'ev, 1978), because for Leont'ev (1978) the activity positions the participants and provides the initial meaning and motivation. Sense is personal intention, whilst meaning is public and agreed socially. Sense determines the relationship between objects and the needs of the individual (Gal'perin, 1973).

Activity theory draws on Vygotsky's (1978) semiotic approach (Lerman, 1996), which for me only parallels the postmodernist study of signs, where there is a sea of signifiers and multiple interpretations, none which can claim to be the truth (Derrida, 1977).

I am quite conscious that I am researching myself and that as a postmodernist it is important that any framework takes into account my cultural diversity (Cole, 1988). This may be possible with a network of interacting activity systems, but the conceptual tools need developing that would allow me to understand this network and the multiple selves.

Action Research (Lewin, 1946), which has been seen as a form of *'collective self-reflective inquiry'* (Kemmis & McTaggart, 1988: 5) offers a paradigm rooted in critical theory (Weiskopf & Laske, 1996: 123), which for a Postmodernist, like myself, poses great problems if used as a framework because it assumes that consensus is achievable and neglects the issue of power, which prevents some participants becoming empowered (Grundy, 1996: 111).

The *'Discipline of Noticing'* (Mason, 2002), which seems to take on the postmodernist view that the *'event'* is the change in perspective which occurs after a significant occurrence (Lyotard, 1979) seems more fruitful. For Mason, the event is the stories that participants tell; the story they tell depends on the significance they give to incidents (Mason, 1994). Noticing is then picking up on distinctions, sensitivities and significances.

In taking a qualitative stance with the *'Discipline of Noticing'* (Mason, 2002), many problematic issues arise with the 'accounts of' and 'accounts for'. If we see only what we are prepared to see how do we see anything new, but this view appears very circular, very cause-and-effect. You could break the circle and have cause and effect on opposite poles, but I feel it is more productive and postmodern to accept the circularity and see co-emergence of cause and effect (Maturana, 1988), which only mirrors the concept of karma in Indian philosophy, where cause and effect are combined together as intentional action. As a postmodernist to see something new is only new to you. There is nothing new in ideas (Lyotard, 1979) and hence I must also deny that there exists only one mechanism to explain how new ideas appear (Feyerabend, 1993).

As the data is constructed by the researcher it is natural to assume as a researcher changes continually, their record will change of the same situation; they, the researcher, effectively notice different things. The use of triangulation (Elliott and Adelman, 1976) means the actual event is negotiated between different observers, which for the discipline of noticing means negotiating accounts. But even with triangulation, there is an issue with fidelity (Mason, 2002): the faithfulness of the account to the record. In the discipline of noticing the experience is the data; the account gives access to the experience for the reader. It is a successful account if it resonates with other readers.

In creating accounts I must observe, describe and analyse, which allows me to select what I observe, describe and analyse. Analysis creates data and could potentially mask the original experience, which is why it is important for researchers to reveal the purpose of what has been recorded and what was not. Description or observation involves interpretation, so the researcher is part of the data. There is, hence, an issue of objectivity, which requires the researcher to separate the phenomenon being analysed from the analysis, which occurs in the distinction between account-of and account-for. This itself is difficult because we are looking at human behaviour, which is the only thing that can be agreed by independent researchers. In looking at human behaviour we are drawn into discussing beliefs, motivations, emotions, cognitions, thoughts and reasoning. Many of these will be constructed by researchers in trying to account for observations. I think it is better to see belief and behaviour as co-emergent (Mason, 2002). Two sides of the same coin. Belief encourages a certain behaviour and behaviour generates certain beliefs.

In forming accounts we will naturally introduce labelling but there is a danger you label the behaviour of others, which pushes the focus of your own enquiry away from yourself onto others. In labelling others it must be symptomatic of the thing you are looking at yourself. In conclusion, generalisability is going to be a concern given all these issues but noticing is *'seeking generalisability in the awareness, in sensitivity to notice and opportunities to act, not in the applicability of assertions'* (Mason, 2002: 242)

Jaworski (1994) has developed the framework of 'reflective practice': *'the making explicit of teaching approaches and processes so that they can become the objects of critical scrutiny'* (Jaworski, 1998: 7). It requires evaluation of what occurs and feeding into future planning without a need for critical knowledgeable action. There are similarities with the 'discipline of noticing' (Mason, 2002) as 'noticing' significant events is similar to 'reflecting-on-action' (Jaworski , 1994), whilst 'marking' issues in future practice leads to choices in future activity. This is similar to reflecting-in-action (Jaworski, 1994), where teachers become aware of actions, decisions and judgements as they occur in their teaching practice and hence have more choices in their future actions. It should not be seen as similar to action research because any reflective practice contains only elements of the action research cycle as described by Kemmis' (1995) *'cycles of* *planning, acting, observing, reflecting, replanning, further action, further observation, and further reflection'* (p.156).

For my research question, which centres on my development and issues raised for me it is sensible to use the 'reflective practice' framework (Jaworski, 1994), because I am looking at my own development and reflecting on it. In the *'Discipline of Noticing'* (Mason, 2002), there is a conscientious effort made to reduce the gap between action and reflection. My looking into myself is a much more slower and reflective process than the *'Discipline of Noticing'* framework (Mason, 2002). I can see there are similarities between Jaworski's (1994) reflecting and accounting for (which uses critical analysis of the teacher) and Mason's (2002) accounts of and for.

In summing up my own stance, I feel comfortable with using this 'reflective practitioner' qualitative research paradigm (Ernst, 1998) to explore the features surrounding a particular case (Erlwanger, 1973). I hope the particular will illustrate the general.

3.3 Research Methods

I will first consider particular methods, which I could use as a reflective practitioner (Jaworski, 1994) and highlight tensions, which my postmodernist stance causes. I will then go on to discuss what I actually did with these tensions to try and answer my research question because I have to commit myself to both a framework and a methodology to produce a dissertation.

3.3.1: Particular methods

(a) Journal writing

This seems a valuable tool for the 'reflective practice' because it allows significant events to be written down in a journal as an aid to re-open that experience when you come to write the brief-but-vivid-accounts-of. Validity lies in the ability of the journal to represent the researcher's subjective world, which can be improved as Plummer (1983) points out by getting the researcher to autocritique his own journal and comparing the journal with other journals, for example student journals on noticings in the classroom or even interviewing students to get a comparison. The journal though still has problems for a postmodernist in terms of what of the reality is captured, how it is interpreted and what voices are heard (Linstead, 1993). There are a large number of possible representations of reality and a journal will tend to concentrate on linguistic actions. It is important to be sensitive to both the meaning and character of language seen, but also to accept that language is not a mirror of reality. I would hence doubt that reality can be captured (Rorty, 1980). This is because the sea of signifiers, which constitute language exist within a closed system divorced from reality (Alvesson & Sköldberg, 2000). Postmodernists hence see individuals who are constituted from language as ambiguous, equivocal and inconsistent. These problems highlight the difficulty in capturing and identifying the 'self' within the journal.

(b) Questionnaire

Questionnaires like interviews can be structured, semi-structured or unstructured. It is generally agreed (Cohen, Manion & Morrison, 2000) the larger the size of the sample, the more structured, closed and numerical the questionnaire may have to be, and the smaller the size of the sample, the less structured, more open and word based the questionnaire may have to be.

Closed questions prescribe the range of responses for the respondent they are quick to complete and easier to code and do not rely on the articulation skills of the respondent (Wilson & McLean, 1994: 21). Whilst I can see that open questions allow remarks, qualifications and explanations to the questions (Oppenheim, 1992: 115), but the responses will be difficult to code and classify. The issue really is one of 'fitness for purpose' (Cohen, Manion & Morrison, 2000).

In designing closed questions there are mainly three styles: dichotomous, multiple choice and rating scales. Dichotomous questions with a *'yes/no'* response struggle because even though it pushes people 'off the fence' they tend mainly to put a *'yes'* response (Youngman, 1984: 163), which highlights the tendency of respondents to agree with a statement. Multiple choice questions seem to have an advantage over dichotomous questions because they give a range of responses, which would have to be discrete. Immediately you get into problems with the interpretation of meaning of words for respondents, which means that whatever was intended by a question may not formulate within the mind of the respondent. As a postmodernist I feel you cannot avoid this. In using rating scales like the Likert scale you build in a degree of sensitivity and this allows you also to differentiate the responses and still allow you to generate figures, but this still does not avoid the problem with the meaning of words. The scale, which has: Strongly disagree, disagree, agree and strongly agree has an advantage because it gets people 'off the fence', but will still have some problems of interpretation (Cohen, Manion & Morrison, 2000). As one person's 'agree' is another person's 'strongly agree'. The semantic differential scale is where an adjective is put at one end of a numerical scale and its opposite at the other end (Osgood et al, 1957), a scale which I feel leaves the respondent to guess what the numbers signify and hence quantify the meaning of words. I see it as leading to potentially greater problems in interpretation. Both scales suffer from problems of honesty from respondents, interpretation of intensity of feelings, so that strongly agree should not be taken as twice as intense as agree or that even they have the same relationship with disagree and strongly disagree (Oppenheim, 1992: 190-5). Respondents will tend to ignore the extreme options preferring not to be seen as extremist (Cohen, Manion & Morrison, 2000). A central problem in questionnaires is that it does not allow different positions to appear, which is apparent from the questions set. These are inevitably created by the researcher and not by the respondents. There is neither scope for respondents to present themselves in a variety of ways, which will be clear from how their answers are sort (Alvesson & Sköldberg, 2000). This will be more apparent with closed questions than open questions, but questionnaire tend to restrict themselves to linguistic answers be they to interpret words or write some words down. The lack of pluralism is a significant tension for postmodern researchers using questionnaires.

Any wording in a questionnaire needs to be both clear and bold to draw the participant's attention to important features (Verma & Mallick, 1999: 121). The questionnaire should be piloted because it increases practicability (Oppenheim, 1992), identifies questions which are misunderstood or not completed (Verma & Mallick, 1999: 120) and questions, which offer little discrimination (Youngman, 1984: 172).

It is important to not forget that words in the questionnaire have no fixed meaning, so hence the interpretation by respondents can be ambiguous and variable, which ultimately means to solely rely on it is unreliable (Alvesson & Sköldberg, 2000). Piloting will not necessary eliminate all misunderstanding or non-completion because the ambiguity and variability is inherent in language interpretation. It is important to allow both the different voices within an individual and different individuals to express their positions. This multiplicity is a difficult tension to grapple with within the inherent structure of a questionnaire.

(c) Interviewing

Interviewing, I do feel, can offer insights into mathematics learning. The number of types of interviews varies from author to author but structured, group, unstructured and semi-structured (Fontana & Frey, 1994) interviews are an accepted common classification for interviews.

Structured interviews involve the interviewer asking each interviewee a series of pre-written questions with a limited range of responses. It has the potential to create an environment where responses are forced or given to please the interviewer (Cohen, Manion & Morrison, 2000). My own use of a structured interview in my mini-research project involved using *'performance questions'* (Zazkis & Hazzan, 1999), which are designed to elicit their understanding of a topic by creating questions which they have to answer. I did not just leave them to answer the question, but sought clarification to get more detailed information on what is going on. My experience highlights the fact that structured interviews can struggle if they are too rigid because they prevent students explaining their ideas. This reduces their voice and does not allow their position or ideas to become public. This is a serious tension for a postmodernist who is trying to encourage pluralism.

In contrast unstructured interviewing involves interviewing without pre-set questions and can be useful at the beginning of a study to provide insight (Mouly, 1970). Unstructured interviews potentially have the advantage of providing greater depth than structured interviews. For this form of interviewing the topic of discussion will probably be formed by the researcher, so even though it is far more fluid than a structured interview the insight and depth will probably not allow positions or special interests of the interviewees to appear (Alvesson & Sköldberg, 2000).

Semi-structured interviews are where topics and open-ended questions are prepared but the exact sequence and wording does not have to be followed with each interviewee.

Group interviewing is the *'systematic questioning of several individuals simultaneously in formal or informal settings'* (Fontana & Frey, 1994: 364), which can be done in a structured, semi-structured or unstructured way.

Watts and Ebbutt (1987) have highlighted the advantages of a discussion developing, it's more timesaving and less intimidating for students than individual interviews. It is not appropriate or possible to discuss personal matters and it is difficult to code up the responses after a group interview (Lewis, 1992). Group sizes of six or seven are seen to be optimum because too few can put undue pressure on individuals, too large and the group can fragment and lose focus (Lewis, 1992).

In choosing who to interview, it is important not to pick students who particularly interact well with me. The population must be accessible because any reluctance, by students, to take part can affect the representativeness of the sample. I have tended to use a purposive sampling method (Schofield, 1996), where I pick the cases to be included for my own purpose. I ended up choosing more talkative students, which I can see limits the ability to generalize to a wider population because the sample cannot be seen to be representative. It does though allow me to pick on students who can articulate changes they have noticed.

In conclusion to ensure validity within interviewing leading questions need to be eliminated (Morrison, 1993:66-67). The issue of *power* between interviewer and interviewees (Scheurich, 1995: 246) needs to be understood because it will affect the data. This could mean considering experimenting with interviewing and with ways to represent interviews that bring out the indeterminacy of interview interactions and possibly allow for the uncontrollable play of power within the interaction (Scheurich, 1997).

The use of a teacher, as the researcher, to interview pupils may lead to the interviewees giving answers which they think the researcher wants to hear (Hitchcock & Hughes, 1989).

It is also important for researchers to realise that the data produced from interviews tend to agree with their own personal thoughts (Mouly, 1970). Hence it is important to realise this and use the data as part of triangulation, with data gathered in other ways, to demonstrate concurrent validity (Campbell and Fiske, 1959).

I can see that like questionnaires for a postmodernist there is no stable reality or meaning that can be represented in an interview. The interaction is indeterminate because there is a complex play of feelings, powers, desires, fears, needs and thoughts for both interviewee and interviewer, which cannot be represented (Scheurich, 1997). Representation will tend to focus on linguistic actions. The indeterminacy is compounded by the fact that how interviewees represent reality in an interview situation has less to do with how reality is, but more to do with their temporal subjectivity that is created by the local discursive context of an interview (Alvesson & Sköldberg, 2000). Once again it is difficult to deal with the tension of allowing both the multiple voices within individuals and the different individuals to be heard.

(d) Videoing

Videoing I feel can show me and the pupils interacting with the mathematics (Yerushalmy, 1999), which an audio recording would struggle with as it fails to record visual data (Wragg, 1994). It has the potential to record beyond the observer's view or just frequently occurring events (Erickson, 1992: 209-210). It has been noticed by Brown (1990) that the camera operator, who becomes the editor, can remain fixed for large periods of time on the dominant event, which tends to be the teacher or whiteboard. This problem is called reactivity (Morrison, 1993: 91), it can be combated by offering some limited direction to the camera operator. Jaworski (1990) offers the solution of having two cameras in combination, which clearly would increase the field of visual data recorded.

In coming to analyse the video there is an opportunity of repeatedly viewing the tapes which as Mehan (1992) points out allows the researcher to *'see and hear a different version of social life than is otherwise possible'* (p. 93). It would be foolish to assume repeated viewings would eventually give a definitive account of the event. It is probably true that the *'event'* never existed to be caught on camera, because a videotape cannot capture the original classroom event itself because it only records sound and images from where it is pointing, not the smells or other camera angles (Pimm, 1993).

We can individually view a videotape and each time it creates a '*resonance*' in the viewer, which allows reflection on his or her practice (Schon, 1983). Group viewing of the tape can allow after discussion of the resonances created, a consensual resonance unique to the group (Kieran & Towers, 1998). This approach favours Habermas's (1981) view that we legitimise knowledge by consensus through discussion, which Lyotard (1979), rightly rejects, because there is a crisis in legitimising knowledge, which is the '*Postmodern Condition'*.

The disadvantages associated with using videotaping includes the fact the camera will only ever give a selective view of the action; the analysis of the tape is both complex and time consuming and there will be problems of interpretation (Tilstone, 1998). Jaworski and Gates (1987) highlight the fact people see different things when watching videos. This could be because they see what they want to see because of pre-conceived ideas about what should be happening. Students themselves may play up to the camera, which may need students to be exposed to a camera for a certain period of time for them to get used to the camera and behave naturally in front of it (Tilstone, 1998).

Cobb and Whitenack (1996) have used *'grounded theory'* (Glaser and Strauss, 1967), in analysing videoed mathematics lessons, where the data suggests the theory. This is immediately problematic, because it assumes the observer has no theories or pre-conceived ideas about what he or she thinks is going on. It is equally possible to argue that the theory suggests the data you see. Alternatively you could see data and theory suggesting each other. Emerging data changes theory and new theories change what data you see. In using conjecture and refutation (Popper, 1990) to construct the *grounded theory* they see *theory as a process* (Glaser and Strauss, 1967), but Feyerabend (1993) rejects the falsification principle (Popper, 1990), which sees multiple theories as acceptable as conjectures until they are refuted or falsified by new evidence. Theory develops as a process here because it is always in process and never static. It is not a search for truth in science but better and better theories. Feyerabend (1993) stresses the need for pluralism in science. In an attempt to achieve greater validity there is a need for *saturation* (Glaser and Strauss, 1967), which is simply the situation where hypothesis or categories generated from observation is tested repeatedly against the data. I can see also the value of keeping a record of the process of developing an analysis as a way of showing its grounding (Gale & Newfield, 1992) and the importance of allowing my analysis to be criticized by other researchers (Cobb & Whitenack, 1996).

If we treat video footage as text, then it is once again impossible to capture reality. It becomes an expression of the predispositions and creativity with which the viewer or reader approaches it (Brown, 1990). The video is open immediately to different interpretations. This means it is important to emphasise what has been excluded, being sensitive to the language being used and the predispositions of the camera man. The video footage must be viewed from different angles and a careful reflection of the process of exclusion taken. The researcher must have good knowledge of different theoretical perspectives and use this openly to interpret the video (Alvesson, 1993). They must reinforce a weak voice or compensate for an absent point of view. It is imperative they are self-critical with regard to theoretical frames of reference. In recognising the subtleties of language and the politics of the text ensures the tensions of dealing with pluralism and exclusion are dealt with sensitively. Hence multiple voices, multiple reality, pluralism and ambiguity need to be emphasised for a postmodernist

(e) Audio-recording

Even though audio recording fails to record visual data (Wragg, 1994) it does have the advantage that it is less obtrusive and easier to undertake in the classroom. As a research method it suffers similar problems to video-recording, just because there seems to be less data does not imply that any of the problems discussed in the previous section are any less reduced. It may be possible to deal with the dilemma of multiple voices and ambiguity by allowing bolder readings to be preceded or followed by alternative readings. This could be aided by representing the data in different ways. Consequently it should allow a pluralistic interpretation to develop (Alvesson & Sköldberg, 2000).

(f) Observation

Observation is where the researcher is observing in situ rather than at second hand (Patton 1990:203-5). The researcher role can be seen on a continuum as *complete participant*, to *participant-as-observer*, onto *observer-as-participant* and finally to the *complete observer* (Gold, 1958). Within my framework of the 'reflective practitioner' I am researching from the inside or conducting extra-spective research, which means I am researching my own practice from the inside. I am trying to observe my own practice whilst teaching. This will inevitably highlight linguistic actions because social interaction revolves mainly around speech. It will be a struggle to get a definite observation because of the indeterminate nature of language and the large number of possible representations. I could be more sensitive to the meaning and character of language in my interpretations (Alvesson & Sköldberg, 2000).

3.3.2: Methods used

I discuss in this section what I actually did given the reservations as a postmodernist I had with the research methods. The constraints of writing a dissertation meant that I had to decide to do something. As a reflective practitioner I see my journal as being central in identifying significant events this year because it allows significant events to be written down as an aid to re-open that experience when I came to write the brief-but-vivid-accounts-of. This is an important research method for my research question as it will allow me to record how the notion of play is changing the thinking behind my practice.

I felt that videoing and audio-recording would give more data to analyse especially when it is of a visual or auditory nature respectively and allow me to discuss those significant events further. The significant events I caught in hindsight will be marked by changes in perspective that take place after significant changes in my practice (Lyotard, 1979). I have videod and audio recorded five lessons each. The videoed lessons (7.10.05, 17.10.05, 11.11.05, 18.1.06 and 24.3.06) were chosen because they gave a chance to record data following the four mini-shifts: starting with a problem (7.10.05), generalisation (17.10.05), room layout (18.1.06) and discussion (24.3.06). I felt video-recording was valuable because it allowed me to look more carefully at the play which was occurring. It was important to be selective on what was recorded, because vast amounts of data was quickly collated but this took even longer to process and did not always illustrate the focus of my inquiry. I tried to capture significant shifts, which I sometimes caught in hindsight because shifts occurred some time unexpectedly. I tried to remain open to the data suggesting categories, hypothesis and theories. The audio-recorded lessons (14.10.05, 10.11.05, 13.12.05, 16.1.06 and 7.2.06) were chosen because I was researching the verbal play that occurs with the language games of mathematics. I recognised that my audio-recordings lost important visual clues and the sound quality was variable because the recording device tended to be fixed. (Wragg, 1994). I was able to identify the person speaking and I felt that for my research question it focused me on the speech occurring and this heighten my awareness of what was being said. This was important when I was looking at the changing dialogue within my classroom, as I tried to encourage pupils to discuss and play with the questions. A video-recording was always more difficult to set up and picked up on many things. I audio-recorded discussions, which did not heavily rely on visual imagery.

I have already proposed in section '2.12 Research Question' a series of mini-shifts that I felt would help me develop a classroom where generalisation and play, are more prevalent. The motivation behind introducing small white boards, starting with a problem, using the idea of generalisation, changing room layout and concentrating on discussions is centred on the belief that as an enactivist it is important to play the language games of mathematics (Wittgenstein, 1956) with the pupils, so giving them the opportunity to experience their world as mathematical (Fleener et al., 2004). The shifts were about creating the environment, where there was an opportunity to play the language games of mathematics. Hence the introduction to problem solving and generalisation two concepts central to what mathematics is and the use of small white boards, changing room layout and concentrating on discussion was about encouraging students to participate in these language games of mathematics.

I tried to use all of my classes, because I changed my practice with all of them. The topics described in the timetable in the next section are not as significant as the fact that it was the first time I either used some technique or hoped to notice something significant. The year 7 and 12 class are mixed ability. The year 9 class is the middle set of 5. The year 10 class is the top set of 5 and the year 11 class is a fourth set out of 5. I will use pseudonyms when I refer to pupils.

A questionnaire (Appendix 1) was then set to see how pupils perceived the changes which had occurred in my teaching. In taking a postmodernist stance there is a tension between allowing both the different voices and different positions to be represented. If I allow as many different positions to surface I feel their voices need to be limited, so I can analyse what is said. Alternatively if I allow all their voices to be heard then I need to limit the number of positions explored. A questionnaire cannot allow both plurality in positions and voices to appear, because of there nature. I have hence deliberately ignored their own interpretations and feelings about the changes in my teaching. In looking centrally at myself in this dissertation I have focussed on my position. I questioned 15 students in total all of whom I taught last year and still teach: two year 10 students and thirteen year 11 students. The questions were chosen to help me form interview questions. A few of them were set specifically to elicit behaviour changes in my students in terms of engagement (Qn.1); student control of where the lesson goes (Qn.3); frequency of asking questions (Qn.5); freedom to ask questions (Qn.5, 8 &14); motivation (Qn.16) and enjoyment (Qn.18). I also felt I could seek their views on issues raised for me by setting questions about misconceptions being tackled (Qn.4 & 17); starting with a problem (Qn.6); seeking generalisations (Qn.12); using the notion of play in the classroom (Qn.14) and the level of student discussion (Qn.9 &13).

I used closed questions on a Likert scale with four options (see Appendix 1). This is problematic because it is reducing the possibility of their voices being heard and their positions being represented. The Likert scale assumes students understand words in exactly the same way. In using only four options 'strongly disagree, disagree, agree and strongly agree' I am not allowing them to be undecided. I do not feel the Likert scale is satisfactory with my postmodern tension of the need for plurality, but it does allow me to quickly ascertain how students perceive the changes in my teaching. It pragmatically allows me to move onto an interview. This questionnaire was given to all the students I had taught last academic year 2004/05 and this academic year 2005/06. It focussed on the issues of starting with a problem; the level of discussion; the type of discussion; the amount of freedom in the lessons for pupils.

Validity in questionnaires can be improved by interviewing because they will allow the accuracy to be checked, which I did after the questionnaire to elicit points raised from the questionnaire. In comparison to interviewing; questionnaires are more reliable because they can allow anonymity, which can encourage honesty; more economical in terms of time and money when you have a small population. On the other hand in interviewing the interviewee can seek clarification and go at his or her own pace, whilst questionnaires are normally done in a hurry (Cohen, Manion & Morrison, 2000).

My research question is looking at how the notion of play is changing my practice. For me individuals exist as a social bond (Lyotard, 1979) and so the play I am interested in occurs in a classroom setting. I felt group interviewing was the most appropriate form of interviewing because an individual interview would not encourage the same level of discussion. I could see problems with group interviewing as being dominated by one person and hence suppressing individual expression. My beliefs with play see it occurring within a social setting, so I am not overtly concerned with individual expression because I do not see individuals as existing alone. Within the group interview I thought it was important to have a semi-structured interview so it allows flexibility in the ordering of questions and allows flexibility in the words used. This I felt could allow me to respond to the indeterminate nature of the discussion, I did the interview after doing a questionnaire, after three half terms, so I had time to consider what I am looking for. Hence an unstructured interview is inappropriate, whilst a structured interview assumes the researcher is aware of what he or she does not know so questions are devised to elicit the information (Lincoln & Guba, 1985). I did not feel that I was totally aware of exactly what I need to find out, because play is such an abstract concept and means different things to different people being so rigid would detract from their own feelings about play and how it has evolved or not evolved in my teaching this year.

The follow up interview (Appendix 2) was designed to further investigate the results, which came out of the questionnaire. It undoubtedly produced students' accounts-of their experiences. Once again their positions are not being explored. The voices which are heard concentrate on providing essentially accounts-of rather than allowing them to explore accounts-for. In a sense there is a dilemma, as for questionnaires, to allow a plurality in both positions and voices to appear. This is not possible, so pragmatically within this small study I have concentrated on looking at questions from my perspective. I had a group interview (3.3 Research Methods) with one out of two of the year 10 students and five out of thirteen year 11 students, which I pick randomly from set lists. A group interview was conducted with an audio-recording made of the session. It is critical to realise that individuals do not speak with a single voice, but may represent different voices; for example thoughts, feelings and evaluations. Hence within a group interview it is important to be aware how individuals are represented and excluded, but also how the different voices within individuals are represented and excluded (Alvesson & Sköldberg, 2000). Questions were chosen to highlight issues raised from the questionnaire. In terms of my research question: questions 2, 4, 6 and 7 in appendix 2 deal specifically with behaviour changes of the students in terms of control, copying, play and motivation. Question 1 deals with starting lessons with a problems and seeing if students feel it allows me to see where they are. Question 3 is eliciting the fact that as I encourage discussion are problems being dealt with an issue for me in developing generalisation and play. In question 5, I am seeing if I still control the lesson far too much that I am discouraging them playing.

3.4 Research timetable

In this section I outline a record of events for my research. I conducted this research within a selective school with pupils whose ability for their age ranges is in the top fiftieth percentile:

- 7.9.05: Started journal today recording significant events, which occur on each school day.
- 9.9.05: Introduced 'show me' boards to Year 9 first as soon as possible, as it is a small group of 18.
- 12.9.05: Rolled out use of 'show me' boards to other years.
- 7.10.05: Started video-recording lessons with a visual component. I started with year 9 on starting with a problem, which will represent my second shift.
- 14.10.05: Audio-recorded a Year 7 discussion lesson on generalisation. A class size of 23, involved in looking at one particular question. I felt it was an ideal opportunity to concentrate on the dialogue.
- 17.10.05: Video-recorded a year 11 lesson on generalisation, as this lesson gave an ideal opportunity to record the visual aspects of students working on algebraic fractions.

- 10.11.05: Audio-recorded a year 10 discussion on fraction multiplication and division. I recorded a serious discussion.
- 11.11.05: Video-recorded a year 12 lesson investigating graph transformations; a very visual lesson where the recording captured their sketches.
- 13.12.05: Audio-recorded a year 7 lesson on solving equations.
- 16.1.06: Audio-recorded a year 11 lesson on limits of accuracy
- 18.1.06: Video-recorded a year 11 on a major shift in room layout. I saw that they had the confidence to make this shift successful.
- 7.2.06: Audio-recorded a year 7 lesson on the area of a triangle by investigation.
- 20.3.06: Set questionnaire (Appendix 1) to all the students I had taught last academic year 2004/05 and this academic year 2005/06.
- 24.3.06: Videod a Year 9 lesson on introducing Pythagoras in circle time. This lesson brought all the shifts together.
- 27.3.06: Followed up questionnaire with an interview (Appendix 2).

3.5 Analysis

This section is a reflection on what I did with my research data and how I analysed it to inform me of what to do next and what to present as results. My journal was central to me identifying significant events. Lyotard (1979) sees the significant event as the change in perspective, which occurs after a significant occurrence. I had already decided some potential significant occurrences when developing my research question (Section 2.12), so once the journal was finished I retro-spectively looked back at it seeking significant events which showed me a dynamic change over time. These were heavily influenced by the significant changes I was making to my practice and for each change there were many events, which I could have potentially used but in my mind they were not as vivid for me as the events I chose. I also conscientiously decided for the sake of succinctness to choose just one event, which was representative of the significant changes occurring at that time. There were other events that I have not reported on which highlight other significances, but these I did not feel added anything to my research focus on play. I decided to group significant events around themes, which were significant to me and showed a dynamic change over time. The themes came dually from the changes I was instigating within the lessons (Section 2.12) and similarities I was noticing in journal entries over a certain time period. I decided to look at them chronologically looking at my journal and thinking about ideas, which I had before the episode: *pre-incident*. The *incident*, which dealt with describing the significant episode. This was in turn broken down into *accounts-of* and *accounts-for* and finally *post-incident*, which reflected on how my thoughts had changed since the episode. This structure helped to identify why certain episodes were significant. They caused an emotional reaction, which can be explained to some extent by looking at the reasons behind what was happening around these significant episodes. The structure of reporting these episodes helped to identify these reasons.

Analysis of the videos and the audio-recordings did not use *'grounded theory'* (Glaser and Strauss, 1967), but relied on picking up on significances in the journal and looking at either video or an audio-recording to explore this significance more. I tried to remain open to what both mediums provided. They essentially allowed my episodes to be more vivid and helped me to re-enter events more clearly.

The questionnaire was devised after my analysis of the journal. I decided to code it by assigning a value of –2 for a strongly disagree response, -1 for a disagree response, +1 for an agree response and +2 for a strongly agree response. A non-response or writing 'same' as a response scored 0. I know that this valuation is a very crude measure and as a postmodernist this assigning of values is unsatisfactory, but gave me an indication of what my students think by looking at the total score for each question. The quantification of words is problematic because one language game 'words' is incommensurable with the other language game 'numbers', but it does allow an alternative presentation of the phenomena (Alvesson & Sköldberg, 2000). The tension is essentially quantification enables conclusions to be drawn more easily, but the assigning of a numerical scale is making a value judgement about words. I cannot see how this tension can be avoided even looking at the results of the questionnaire without assigning numerical values will involve the assigning of some value system, which much be made transparent. As a postmodernist this process of analysing the questionnaire will always be problematic because it means a metanarrative is chosen over other metanarratives (Lyotard, 1979). Once I collated the total scores I looked at the questions, which showed a mixture of agree and disagree responses. These had total scores closer to 0. I then thought about my significant episodes and tried to devise questions which highlighted both their contrasting opinions (Appendix 1: Questions: 2-4,12, 14,15 and 17) and issues raised for me in my episodes: copying, enjoyment, games, misconceptions, starting with a problem, motivation, discussion and control (Appendix 2).

In analysing the interview I first created a transcript, so it was easier to get hold of everything that was said. I looked at their responses having been already aware of significant issues, which arose from my analysis of episodes. I used comments, which either supported my findings or contrasted them.

3.6 Summary

I have looked at a variety of research methods and reflected on their relevance in relation to my research question and methodology. I have justified my use of video and audio recording to pick up on behaviour changes in my students and allow me to consider issues that are raised for me. A journal was kept to record 'reflecting-on-action' (Jaworski, 1994) and a relevant questionnaire and a group interview was done to assess what changes have occurred. I have made explicit my research timetable and discussed my analysis techniques.

4. Results

4.0 Introduction

In this chapter I will firstly explore five significant incidents I noticed in the academic year 2005/06. This will develop a story of how the notion of play has impacted on my teaching. I will present them in a diary format in chronological order breaking each incident down into pre-incident thoughts. Ideas I had before the incident about that aspect of teaching. Incident, which deals with a particular significant episode which occurred. This is itself further broken down into account of and account for and post-incident analysis, which reflects on how my thoughts have changed since the episode. Secondly I will explore what the questionnaires and the interview revealed.
Throughout this chapter I will highlight in italics the research data and state the source or research method used.

4.1: Episode 1

Pre-incident:

I have felt for a long time that my teaching style was predominantly me standing at the front and lecturing pupils with question and answer sessions dispersed evenly throughout the lecture, to make sure they understood what was going on, but this was limited to a few that I choose to seek answers from. This style I have felt has not been particularly effective because when routine questions were set many in the class would struggle to get started. I feel this is because their own ideas were not being accessed, developed or even examined. I assumed just by talking they would gain understanding. In hindsight I had a 'transmission model' of teaching and realised by August 2005 that I needed to access student ideas and I decided to explore the use of individual whiteboards to help me achieve this.

Incident: Use of whiteboards

Account of:

On 9.9.05 I decided to use *individual whiteboards or 'show-me' boards for the first time with Year 9 (Set 3 out of 5) on the topic of indices (Journal: 9.9.05).* The classroom was in a typical forward facing desk organisation. All the children had a board and pen and after a brief discussion with the class about the fact we seem to add the powers when we times expressions written in index form together. I asked a series of questions related to multiplying and dividing indices. Students seem to enjoy writing on these boards and they were encouraged to have a go or leave a question mark. I picked on students who had got it wrong or left their boards blank to try and explain what is going wrong for them. Others were encouraged to offer solutions. By 14.9.05 with *Year 10* I used the whiteboards for the first time to develop a metaphor on *expanding brackets (Journal: 14.9.05).* I started with the familiar grid method of calculating $x(x+2)$ by showing the following grid and explaining that x and 2 are the dimensions of the rectangles and I stressed the fact you can work out the area and this expands the brackets:

	x	2
x		

I filled in the rectangles with x^2 *and* $2x$ *respectively to show how the area metaphor works, they were then asked to explore* $(x+1)(x+2)$ *using this metaphor. Almost all drew: (Journal 14.9.05)*

	x	2
x		
1		

And they filled in the grid correctly and could see the need to simplify expressions. They were confident in working out and simplifying $(x+3)(x+4)$ and *appreciated the geometrical significance of the grids having to be the same length if they are x by x (Journal 14.9.05)*, not drawn accurately above. $(x-1)(x+2)$ was also dealt with successfully, but many queried the concept of a negative length. I explained that mathematical metaphors work in a world of their own. *Some students were not convinced by this (Journal 14.9.05)* but like on my initial use they very much enjoyed the novelty of working on small white boards. On the 16.9.05 I used *the grid method in reverse to show how to factorise* $2x^2 - x - 6$ *with the Lower sixth as shown below (Journal: 16.9.05):*

	?	?
?	$2x^2$	
?		-6

They seem to like the idea that they could play with the values of the question marks to find the correct answer.

Account for:

This initial foray into using small whiteboards was a significant shift for me because I begun to realise with the initial use with Year 9 that I could for the *first time access every one's answer as they all had to write something down or at least try to (Journal: 9.9.05).* In the past I have tended to ask students for their answers and these have tended to be students with their hands up. I feel it has opened up a new world of possibilities for me because I now have a multitude of ways for the lesson to progress. The control has shifted towards them in terms of the lessons are not following a pre-determined lesson plan. I still see this exchange with year 9 as a typical question and answer session very much training behaviour (Mason, 2002), but with year 10 I offered them a chance to *play with the metaphor offered (Journal 14.9.05)*, to suggest a way to tackle expanding two brackets which only highlights Holton's (2001) view that mathematical play should be a solver-centred activity with the solver in charge; uses the solver's current knowledge and develops links between the solver's current schemata. The year 10 lesson was poignant because it started really from a problem: expanding two brackets and this I feel gives an opportunity to initiate mathematical activity by starting with a question (Halmos, 1994). Personally, I now realise mathematics is about problem solving (Halmos, 1975) and not about learning techniques, which only encourages behaviour to be trained. Setting problems and discovering what students make of them is important because, I feel, it gives you access to their ideas and possibly a view of their schema (Sfard, 1991), which informs your teaching.

Post-incident:

After these incidents I felt excited by the fact that everyone is involved. I noticed from the *video (Video: 7.10.05)* of the Year 9 lesson that using the whiteboards slowed down the pace of the lesson, so now typically there is pupil-teacher interaction for a full 50 minutes, compared to older videos I have of my teaching which generally start with long expositions, which move quite rapidly. The average exposition being 20 minutes. This is because the old videos show me following the keenest in the class. The whiteboards allow assessment to occur more frequently and allow the natural powers students have to be made public and exploited by me (Mason, 2004). It still raises the question what the purpose is of the whiteboard. It could be used to do exercise type questions like in the indices lesson above (9.9.05) or be used to allow a metaphor to be developed like in expanding brackets lesson above (14.9.05). In a latter lesson recorded on *audio-tape (Audio: 7.2.06)* on area of a triangle (7.2.06), Calliope said *'Sir, can we move on. This is* *all the same'*. This highlights the important point that students who grasp the concept being taught may get bored easily if the level of the questions do not get more difficult. There is clearly a tension between the individual and the class. The more I focus on concentrating on helping one person, the less attention I pay to the overall progress of the class. The more I concentrate on the class's progress, the less attention I pay to individual problems. As an enactivist it is important to reach a balance between the individual and the class, because the self is always changing influenced by their environment.

4.2: Episode 2

Pre-incident:

I can see that I am accessing student ideas, but only ones that I have put in their heads because of my teaching. There must be a way to get their ideas without me teaching anything, which I feel can only happen by setting a problem.
Halmos (1980) and Polya (1962) have famously linked mathematical thinking to problem solving. I feel it would be worthwhile starting with a problem rather than teaching a technique to gauge where the pupils are and build up from their knowledge. This agrees with my enactivist position because all knowing is about doing. Hence by setting a problem students will start doing something, but also require students to accommodate within their existing schema. The problem will have to be set at the right level with the right amount of novelty, too difficult and the trust and the attention of the pupils may be lost.

Incident: Starting with a problem

Account of:

With my Year 9's on the 7.10.05. *I started the lesson by just writing (Journal: 7.10.05):*

$$\begin{array}{ll} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ 10^1 & 10 \\ 10^2 & 100 \\ \cdot & \cdot \\ \cdot & \cdot \end{array}$$

I asked them to complete the pattern above and below. *They managed to continue the pattern vertically in both columns forward and backwards (Journal: 7.10.05)*, which I felt was as a good way to introduce standard form. *I noticed they could see the need to multiply and divide by 10 in both columns. They managed to complete the powers of 10:* $10^2, 10^1, 10^0, 10^{-1}, 10^{-2}$ *some did struggle to get* 10^0. *I pushed the discussion onto discussing how 0.02 is '2 lots of 0.01' and suggested* 2×10^{-2} *as an appropriate form to write 0.02, which we call standard form. I asked them to attempt 0.022 and some suggested* 22×10^{-3}, *which is clearly a form but not the standard form. I was really struggling to convince all of them that it is* 2.2×10^{-2} *(Journal: 7.10.05).* I resorted to talking about moving the decimal place twice to turn 0.022 into a number between 1 and 10 and discussing the fact that to reverse this we have to $\times 0.01$ or $\times 10^{-2}$. *They were more successfully with 6,000 (Journal: 7.10.05).*

On 31.10.05 I used a similar approach with Year 10 in drawing on their ideas. I started with these list of expressions and I put question marks down as tasks *they had to evaluate.* $\times$? *(Journal: 31.10.05),* referred to the multiplying factor between answers on the left. Ordinary question marks (?) referred to a single number, which was the answer to the indices evaluated. *Most realised '$\times$? ' was 64.*

$64^1 = 64$

$\times ?$

$64^2 = ?$

$\times ?$

$64^3 = ?$

$\times ?$

$64^4 = ?$

I then put up a series of powers of two:

$2^{\frac{0}{1}} = ?$

$\times ?$

$2^{\frac{1}{1}} = ?$

Most got this pair. I developed this to look at:

$$4^{\frac{0}{2}} = 1$$

$$\times ?$$

$$4^{\frac{1}{2}} = ?$$

$$\times ?$$

$$4^{\frac{2}{2}} = 4$$

and then onto:

$$8^{\frac{0}{3}} = 1$$

$$\times ?$$

$$8^{\frac{1}{3}} = ?$$

$$\times ?$$

$$8^{\frac{2}{3}} = ?$$

$$\times ?$$

$$8^{\frac{3}{3}} = 8$$

I then created:

$$16^{\frac{0}{4}} = 1$$

$$\times ?$$

$$16^{\frac{1}{4}} = ?$$

$$\times ?$$

$16^{\frac{2}{4}} = ?$

$\times ?$

$16^{\frac{3}{4}} = ?$

$\times ?$

$16^{\frac{4}{4}} = 16$ *(Journal: 31.10.05)*

I introduced the concept of square root and cube root being powers of a half and a third respectively (Journal: 31.10.05) and hence introduced an algebraic way of solving these problems using $x^{\frac{m}{n}} = (\sqrt[n]{x})^m$ as a definition. *They easily coped with answering* $25^{\frac{3}{2}}$, $64^{\frac{2}{3}}$ and $125^{\frac{1}{3}}$ *using the earlier method rather than the algebraic approach I stressed (Journal: 31.10.05).*

Account for:

The significance for me of the above is that I started with an incomplete pattern and asked them to figure out the rest of the pattern, which for me was starting the lesson with a 'problem'. It is student centred and for the first time in my teaching career I am not telling them how to do something. I suspect the problem with 0.022 stems from them seeing it as 22 lots of 0.001, which only follows my argument that *0.02 is 2 lots of 0.01 (Journal: 7.10.05).* Hence they write 2×10^{-2} for 0.02 and 22×10^{-3} for 0.022. In hindsight this makes me feel that the method offered needed to be expanded to include a look at the variety of ways 0.022 could be represented for example:

$0.022\times10^{0}, 0.22\times10^{-1}, 2.2\times10^{-2}$.

This could then possibly offer a way to bring in the standard form. I can now see there was opportunity to play with this question, by looking at the variety of ways of playing with the powers of 10 used, which was lost. In struggling to get the concept across *I revert to an exposition of a standard method I have taught in the past (Journal: 7.10.05)*, which I feel is training behaviour rather than educating awareness (Mason, 2002). There is something arbitrary (Hewitt, 1999) about learning to write numbers in standard form, but this need not be explained until students are comfortable with writing numbers in a variety of powers of 10. I can now see that I should not be surprised that 6,000 was written comfortably in standard form, but if I had set 6600 or 6660 or even 6666 I may have encountered similar problems to 0.022. The prevalence of non-zero numbers possibly being an issue for them.

The year 10 lesson on fractional indices starts with me stressing a constant multiplying factor between statements and then setting a series of questions to explore this link. I counted *22 out of 24 regularly getting the answers right (Journal: 31.10.05)*, using the metaphor developed. In contrast the Year 9 lesson on 7.10.05 on standard form showed only *8 out of 18 could write 0.022 (Journal: 7.10.05).* It could be possible that in this Year 10 lesson by directing students noticing to the key points in the pattern, which is this constant multiplying factor between indices questions that their understanding remains context understood because the more algebraic method failed to be used.

Post-incident:

I have tried to start with a problem, but have now come to realise that problems in mathematics can come in a variety of forms and not necessarily ones which have one answer, for example solving algebraic problems. But also discovering patterns, creating formulae, in reality wherever there is mathematical thinking present there is problem solving occurring (Polya, 1962). I am beginning to see mathematical thinking as any application of logical thinking to a problem. What the problem is depends on what area you are working on. It could be a murder investigation, but there is still mathematical thinking occurring.

4.3: Episode 3

Pre-incident:

I feel it is natural to see starting with a problem as extending into seeking generality or using generality to find a solution (Mason, 1986). I have already discussed its advantages in developing schema (Tall, 1991) in section 2.10. Seeing the general through the particular and seeing the particular in the general (Mason, 1996) seems an ideal way to get students to understand a problem because as Polya (1957) pointed out if there is a difficult problem you can't solve then there is an easier problem within the most difficult one which you can solve. By generalising the easier problem you can solve the harder one. This also works in reverse that you should be able to see the solution of the simpler problem within the general problem.

Incident: Generalisations

Account of:

By the 14.10.05 I was confidently using whiteboards with a Year 7 class when as the initial starter I decided to try and develop a lesson which introduced the concept of generality especially the issue of an algebraic letter representing generalised numbers. I initially posed the problem *simplify* $x + x = ?$ *(Journal: 14.10.05)* to gauge their knowledge. I counted *20 out of 23 had written 2x (Journal: 14.10.05)*. I asked them why they had done this and Clio responded *'one sweet plus one sweet is two sweets' (Audio: 14.10.05)*. Initially I felt that the students were treating the letters as physical objects rather than measures of physical objects, so I immediately asked them to *work out (Journal: 14.10.05)* the questions below, in an attempt to confront this view:

$1 + 1 =$
$2 + 2 =$
$3 + 3 =$
$4 + 4 =$
$0.5 + 0.5 =$
$x + x =$

(Journal: 14.10.05)

only *4 out of 23 when questioned* as a class *said they noticed the connection between the numerical examples and the algebraic question (Journal: 14.10.05)*. All when asked said *they noticed the answers go up in 2, for the first four questions (Audio: 14.10.05)*. Clearly the 0.5 question is not for all of them making them notice something different. I tried a *similar simplification with* $5x - 2x =$ *, all got 3x, but they struggled initially to come up with numerical examples, to illustrate* $5x - 2x = 3x$ *(Journal: 14.10.05)*: particulars to illustrate this generality (Mason, 2002) until I stressed it means *'5 times something minus 2 times something....' (Audio and Journal: 14.10.05)* and then someone introduced the sweet metaphor again *'5 sweets – 2 sweets = 3 sweets' (Audio and Journal: 14.10.05)*, which I tried to explain was a poor metaphor because I noticed 'x' as a object is beginning for some of them to take a physical existence.

By the 17.10.05 with Year 11 I tried a similar approach as I did with year 7 on the 14.10.05, where I first start with an algebraic expression to be simplified and then devised numerical examples or particulars to get the students to understand the generality. I started with a problem $\frac{2}{x} + \frac{3}{x+1}$ *(Video and Journal: 17.10.05), 15 out of 16 struggled to come up with a common denominator (Video: 17.10.05)* even though they *understood how to combine* $\frac{1}{x} + \frac{1}{2x}$ *from a previous lesson (Journal: 12.10.05).*

In trying to allow them to see the general through the particular, I put up a series of examples related to the original question for them to have a go on their whiteboards:

$$\frac{2}{5}+\frac{3}{6}=\frac{?}{?}+\frac{?}{?}$$
$$\frac{2}{6}+\frac{3}{7}=\frac{?}{?}+\frac{?}{?}$$
$$\frac{2}{7}+\frac{3}{8}=\frac{?}{?}+\frac{?}{?}$$
$$\frac{2}{x}+\frac{3}{x+1}=\frac{?}{?}+\frac{?}{?}$$

(*Video: 17.10.05)*

All of them correctly could do the first three questions, but struggled with the last one (Video: 17.10.05). I could see they needed some hint, so I wrote on the board:

$$\frac{2}{5}+\frac{3}{6}=\frac{?}{5\times 6}+\frac{?}{5\times 6}$$
$$\frac{2}{6}+\frac{3}{7}=\frac{?}{6\times 7}+\frac{?}{6\times 7}$$
$$\frac{2}{7}+\frac{3}{8}=\frac{?}{7\times 8}+\frac{?}{7\times 8}$$
$$\frac{2}{15}+\frac{3}{16}=\frac{?}{15\times 16}+\frac{?}{15\times 16}$$
$$\frac{2}{115}+\frac{3}{116}=\frac{?}{115\times 116}+\frac{?}{115\times 116}$$
$$\frac{2}{x}+\frac{3}{x+1}=\frac{?}{?}+\frac{?}{?}$$

(Video: 17.10.05)

Analysis of the *video showed 12 out of 16 pupil whiteboards felt that* $x\times x+1$ *is the common denominator. In the ensuing discussion within the class many felt writing it as* $x(x+1)$ *did not feel natural to them. 13 out of 16 when questioned felt the brackets needed to be expanded (Video: 17.10.05).*

Account for:

It is apparent to me now that I am focusing on the general immediately, which previously in my teaching I had not even considered. The way in which generality is introduced in these lessons I feel is confusing because it does not go from the particulars to the generals. The first part of the generality was thrown at them and then when this generally failed, particulars where explored. For example with the year 7 class I needed to introduce examples like: $-1+-1=?$, $0.28+0.28=?$ and hence show a variety of numbers in no particular order. I should not be surprised that my examples:

$1+1=$
$2+2=$
$3+3=$
$4+4=$
$0.5+0.5=$
$x+x=$

(Journal: 14.10.05)

produced the response that the answers go up by 2, because they seem to be looking at the problem vertically in terms of the connection amongst the answers. A random selection of examples with no connection between them using negatives and decimals may get them to focus on the fact that $x+x=2x$ means add anything to itself and you make two lots of it. In coming up with examples to illustrate $5x-2x=3x$, I can now see that maybe from their responses in the lesson that the individual expressions 5x, 2x and 3x, were perceived as physical objects like sweets . I suspect they do not even see the process $5 \times x$ for $5x$, let alone the process-product duality demanded by Sfard (1991) for understanding. Hence they struggled as a whole to devise numerical examples to illustrate the generality, which highlights the fact that they did not understand the concept of ax, where a is a constant. The Year 11 lesson showed similar failings because it started with a problem too difficult for the pupils because they seemed to struggle to see the local pattern when one denominator is one more than the other. That had to be made explicit for them and only then could they begin to see what the denominator could be. I think the generality would have been clearer if examples like these were offered:

$$\frac{2}{5}+\frac{3}{x}=$$
$$\frac{2}{5}+\frac{3}{x+1}=$$

after offering:

$$\frac{2}{5}+\frac{3}{6}=\frac{?}{5\times 6}+\frac{?}{5\times 6}$$
$$\frac{2}{6}+\frac{3}{7}=\frac{?}{6\times 7}+\frac{?}{6\times 7}$$
$$\frac{2}{7}+\frac{3}{8}=\frac{?}{7\times 8}+\frac{?}{7\times 8}$$
$$\frac{2}{15}+\frac{3}{16}=\frac{?}{15\times 16}+\frac{?}{15\times 16}$$
$$\frac{2}{115}+\frac{3}{116}=\frac{?}{115\times 116}+\frac{?}{115\times 116}$$

(Video: 17.10.05)

because there is a gradual move to algebra or generalisation.

I have started in these lessons with the generality, seen what they can produce and then pushed pupils to see particular examples. There does seem too much emphasis on seeing the particular in the general, looking for numerical examples to illustrate the general case. I should have allowed pupils to see the general through the particular, allowing pupils to see and make up specific numbers as illustrative of the general case. This would have allowed concepts to develop from student ideas (Sfard, 2003).

Post-incident:

I can see that I am doing more of the playing because I am the one who is doing all the thinking of questions, figuring out what they write and trying to reason what they do. Whitton (1998) sees this as meaningful play, but I need to encourage my students to get more involved and construct their own examples with a view to eventually constructing generic and extreme examples (Watson & Mason, 2005). Jarrell (1998) offers arranging the physical environment to encourage this high level thinking.

4.4: Episode 4

Pre-incident:

The set up of the room with desks facing forward and in rows limits their ability to engage with each other and set the pace and direction of the lesson. This is something I could change to encourage higher level thinking (Jarrell, 1998).

Incident: Room Layout

Account of:

On Friday the *13.1.06 I decided to arrange the desks in a horseshoe shape with the shoe pointing towards the large fixed whiteboard (Journal: 13.1.06).*

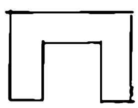

Figure 1: Drawing of a horseshoe shape

I handed out the white boards or 'show me' boards, pens and rubbers one to each student. *I introduced a simple linear equation* '$2x+1=3$' *on the main board and asked pupils to work on it and show their answers to each other. I attempted the questions myself. I played with the question by changing the numerical values to develop steadily more difficult questions for them to have a go at:*

'$2x+1=4$', '$2x-3=4$' *and* '$3x-3=4$' . *Students who sat in the same line initially worked well but became restless because they couldn't see all of each other's solutions. In the ensuing discussion, students who were stuck were encouraged to pick students whose solutions they could see were correct by comparing it to mine and realising this* *solution was also common with some members of the class. The stuck student would then ask his chosen pupil how he answered it. I did encourage stuck students to evaluate their own answers and ask questions more specifically about the method used (Journal: 13.1.06).*

By the following Monday 16.1.06 I got my Year 10's to put their desks in a diamond shape with a gap facing the main whiteboard (Journal: 16.1.06).

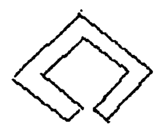

Figure 2: Drawing of a diamond shape

The room became very cramped and it was an oblong diamond that was created. This did allow more of the pupils to see each other.

I explained to a member of the OPAL (Dyslexic and Dyspraxic) department my problem with arranging desks in a way that every student could see each other. I asked if we had a circular table in the school on the 16.1.06. He said *'we don't but why don't you put the kids in a square' (Journal: 16.1.06).* I pointed out *'they couldn't see each other when using the whiteboards' (Journal: 16.1.06). He suggested a circle of chairs, which with the whiteboards seemed a good idea (Journal: 16.1.06).*

The following day 17.1.06 I asked the Year 9's to move the desks in the middle to the wall, which traditionally in the mathematics department face forward in pairs of three columns and four rows. *I then asked them to put the chairs in a circle in the middle, without a desk, with a chair for me (Journal: 17.1.06).* I distributed whiteboards, pens and rubbers for everyone including myself and sat in the circle. I conducted the lesson using my small whiteboard to introduce mixed fraction questions involving the four operations. The pupils remained attentive, well behaved for the next 75 minutes. *Many misconceptions were explored because we all had a go at the question (Journal: 17.1.06).* The pupils actively asked each other to justify their answers and occasionally asked me. I was conscious that I tended to exhibit the most 'play' like behaviour because I posed more problems for the group to have a go at. Some pupils did offer some questions, which I occasionally vetoed because of the perceived difficulty.

On the 18.1.06 the round circle approach was used for the first time with Year 11 on limits of accuracy. *They had struggled initially to settle down before the circle was created. In the circle they mainly sat next to their friends and when a discussion was begun about a question students tended to call out, which Polymnia pointed out led to important points being missed and a few students; Melpomene and Mnemosyne were just discussing the work amongst themselves. Zeus suggested calling out was good or otherwise spontaneity is lost (Video: 18.1.06).*

Account for:

I was aware of a significant shift in my teaching in the initial two day period. Physically you can see the room starts off with the desks all facing forwards and an implication that the teacher is at the front. The teacher is the only person in the room who can communicate to everyone, verbally and visually. By the 13.1.06 you could see in the horseshoe that each student could communicate to two-thirds of the class, but still the teacher is the only one who can communicate to everyone. The diamond shape allows the main board to be brought into play and allows each pupil to see three-quarters of the class.

Two issues are raised for me with these lessons. Firstly students are being empowered to become teachers to the class and secondly there is far more sharing of ideas and problems. Hence more opportunities for discussion.

The circle (17.1.06) with me part of it and active in it allowed everyone in the room to communicate verbally, which was always happening and most importantly now visually with the boards. For the first time in my teaching career everybody had face-to-face communication with everyone else. I feel strongly that with the older pupils they are just as confident in posing questions. All years are keen to have a go at explaining questions, which makes them think twice. Firstly when doing an initial problem and secondly when trying to come up with an explanation to help the pupils who are struggling. Research by Lown (2002) has shown circle time taps into the following psychological processes: self-esteem, language and interactive skills, intellectual development, social awareness, cooperation, group functioning, problem-solving, decision-making and motor development. These attributes have been taken from research where the activity is primarily trying to develop children's personal, social and emotional skills but there are similarities with features highlighted by 'play researchers'. The evidence-based research to back up the effectiveness of circle time is limited (Kelly, 1999). It is difficult to see what is creating the perceived enthusiasm for circle time; is it my personality, the activity, the structure or something else. One of the key features of circle time is that it encourages socially mediated learning (Vygotsky, 1962), which as an Enactivist I am trying to encourage.

The circle time seems to create a tension in *who is playing, in terms of devising questions (Journal: 13.1.06)*. The more I dictate what goes on in the circle, in terms of (a) putting their hands up, if they want to make a contribution; (b) never talking when another person is talking and (c) interjecting when I feel it is necessary and posing most of the problems in a 'play' like way seems to reduce their opportunities to play or devise questions. They do occasionally suggest modifications. I am very much a 'control freak' and I am worried that giving more control to them will lead them to *play with the 'classroom management' issue (Journal: 16.1.06)* and their problems posed will be too difficult and lead to chaos and little learning as the louder members of the group will probably dominate to the detriment of the quieter students.

My experience would show that this is likely. I feel some ground rules are required for circle time with older pupils, which pupils work with me to devise to give them some say in what is going on. I suspect they will want to keep the veto to call out when they feel they have an important point to make. The physical shift in the room is mirroring the shift in who plays the role of the teacher. In circle time *everyone is a teacher and everyone is a pupil (Journal: 13.1.06)*. I have suggested *pupils see me as the expert (Journal: 13.1.06)* who can make mistakes. It has pushed who controls what happens in the room to everyone because I only interject when I feel one pupil does not understand and make the others explain or if we need to move a discussion on.

Circle time is pushing me to re-examine my role in the classroom, which I envisage as creator of some activity or problem and the policing of 'circle time' rules. Students must take on greater responsibility and feel confident to play with the material being offered. I need to sit back more and allow them to play. I feel circle time creates an issue on who does the most effective playing the students or me. It seems as a teacher I must put restrictions on the play to ensure the relevant curriculum is covered.

Post-incident:

I can see how circle-time is supporting pupils who struggle to solve the problem within the classroom, because the discussion focuses on their problems. In consequence there will be a lack of differentiation for the pupils who can solve the problem. My videos showed them bored at times, because they could easily do it. One particular thing I have noticed in my journal writing since the 17.1.06 with year 7, in particular, is pupils looking at other boards before writing down their answers or changing their answer once they realise what the common answer was. I could see some of them had realised their errors, from their facial expressions, but it was difficult to catch the ones who were copying. There is an issue here of honesty.

4.5: Episode 5

Pre-incident:

I cannot see how I am going to stop copying, except by encouraging meaningful discussion I may educate the awareness of copiers. Discussion is seen as a natural outcome of circle time because it allows children to experiment with new ideas (Housego & Burns, 1994).

Incident: Discussion

Account of:

I am going to give an account of two incidents because I feel they are related.

Significantly on the 13.12.05 I allowed Clio, Year 7, to come to the board to explain to the class how to write algebraically *'13 equals 4 plus 3 times a number' (Audio: 13.12.05)* and to solve it:

Clio: ' I am going to write down the question Mr Uppal told us'

He writes down $13 = 4 + 3 \times \mathrm{a}$ on the main whiteboard.

Clio*: 'Ok, any problems with this or questions?'*

Thalia: 'You put three times 'a', why don't you put down 3a or 3x'

Clio: 'No, But this is the question'

Pandora: 'Where did the 'a' come from?'

Clio*: 'But this is the question, thirteen equals four plus three times a number'*

Pandora: 'Yeh, but it could be any number'

Thalia: 'You could choose any number'

Calliope: 'It's a question mark, a particular number look what happens when you solve it'

Thalia: 'Oh, yeh...'

(Audio: 13.12.05)

On the 16.1.06 with Year 11, Euterpe found it difficult to understand that the maximum of $\ell - w$, when ℓ is 25cm to the nearest centimetre and w is 10 cm to the nearest centimetre.

I started the lesson by setting this problem, on the small whiteboards:

Teacher: 'Terpsichore, why did you get 16cm ?'

Terpsichore: ' I took the largest that ℓ could be, which is 25.5cm from the smallest that w could be, which is 9.5 cm'

Euterpe: 'Why is it not 25.5cm minus 10.5cm ?, subtract the largest from the largest '

Polymnia: 'Look at my board, here is a number line. Here is 10 and here is 25. , Where can 10 be to the nearest centimetre?

Draws:

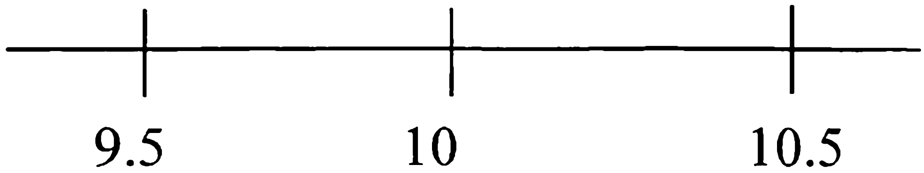

Figure 3: Polymnia's drawing of a number line

Euterpe: 'Ah, between 9.5 and 10.5'

Polymnia: 'What about 25?'

Euterpe: ' 24.5.25.5, that's easy!'

Teacher: 'Now Euterpe. *Imagine a line, which connects any point within 10cm, to any point in 25cm. What is the longest line?'*

Euterpe: 'Ugh!'

Melpomene: ' If I pick the points further apart at 9.5 and 25.5 you must get the maximum, look at my board Euterpe'

Melpomene draws lines to connect points from one region to the other, to illustrate what he means.

Euterpe: 'Oh, yeh'

(Audio: 16.1.06)

Account for:

The year 7 lesson extract above shows well the scientific debate encouraged by Legrand (1995). Pandora proposes the conjecture that 'a' could be any number, which Thalia also formulates, but Calliope challenges with her ability to see that if you solve it you only get one solution. It represents an unknown, hence I suspect her use of a question mark. Research would show students often confuse the different usages of letters, sometimes they represent unknowns in mathematics, but sometimes they represent variables (Kűchemann, 1981) . The year 11 lesson has a discussion, which also involves me. This is an example of a positive-driven discussion (Michaels & Sohmer, 1999) where students are taking up a position that the answer is 16cm and trying to find evidence to support their conclusion. Polymnia clearly had the strongest position because his idea eventually led to Euterpe understanding the question. Other contributions by me and Melpomene illustrate that we are taking his idea to explain it to others. It is interesting to see that in both these discussions only a maximum of 4 or 5 people are involved. These individuals feel they are gaining something from the discussion, which they have reported in the questionnaire (4.7 Questionnaires) and following discussion, but others do sit quietly.

Post-incident:

The evolution of circle time and discussion have really occurred simultaneously. Circle time encourages discussion. Discussion encourages circle time. This does not surprise me given the fact that the best way to have a discussion is to face each other.

4.6 Summary of episodes

These five episodes represent a significant change in my practice. The use of whiteboards opens up the network of 'selves' in the classroom, so as an enactivist by getting them to use the whiteboards I am ensuring they are doing something, other than listening, which I can never verify is actually happening. Starting with a problem, my second significant shift, has arisen to access student ideas, to set a baseline from which the lesson can progress. This is significant for me, because I am now starting from where the pupils are, which in hindsight must be important to ensure effective learning. It has also changed my perspective of mathematics from being about learning techniques and applying them, to being centrally about solving problems, which Holton (2001) has argued encourages play. Educationally it develops links between the students' current schema and by reflecting on action (Sfard, 1991), reflecting on the activities of the mind, new schema can be developed.

It has become apparent to me setting problems does not in itself encourage schema development. Problems must be developed to encourage students to discover patterns, investigate changes in structure and develop formulae, rather than just solving an isolated problem. Any problem must be seen in the context of other problems. Lakatos's (1976) and Wheeler's (1982) model of mathematical thinking encourages play and schema development because there is continual change in conjecture or mathematical model development. Generalisation is a good term to describe this process, which is continually sought in mathematics. Any general result in higher mathematics may not hold for a different mathematical world. Results are seen to hold for specific mathematical worlds.

The shift in my view of mathematics, which occurred because of looking at generalisation highlights the need to concentrate on seeing the general through the particular: the natural way that generalisations develop rather than the other way round. Professionally I need to put more thought into question setting, so generalisations are constructed which develop the appropriate schema. A lesson plan for me now is about developing a sequence of questions or problems to help form the appropriate schema rather than a proof and a set of worked examples.

The last two shifts really occurred in parallel: changing room layout and increasing the level of discussion has led to circle time. Significantly, previously I was doing most of the 'playing', but now students have an input as they ask each other questions and begin to set questions. Discussions have been mainly positive-driven discussions (Michaels & Sohmer, 1999) in format, rather than in a scientific debating format (Legrand, 1995). Personally the Lakatos (1976) model of debate as I see in Legrand (1995) has been difficult to develop because I have concentrated on using discussion to teach particular mathematical topics, rather than extending it to go further to seek generalities in these topics. The Wheeler (1982) model was never really developed because no real problems were introduced to be modelled.

Overall, I see how the shifts have allowed the 'selves' to come out, so the network of relationships between the 'selves' and the mathematics can be seen. This has affected who does the playing, because I empower the students and make the lesson more explicit in terms of using whiteboards and discussion. Students, now, are doing more playing. My view of mathematics has changed from a static collection of theorems to more dynamic ever changing theorems, which mirrors the way the shifts have made the lessons more dynamic. One theme I can see in all the episodes is how the notion of play is becoming more embedded in the lessons.

I wanted to find out what the students felt about these shifts, so I created a questionnaire, which is discussed in the next section.

4.7 Questionnaire

Having conducted my teaching research. I was keen to find out what the students felt about the shifts I had tried, so I set a questionnaire (see Appendix 1) on the 20.3.06 to all the students I had taught last academic year 2004/05 and this academic year 2005/06. This was 15 in total. I followed this up with an interview on the 27.3.06 with one out of two year 10 student and five out of thirteen Year 11 students, which I picked randomly from set lists. The interview was designed to investigate further the responses, which came out of the questionnaire.

I will start this section by looking statistically at the questionnaire results. I will develop this further by discussing particular themes, which have been significant to my change in practice and which have also come from the questionnaire and the follow up interview. Firstly I present a list of statements used in the questionnaire.

Questions from the questionnaire (Appendix 1):

1. I feel more engaged with my mathematics lessons this year compared to last year.
2. I feel that at the start of the lessons Mr Uppal begins at a point where I understand the topic compared with last year
3. I feel I have some control of where the lessons go this year compared with last year
4. My misconceptions are tackled more in the lessons now compared with last year
5. I ask less questions now compared with last year
6. Lessons start with a problem now
7. Lessons reinforce my understanding of the subject now compared with last year
8. I feel more free enough to express my opinions without worrying if I am right or wrong now compared with last year
9. I feel there is more discussion in lessons this year compared with last year
10. I feel more confident with my mathematics this year compared with last year
11. My teacher is asking more questions this year than last year

12. I am told 'what to do' more in lessons this year compared with last year

13. The students do more talking than the teacher in the lessons

this year compared to last year.

14. I am allowed to make up questions for the class to consider now more than I was allowed last year

15. I feel I have more freedom to change a particular part of the question and see what happens this year than I did last year

16. I am more motivated in mathematics lessons now compared with last year

17. I feel lessons are more meaningful to me now than they were last year

18. I enjoy the lessons more this year than I did last year

19. I feel I am more active and involved in the lessons now compared with last year

20. I can see more clearly the rules that govern each new topic I learn in mathematics now compared with last year

21. This year I can see clearer the connection between a new topic and my mathematical knowledge compared with last year

22. I feel more confident in solving problems this year compared with last year

I tallied the responses, as in the table below, and then coded it by assigning a value of –2 for a strongly disagree response, -1 for a disagree response, +1 for an agree response and +2 for a strongly agree response. A non-response or writing 'same' as a response scored 0. To allow an easy comparison I calculated the total score for each question.

Results from the questionnaire (Appendix 1):

Question	Strongly Disagree	Disagree	Agree	Strongly Agree	Score
1	1	1	10	2	+11
2	0	4	8	1	+6
3	0	4	9	1	+7
4	0	5	8	1	+5
5	2	5	5	2	+0
6	1	3	9	1	+6
7	0	2	9	2	+11
8	0	3	6	3	+9
9	0	0	7	6	+19
10	0	2	9	3	+13
11	0	2	9	2	+11
12	0	7	4	1	-1
13	0	2	7	5	+15
14	1	7	4	2	-1
15	0	5	8	1	+5
16	1	1	6	4	+11
17	0	3	3	3	+4
18	1	3	7	1	+4
19	0	2	11	1	+11
20	0	3	7	2	+8
21	0	3	7	2	+8
22	0	1	8	2	+11

Table 2: Results from the questionnaire

I am going to refer to this table as I develop particular themes, which I feel are significant to the changes in my practice and which have also come out from the questionnaire and interview responses.

4.7.1 Starting with a problem

The interview questions (Appendix 2) highlight the important issues that arose from this first questionnaire. Question 2 in the Questionnaire picked up what effect 'starting with a problem' is having: four out of fifteen disagreed with this statement, but the others felt it was generally true that questions started from their own understanding. I think this is aided by having easy problems posed when starting a new topic. In the interview Erato said *'it was impossible to ensure the lesson started from everyone's understanding because people were at different points' (Interview: 27.3.06)*. Melpomene developed this further *'it would help if key facts were written down' (Interview: 27.3.06)* in response to Zeus's comment *'if pupils are lost one lesson then they remain lost in the next lesson' (Interview: 27.3.06)*. Overall I accepted that it is difficult to ensure the initial problem is posed at the right level, however starting with questions or a problem to solve allows greater accessibility than when I used to begin with a statement and a proof.

4.7.2 Control

After initially setting a problem I have tried to encourage discussion, which gives the students a chance to play with the questions, which from question 3 in the questionnaire my students generally agree with me. I find myself wondering whether it is due to the use of small whiteboards, because they allow pupils the option of writing on the boards their attempt or even just 'I don't understand'. This seems to show the voluntary aspect of play, which Huizinga (1955) points out is essential for play. Students decide how much involvement they put into a task. The accompanying interview bought up a poignant point by Terpsichore *'we do have a certain amount of control, but if we do not understand it we are always going back to you, so we don't really have that much control' (Interview: 27.3.06).* I feel the learning environment being created is not independent of me, because they are not *encouraged to seek techniques within a textbook or from another resource (Journal: 13.1.06).* I am the only resource available to them, as an expert. Hence the degree of control they have is lowered.

Many in the interview highlighted the fact that the louder people dominated and controlled discussions to the detriment of others. Urania raised the point in the interview of pupils being put under pressure to explain concepts, ideas or methods to others and this was particularly acute when only one person within circle time understood it. I was deliberately trying to avoid interjecting, in these situations, so any explanation was done in 'student speak'. In conclusion lessons are being controlled by the louder, more dominant characters but I as the only resource am dictating where the lesson is going.

Interestingly, even though students feel they have more control over where the lessons go, they do not feel they are allowed to make up questions (Question 14). In the interview Polymnia pointed out *'if we were to write our own questions we would either make them really easy or incredibility hard' (Interview: 27.3.06).* Zeus felt this would demotivate pupils because they would give up. I feel it is in hindsight difficult to get pupils to devise questions unless I am guiding them. This I have done by asking pupils to change one particular part of a question, for example a numerical value. In the interview Melpomene stressed the fact: *'that you (teacher) makes up the questions' (Interview: 27.3.06).* I responded with the fact that I tried to get pupils to devise questions and Zeus pointed out *'that if they don't change it to want you want, you change it yourself' (Interview: 27.3.06).* Urania responded *'it depends on the topic, because you may move onto a different topic, when you may just want to change a small bit (to stay with that topic)' (Interview: 27.3.06).* I admitted that I was scared that the question would be changed so much that nobody could do it. There is some limited control I have passed to them, but it is obvious to them and me that I am careful where discussions go.

4.7.3 Discussion and dealing with problems

In terms of the quality of discussions, students feel they are doing more talking (Question 13). I think this may be because of the fact that there is so much more circle time now and with it more discussion. Past analysis of a third year lesson (24.5.04) in my original portfolio on graphs and inequalities shows a style of teaching which has 15 minutes of exposition out of a lesson which lasts 40 minutes with some student questioning at the beginning then students work on a task, but very little actual discussion. A recent third year lesson *(Video: 24.3.06)* on introducing Pythagoras in a circle time format showed only 10 minutes of direct teacher speaking, but this was intermittent because students were involved in a task or explaining ideas to others and far more actual discussion. They had two years ago said very little to the group no more than 10 minutes in a period lasting 40 minutes. Now it is common to hear 20 minutes of student speak because of circle time encouraging discussion. Students themselves claim in these discussions that they feel free enough to express their opinions (Question 8), Polymnia, Year 11, confirmed this in the interview, when he said *'that most people felt confident enough to say that they did not understand what was going on' (Interview: 27.3.06)*. I still suspect, which many in the interview agreed with, that there is a minority who are quieter who do not express their opinions because others seem to know what is going on. One of the problems highlighted by Melpomene, in the interview, is that students are not all totally honest and confident enough to admit they do not understand and some will copy other small whiteboards to avoid the issue. This can clearly hinder any attempt to deal with problems. It is important to create an environment, which encourages honesty amongst the pupils.

Personally I feel students were generally honest because I spend time discussing with the class their problems, which a majority agree is being tackled (Question 4). Zeus highlighted the fact *'that once a majority understand you tended to move on' (Interview: 27.3.06)*. I feel there is a tension in moving the majority on or dealing with the problems of a minority, but this tension exists because the whole class is taught together; within individual pre-selected groups this tension may not exist but others will naturally arise.

4.7.4 Motivation

An overwhelming majority agreed with finding the lessons more motivating, meaningful and enjoyable (Questions 16-18), which was captured in the interview by Erato *'you're more involved it's not just copying off the board and doing exercises and it's more fun' (Interview: 27.3.06)* (Rothlein et al., 1988). Mnemosyne highlighted the fact *'you get to* *see how people are doing it and put it into practice' (Interview: 27.3.06).* I think the motivation may be coming from the fact all pupils are engaged and they are supporting each other in circle time.

Melpomene highlighted a need for the material to be more relevant to the real world, when asked: '*what is stopping lessons being meaningful ?*' (Question 9: Appendix 2) Zeus felt the real applications of the topics might only be relevant to a few people. It is clear some see the meaning from applications, whilst Terpsichore sees the meaning coming from the fact it is part of getting a successful GCSE result. I suspect the others found it meaningful because they could engage with it.

4.8 Summary

This chapter has presented evidence, which centres around five significant episodes: Use of whiteboards, starting with a problem, Generalization, Room Layout and Discussion. It has been argued in the summary of episodes how these shifts have created an atmosphere where the network of 'selves' are more visible and how students are beginning to play with the questions. The first questionnaire (Appendix 1) and interview (Appendix 2) highlight how positively the evolution to circle time has been received by the students. Interestingly new tensions have arisen in this circle time, which is trying to encourage play: firstly the needs of minority to stay with a discussion, so that they understand the problem or question in front of them against the needs of the majority to move on to a more difficult problem. As an enactivist I see this as the tension between the self and the community. Paradoxically there is no permanent self (Davis, 1996) but an every changing self, which is transformed through play. In stopping a particular discussion some 'selves' are not transformed by the mathematics. Secondly, I have clearly been pushing circle time to get the students to play with the questions. Students feel I have restricted this, which Urania highlighted was important to stay within the topic. Terpsichore highlighted that in the end students came back to me, when they do not understand it. Underlying both these comments is the fact that I am trying to follow a curriculum and I am the only one in the room who knows what the curriculum is. Students could play if the curriculum is made more transparent and there are more resources available to the students like the internet, older pupils and different textbooks. It could allow students to be more astute to changes in a question.

5. Reflections

5.0 Introduction

In this section I will seek to answer my research question: *"As I change some factors in the way I go about teaching, what behaviours change in my students and what issues are raised for me in trying to develop a classroom where generalisation and play are more prevalent?"* and discuss whether the research methods that I chose gave me useful data, changes I might have made in hindsight and implications for my own future practice as a teacher and continuing reflective practitioner. I finish by looking at my postmodernist reflections.

5.1 Reflections

The five significant shifts: Use of whiteboards, Starting with a problem, Generalization, Room Layout and Discussion have had a significant shift in student behaviour. It is felt by students that they are more engaged with what is going on, they see mathematics to be about problem solving. They do not necessarily see the investigative and generalization nature of the subject. This could be because of a failure by me to introduce these terms more formally in to their language but students are more confident and are able to see connections.

I have personally seen more confident students in trying these shifts and sense that mathematics is becoming more about them and their thoughts, because they are being more mathematical. Some students seem unwilling to do the hard work to gain this understanding of mathematics and would prefer me to teach a technique. This, I see, as the tension between the demands of students who seek an algorithm and others who seek to understand.

My role seems to have diminished as they have taken charge of the discussion and decide who speaks next. I feel I have elevated them to teachers once they understand because they are given the freedom to teach others. I have developed more independent learners or student-centred learners limited by the tension to get through the curriculum. The rise of the use of generalisation as a tool to teach mathematics has emerged slowly and in a manner which has started with a problem and looking for simpler problems to generalise from to solve the original problem. I feel it would have been better to develop from particulars to the general; the way Lakatos (1976) would see as the development of mathematics. Personally my perception of what mathematics is has changed since exploring this idea and it has opened up more possibilities in which a lesson can develop once an initial investigation or problem is set. I feel there should be an opportunity for classes to form *'little narratives'* (Lyotard, 1979): groups which come together to achieve limited, short-term objectives. These objectives should encompass gaining understanding of the relevant mathematics curriculum: the narrative.

Play has been a difficult idea to develop because of a reluctance, by me, to play games with little mathematical content (Ainley, 1988) or difficulty in devising and finding appropriate games. I still have made some progress in terms of developing discussion as a play format, but my willingness to give more control to them in setting questions has been limited and severely censored by me, because of the tension of getting through the curriculum.

In the learning of mathematics I have realised early on that mathematics is about problem solving. The use of whiteboards has pushed me to see that mathematical activity should start with a problem because that seems an obvious point to begin to see where the students are. Immediately it is important to set initial problems which give students a hold on the topic. In my teaching of mathematics more has been done this year to empower students to become teachers. This has not been in an attempt to get students to take over, but give them a chance to explain concepts to others or to have a go at a technique which has just been explored. It started with the use of whiteboards which allowed me to access student ideas; pupils coming to the front to explain their method to the class, which I realised was slow because pupils could not see each other's whiteboards and eventually through rearranging the classroom to 'circle time'. Circle time has been a playful way to conduct mathematics lessons because each pupil has the ability to ask others questions about problems set. In the circle pupils do not seem to be intimidated in playing the teacher. A thread which emerges here has been my desire to empower students to discuss more the ideas they are learning and going at the pace that suits them. These are play like characteristics, which Genova (1995) sees occurring in language games, which I feel has been possible because of circle time.

I feel a common theme in both the teaching and learning of mathematics is my development in providing students with an experience of doing mathematics in a playful manner. Admittedly I do most of the playing for them, but they are beginning to emerge as better players even with the tensions I have described.

An implication of this research is a need as a teacher to continue to encourage students to play. Games may be the area from which the next shift occurs, an area which I have found difficult to develop because of the need for them to be mathematical. This could be the shift which gives them the confidence to change mathematical questions gradually and to seek generalisation and hence be mathematical.

The research methods themselves swamped me in one sense with too much data for this small study. The journal was the most productive in picking up on themes as a reflective practitioner. A lot of the audio and video recordings was not used. I can not see in hindsight how you can avoid this when significances are not pre-determined. The questionnaire deliberately did not take into account students' interpretations and feelings about the changes they were witnessing in my teaching. The use of the likert scale in their responses and the quantification of the results are all issues, which are problematic to a postmodernist. These were all done to allow a purposeful interview to occur, which inevitably made students' produce accounts-of their experiences. All the methods I found problematic as a postmodernist, but seem necessary to produce a small research piece. The heart of the tension is that in formulating a research question, which centres around me. I am forced to conduct questionnaires and interviews, which focus on my noticings and using formats within the methods, which inform me of actions. Paradoxically if I was not very specific about a research question, then I could be far more pluralistic in my research methodology

As a reflective practitioner there is a need to define mathematical play more accurately than I have managed. To establish the level at which it promotes learning and understanding. To find out whether it promote these things for all students.

Davis (1996:211) says *'Within mathematics education, the importance of play has not been overlooked, although it might be argued to have been undervalued'.*

There is clearly a need for a lot more research in this area for secondary school pupils.

5.2 Postmodern reflections

Postmodernists like myself will always struggle to produce research within the structure and expectations of a modernist dissertation process. I have tried to present the unpresentable (Lyotard, 1979). These are the crisis of identifying both the researcher and the subject of research; the crisis of methodological certainty; the crisis in validity and the crisis in representation (Scheurich, 1997). All these crises have manifested themselves in my work because I struggled to really connect 'play' and 'mathematics'. The models of mathematical thinking were far too limited, because I could not imagine other ways of thinking. The frameworks all had pitfalls, which any conceivable framework will always have because they are by nature structuralist. Validity and reliability were inappropriate given that postmodernist assume that there is no 'reality' out there to be investigated, described or catalogued. It is seen to be socially constructed, hence the novel is now seen as a possible research methodology (Slattery, 1997). The research rigour then being decided by aesthetics and ethics. I do not feel as a postmodernist even this methodology would eliminate the central problem that postmodernist have in that there is a crisis in legitimising the narrative. I can see that I have cycled initially through postmodernism, when I take my enactivist stand, but then cycle through a modernist track in the main bulk of my research until now returning to Postmodernism. Lyotard (1979) saw these cycles alternating throughout the course of history. This dilemma is why I had to depart from my postmodernist stance, so that I could produce some data to research. The problem of being a postmodern education researcher is that most researchers in this area are more articulate about what they are against than what they are for (Alvesson and Sköldberg, 2000). In reality I feel I am not comfortable with any research methodology or framework at all. To me what is equally important to what has happened to the subjects or objects ('texts') of this research is how I have changed in doing this research as a teacher (Foucualt, 1988). This dissertation should be read as a modernist text, which is trying to grapple with postmodern ideas.

Appendix 1

Questionnaire

Please Circle your year group

Year 6/7/8/9/10/11/12/13

Please circle the response which best agrees with your feelings.

1. I feel more engaged with my mathematics lessons this year compared to last year.

Strongly Disagree	Disagree	Agree	Strongly Agree

2. I feel that at the start of the lessons Mr Uppal begins at a point where I understand the topic compared with last year

Strongly Disagree	Disagree	Agree	Strongly Agree

3. I feel I have some control of where the lessons go this year compared with last year

Strongly Disagree	Disagree	Agree	Strongly Agree

4. My misconceptions are tackled more in the lessons now compared with last year

Strongly Disagree	Disagree	Agree	Strongly Agree

5. I ask less questions now compared with last year

Strongly Disagree	Disagree	Agree	Strongly Agree

6. Lessons start with a problem now

Strongly Disagree	Disagree	Agree	Strongly Agree

7. Lessons reinforce my understanding of the subject now compared with last year

Strongly Disagree	Disagree	Agree	Strongly Agree

8. I feel more free enough to express my opinions without worrying if I am right or wrong now compared with last year

Strongly Disagree	Disagree	Agree	Strongly Agree

9. I feel there is more discussion in lessons this year compared with last year

Strongly Disagree	Disagree	Agree	Strongly Agree

10. I feel more confident with my mathematics this year compared with last year

Strongly Disagree	Disagree	Agree	Strongly Agree

11. My teacher is asking more questions this year than last year

Strongly Disagree	Disagree	Agree	Strongly Agree

12. I am told 'what to do' more in lessons this year compared with last year

Strongly Disagree	Disagree	Agree	Strongly Agree

13. The students do more talking than the teacher in the lessons this year compared to last year.

Strongly Disagree	Disagree	Agree	Strongly Agree

14. I am allowed to make up questions for the class to consider now more than I was allowed last year

Strongly Disagree	Disagree	Agree	Strongly Agree

15. I feel I have more freedom to change a particular part of the question and see what happens this year than I did last year

Strongly Disagree	Disagree	Agree	Strongly Agree

16. I am more motivated in mathematics lessons now compared with last year

Strongly Disagree	Disagree	Agree	Strongly Agree

17. I feel lessons are more meaningful to me now than they were last year

Strongly Disagree	Disagree	Agree	Strongly Agree

18. I enjoy the lessons more this year than I did last year

Strongly Disagree	Disagree	Agree	Strongly Agree

19. I feel I am more active and involved in the lessons now compared with last year

Strongly Disagree	Disagree	Agree	Strongly Agree

20. I can see more clearly the rules that govern each new topic I learn in mathematics now compared with last year

Strongly Disagree	Disagree	Agree	Strongly Agree

21. This year I can see clearer the connection between a new topic and my mathematical knowledge compared with last year

Strongly Disagree	Disagree	Agree	Strongly Agree

22. I feel more confident in solving problems this year compared with last year

Strongly Disagree	Disagree	Agree	Strongly Agree

Appendix 2

Interview questions

1. Do you feel it is possible to start lessons where you are ?
2. Do you feel you have control over the lesson in circle time ?
3. What is stopping misconceptions being delt with ?
4. Do you feel circle time is encouraging copying ?
5. Are you really still told what to do ?
6. Why do you feel you are not willing to change the questions or play with them ?
7. What is stopping you enjoying the lessons ?
8. When do we move on, when one person doesn't understand the topic ?
9. What is stopping lessons being meaningful ?
10. How do you feel about playing games ?

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