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Modeling Newsvendor Behavior: A Prospect Theory Approach

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Problem definition: Studies have shown that the behavior of subjects in newsvendor experiments is not consistent with expected profit maximization – an assumption that is often made in operations management literature. Although prospect theory has been established as a popular model of behavioral decision making under uncertainty, it was considered to be inconsistent with observed newsvendor behavior (in particular, the pull-to-center effect) until a recent study proposed a prospect theory model that is consistent with the pull-to-center effect; however, this model's ability in representing newsvendor behavior compared to other plausible prospect theory models is unexplored in the literature. This paper takes a more comprehensive approach in building several prospect theory-based newsvendor models, and evaluates their competence in representing the observed newsvendor behavior. An important feature of these models is that they are not only consistent with the pull-to-center effect, but they can also, in accordance with the findings from recent research, accommodate individual-level heterogeneity in order quantities.

Academic/Practical Relevance: Designing effective supply chain processes and inventory systems requires that the underlying models represent the observed newsvendor behavior reasonably well, especially in settings where most decisions are made by individuals. Our paper provides a rigorous basis for choosing a model when characterizing the decision making process of a newsvendor. Moreover, our novel approach to model building and testing could serve as a template for selecting appropriate prospect theory models in contexts other than the newsvendor problem.

Methodology: Motivated by different types of reference points studied in the decision theory literature, we first build several newsvendor models that can theoretically accommodate individual level heterogeneity in order quantities. Thereafter, using a multipronged approach based on theoretical criteria, goodness of fit, and empirical validity, we evaluate these models to determine the most appropriate model.

Results: The model with mean demand as the stochastic reference point consistently outperforms other models, reducing the prediction error by as much as 31% on the experimental data used for this study. Moreover, all the empirical regularities considered in our paper are consistent only with this model. This suggests that mean demand is more likely to be adopted by experimental subjects as a reference point – perhaps because of its greater salience than the other plausible reference points considered.

Managerial Implications: Since decisions are predominantly made by human retailers in the emerging markets, we represent their behavior by the model with mean demand as reference point, and identify settings in which they could benefit from investing in decision support systems. We also demonstrate the benefits to a supplier from approximating his retailers' behavior with this model relative to him using the other prospect theory models considered in this paper.

1. Introduction

The newsvendor model is at the core of inventory and supply chain management. It has been used extensively in the design of optimal inventory systems and supply chain contracts. Most of the literature on these topics assumes that the newsvendor is either an expected profit-maximizing agent or a risk-averse agent. Such assumptions are reasonable in many settings; however, when newsvendor decisions are made by humans, certain behavioral factors may systematically lead to an outcome that differs from that predicted by these models, thus rendering conventional models ineffective in terms of representing the underlying behavior. This is especially likely in emerging markets, where most decisions are made by individuals, unassisted by standard decision support systems. To operate effectively in such environments, it is important to understand the decision biases of humans in a newsvendor context, to build formal models that incorporate that behavior, and then to apply those models to the design and analysis of systems and incentive structures.

In their seminal paper, Schweitzer and Cachon (2000) identified an important decision bias – called the *pull-to-center* (PTC) effect – in newsvendor decisions: for products with high (resp., low) profit margins, the average order quantity is greater (resp., less) than the mean demand but less (resp., greater) than the expected profit-maximizing quantity. (The interval between the mean demand and the expected profit-maximizing quantity is known as the *PTC zone*.) This effect has been replicated in subsequent experiments by, among others, Bolton and Katok (2008), Bostian et al. (2008), de Véricourt et al. (2013), and Rudi and Drake (2014). The behavioral operations management literature, thereafter, has proposed several models to explain the PTC effect; Nagarajan and Shechter (2014) comprehensively summarize this literature. Following Kahneman and Tversky (1979) and Tversky and Kahneman (1992), prospect theory (PT) has figured prominently in the literature on individual decision making. It is hardly surprising, then, that PT has been much discussed as a possible explanation for the PTC effect. Before we dive into PT in the context of the newsvendor problem, in the next paragraph we provide a brief overview of PT.

Prospect theory is a well-established descriptive theory of decision making that has been used to explain several phenomena of interest in various fields; see Camerer (2000) and Barberis (2013) for a comprehensive review of PT applications. This theory deviates from expected utility theory in two important ways. First, utility under PT is assigned to gains and losses with respect to a reference point and not to the final asset; this feature is known as *reference dependence*. Second, losses "loom larger" than do gains of the same size; this phenomenon is known as *loss aversion*. (A third important deviation, one that we do not consider here, is the nonlinear weighting of probabilities; however, numerical analysis suggests that the insights from our paper remain unaffected if nonlinearly weighted probabilities were used.) Reference point is an important feature of prospect theory. It determines how a decision maker perceives gains and losses under a prospect, and hence plays a crucial role in explaining the decision maker's attitude toward that prospect.

Decision models based on PT had been ruled out earlier as a potential explanation for newsvendor behavior. By considering zero profit as the reference point, Schweitzer and Cachon (2000) argue that PT cannot predict the PTC effect when the newsvendor can only make gains (i.e., receive only positive payoffs) as a result of her quantity decisions. These authors reject PT by conducting experiments in the "gains" domain only and showing the PTC effect's persistence in these experiments. Nagarajan and Shechter (2014) account for nonlinear weighting of probabilities and show that Schweitzer and Cachon's insights remain unchanged. In contrast to these studies, Long and Nasiry (2015) show that if the reference point for an order quantity is assumed to be the "weighted average of the maximum and minimum profits that could be obtained with that quantity", then the resultant model *can* predict the PTC effect. The important contribution of their study is that it highlights the central role played by the reference point in predicting newsvendor behavior.

However, Long and Nasiry's model relies on an arbitrarily chosen reference point, which is neither theoretically motivated nor empirically validated. Given that the reference point is crucial in a prospect theory model (Barberis 2013), a rigorous evaluation of their model along with other plausible PT models is needed to determine its appropriateness in modeling newsvendor behavior. In this paper we take a more comprehensive approach in building and evaluating PT models. While building these models we not only use consistency with the PTC effect as an evaluation criterion, but also consider these models' ability to accommodate individual-level heterogeneity in order quantities. A detailed discussion on the importance of incorporating heterogeneity as a criterion for model evaluation in the newsvendor context follows.

Importance of predicting heterogeneity. In the aforementioned newsvendor experiments, the PTC effect was observed at the *aggregate* level; yet research thereafter has assumed that it is a characteristic property of *individual*-level behavior. Lau et al. (2014) challenge this assumption. These authors establish that individual behavior varies widely in newsvendor experiments and that a significant share of the subject pool does *not* exhibit the PTC effect in both low- and high-margin settings. Across four experiments analyzed in their paper – that is, the low- and high-margin experiments of Bolton and Katok (2008) and their replication by Lau et al. (2014) – 28%–45% of subjects have mean order quantities outside the PTC zone, 39%–61% have median orders outside the PTC zone, and 56%–76% of subjects have modal orders outside the PTC zone (Lau et al. 2014, p. 71). The authors conclude that the PTC effect is only an aggregate-level phenomenon and that it "does not accurately describe individual behavior" (p. 73). We must consequently bear in mind that a model should not be accepted as capturing newsvendor behavior, or rejected as being unrepresentative, based solely on the PTC effect.

Any model that plausibly represents newsvendor behavior should predict the existence – under both low- and high-margin settings – of individuals who exhibit the pull-to-center effect (i.e., who order quantities inside the PTC zone) *and* of individuals who do not exhibit this effect (i.e., who order quantities outside the PTC zone). One might ask why is there a need for new prospect theory-based models when such predictions could be made by a simple expected utility model with individuals who differ in their attitudes toward risk: if an experiment includes both risk-averse and risk-seeking subjects, then we will observe both underordering and overordering (with respect to the expected profit-maximizing quantity) in the experimental data. Under this model, however, no single subject will underorder in a high-margin experiment *and* overorder in a low-margin experiment. Yet that prediction is inconsistent with the evidence from within-subject experiments (e.g., Käki et al. 2015), where the same individual places orders in both low- and high-margin settings, that an irrefutable proportion of subjects exhibit the PTC effect in both of these margin settings.[1] This suggests that the expected utility model is probably not an appropriate representation of observed newsvendor behavior. Similarly, models of waste aversion, stockout aversion, and underestimated opportunity costs (Schweitzer and Cachon 2000) – which predict either underordering or overordering for *all* individuals – also cannot predict the PTC effect consistently across low- and high-margin settings.

In contrast, the most prominent explanations for the PTC effect are consistent with the empirical finding just described. Among these are minimizing ex post inventory error, anchoring on mean demand and insufficiently adjusting toward the expected profit-maximizing quantity (Schweitzer and Cachon 2000), bounded rationality (Su 2008), and overconfidence bias (Ren and Croson 2013). However, these explanations predict a PTC effect only for *all* individuals and so cannot accommodate the existence of individuals who do not exhibit the effect.[2,3]

As a result, using any of the foregoing models for analysis ignores the behavior of a significant proportion (sometimes as much as half) of the population, which, therefore, necessitates building models that better represent newsvendor behavior. This paper builds and evaluates PT-based models that can consistently predict the PTC effect across low- and high-margin settings *and* that also accommodate individuals who do not exhibit the PTC effect. Hereafter, we refer to such models as those that *accommodate* (or *predict*) *heterogeneity* in newsvendor behavior. We believe that these models better represent observed newsvendor behavior than models in the existing literature.

Prospect theory models in this paper. By considering several different types of reference points, we first build models in Section 2 that can theoretically predict heterogeneity in newsvendor behavior. In theory, there are infinite possible reference points. So following Kahneman's (1992) argument that a reference point is a *salient point* within an individual's cognitive norm, we choose the candidate reference points based on salience. Under the "traditional" PT framework of Kahneman and Tversky (1979), we consider salient payoffs in a newsvendor experiment such as the

[1] We obtained the control group data from Käki et al. (2015), which is a replication of Experiment 1 in Schweitzer and Cachon (2000). If we say that a subject exhibits the PTC effect if her mean order quantity lies in the PTC zone, then our analysis reveals that 68% of subjects exhibit this effect in both the margin settings.

[2] Strictly speaking, in this setting there are two types of order quantity variations: the variation *across* individuals' orders and the variation *within* an individual's orders. We are interested in variation of the former type. Many standard decision-making models do not account for variation of the latter type: the resultant decision is deterministic for a given set of problem parameters. However, they can be easily modified to accommodate such variation by making the choices stochastic, where the probability of choosing an option is increasing in the valuation of that option. Such models are popular in cognitive sciences; see, for example, the stochastic version of PT used in Erev et al. (2010).

[3] In the newsvendor context, the bounded rationality model of Su (2008) allows for variation within an individual's order quantities. Since the underlying valuation model relies on expected profits, it follows that the modal order quantities predicted by this model under the uniform demand distribution are either the endpoints of the support or the expected profit-maximizing quantity. Yet these predictions are empirically inconsistent, as shown by Katok (2011) and as evident in Figure 2 of our paper. Moreover, by Proposition 3 of Su (2008) this model can predict only a PTC effect for all the individuals and so cannot accommodate individuals who do not exhibit this effect.

minimum payoff, the maximum payoff, and the sure-shot payoff as the reference points. Because the information in newsvendor experiments is usually presented in terms of demand distribution and not of payoff distribution, we also consider the salient pieces of demand-related information (such as the endpoints and the mean of the distribution) as the stochastic reference points (Sugden 2003, Kőszegi and Rabin 2006, De Giorgi and Post 2011). In addition, we consider the disappointment model of Bell (1985), wherein a prospect's reference point is its expected value. As in Long and Nasiry (2015), it is a prospect-dependent reference point; however, it has been axiomatically established and used in decision sciences literature (see, e.g., Loomes and Sugden 1986, Gul 1991).

After we build models that predict heterogeneity in newsvendor behavior, to determine the most appropriate model, we further evaluate these models using on a multipronged approach based on some theoretical criteria, goodness of fit to the experimental data, and the validity of their predictions vis-à-vis some empirical regularities. Section 3 presents the methodologies and results of our analysis to compare the different models. We find that the model of Long and Nasiry (2015) falls short on almost all theoretical and empirical criteria as compared to other models in this paper. Although the predictive performance of other models varies across comparison criteria, the model with mean demand as reference point consistently outperforms those other models: it reduces prediction error by as much as 31% over competing models on the experimental data, and it is consistent with all the empirical regularities examined in our paper. This suggests that mean demand is more likely to be adopted by experimental subjects as a reference point – perhaps because mean demand is more salient than the other plausible reference points considered.

We then extrapolate our insights from lab experiments to the retail industry in emerging markets in Section 4. We represent the behavior of human retailers by the model with mean demand as reference point, and identify settings in which they could benefit from investing in decision support systems. We also demonstrate the benefits to a supplier from approximating his retailers' behavior with this model relative to him using the other models.

Our paper makes three main contributions. First, we highlight the importance of accommodating heterogeneity in newsvendor behavior, after which we build PT models that predict heterogeneity. Second, we use several theoretical and empirical criteria to test the relative performance of these models, and then adopt the model most likely to derive some empirically consistent predictions and prescriptive insights. The criteria developed in this paper could be used hereafter to evaluate the behavioral newsvendor models. Third, the models discussed in this paper emphasize the importance of accounting for loss aversion and reference dependence in the newsvendor scenario. These concepts have been successfully applied by economists in several contexts, but they have yet to penetrate the OM literature. The approach we take can serve as a template for identifying and selecting appropriate models with loss aversion and reference dependence – and, it is hoped, thus increasing their application in the OM literature.

2. Alternative Newsvendor Models

We start by presenting the general notation and assumptions that will be used in the analysis. The decision maker is assumed to face a newsvendor problem with unit cost c, unit revenue r, and no salvage value. Demand $x \in \mathcal{X}$ is distributed with density function f, distribution function F, and survival function $\bar{F}$, all of which are known to the decision maker. Let μ represent the mean of this distribution, and define $\alpha = c/r$ as the *experimental condition.* The product is considered to be high margin if $\alpha < 0.5$ or low margin if $\alpha > 0.5$. It is well known that the expected profit-maximizing quantity $\tilde{q}_{\text{EP}}$ satisfies $F(\tilde{q}_{\text{EP}}) = 1 - \alpha$. In some contexts, we refer to $\tilde{q}_{\text{EP}}$ as $\tilde{q}_{\text{LM}}$ and $\tilde{q}_{\text{HM}}$ – and the corresponding α as α_{LM} and α_{HM} in (respectively) the low-margin and high-margin conditions – when needed to indicate specifically which margin condition is under discussion.

We assume that the newsvendor is a PT value-maximizing individual characterized by the utility function v over payoffs. The degree of a newsvendor's loss aversion is captured by the parameter λ, where $v(-x) = -\lambda v(x)$ for $x > 0$. We restrict our analysis to power utility functions: $v(x) = x^{\delta}$ with $0 < \delta \leq 1$. We also follow Long and Nasiry (2015) in restricting analysis to the uniform distribution – that is, $X \sim \text{U}(d_{\min}, d_{\max})$ – which is the most commonly used distribution in newsvendor experimental studies. However, an extensive numerical analysis reveals that the results presented in this section remain unchanged when we use distributions that have increasing hazard rate function and log-concave cumulative distribution function (e.g., normal, gamma, log-normal distributions).

Different types of reference points. To understand the differences between various types of reference points considered in our paper, we first write the valuation of order quantity q under a generic reference-dependent model. We define

$$\tau(x, q) = r \min\{x, q\} - cq, \tag{1}$$

which is the actual payoff when the realized demand is x and the ordered quantity is q. Let $\mathcal{P}(x, q)$ be a generic reference point that could depend on both the realized demand x and the order quantity q. Define $\mathcal{X}_>$ as that part of the domain where the demand realizations lead to a payoff which is higher than the reference payoff, i.e., $\mathcal{X}_> = \{x \in \mathcal{X} : \tau(x, q) > \mathcal{P}(x, q)\}$. Hence, $\mathcal{X}_>$ is the gains domain. We can similarly define the losses domain $\mathcal{X}_<$, such that $\mathcal{X} = \mathcal{X}_> \cup \mathcal{X}_<$ and $\mathcal{X}_> \cap \mathcal{X}_< = \phi$. Then the reference-dependent valuation of order quantity q is given by

$$V(q) = \eta \int_{x \in \mathcal{X}} \tau(x, q)\, dF(x) + \left\{ \int_{x \in \mathcal{X}_>} \big(\tau(x, q) - \mathcal{P}(x, q)\big)^{\delta} dF(x) - \lambda \int_{x \in \mathcal{X}_<} \big(\mathcal{P}(x, q) - \tau(x, q)\big)^{\delta} dF(x) \right\}, \tag{2}$$

where the first term is called the *consumption utility* (also called the rational part of the utility) and the second term is called the *gain-loss utility* (also called the psychological part of the utility). The former is simply the expected profit, whereas the latter is the expected utility of gains and losses perceived with respect to the reference point, where losses are weighed by the loss aversion parameter λ. The relative weight placed by the decision maker on the consumption utility is characterized by the parameter η.[4] Using this generic framework, we can now define the following reference points:

(i) A fixed reference point (FRP) model is the one where the reference point $\mathscr{P}(x,q)$ is independent of both x and q – it is a *fixed* payoff in the outcome space.

(ii) Under a stochastic reference point (SRP) model, $\mathscr{P}(x,q)$ is independent of q but depends on x. Because of this dependence on x, which is the only stochastic element in this setting, the reference point is called *stochastic*.

(iii) For a prospect-dependent reference point (PRP) model, $\mathscr{P}(x,q)$ is independent of x but depends on q. Since the order quantities are prospects for the newsvendor, the reference point is termed *prospect-dependent*.

We rewrite (2) in the subsequent sections where we analyze specific reference points of the above types in the newsvendor context. Consistent with the definitions used in papers which first proposed FRP and SRP models (Kahneman and Tversky 1979, Tversky and Kahneman 1992, Sugden 2003), we do not incorporate consumption utility, and set $\eta = 0$ for these models in this section. This also showcases the ability of these models to parsimoniously predict heterogeneity in newsvendor behavior without relying on parameter η. In contrast, PRP models require consumption utility to predict heterogeneity (as we shall see in Section 3.1); therefore, we retain η for these models.

Predicting heterogeneity. Now we formalize what it means for a model to predict heterogeneity in newsvendor behavior. To this end, we distinguish between two types of parameters: (i) environment-level parameters, α, r and D, that are independent of models and individuals, and (ii) model-level parameters, such as λ and δ, that characterize the individuals in the population. We denote the universal set of individuals as $\mathscr{I}$, which is simply the set of model parameters used to characterize those individuals. Let us define $\mathscr{I}_{\mathrm{P}}^{\mathscr{m}}(\alpha, r, D) \subset \mathscr{I}$ as the set of individuals who are predicted to exhibit pull-to-center effect by model $\mathscr{m}$ (i.e., their optimal order quantity, as predicted by $\mathscr{m}$, lies inside the PTC zone) when the environment-level parameters are α, r and D. Similarly, $\mathscr{I}_{\mathrm{N}}^{\mathscr{m}}(\alpha, r, D)$ is defined as the set of individuals who are predicted to not exhibit PTC effect by model $\mathscr{m}$ (i.e., their optimal order quantity, as predicted by $\mathscr{m}$, lies outside the PTC zone).

As discussed in Section 1, a model accommodates heterogeneity if and only if it can predict the existence of individuals who exhibit the PTC effect *as well as* of individuals who do not exhibit that effect. Such a model should also, in line with the evidence from within-subject experiments,

[4] The existing literature defines η as the weight on the gain-loss utility. However, without loss of generality, we redefine it as the weight on the consumption utility. This is helpful while estimating the parameters of FRP and SRP models in Section 3.2 because these models tend to put very low weight on the consumption utility; therefore, the estimation process converges quickly with this redefinition.

predict the existence of individuals who exhibit the PTC effect under both low- and high-margin settings. These conditions can be formally written as:

(H1) $\mathcal{G}_{\mathrm{P}}^{m}(\alpha, r, D) \neq \varnothing$ and $\mathcal{G}_{\mathrm{N}}^{m}(\alpha, r, D) \neq \varnothing$ for all $\alpha \in [0,1]$, and

(H2) $\mathcal{G}_{\mathrm{P}}^{m}(\alpha_{\mathrm{LM}}, r, D) \cap \mathcal{G}_{\mathrm{P}}^{m}(\alpha_{\mathrm{HM}}, r, D) \neq \varnothing$ for all $\alpha_{\mathrm{LM}} \in (0.5, 1]$ and $\alpha_{\mathrm{HM}} \in [0, 0.5]$,

where $\varnothing$ represents the null set. Recall that theories such as expected utility maximization, waste aversion, stockout aversion, and underestimated opportunity costs satisfy condition (H1) but not (H2). The extant models in behavioral OM literature that are proposed to explain the PTC effect (such as bounded rationality, ex post inventory error minimization, overconfidence bias) satisfy (H2) but not (H1). Here, we explore prospect theory-based models that satisfy both (H1) and (H2).

To show that a model satisfies both (H1) and (H2), we follow the method of proof by construction. For a given (α, r, D), we construct *nonempty* sets $\mathscr{G}_{\mathrm{P}}^{m}(\alpha, r, D)$ and $\mathscr{G}_{\mathrm{N}}^{m}(\alpha, r, D)$ such that individuals in the former set are predicted by m to exhibit PTC effect, whereas those in the latter set are predicted to not exhibit that effect. The construction of these sets also sheds light on the behavior of the optimal order quantity predicted by m, and on the important role played by loss aversion in predicting heterogeneity. Since $\mathcal{G}_{\mathrm{P}}^{m} \supset \mathscr{G}_{\mathrm{P}}^{m} \neq \varnothing$ and $\mathcal{G}_{\mathrm{N}}^{m} \supset \mathscr{G}_{\mathrm{N}}^{m} \neq \varnothing$, (H1) is satisfied. Thereafter, for arbitrarily given α_{LM} and α_{HM}, we show that sets $\mathscr{G}_{\mathrm{P}}^{m}(\alpha_{\mathrm{LM}}, r, D)$ and $\mathscr{G}_{\mathrm{P}}^{m}(\alpha_{\mathrm{HM}}, r, D)$ are *overlapping*. Since $\mathcal{G}_{\mathrm{P}}^{m}(\alpha_{\mathrm{LM}}, r, D) \cap \mathcal{G}_{\mathrm{P}}^{m}(\alpha_{\mathrm{HM}}, r, D) \supset \mathscr{G}_{\mathrm{P}}^{m}(\alpha_{\mathrm{LM}}, r, D) \cap \mathscr{G}_{\mathrm{P}}^{m}(\alpha_{\mathrm{HM}}, r, D) \neq \varnothing$, (H2) is also satisfied. With this understanding, we now proceed with analyzing specific reference point models.

2.1. Prospect-dependent Reference Point (PRP) Models

The first PRP model we discuss is that of Long and Nasiry (2015). Second, we discuss the more popular disappointment model of Bell (1985).

2.1.1. Weighted Average of Maximum and Minimum Profits as PRP. The reference point for an order quantity q in the model of Long and Nasiry (2015) is the weighted average of the maximum and minimum profits associated with ordering q:

$$\mathscr{P}(x, q) \equiv \Omega(q) = \beta(r-c)q + (1-\beta)(r d_{\min} - cq) \quad \text{for all } x, \tag{3}$$

where $\beta \in [0,1]$ is the newsvendor's level of optimism. The newsvendor has high expectations for the final outcome if the value of β is high, whereas a low β implies that she anchors more on the worst-case scenario. With this reference point, (2) can be rewritten as

$$\begin{aligned} V(q) = \eta\left\{\int_{d_{\min}}^{q} (rx - cq)\, dF(x) + (r-c)q\bar{F}(q)\right\} + \Bigg\{ -\lambda \int_{d_{\min}}^{(\Omega(q)+cq)/r} (\Omega(q) - rx + cq)^{\delta}\, dF(x) \\ + \int_{(\Omega(q)+cq)/r}^{q} (rx - cq - \Omega(q))^{\delta}\, dF(x) + ((r-c)q - \Omega(q))^{\delta} \bar{F}(q)\Bigg\}. \end{aligned} \tag{4}$$

In Proposition 3 of their paper, Long and Nasiry show that the value function $V(q)$ is concave with a unique maximum q^*. Since q^* is above the expected profit-maximizing quantity if $c/r > (\lambda\beta^{1+\delta} + \beta(1-\beta)^\delta)/(\delta(1-\beta)^\delta + \lambda\beta^{1+\delta} + \beta(1-\beta)^\delta)$ and is otherwise below that quantity, the authors conclude that their model can accommodate the PTC effect, which does not necessarily imply that it also satisfies (H1) and (H2). The following proposition states that the model of Long and Nasiry (2015) can actually predict heterogeneity. (The proofs for all propositions are given in Appendix F.) Since an individual here is characterized by tuple $(\eta, \beta, \lambda, \delta)$, the set of all individuals that can be characterized by this model is given by $\mathscr{I} = \mathscr{E} \times \mathscr{B} \times \mathscr{L} \times \mathscr{D}$, where $\mathscr{E} = [0, \infty)$, $\mathscr{B} = [0, 1]$, $\mathscr{L} = [0, \infty)$, and $\mathscr{D} = (0, 1]$ are, respectively, the domains of parameters η, β, λ, and δ. We put $D = d_{\max} - d_{\min}$ and $[z]^+ = \max(z, 0)$.

Proposition 1. *Let $\Omega(q)$ in (3) be the reference point. Then the following statements hold:*

(i) *The value function $V(q)$ is submodular in (q, λ) and unimodal in q.*

(ii) *Assume $rD > 2$. Given (α, r, D), there exists a threshold $\hat{\delta} \in [0, 1]$ and a nonempty interval Λ such that sets $\mathscr{I}_{\mathrm{P}}(\alpha, r, D) = \{(\eta, \beta, \lambda, \delta) \in \mathscr{I} : \lambda \in \Lambda(\eta, \beta, \delta, \alpha, r, D), \delta \geq \hat{\delta}(\eta, \beta, \alpha, r, D), \beta \leq \min\left(1 - 2/(rD), \alpha/(1-\alpha)\right), \eta \leq (1-\beta)^2/[2\alpha - 1]^+\}$ and $\mathscr{I}_{\mathrm{N}}(\alpha, r, D) = \{(\eta, \beta, \lambda, \delta) \in \mathscr{I} : \lambda \notin \Lambda(\eta, \beta, \delta, \alpha, r, D), \delta \geq \hat{\delta}(\eta, \beta, \alpha, r, D), \beta \leq \min\left(1 - 2/(rD), \alpha/(1-\alpha)\right), \eta \leq (1-\beta)^2/[2\alpha - 1]^+\}$ are nonempty. The individuals in $\mathscr{I}_{\mathrm{P}}(\alpha, r, D)$ exhibit PTC effect, whereas those in $\mathscr{I}_{\mathrm{N}}(\alpha, r, D)$ do not exhibit that effect. The model satisfies* (H1).

(iii) *Given α_{LM} and α_{HM}, $\mathscr{I}_{\mathrm{P}}(\alpha_{\mathrm{LM}}, r, D)$ and $\mathscr{I}_{\mathrm{P}}(\alpha_{\mathrm{HM}}, r, D)$ are overlapping. The model satisfies* (H2).

We now explain the structure of this result. Since $V(q)$ is submodular in (q, λ), it follows that $q^*(\lambda)$ is decreasing in λ (Topkis 1998). As the decision maker becomes more loss averse, her reluctance to deviate much from her reference point increases. Here, the deviation is lowest at $q = d_{\min}$. Therefore, $q^*(\lambda)$ starts at $\lim_{\lambda \to 0} q^*(\lambda) \equiv q_0^*$ and, as λ increases, that optimum moves leftward and ends at $\lim_{\lambda \to \infty} q^*(\lambda) \equiv q_\infty^* = d_{\min}$. For a given (α, r, D): if q_0^* is greater than $\tilde{q}_{\mathrm{EP}}$ *and* μ, and if q_∞^* is less than $\tilde{q}_{\mathrm{EP}}$ *and* μ, then there exists an interval Λ such that (i) if $\lambda \in \Lambda$, $q^*(\lambda)$ is between $\tilde{q}_{\mathrm{EP}}$ and μ (and hence in the PTC zone) and (ii) if $\lambda \notin \Lambda$, $q^*(\lambda)$ lies outside the PTC zone.

Although q_∞^* is always below both $\tilde{q}_{\mathrm{EP}}$ and μ, to ensure that q_0^* is above both $\tilde{q}_{\mathrm{EP}}$ and μ, we need conditions $\delta \geq \hat{\delta}$, $\beta \leq 1 - 2/(rD)$, $rD > 2$, $\beta \leq \alpha/(1-\alpha)$, and $\eta \leq (1-\beta)^2/[2\alpha - 1]^+$. (We discuss these conditions in detail in Appendix A.) Then the set $\mathscr{I}_{\mathrm{P}}$, as defined in Proposition 1, constitutes individuals who exhibit PTC effect, whereas those in the set $\mathscr{I}_{\mathrm{N}}$ do not exhibit that effect. The nonemptiness of sets $\mathscr{I}_{\mathrm{P}}$ and $\mathscr{I}_{\mathrm{N}}$ shows that the model satisfies (H1). The remaining individuals are in the set $\mathscr{I}_{\mathrm{R}} = \mathscr{I} - (\mathscr{I}_{\mathrm{P}} \cup \mathscr{I}_{\mathrm{N}})$, which constitutes individuals who exhibit PTC effect, and also those who do not; however, partitioning it between these two types is analytically cumbersome, and, more importantly, does not affect the already proven fact that the model can accommodate both types of individuals.

For given α_{LM} and α_{HM}, $\mathscr{I}_{\mathrm{P}}(\alpha_{\mathrm{LM}}) \cap \mathscr{I}_{\mathrm{P}}(\alpha_{\mathrm{HM}}) = \{(\eta,\beta,\lambda,\delta) \in \mathscr{I} : \lambda \in \Lambda(\alpha_{\mathrm{LM}}) \cap \Lambda(\alpha_{\mathrm{HM}}), \delta \geq \max(\hat{\delta}(\alpha_{\mathrm{LM}}), \hat{\delta}(\alpha_{\mathrm{HM}})), \beta \leq \min\left(1 - 2/(rD), \alpha_{\mathrm{HM}}/(1-\alpha_{\mathrm{HM}})\right), \eta \leq (1-\beta)^2/(2\alpha_{\mathrm{LM}}-1)\}$ is nonempty because $\Lambda(\alpha_{\mathrm{LM}}) \cap \Lambda(\alpha_{\mathrm{HM}}) \neq \varnothing$ (see the proof of Proposition 1). Therefore, the model satisfies (H2).

This result explains the heterogeneity in newsvendor order quantities by appealing to the population's heterogeneity with regard to loss aversion. This emphasizes the important role played by loss aversion in predicting newsvendor behavior. As we shall see, this is a common feature of all the heterogeneity results discussed in this paper.

2.1.2. Expected Profit as PRP. A well-established class of decision models with prospect-dependent reference points are models of disappointment (Bell 1985, Loomes and Sugden 1986, Gul 1991). Under these models, the decision maker forms expectations about uncertain prospects (which serve as reference points) and experiences either elation or disappointment according to whether the realized outcome is better or worse than expected. Here we use the model of Bell (1985), in which the reference point for a prospect is equal to its expected profit:

$$\mathscr{P}(x,q) \equiv \Pi(q) = \int_{d_{\min}}^{q} (rx - cq)\,dF(x) + (r-c)q\bar{F}(q) \quad \text{for all } x. \tag{5}$$

The valuation of quantity q under this model is given by (4) but with $\Omega(q)$ replaced by $\Pi(q)$. Our next proposition states that this model also predicts heterogeneity. Here, $\mathscr{I} = \mathscr{E} \times \mathscr{L} \times \mathscr{D}$.

Proposition 2. *Let $\Pi(q)$ in* (5) *be the reference point. Then the following statements hold:*

(i) *The value function $V(q)$ is submodular in (q,λ) and unimodal in q.*

(ii) *Assume $rD > 8$. Given (α, r, D), there exists a threshold $\hat{\delta} \in [0,1]$ and a nonempty interval Λ such that sets $\mathscr{I}_{\mathrm{P}}(\alpha,r,D) = \{(\eta,\lambda,\delta) \in \mathscr{I} : \lambda \in \Lambda(\eta,\delta,\alpha,r,D), \delta \geq \hat{\delta}(\eta,\alpha,r,D), \eta \leq 3/(8[2\alpha-1]^+)\}$ and $\mathscr{I}_{\mathrm{N}}(\alpha,r,D) = \{(\eta,\lambda,\delta) \in \mathscr{I} : \lambda \notin \Lambda(\eta,\delta,\alpha,r,D), \delta \geq \hat{\delta}(\eta,\alpha,r,D), \eta \leq 3/(8[2\alpha-1]^+)\}$ are nonempty. The individuals in $\mathscr{I}_{\mathrm{P}}(\alpha,r,D)$ exhibit PTC effect, whereas those in $\mathscr{I}_{\mathrm{N}}(\alpha,r,D)$ do not exhibit that effect. The model satisfies* (H1).

(iii) *Given α_{LM} and α_{HM}, $\mathscr{I}_{\mathrm{P}}(\alpha_{\mathrm{LM}},r,D)$ and $\mathscr{I}_{\mathrm{P}}(\alpha_{\mathrm{HM}},r,D)$ are overlapping. The model satisfies* (H2).

The structure here is similar to that in Proposition 1. We therefore desist from further discussion of this result.

2.2. Fixed Reference Point (FRP) Models

We now investigate the PT models with reference points of the form $\mathscr{P}(x,q) = \pi$ for all x and q, where π is an exogenously fixed and constant payoff. Such reference points are compatible with the traditional PT framework (Kahneman and Tversky 1979, Tversky and Kahneman 1992), which accounts only for the gain-loss component of utility. As mentioned earlier, we analyze a set of *salient payoffs* in a newsvendor experiment as plausible candidate reference points. The importance of salience in decision making is widely documented in cognitive psychology (Kahneman 2011) and in behavioral economics (e.g., Chetty et al. 2009, Bordalo et al. 2012).

An obviously salient candidate for the reference point is the *status quo* (i.e., zero payoff). However, Schweitzer and Cachon (2000) show that this reference point is inconsistent with their findings (specifically, the PTC effect) from experiments in which subjects make only positive payoffs. Since this reference point cannot accommodate heterogeneity, we do not explore it further.

Another salient payoff in a newsvendor problem is $(r-c)d_{\min}$. This corresponds to a "MaxMin" reference point (Baillon et al. 2015) – that is, the maximum outcome subjects can *certainly* obtain (which is achieved by ordering $d_{\min}$). Based on experimental and anecdotal evidence from Hershey and Schoemaker (1985) and van Osch et al. (2006), Baillon et al. (2015) argue that "people are looking for security. In a comparison between two prospects, people look at the minimum outcomes of the two prospects and take the maximum of these as their reference point" (p. 10). For this reason, it is also known as a *security-based* reference point. (In the newsvendor setting, this reference point coincides with the MinMax reference point discussed by Baillon et al. (2015). The maximum payoff with a quantity q is $(r-c)q$; its minimum across all quantities is $(r-c)d_{\min}$.) A third salient payoff in the newsvendor setting is $(r-c)d_{\max}$, which is the maximum payoff a subject could earn in the newsvendor experiment. This reference point could serve as a goal (Heath et al. 1999) or as an aspiration level (Diecidue and Van De Ven 2008) of the subjects. Still another salient payoff is $rd_{\min} - cd_{\max}$, which is the minimum payoff a subject could earn in the experiment.

In short, we consider three fixed reference points: (i) $(r-c)d_{\min}$, (ii) $(r-c)d_{\max}$, and (iii) $rd_{\min} - cd_{\max}$. The following proposition states that the latter two candidates cannot predict heterogeneity.

Proposition 3. *Let q^* be the optimal order quantity of $V(q)$.*

(i) *If $\pi = rd_{\min} - cd_{\max}$, then $q^* \leq \tilde{q}_{\text{EP}}$ for all α.*

(ii) *If $\pi = (r-c)d_{\max}$, then $q^* \geq \tilde{q}_{\text{EP}}$ for all α.*

When the reference point is $rd_{\min} - cd_{\max}$ (resp., $(r-c)d_{\max}$), all possible payoffs in a newsvendor problem are perceived as gains (resp., losses) and the resultant optimal order quantity q^* is always less (resp., greater) than the expected profit-maximizing quantity $\tilde{q}_{\text{EP}}$. These two reference points fail to satisfy (H2); therefore, they cannot accommodate heterogeneity, and are not explored further in this paper. As we show next, however, this is not the case when $\pi = (r-c)d_{\min}$. Because the FRP models characterize individuals using parameters λ and δ, here $\mathcal{I} = \mathcal{L} \times \mathcal{D}$.

Proposition 4. *Let $\pi = (r-c)d_{\min}$. Then the following statements hold:*

(i) *The resultant value function $V(q)$ is submodular in (q, λ) and unimodal in q.*

(ii) *Given* (α, r, D)*, there exists a threshold* $\hat{\delta} \in [0,1]$ *and a nonempty interval* Λ *such that sets* $\mathcal{I}_{\mathrm{P}}(\alpha, r, D) = \{(\lambda, \delta) \in \mathcal{I} : \lambda \in \Lambda(\delta, \alpha, r, D), \delta \geq \hat{\delta}(\alpha, r, D)\}$ *and* $\mathcal{I}_{\mathrm{N}}(\alpha, r, D) = \{(\lambda, \delta) \in \mathcal{I} : \lambda \notin \Lambda(\delta, \alpha, r, D), \delta \geq \hat{\delta}(\alpha, r, D)\}$ *are nonempty. The individuals in* $\mathcal{I}_{\mathrm{P}}(\alpha, r, D)$ *exhibit PTC effect, whereas those in* $\mathcal{I}_{\mathrm{N}}(\alpha, r, D)$ *do not exhibit that effect. The model satisfies* (H1)*.*

(iii) *Given* α_{LM} *and* α_{HM}*,* $\mathcal{I}_{\mathrm{P}}(\alpha_{\mathrm{LM}}, r, D)$ *and* $\mathcal{I}_{\mathrm{P}}(\alpha_{\mathrm{HM}}, r, D)$ *are overlapping. The model satisfies* (H2)*.*

Although the structure here is similar to that of Propositions 1 and 2, the model with $(r-c)d_{\min}$ as the reference point does not rely on consumption utility (and parameter η) to predict heterogeneity; incorporating it, however, does not affect the heterogeneity prediction of the model. Thus, it is a more parsimonious representation of newsvendor behavior than are its PRP counterparts.

Remark 1. Using the model in their paper, Long and Nasiry (2015) conclude that prospect theory can explain the pull-to-center effect. Strictly speaking, however, the claims made by Schweitzer and Cachon (2000) and Nagarajan and Shechter (2014) are based on the traditional PT model; these claims are not invalidated by Long and Nasiry's results because that model does *not* fit into the traditional PT framework. The model with $(r-c)d_{\min}$ as a fixed reference point actually reconciles the traditional prospect theory model and the PTC effect.

2.3. Stochastic Reference Point (SRP) Models

The information in several newsvendor experiments is presented in terms of demand distribution rather than payoff distribution. It is therefore important to consider the effect of salient pieces of demand-related information on newsvendor decisions. In the case of a uniform distribution, subjects are presented with the two endpoints of the support. By the focusing effect (Kahneman et al. 1982), such a representation makes these quantities explicitly salient. Another quantity that is implicitly salient in this setting is the mean demand, since it is the midpoint between the two reported endpoints. In this section, we explore $d_{\min}$, $d_{\max}$ and μ as our candidate reference points.

An important distinction here, however, is that these reference points (unlike those in Section 2.2) are prospects themselves. We can accommodate a prospect as reference point by employing the stochastic reference point framework (Sugden 2003, Kőszegi and Rabin 2006, De Giorgi and Post 2011). In particular, we use the model developed by Sugden (2003), under which any given prospect is evaluated based on its relative gain-loss utility with respect to the reference prospect.[5] We next derive the gain-loss utility for the above discussed reference points.

[5] Formally, the model here is obtained by setting $v(x,z) = (x-z)^{\delta}$ in (1) of Sugden (2003). Other formulations of SRP models (e.g., Kőszegi and Rabin 2006, De Giorgi and Post 2011) usually include consumption utility along with gain-loss utility, much as in the PRP models. Including consumption utility in our paper's SRP models does not change their theoretical predictions. It is also typical for papers that discuss SRPs to discuss also the concept of personal equilibrium (PE), which is seen as endogenously determining the reference point (see Kőszegi and Rabin 2006 and De Giorgi and Post 2011 for the definition of PE). Although PE is normatively appealing, from an empirical standpoint it is not robust (Wenner 2015, Sprenger 2015).

2.3.1. Maximum Demand as SRP. The gain-loss utility of an order quantity q is defined as the relative gain or loss experienced by the newsvendor when she orders that quantity as compared with when she orders her reference quantity (here, $d_{\max}$). Formally, $\mathscr{P}(x,q) = \tau(x, d_{\max})$ for all q, where $\tau(x,q)$ is as defined in (1). Therefore, contingent on the realized demand x, the relative gain-loss is equal to: (a) $(rx - cq) - (rx - cd_{\max}) = c(d_{\max} - q) > 0$ when $d_{\min} \le x < q$; (b) $(rq - cq) - (rx - cd_{\max}) > 0$ when $q \le x < \alpha d_{\max} + (1-\alpha)q$; and (c) $(rq - cq) - (rx - cd_{\max}) \le 0$ when $\alpha d_{\max} + (1-\alpha)q \le x \le d_{\max}$. The resultant value function is given by

$$V(q) = \int_{d_{\min}}^{q} c^{\delta}(d_{\max} - q)^{\delta}\, dF(x) + \int_{q}^{q_{\bar{\alpha}}} r^{\delta}(q_{\bar{\alpha}} - x)^{\delta}\, dF(x) - \lambda \int_{q_{\bar{\alpha}}}^{d_{\max}} r^{\delta}(x - q_{\bar{\alpha}})^{\delta}\, dF(x), \tag{6}$$

where $q_{\bar{\alpha}} = \alpha d_{\max} + (1-\alpha)q$. This leads to our next proposition. Note that $\mathscr{I} = \mathscr{L} \times \mathscr{D}$.

Proposition 5. *Let $\tau(x, d_{\max})$ be the reference point. Then the following statements hold:*

(i) *The value function $V(q)$ is supermodular in (q, λ) and unimodal in q.*

(ii) *Given (α, r, D), there exists a threshold $\hat{\delta} \in [0,1]$ and a nonempty interval Λ such that sets $\mathscr{I}_{\mathrm{P}}(\alpha, r, D) = \{(\lambda, \delta) \in \mathscr{I} : \lambda \in \Lambda(\delta, \alpha, r, D), \delta \ge \hat{\delta}(\alpha, r, D)\}$ and $\mathscr{I}_{\mathrm{N}}(\alpha, r, D) = \{(\lambda, \delta) \in \mathscr{I} : \lambda \notin \Lambda(\delta, \alpha, r, D), \delta \ge \hat{\delta}(\alpha, r, D)\}$ are nonempty. The individuals in $\mathscr{I}_{\mathrm{P}}(\alpha, r, D)$ exhibit PTC effect, whereas those in $\mathscr{I}_{\mathrm{N}}(\alpha, r, D)$ do not exhibit that effect. The model satisfies* (H1).

(iii) *Given α_{LM} and α_{HM}, $\mathscr{I}_{\mathrm{P}}(\alpha_{\mathrm{LM}}, r, D)$ and $\mathscr{I}_{\mathrm{P}}(\alpha_{\mathrm{HM}}, r, D)$ are overlapping. The model satisfies* (H2).

The structure seen here is again similar to previous propositions except that $V(q)$ is now supermodular in (q, λ), from which it follows that $q^*(\lambda)$ increases with λ (Topkis 1998). The reason is that, as her loss aversion increases, the decision maker becomes more reluctant to deviate much from her reference point (here, equal to $d_{\max}$) and so orders close to that reference point.

2.3.2. Minimum Demand as SRP. Here, $\mathscr{P}(x,q) = \tau(x, d_{\min})$ for all q. Because a newsvendor who orders quantity $d_{\min}$ earns $(r-c)d_{\min}$ in every state of the world, i.e., $\tau(x, d_{\min}) = (r-c)d_{\min}$ for all x, it follows that this model is equivalent to the model with $(r-c)d_{\min}$ as FRP, which we analyzed in Section 2.2.

2.3.3. Mean Demand as SRP. Under this model, we have $\mathscr{P}(x,q) = \tau(x, \mu)$ for all q. The relative gain-loss when $m(x)$ is the reference point can be written using the same technique as in Section 2.3.1. The value obtained from ordering a quantity $q < \mu$, denoted $V_{<\mu}(q)$, is given by

$$\begin{aligned} V_{<\mu}(q) = {} & \int_{d_{\min}}^{q} c^{\delta}(\mu - q)^{\delta}\, dF(x) + \int_{q}^{q_{\bar{\alpha}}} r^{\delta}(q_{\bar{\alpha}} - x)^{\delta}\, dF(x) \\ & - \lambda \int_{q_{\bar{\alpha}}}^{\mu} r^{\delta}(x - q_{\bar{\alpha}})^{\delta}\, dF(x) - \lambda \int_{\mu}^{d_{\max}} (r-c)^{\delta}(\mu - q)^{\delta}\, dF(x), \end{aligned} \tag{7}$$

where $q_{\bar{\alpha}} = \alpha\mu + (1-\alpha)q$. When a quantity $q > \mu$ is ordered, the value is

$$\begin{aligned} V_{>\mu}(q) = & -\lambda \int_{d_{\min}}^{\mu} c^{\delta}(q-\mu)^{\delta}\, dF(x) - \lambda \int_{\mu}^{q_\alpha} r^{\delta}(q_\alpha - x)^{\delta}\, dF(x) \\ & + \int_{q_\alpha}^{q} r^{\delta}(x-q_\alpha)^{\delta}\, dF(x) + \int_{q}^{d_{\max}} (r-c)^{\delta}(q-\mu)^{\delta}\, dF(x); \end{aligned} \tag{8}$$

here $q_\alpha = (1-\alpha)\mu + \alpha q$. We use $V(q)$ to denote the overall value function; it is equal to $V_{<\mu}(q)$ for $q < \mu$ or to $V_{>\mu}(q)$ otherwise. Note that $V(q)$ is continuous. The following proposition characterizes the shapes of $V_{<\mu}(q)$ and $V_{>\mu}(q)$ and also shows that this model predicts heterogeneity.

Proposition 6. *Let $\tau(x,\mu)$ be the reference point. Then the following statements hold:*

(i) *$V_{<\mu}(q)$ is supermodular in (q,λ). There exists a threshold $\hat{\lambda}_{<\mu}$ such that $V_{<\mu}(q)$ is unimodal (i.e., has a unique interior mode) if $\lambda < \hat{\lambda}_{<\mu}$, otherwise it is increasing in its domain $[d_{\min}, \mu)$.*

(ii) *$V_{>\mu}(q)$ is submodular in (q,λ). There exists a threshold $\hat{\lambda}_{>\mu}$ such that $V_{>\mu}(q)$ is unimodal if $\lambda < \hat{\lambda}_{>\mu}$, otherwise it is decreasing in its domain $(\mu, d_{\max}]$.*

(iii) *Given (α, r, D), there exists thresholds $\hat{\lambda}_{\mathrm{P}} \geq 0$, $\hat{\lambda}_{\mathrm{N}} \geq 0$ and $\hat{\delta} \in [0,1]$ such that sets $\mathcal{G}_{\mathrm{P}}(\alpha,r,D) = \{(\lambda,\delta) \in \mathcal{G} : \lambda \geq \hat{\lambda}_{\mathrm{P}}(\delta,\alpha,r,D), \delta \geq \hat{\delta}(\alpha,r,D)\}$ and $\mathcal{G}_{\mathrm{N}}(\alpha,r,D) = \{(\lambda,\delta) \in \mathcal{G} : \lambda \leq \hat{\lambda}_{\mathrm{N}}(\delta,\alpha,r,D), \delta \geq \hat{\delta}(\alpha,r,D)\}$ are nonempty. The individuals in $\mathcal{G}_{\mathrm{P}}(\alpha,r,D)$ exhibit PTC effect, whereas those in $\mathcal{G}_{\mathrm{N}}(\alpha,r,D)$ do not exhibit that effect. The model satisfies* (H1).

(iv) *Given α_{LM} and α_{HM}, $\mathcal{G}_{\mathrm{P}}(\alpha_{\mathrm{LM}},r,D)$ and $\mathcal{G}_{\mathrm{P}}(\alpha_{\mathrm{HM}},r,D)$ are overlapping. The model satisfies* (H2).

We can use this result to characterize the shape of $V(q)$. First, we define $\hat{\lambda}_1 = \min\left(\hat{\lambda}_{<\mu}, \hat{\lambda}_{>\mu}\right)$ and $\hat{\lambda}_2 = \max\left(\hat{\lambda}_{<\mu}, \hat{\lambda}_{>\mu}\right)$. Then Proposition 6 shows that for $\lambda < \hat{\lambda}_1$, both $V_{<\mu}(q)$ and $V_{>\mu}(q)$ are unimodal, hence $V(q)$ is bimodal. As λ increases, the modes of both these functions approach μ owing to supermodularity of $V_{<\mu}$ and submodularity of $V_{>\mu}$. (This is because a subject's reluctance to deviate strongly from the reference point increases with her loss aversion.) For the newsvendor in a low-margin setting, the mode of $V_{>\mu}$ is exactly equal to μ when $\lambda = \hat{\lambda}_1$. For $\hat{\lambda}_1 < \lambda < \hat{\lambda}_2$, $V_{>\mu}(q)$ is decreasing in its domain whereas $V_{<\mu}(q)$ remains unimodal in its domain. Thus $V(q)$ is unimodal for these values of λ with its mode equal to the mode of $V_{<\mu}(q)$. At $\lambda = \hat{\lambda}_2$, the mode of $V_{<\mu}(q)$ is exactly equal to μ. Hence for $\lambda > \hat{\lambda}_2$, we see that $V_{<\mu}(q)$ is increasing and $V_{>\mu}(q)$ is decreasing in their respective domains; therefore, $V(q)$ is again unimodal but with mode now at μ.

The condition $\delta \geq \hat{\delta}$ in part (iii) ensures that the mode of $V_{<\mu}(q)$ is below $\tilde{q}_{\mathrm{LM}}$ when $\lambda = 0$. Then, there exists a unique threshold $\hat{\lambda}_{\mathrm{N}} \geq 0$ such that when $\lambda \leq \hat{\lambda}_{\mathrm{N}}$, $V(q)$ is either bimodal or unimodal with mode(s) *outside* the PTC zone; and when $\lambda \geq \hat{\lambda}_{\mathrm{P}} \equiv \max(\hat{\lambda}_{\mathrm{N}}, \hat{\lambda}_1)$, $V(q)$ is unimodal with mode *inside* the PTC zone. Therefore, subjects in the set $\mathcal{G}_{\mathrm{P}} = \{(\lambda,\delta) \in \mathcal{G} : \lambda \geq \hat{\lambda}_{\mathrm{P}}, \delta \geq \hat{\delta}\}$ exhibit the PTC effect whereas those in the set $\mathcal{G}_{\mathrm{N}} = \{(\lambda,\delta) \in \mathcal{G} : \lambda \leq \hat{\lambda}_{\mathrm{N}}, \delta \geq \hat{\delta}\}$ do not. Because these sets are nonempty, the model satisfies (H1) in low-margin settings. We can similarly interpret the result in a high-margin setting. Now for arbitrarily given α_{LM} and α_{HM}, $\mathcal{I}_{\mathrm{P}}(\alpha_{\mathrm{LM}}) \cap \mathcal{I}_{\mathrm{P}}(\alpha_{\mathrm{HM}}) = \{(\lambda, \delta) \in \mathcal{I} : \lambda \geq \max(\hat{\lambda}_{\mathrm{P}}(\alpha_{\mathrm{LM}}), \hat{\lambda}_{\mathrm{P}}(\alpha_{\mathrm{HM}})), \delta \geq \max(\hat{\delta}(\alpha_{\mathrm{LM}}), \hat{\delta}(\alpha_{\mathrm{HM}}))\}$. Since this set is also nonempty, the model satisfies (H2), and hence accommodates heterogeneity in newsvendor behavior.

3. Model Evaluation

In Section 2 we studied models that, in theory, predict heterogeneity in the newsvendor problem. These models were based on prospect theory (and its extensions) but differed in their reference point assumptions. Now we investigate which of these models better represents the observed newsvendor behavior. For this purpose we adopt a multipronged approach described below.

Model	Reference point	$\mathcal{P}(x,q) =$	Type
M1	Weighted avg. of max. and min. profits	$\Omega(q)$	PRP
M2	Expected profit	$\Pi(q)$	PRP
M3	Sure-shot profit	$(r-c)d_{\min}$	FRP
M4	Maximum demand	$\tau(x, d_{\max})$	SRP
M5	Mean demand	$\tau(x, \mu)$	SRP

Table 1 Competing models.

Since our objective is to investigate models of newsvendor behavior that could be used for analysis in broader settings such as inventory and supply chain management, we first evaluate models based on some *theoretical criteria*. The fundamental theoretical criterion in this paper, which is already imposed on the models in Section 2, is that they should predict heterogeneity in newsvendor behavior. Table 1 lists the models that satisfy this criterion. In addition, we consider a model to be better (i) if its predictions in certain corner cases are consistent with what we would expect to happen intuitively in those cases, and (ii) if it makes these predictions as parsimoniously as possible. In Section 3.1 we compare the competing models based on these criteria. This exercise also reveals some limitations of PRP models in modeling newsvendor behavior.

In addition to being theoretically sound, we expect a good behavioral newsvendor model to fit the actually observed behavior well. In Section 3.2, using the experimental data from existing studies, we evaluate the models based on several *goodness-of-fit* measures both (i) at the population level, and (ii) at the individual level. Evaluating a model based on quality of fit is popular both in the behavioral economics literature (e.g., Hardie et al. 1993, Baillon et al. 2015) and in the OM literature (e.g., Su 2008, Hasija et al. 2010, Elmaghraby et al. 2015). The population-level analysis fits the data assuming that all the subjects share the same reference point, hence the corresponding inferences will be useful in settings where we approximate the population with a single reference point model. In contrast, the individual-level analysis fits each subject's data separately, thereby revealing the extent of reference point heterogeneity in the population. Both analyses complement each other and reveal to what extent the model performance replicates over the two levels of analysis. We also show an application of the population-level and individual-level empirical analyses in a supplier-retailer relationship setting in Section 4.2.

Finally, we test the *empirical validity* of the models by validating their predictions vis-à-vis some observations from the data (much in the spirit of Abeler et al. 2011, Wenner 2015, Sprenger 2015). To that end, in Section 3.3 we identify three empirical regularities in the newsvendor experiments: (i) the aggregate-level order quantity is convex-concave shaped in α, (ii) the modal decision in several experiments tends to be mean demand, and (iii) the performance gap (defined as the difference between the optimal expected profit and the actual realized profit) follows an M-shape in α. Not all competing models yield predictions that are consistent with these empirical observations, so they are useful in invalidating some models.

3.1. Credible Corner Case Predictions and Model Parsimony

Consider a newsvendor with $\delta = 1$. We now analyze the behavior of the optimal order quantity q_m^* predicted by model m for this newsvendor when some parameters are taken to their extreme values. For a given unit revenue r, we would expect a credible newsvendor decision model to predict very low order quantities when the product's cost is very high, i.e., $\lim_{c\to r} F(q_m^* \mid \delta = 1) = 0$. We also would expect such model to predict very high order quantities when the product's cost is very low, i.e., $\lim_{c\to 0} F(q_m^* \mid \delta = 1) = 1$. Next consider a newsvendor who – in addition to being risk neutral ($\delta = 1$) – is also loss neutral ($\lambda = 1$). Since nonneutral attitudes toward risk and reference effects (which play a role through loss aversion) are the only behavioral features accounted for in the models considered in our paper, these features should disappear – and hence we should recover the expected profit-maximization – when we set $\lambda = 1$ and $\delta = 1$. We, therefore, expect this newsvendor to order the expected profit-maximizing quantity, i.e., $\lim_{\lambda\to 1} F(q_m^* \mid \delta = 1) = 1 - \alpha$. Table 2 displays the values of the three limits just discussed for models M1–M5 along with the values we would intuitively expect them to take.

	Expected prediction	Prediction of model m				
		M1	M2	M3	M4	M5
$\lim_{c\to r} F(q_m^* \mid \delta = 1)$	0	$\dfrac{(1-\beta)}{\eta+1+(\lambda-1)\beta^2}$	0	0	0	0
$\lim_{c\to 0} F(q_m^* \mid \delta = 1)$	1	$\dfrac{\eta+(1-\beta)}{\eta+1+(\lambda-1)\beta^2}$	$1-\sqrt{\left[1-\dfrac{2\eta}{[\lambda-1]^+}\right]^+}$	1	1	1
$\lim_{\lambda\to 1} F(q_m^* \mid \delta = 1)$	$1-\alpha$	$\dfrac{\eta(1-\alpha)+(1-\beta)}{\eta+1}$	$1-\alpha$	$1-\alpha$	$1-\alpha$	$1-\alpha$

Table 2 Corner case predictions of the competing models when $\delta = 1$.

The predictions of models M3–M5 are consistent with what we would expect intuitively in all three cases, whereas the predictions of models M1–M2 are inconsistent in some cases. In fact, depending on the values of the parameters, the limits predicted by M1 and M2 could be significantly different from their intuitively expected counterparts. For instance, if $c = r$, $\lambda = 2$, and $\beta = 1/3$, then, for any $\eta > 4.5$, q_m^* predicted by M1 is greater than mean demand; if $c = 0$, $\lambda = 2$, and $\beta = 1/2$, then, for $\eta > 4$, M1 predicts q_m^* that is less than μ. These predictions are clearly unrealistic.

Why, then, do both the PRP models give such unrealistic predictions? As evident from (4), the reference dependence of $V(q)$ under M1 and M2 is only through terms $\Omega(q) + cq$ and $\Pi(q) + cq$ respectively (only these terms appear in the integrand and in the integration limits). Yet from (3) and (5) it follows that $\Omega(q) + cq = \beta rq + (1 - \beta)rd_{\min}$ and $\Pi(q) + cq = \int_{d_{\min}}^{q} rx\, dF(x) + rq\bar{F}(q)$, which are *independent* of c. As a result, the reference effects in these models are independent of the economic trade-off involved in the newsvendor problem, which leads to the unrealistic predictions shown in Table 2. In general, the reference effects are independent of cost under any PRP model in which $\mathscr{P}(x, q) + cq$ is independent of c. Although such PRP models may be appropriate in other contexts, our analysis indicates that they may be inappropriate in the newsvendor context.

The dependence of $V(q)$ on product cost in these models is *only* through the consumption component of utility. Hence this component is necessary if q_m^* is to depend on c. In fact, the newsvendor with $\eta \to 0$ gives so little weight to the consumption component that her optimal order does not depend on c. Because of this dependence on consumption utility – and thus on η, which balances the weight between consumption and gain-loss components – M1 and M2 are a less parsimonious representation of newsvendor behavior than are M3–M5. M2 is more parsimonious than M1 because the latter uses an additional parameter β to model the newsvendor behavior.

3.2. Goodness of Fit to Experimental Data

Now we use the maximum likelihood estimation technique to evaluate the models in Table 1 based on the quality of their fit to the *actual* heterogeneity in the newsvendor experimental data. (Since PRP models yield unrealistic predictions only when the parameters approach corner cases, and not when they take intermediate values, we include them in this analysis.) We use two sources of experimental data to test the fit of the candidate models. The first is Study 1 of Bolton and Katok (2008), in which 20 subjects were presented with a low-margin newsvendor problem and 18 with a high-margin problem – both for 100 seasons of demand. In the low-margin case, revenue $r = 12$, cost $c = 9$, and demand $X \sim \mathrm{U}(50, 150)$; in the high-margin case, $r = 12$, $c = 3$, and $X \sim \mathrm{U}(0, 100)$. Our second data source is a replication of the first by Lau, Hasija, and Bearden (2014), where 94 subjects were presented with a low-margin problem and 80 subjects with a high-margin problem, both for 20 seasons of demand. We shall refer to these studies (and the corresponding data sets) as BK and LHB, respectively. For summary statistics of the two studies, see Lau et al. (2014).

We chose these particular data sets for two main reasons. First, BK is a controlled lab experiment whereas the subjects in LHB were recruited from Amazon Mechanical Turk. Thus we can check the extent to which our results can be replicated across such diverse environments. Second, both these studies use a uniform demand distribution; that directly fits the analyses in Long and Nasiry (2015) and in this paper.

We incorporate consumption utility for all the models in our empirical analysis. Although M3–M5 do not require consumption utility to theoretically predict heterogeneity in newsvendor behavior, including it (and hence the parameter η) makes the estimation process consistent, and the comparison fairer across the models. As a robustness check, we also estimated M3–M5 with η set to zero, but this does not affect the conclusions drawn in this section.

3.2.1. Population-Level Estimation. In this section we compare models based on their fit to the data from the entire subject pool. The objective of this analysis is to investigate which reference point model best fits the observed newsvendor behavior at the *population level*.

Likelihood functions. We represent the data set as the set of order quantities $\{q_{ij}\}$ placed by subjects $i = 1, \ldots, m$ in seasons $j = 1, \ldots, n$. These values are $m = 38$ and $n = 100$ for BK and $m = 174$ and $n = 20$ for LHB. To write the likelihood function for model $\mathcal{M} \in \{\text{M1}, \text{M2}, \text{M3}, \text{M4}, \text{M5}\}$, we assume that $q_{ij} = q^*_{i,\mathcal{M}} + \varepsilon_{ij,\mathcal{M}}$, where $q^*_{i,\mathcal{M}}$ is the optimal order quantity predicted by $\mathcal{M}$ for subject i and where $\varepsilon_{ij,\mathcal{M}} \sim \text{N}(0, \sigma^2_{\mathcal{M}})$ is an independent and identically distributed (i.i.d.) zero-mean normal error term. For reliable estimation based on the limited experimental data available, we reduce the number of parameters to be estimated by assuming that all subjects have a linear utility function (i.e., $\delta = 1$) and the error variance $\sigma^2_{\mathcal{M}}$ is homogeneous across the subject pool.

Given the experimental condition α_i, $q^*_{i,\mathcal{M}}$ for subject i is a function of her loss aversion parameter λ_i. Since the λ_i are not given exogenously for experimental subjects, we must estimate those values endogenously from the data set. Yet that approach would result in too many parameters to be estimated; therefore, instead of estimating subject-level λ_i values, we assume that these λ_i are drawn from a distribution and then estimate the parameters of this distribution. More specifically, we assume that $\lambda_i \sim \text{Gamma}(a_{\mathcal{M}}, b_{\mathcal{M}})$, where $a_{\mathcal{M}}$ and $b_{\mathcal{M}}$ are (respectively) the shape and inverse scale parameters. If we now use the i.i.d. property of ε_{ij} and assume that – given the set of parameters $\mathbb{P}_{\mathcal{M}}$ of model $\mathcal{M}$ – the order quantities of a subject are conditionally independent of others' order quantities, then the data set's likelihood function is given by

$$\mathcal{L}_{\mathcal{M}}(\{q_{ij}\} \mid \{\alpha_i\}, \mathbb{P}_{\mathcal{M}}) = \prod_{i=1}^{m} \int_0^{\infty} \frac{1}{\left(\sqrt{2\pi}\sigma_{\mathcal{M}}\right)^n} \exp\left\{ -\sum_{j=1}^{n} \frac{(q_{ij} - q^*_{\mathcal{M}}(\lambda \mid \alpha_i, \mathbb{P}_{\mathcal{M}}))^2}{2\sigma^2_{\mathcal{M}}} \right\} f_\gamma(\lambda \mid a_{\mathcal{M}}, b_{\mathcal{M}})\, d\lambda.$$

Here f_γ is the gamma distribution's density function, and $q^*_{\mathcal{M}}(\lambda \mid \alpha_i, \mathbb{P}_{\mathcal{M}})$ is the optimal order quantity predicted by $\mathcal{M}$ under experimental condition α_i when loss aversion is equal to λ. (Because the value function under M5 could be bimodal, we use the mode that maximizes the subject's likelihood.) The set $\mathbb{P}_{\mathcal{M}} = \{\eta_{\mathcal{M}}, \sigma_{\mathcal{M}}, a_{\mathcal{M}}, b_{\mathcal{M}}\}$ for $\mathcal{M} \in \{\text{M2}, \text{M3}, \text{M4}, \text{M5}\}$, whereas $\mathbb{P}_{\text{M1}} = \{\eta_{\text{M1}}, \beta_{\text{M1}}, \sigma_{\text{M1}}, a_{\text{M1}}, b_{\text{M1}}\}$. For estimation purposes, we assume that β_{M1} and $\eta_{\mathcal{M}}$ are also homogeneous across the subject pool.

Estimation. For all the competing models, the parameters in $\mathbb{P}_m$ cannot all be estimated simultaneously by maximizing $\mathcal{L}_m$ because the corresponding systems of first-order conditions are overdefined. (Detailed analysis is available on request.) With an exogenously determined η_m, however, the systems are perfectly defined. We therefore employ the method of *cross-validation* – a technique heavily used in the machine learning literature to estimate "hyperparameters" (Hastie et al. 2001) – for estimating the parameters of these models. We randomly split the data set into a training set, a cross-validation set, and a test set in respective proportions of 60%, 20%, and 20%. (We pool the low- and high-margin data for estimation.) The result is 22-8-8 split among subjects across the training, cross-validation, and test sets for BK, and a 104-35-35 split for LHB. We first fix a value of η_m; next we estimate the remaining parameters $\mathbb{P}_m - \{\eta_m\}$ by maximizing the log-likelihood on the training set. Using these estimates, we then compute the log-likelihood on the validation set. Note that this log-likelihood is a function of the initially chosen η_m. We repeat this process by varying η_m and then choose the value that maximizes the log-likelihood on the validation set. Using this *optimal* value of η_m and the corresponding estimates of remaining parameters, we compute the log-likelihood on the test set which is used for the purpose of comparison across the models.

All the estimation steps just described are taken for a single random split of the data set. Because this method yields only one comparative log-likelihood value, we repeat the process of randomly splitting the data set 1,000 times and obtain the sampling distribution of $\log \mathcal{L}_m$ using the 1,000 log-likelihood values computed on randomly chosen test sets. (We use the same random splitting of the data set for all the models.) Since these test sets are chosen randomly, the corresponding log-likelihood is a random variable and the distribution obtained is the distribution of this random variable. The resultant sampling distributions form the basis of our model comparison exercise.

Since η_m is unbounded from above, in theory we should explore all the values of η_m in $[0, \infty)$ and choose the one that maximizes the likelihood on the validation set. However, computational limitations require that we restrict the analysis to η_m in $[0, 5]$ with steps of size 0.1. This restriction does not significantly hinder our analysis: over 1,000 random splits, the proportion of optimal η_m values that lie below 4.5 is greater than 94% in BK and 98% in LHB across all the models.

Results. Since we compare the models based on the *out-of-sample* log-likelihoods computed on test sets, we do not need the Akaike (or Bayesian) information criterion (which account for parsimony to avoid overfit) for model comparison. Table 3 presents the mean log-likelihood estimated using BK and LHB for all the models. (The corresponding parameter estimates are presented in Tables 6–10 in Appendix B.) Using a one-sided paired t-test for all possible model pairs (with Holm–Bonferroni correction for multiple testing) and a significance level of 0.01, we infer the ordering M5 > M1 $\approx$ M3 > M2 $\approx$ M4 on BK (all the adjusted p-values are less than 0.001 except for the

	$\log \mathcal{L}_{\text{M1}}$	$\log \mathcal{L}_{\text{M2}}$	$\log \mathcal{L}_{\text{M3}}$	$\log \mathcal{L}_{\text{M4}}$	$\log \mathcal{L}_{\text{M5}}$
BK	−3291.80	−3297.59	−3292.48	−3300.55	−3285.34
	(127.05)	(133.45)	(129.28)	(128.77)	(128.62)
LHB	−3021.93	−3029.01	−3034.50	−3040.01	−3015.13
	(43.79)	(40.43)	(41.70)	(38.26)	(40.11)

Table 3 Means and standard deviations (in parentheses) of the sampling distributions of log-likelihood for the competing models.

comparisons M1 > M3 and M2 > M4, which result in adjusted p-values of 0.154 and 0.097), and the ordering M5 > M1 > M2 > M3 > M4 on LHB (all the adjusted p-values are less than 0.001).

We see that, although the performance of other models varies across the data sets, M5 consistently fits best. This result continues to hold with other goodness of fit measures presented in Appendix C. First, we find that the cumulative distribution function (CDF) of the sampling distribution of the log-likelihood under M5 stochastically dominates (in the first order) the CDFs under other models. In other words, M5 dominates the other models not only based on the mean log-likelihood, but also based on the log-likelihood as a random variable. Second, we evaluate the models using a more intuitive measure (called root-mean-square error) that is based on the distance between the observed and predicted order quantities. We find that the percentage improvement in the prediction error of M5 with respect to other models is 5%–31% on BK and 5%–20% on LHB. Finally, we estimate the models separately for low- and high-margin conditions of both BK and LHB. Consistently with the above analysis, M5 performs the best on all the four conditions.

3.2.2. Individual-Level Estimation. So far our analysis implicitly assumed that the decision making process of all subjects is represented by the same PT model. However, we can go a level deeper and reanalyze the data by assuming that subjects may be heterogeneous in terms of the PT model that best represents their decision making process. To that end, we estimated the model parameters at the individual level. The estimation procedure and results are presented in Appendix D. We find that – although the subject pool is heterogeneous in terms of reference points – a significant majority (50% on BK and 56% on LHB) is classified to be consistent with M5 (i.e., their log-likelihood under M5 is higher than that under other models). Thus, as in the population-level analysis, M5 dominates other models in the individual-level analysis as well.

3.3. Consistency with Some Empirical Observations

Here we test the empirical validity of the competing models with regard to three observations in the newsvendor experiments at the aggregate level.

3.3.1. Pattern of Aggregate-Level Order Quantities. Ockenfels and Selten (2014) identify a unique pattern of aggregate-level order quantities across experimental conditions. The authors conduct a newsvendor experiment with 11 treatment conditions by varying α from zero to one in steps of 0.1, demand $X \sim \mathrm{U}(0,100)$, 340 subjects in total, and 200 rounds per subject. Figure 1(a) shows the normalized order quantities for each α, averaged across rounds and subjects (reported in Table B.1 of Ockenfels and Selten 2014). Figure 1(a) also shows the spline that best fits these order quantities, and as we can see, it is convex-concave in shape. In other words, starting from $\alpha = 0.5$ the average order quantities deviate convexly (resp., concavely) from the expected profit-maximizing orders for $\alpha < 0.5$ (resp., $\alpha \geq 0.5$), and the deviations are close to zero at the extremes $\alpha = 0$ and $\alpha = 1$. Ockenfels and Selten (2014) use this convex-concave pattern (hereafter referred to as cx-cv pattern) to compare several models in their paper; here, we examine whether or not the competing PT models can predict the cx-cv pattern for the average order quantities.

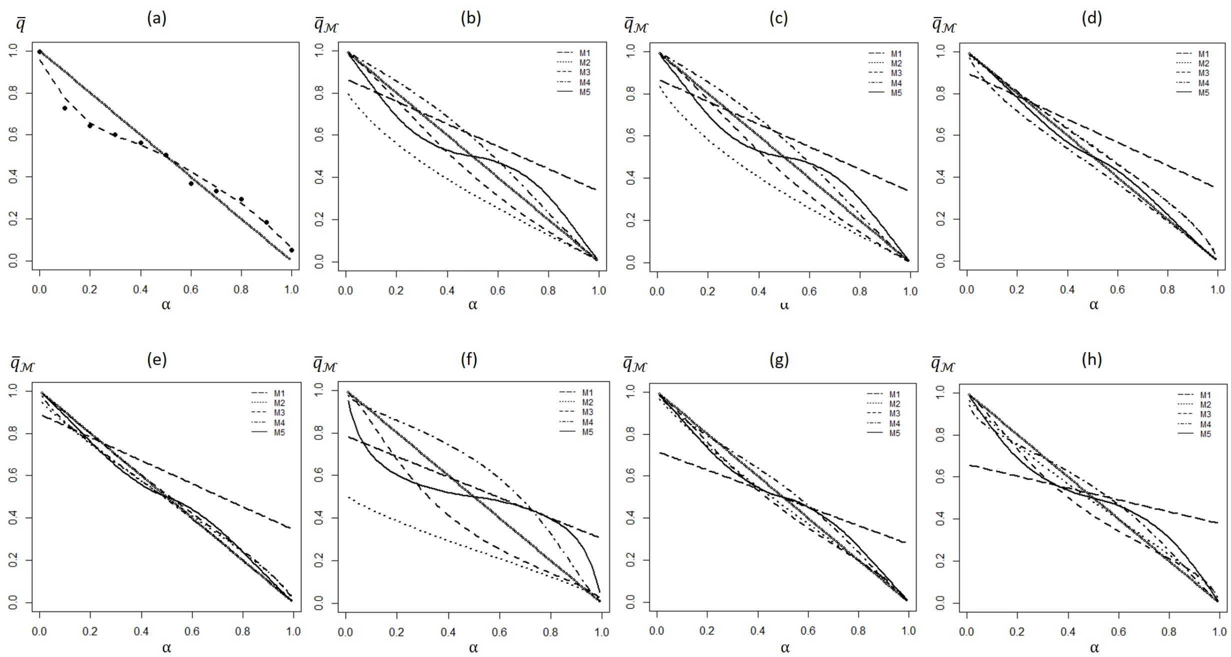

Figure 1 (a) Pattern of average order quantities reported by Ockenfels and Selten (2014). (b)–(f) Patterns predicted by the competing models under the loss aversion distribution elicited by Abdellaoui et al. (2007) using the definition of Kahneman and Tversky (1979), Wakker and Tversky (1993), Bowman et al. (1999), Neilson (2002), and Köbberling and Wakker (2005) respectively. (g)–(h) Patterns predicted by the models under the loss aversion distribution obtaied from BK and LHB respectively.

We define the normalized aggregate order quantity $\bar{q}_{\mathcal{M}}(\alpha)$ predicted by model $\mathcal{M}$ under the experimental condition α as $\bar{q}_{\mathcal{M}}(\alpha) = \frac{1}{D}\int_0^\infty q^*_{\mathcal{M}}(\lambda \mid \alpha, \mathbb{P}_{\mathcal{M}})\, dF_{\mathrm{LA}}(\lambda \mid \mathcal{M})$, where F_{LA} is the distribution of loss aversion, and $q^*_{\mathcal{M}}$ is computed assuming demand $X \sim \mathrm{U}(0,100)$ and $D = 100$ such that $\bar{q}_{\mathcal{M}} \in [0,1]$. We estimate F_{LA} using two methods. First, we follow Abdellaoui et al. (2007), who elicit the individual-level loss aversion parameters using five different definitions of loss aversion from five different studies (Kahneman and Tversky 1979, Wakker and Tversky 1993, Bowman et al. 1999, Neilson 2002, Köbberling and Wakker 2005) and report the estimates in Appendix C of their paper. To these individual-level estimates we fit Gamma(a,b) by maximizing the likelihood. The corresponding best-fitting estimates $(\hat{a},\hat{b})$ are $(2.61,1.21)$, $(3.27,1.63)$, $(1.46,1.96)$, $(1.37,1.28)$, and $(0.55,0.07)$. Second, we use the distributions of λ obtained in Section 3.2.1 with BK and LHB. In total, we have seven different $F_{\rm LA}$ distributions, where the first five are determined exogenously from the existing studies and are independent of models, whereas the remaining two are model-dependent and are determined by fitting the models to the observed order quantities.

For models M1–M5, Figures 1(b)–1(h) plot $\bar{q}_m(\alpha)$ using these seven $F_{\rm LA}$ distributions. In the first five cases we only obtained a and b values exogenously, but computing q^* under M1 and M2 also require η and β values. Therefore, we use the η and β values we obtained from BK for M1 and M2 in Figures 1(b)–1(f). However, the patterns under these models look similar when we use the η and β values obtained using LHB. We set $\eta=0$ for M3–M5 in Figures 1(b)–1(f).

We make the following observations from Figure 1. The average order quantity predicted by M1 is linear in α, which is inconsistent with the cx-cv pattern. Moreover, since the average order is close to $d_{\max}$ when $\alpha=0$, and it is close to $d_{\min}$ when $\alpha=1$, any model that yields a linear relationship cannot simultaneously capture both phenomena, and hence will be inconsistent with the cx-cv pattern. M2 and M3 predict average order quantities which mostly lie below the diagonal, whereas M4's average order quantity predictions mostly lie above the diagonal. Hence, these models are also inconsistent with the cx-cv pattern. Only M5 predicts the desired pattern consistently across all the seven cases. This observation, therefore, provides some empirical consistency for M5.

Remark 2. Predicting the cx-cv pattern in Figure 1(a) encompasses predicting PTC effect at the aggregate level: for all $\alpha<0.5$ the aggregate order quantities lie below the diagonal, whereas for all $\alpha\geq 0.5$ (except for $\alpha=0.6$) they lie above the diagonal.[6] Therefore, M5 offers a reconciliation between the heterogeneity in order quantities observed at the individual level and the PTC effect consistently observed at the aggregate level. The explanation relies on a fact well established in decision theory (Kahneman and Tversky 1979, Abdellaoui et al. 2007): the distribution of λ is right skewed and the population contains more loss-averse individuals (i.e., ones with relatively high values of λ) than gain-seeking individuals (i.e., ones with relatively low values of λ). Recall from Proposition 6 that the former tend to order closer to mean demand than the latter, and hence the former tend to exhibit the PTC effect whereas the latter do not. It follows that the aggregate order quantity averaged over the entire population tends to lie inside the PTC zone.

[6] For $\alpha=0.6$, Ockenfels and Selten (2014) note in their footnote 12 that the observed average order quantity may or may not be an anomaly: "for $z=0.6$ the average order is on the opposite side of the optimal order than predicted by the pull-to-center bias. Clearly more evidence is needed for robust conclusions" (p. 241). For the purposes of our paper, we simply assume that it is an anomaly, and that the cx-cv pattern implies PTC effect at the aggregate level. The spline in Figure 1(a) is drawn by excluding the observation at $\alpha=0.6$.

3.3.2. Mean Demand as Modal Decision. As we just discussed, the pull-to-center effect relates to the consistent observation – notwithstanding individual-level heterogeneity – that the *mean* order quantity across all experimental subjects lies in the PTC zone (Lau et al. 2014). Another such empirical regularity relates to the *modal* order quantity: at the aggregate level, mean demand is consistently observed as the modal decision across several independent studies. Figure 2 presents the histograms of orders placed by subjects in five different studies. We see that mean demand continues to be the modal decision regardless of whether the margin is high or low, regardless of whether demand is uniformly or normally distributed, and irrespective of whether the subjects are undergraduates, MBA students, or individuals laboring via Amazon Mechanical Turk. (The lone exception is the BK low-margin case, where mean demand is the second mode.)

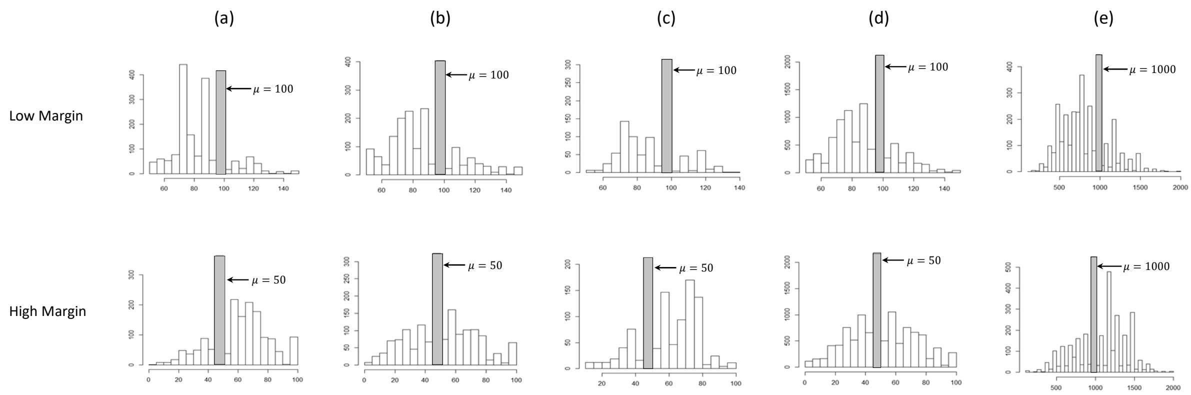

Figure 2 **Histograms of order quantities from (a) Study 1 of Bolton and Katok (2008), (b) its replication by Lau et al. (2014), (c) Study 1 of de Véricourt et al. (2013), (d) Study 2 of de Véricourt et al. (2013), and (e) main study of Rudi and Drake (2014).**

Of all the models we consider, only M5 can *theoretically* predict mean demand as the modal decision across both margin settings. Recall from the discussion following Proposition 6 that for $\lambda > \hat{\lambda}_2$, M5 predicts μ as the optimal order quantity. So given the distribution of λ, it follows that $q^*_{\mathrm{M5}}(\lambda)$ is (a function of) a random variable with a continuous distribution that has a mass point only at $q^*_{\mathrm{M5}}(\lambda) = \mu$ (for all $\lambda > \hat{\lambda}_2$); thus μ is predicted to be the modal quantity at the aggregate level. It is easy to verify that if the underlying distribution of loss aversion is unimodal, then other models can predict mean demand as the modal order quantity in either low- or high-margin setting – but not consistently across both margin settings.

Remark 3. Our claim that the mean demand appears as the modal decision in experiments is based on the histograms presented in Figure 2, and on our implicit assumption that they are representative of other experiments. Almost all of the existing papers report only the mean and median order quantities observed in the experiments, they do *not* report modal order quantity. This makes it difficult to conduct a comprehensive study of modal decisions in the existing experiments. We encourage future research to either strengthen or weaken our claim based on further evidence.

3.3.3. Performance Gap Prediction. We define the performance gap in a newsvendor experiment as the difference between the optimal expected profit that could be made in that experiment and the average realized profit made by the subjects in that experiment. Then the performance gap $\Delta_{\mathcal{m}}(\alpha)$ *predicted* by model $\mathcal{m}$ under the experimental condition α is given by

$$\Delta_{\mathcal{m}}(\alpha) = \Pi(\tilde{q}_{\mathrm{EP}}(\alpha)) - \int_0^\infty \Pi\big(q^*_{\mathcal{m}}(\lambda \mid \alpha, \mathbb{P}_{\mathcal{m}})\big)\, dF_{\mathrm{LA}}(\lambda \mid \mathcal{m}), \tag{9}$$

where Π is the expected profit function as given by (5). Without loss of generality, we normalize $r = 1$ such that $c = \alpha$ in (9). The first term on the right-hand side (RHS) is the optimal expected profit, and the second term is the expected profit made by an average individual in the population who behaves as described by model $\mathcal{m}$. Note that $\Delta_{\mathcal{m}}$ is nonnegative.

Figure 3 plots $\Delta_{\mathcal{m}}$ as a function of α for models M1–M5 when the distribution of λ is estimated using Kahneman and Tversky's (1979) definition (see Section 3.3.1), and the various model parameters reported in the figure apply. Extensive numerical analysis reveals that the shapes for several other parameter sets are similar to those shown in Figure 3. We can see from Figure 3 that the models being compared yield different shapes for $\Delta_{\mathcal{m}}$. We argue next that the values of $\Delta_{\mathcal{m}}(\alpha)$ for $\alpha \in \{0, 0.25, 0.50, 0.75, 1\}$ are sufficient to rule out other models in favor of M5.

Even without the support of any experimental data, we would expect a reasonable model to predict $\Delta_{\mathcal{m}}(\alpha)$ close to 0 as α approaches 0 or 1. Models M1 and M2 are inconsistent with regard to this prediction (for the reasons discussed in Section 3.1), and hence can be ruled out. Among the remaining three models, we see from Figure 3(e) that M5 predicts $\Delta_{\mathrm{M5}}(0.50)$ to be lower than both $\Delta_{\mathrm{M5}}(0.25)$ and $\Delta_{\mathrm{M5}}(0.75)$, which is inconsistent with M3 and M4. (Note that also M2 is inconsistent with this prediction.) So if the experimental data supports this prediction of M5, then we could also rule out M3 and M4. (Model M1 could be consistent with this prediction for some parameters; however, it has already been ruled out based on its predictions when $\alpha \to 0$ or $\alpha \to 1$.)

Next, we test this prediction using the low-margin ($\alpha = 0.75$) and high-margin ($\alpha = 0.25$) experimental data of LHB and also the mid-margin ($\alpha = 0.50$) experimental data of Lau and Bearden (2013); we refer to the latter data set as LB. We choose these particular data sets because the subjects in each were recruited from Amazon Mechanical Turk; we therefore expect them to have similar characteristics across the three margin conditions. Furthermore, if the demand distribution is uniform, then Δ is independent of the endpoints of the support and depends only on the support size D; in these data sets, $D = 100$ for all three margin conditions.

Let $S_{0.25} = \{i : \alpha_i = 0.25\}$ be the set of subjects in the LHB high-margin condition; $S_{0.75}$ is defined similarly using LHB. (There is one subject in the high-margin condition for whom we do not have

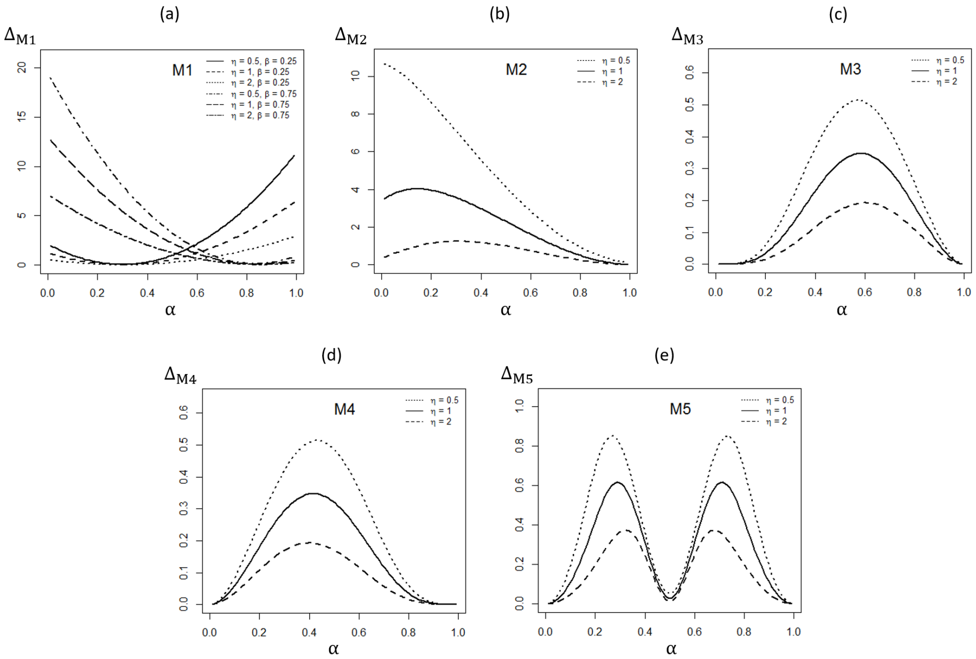

Figure 3 **Δ_m as a function of α plotted with the distribution of λ estimated using Kahneman and Tversky's (1979) definition (see Section 3.3.1), $\delta = 1$, and uniform demand distribution with $D = 100$ for (a) M1 with $\eta_{\text{M1}} \in \{0.5, 1, 2\}$ and $\beta_{\text{M1}} \in \{0.25, 0.75\}$, (b) M2 with $\eta_{\text{M2}} \in \{0.5, 1, 2\}$, (c) M3 with $\eta_{\text{M3}} \in \{0.5, 1, 2\}$, (d) M4 with $\eta_{\text{M4}} \in \{0.5, 1, 2\}$, and (e) M5 with $\eta_{\text{M5}} \in \{0.5, 1, 2\}$.**

the complete realized demand sequence. That subject was therefore removed from our data set.) Let $S_{0.50}$ be the entire subject pool in LB who face the mid-margin condition. For each subject in LHB and LB we use the placed order quantities and the realized demand values to obtain $\hat{\Delta}_i = \Pi(\tilde{q}_{\text{EP}}(\alpha_i)) - \frac{1}{n}\sum_{j=1}^{n}(\min(x_{ij}, q_{ij}) - \alpha_i q_{ij})$; here x_{ij} is the realized demand for subject i in season j. The first term on the RHS is the optimal expected profit under α_i, and the second term is the mean realized profit for subject i. Then the mean *observed* value of $\Delta_m(\alpha)$ is $\bar{\Delta}(\alpha) = \frac{1}{|S_\alpha|}\sum_{i \in S_\alpha} \hat{\Delta}_i$ for $\alpha \in \{0.25, 0.50.0.75\}$. We find that $\bar{\Delta}(0.25) = 4.98$, $\bar{\Delta}(0.50) = 2.40$, and $\bar{\Delta}(0.75) = 3.56$ and that their respective standard deviations are 5.78, 4.13, and 3.55. Using the one-sided two-sample t-test, we find that the mean values of both $\Delta(0.25)$ and $\Delta(0.75)$ are significantly greater ($p < 0.005$) than the mean value of $\Delta(0.50)$, which rules out M3 and M4.

3.4. Model Comparison Summary

Sections 3.1–3.3 rigorously evaluated the competing models based on several theoretical and empirical criteria; our analysis is summarized in Table 4. We find that, although the model of Long and Nasiry (2015) can predict heterogeneity in newsvendor behavior, it yields several unrealistic predictions owing to the prospect dependence of its reference point. Moreover, its fit to data is inferior to some competing models, and its predictions are inconsistent with some empirical observations.

		M1	M2	M3	M4	M5
Theoretical criteria	Heterogeneity in order quantity predictions	✓	✓	✓	✓	✓
	$\lim_{c\to r} F(q_m^* \mid \delta=1)=0$	✗	✓	✓	✓	✓
	$\lim_{c\to 0} F(q_m^* \mid \delta=1)=1$	✗	✗	✓	✓	✓
	$\lim_{\lambda\to 1} F(q_m^* \mid \delta=1)=1-\alpha$	✗	✓	✓	✓	✓
	Parameters in the model (parsimony)*	$\{\eta,\beta,\lambda,\delta\}$	$\{\eta,\lambda,\delta\}$	$\{\lambda,\delta\}$	$\{\lambda,\delta\}$	$\{\lambda,\delta\}$
Goodness of fit	Log-likelihood (BK)†	−3291.80	−3297.59	−3292.48	−3300.55	−3285.34
	Log-likelihood (LHB)†	−3021.93	−3029.01	−3034.50	−3040.01	−3015.13
	RMSE (BK)*‡	17.23	23.74	21.22	21.04	16.42
	RMSE (LHB)*‡	22.01	25.69	26.24	25.65	20.93
	Subject level classification (BK)†‡	16%	16%	13%	5%	50%
	Subject level classification (LHB)†‡	16%	15%	6%	7%	56%
Empirical validity	Convex-concave pattern of average order quantities	✗	✗	✗	✗	✓
	Mean demand as modal decision	✗	✗	✗	✗	✓
	M-shaped performance gap prediction	✗	✗	✗	✗	✓

Table 4 Model comparison summary. Superscript † (resp., *) indicates that the model is better if the corresponding metric is higher (resp., lower); the metrics corresponding to ‡ are discussed in the appendix.

As evident from Table 4, the various methods of comparing different models arrive at much the same result: overall, model M5 represents observed newsvendor behavior better than the competing models. The consistent dominance of M5 over other models could be because of mean demand's greater salience (than the other reference points considered) in newsvendor experiments. The information in newsvendor experiments is usually presented in terms of demand distribution, not payoff distribution, so it is likely that subjects find demand-related information to be more salient than payoff-related information. Moreover, in experiments where demand is uniformly distributed, although the two endpoints of the support are explicitly presented to the subjects, they seem an unreasonable choice in the context of the newsvendor problem where subjects are faced with a decision of balancing the trade-off between over- and underordering; hence their midpoint (i.e., mean demand) is more likely to be adopted as a reference point by the subjects.

4. Some Applications of M5

Having rigorously established that M5 represents newsvendor behavior better than the other models, we put it to use in this section.

4.1. When are Decision Support Systems Beneficial?

As mentioned in Section 1, the decisions in emerging markets are usually made by humans, not by decision support systems (DSS). For some product types, humans can perform as well without such support, in which case there is no need to invest in DSS. However, the performance gap could be significant for other product types; in that case, investment in DSS could be beneficial. Hence this investment decision requires that we understand the relationship between product characteristics and the human–DSS performance gap.

To that end, we extrapolate our insights from lab experiments to the retail industry in emerging markets, where we conjecture that mean demand could be salient for regularly sold products and so M5 might reasonably represent the behavior of human retailers. Then the performance gap between DSS and human retailers can be characterized by Δ_{M5} (defined in (9)) if we assume that the expected profit-maximization represents the ordering behavior of a DSS, and M5 represents the ordering behavior of humans. By gaining more insight into the M-shaped performance gap predicted by M5, we can shed light on when investing in DSS could be beneficial. Hereafter, we denote Δ_{M5} simply as Δ. In Figure 4, Δ is plotted as a function of α for uniform and normal demand distributions – with $\delta = 1$, $\eta = 0$ and with $F_{\mathrm{LA}}(\lambda)$ as estimated previously using Kahneman and Tversky's (1979) definition. (The pattern looks similar for other parameter sets.)

We make the following important observations based on Figure 4. First, Δ is symmetric with respect to $\alpha = 0.5$. This is because, for $\alpha \le 0.5$, we have that $\Pi(\tilde{q}_{\mathrm{EP}}(\alpha)) = \Pi(\tilde{q}_{\mathrm{EP}}(1-\alpha)) + \mu(1-2\alpha)$ and that $\Pi(q^*_{\mathrm{M5}}(\lambda,\alpha)) = \Pi(q^*_{\mathrm{M5}}(\lambda, 1-\alpha)) + \mu(1-2\alpha)$ for all values of λ – as can be easily verified – for both uniform and normal demand distributions; therefore, $\Delta(\alpha) = \Delta(1-\alpha)$. Second, we see that $\Delta \to 0$ as α approaches 0 or 1. Under these extreme cases, both expected profit maximization and M5 yield similar predictions. As a result, the performance gap between these models is close to nil. Third, the performance gap is also small when α is close to 0.5. Since expected profit maximization predicts mean demand as the optimal order quantity as $\alpha \to 0.5$ and the reference point of the individual is itself mean demand, the tendency of the individual to order close to mean demand is high in these cases; again the result is a relatively small performance gap. Finally, with increases in the demand distribution's *variation* (represented here by the support size for a uniform distribution and by the coefficient of variation for a normal distribution), the scope to commit ordering errors increases and so the performance gap becomes larger.

Given these observations, we are now in a position to discuss when it could be beneficial to invest in DSS. When the margin is either very low ($\alpha \to 1$) or very high ($\alpha \to 0$) or close to cost ($\alpha \to 0.5$), performance does not differ appreciably between humans and DSS. However, as α moves away from one of those values, performance deteriorates; it is in such settings that DSS would be beneficial. The need for DSS increases also with variation in the product's demand distribution. Suppose that, instead of using Δ as the performance measure, we use the percentage gap: $\Delta^{\mathrm{P}} = \Delta/\Pi(\tilde{q}_{\mathrm{EP}}) \times 100$ (although this may not be a sensible measure for a normal demand distribution because in that case $\Pi(\tilde{q}_{\mathrm{EP}})$ could be negative). Then, since $\Delta(\alpha) = \Delta(1-\alpha)$ and $\Pi(\tilde{q}_{\mathrm{EP}}(\alpha)) > \Pi(\tilde{q}_{\mathrm{EP}}(1-\alpha)) > 0$ for $\alpha \le 0.5$, we have $\Delta^{\mathrm{P}}(\alpha) < \Delta^{\mathrm{P}}(1-\alpha)$. Thus the percentage effect on performance is greater for low-margin products, which also calls for DSS investment in such settings.

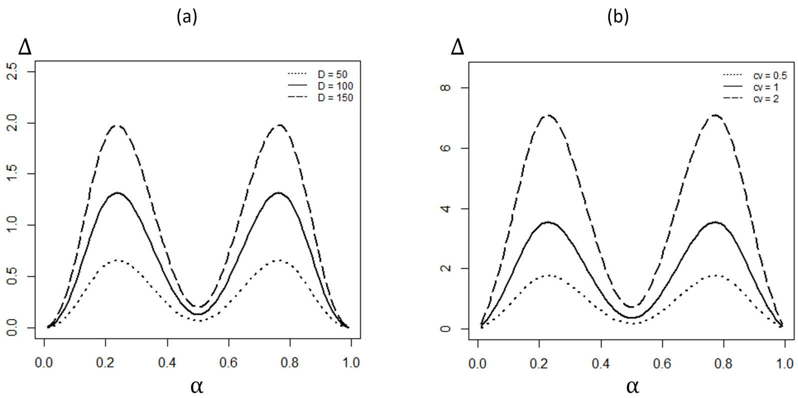

Figure 4 Δ **as a function of** α **plotted with the distribution of** λ **estimated using Kahneman and Tversky's (1979) definition and with** $\delta = 1$ **and** $\eta = 0$**, for: (a) uniform demand distribution with** $d_{\min} = 0$ **and** $D \in \{50, 100, 150\}$**; and (b) normal demand distribution with** $\mu = 100$ **and coefficient of variation in** $\{0.5, 1, 2\}$**.**

4.2. Benefits to Supplier from Using M5

We now discuss how our results could be useful to a supplier who interacts with newsvendor-type retailers. We assume that the supplier sells his products to retailers at unit price α, who in turn sell these products in the market to consumers at unit price equal to one (i.e., we normalized $r = 1$ such that $c = \alpha$). In addition, we set the supplier's cost to zero and focus here on his pricing decisions and the resultant revenue. (Extending the analysis to nonzero supplier cost is straightforward, and incorporating it does not qualitatively affect the insights in this section.)

Let $Q(\alpha)$ be the demand generated by retailers when the price set by the supplier is α. Then the supplier's revenue is given by $R(\alpha) = \alpha Q(\alpha)$. Since we expect the demand to decrease as price increases, we see that the higher the value of α, the higher the money made per sale but the lower the quantity sold at that price. Therefore, the optimal value of α balances these opposing effects, and determining it requires an understanding of the demand curve $Q(\alpha)$. The better the supplier can estimate $Q(\alpha)$, the better his pricing decisions will be. Since $Q(\alpha)$ is the sum of order quantities placed by the retailers, a model that better represents the ordering behavior of retailers will lead to a better estimate of $Q(\alpha)$. The analysis in Section 3 indicates that M5 represents the newsvendor behavior better than the other competing models. This suggests that the supplier may be better off if he assumes that all his retailers follow M5 as opposed to other models.

On the other hand, the individual-level analysis in Section 3.2.2 shows that the retailers could be heterogeneous in their reference points (and hence in their decision models). Therefore, an alternative to approximating the decision models of all retailers with a single model is estimating the best models separately for each retailer. Having the information about individual-level models enables the supplier to earn higher revenue because he can now account for this information in the price that he charges all the retailers, or he could charge different prices to retailers depending on their decision models. However, obtaining such information at the individual level could be impracticable for the supplier, especially if he is interacting with hundreds of retailers. Moreover, there could be a market-level constraint that he sell to all retailers at the same price. Under such limitations, it may still be better to assume that all the retailers follow M5 because a significant majority is consistent with this model. Next we numerically demonstrate this claim.

We consider a supplier who sells his products to the subjects (hereafter, retailers) in BK and LHB. We denote the model followed by retailer i as $\mathcal{m}_i$, and the corresponding parameter set as $\mathbb{P}_{i,\mathcal{m}_i}$. (From our individual-level analysis in Section 3.2.2, we know the model $\mathcal{m}_i$ that is compatible with retailer i, and the corresponding maximum likelihood estimate of $\mathbb{P}_{i,\mathcal{m}_i}$, for all retailers in BK and LHB.) At a given price α, let the expected order quantity placed by retailer i be $\bar{q}_{\mathcal{m}_i}(\alpha \mid \mathbb{P}_{i,\mathcal{m}_i})$. Then the expected supplier revenue is given by $R(\alpha) = \alpha Q(\alpha)$, where $Q(\alpha) = \sum_{i=1}^{m} \bar{q}_{\mathcal{m}_i}(\alpha \mid \mathbb{P}_{i,\mathcal{m}_i})$.

If the supplier knows the individual-level models and the corresponding parameter sets $\mathbb{P}_{i,\mathcal{m}_i}$ for all the retailers, then he can set a price $\alpha^{**} = \arg\max_{\alpha \in (0,1)} R(\alpha)$, which earns him the first-best revenue assuming that he charges the same price to all the retailers. However, as discussed above, the supplier may not have a knowledge of the individual level parameters. We assume that the supplier approximates that all his retailers follow model $\mathcal{m}$, and that he knows the corresponding population level parameters $\mathbb{P}_{\mathcal{m}}$ (reported in Tables 6–10). Then the price charged by the supplier is given by $\alpha^*_{\mathcal{m}} = \arg\max_{\alpha \in (0,1)} m\alpha\, \bar{q}_{\mathcal{m}}(\alpha \mid \mathbb{P}_{\mathcal{m}})$, and the resultant expected revenue of the supplier is $R(\alpha^*_{\mathcal{m}}) = \alpha^*_{\mathcal{m}} Q(\alpha^*_{\mathcal{m}})$, where $Q(\alpha^*_{\mathcal{m}})$ is his expected demand when he sets the price to $\alpha^*_{\mathcal{m}}$.

	BK				LHB			
	$X \sim$ U[0,100]		$X \sim$ U[50,150]		$X \sim$ U[0,100]		$X \sim$ U[50,150]	
	α^*	$R(\alpha^*)$	α^*	$R(\alpha^*)$	α^*	$R(\alpha^*)$	α^*	$R(\alpha^*)$
First-best	0.65	995.56	0.73	2334.76	0.65	4402.60	0.78	10621.70
M1	0.81	742.11	0.99	1104.85	0.99	1602.86	0.99	7102.73
M2	0.51	909.38	0.80	2155.37	0.54	4111.16	0.82	10506.48
M3	0.49	890.61	0.84	2251.90	0.52	4027.48	0.88	10175.16
M4	0.55	942.16	0.72	2269.48	0.57	4219.39	0.73	10458.52
M5	0.62	986.03	0.75	2286.74	0.67	4401.58	0.79	10563.79

Table 5 Supplier's price and expected revenue under the first-best scenario, and when he approximates that all his retailers follow model $\mathcal{m}$. The values are reported for BK and LHB, and for demand U[0,100] and U[50,150].

Table 5 shows the values of the price α^* set by the supplier and the corresponding expected revenue $R(\alpha^*)$ for all the competing models, along with the first-best values of price and revenue. These values are reported for both BK and LHB data sets, and for the demand distributions U[0,100] and U[50,150] (which are commonly used in the experiments). We can see that M5 consistently emerges as the best model, when the supplier cannot know the actual model that represents the behavior of each individual retailer. Moreover, the expected revenue from assuming M5 is within

0.96%–2.06% of the first-best revenue on BK and 0.02%–0.55% on LHB, whereas by assuming other models supplier's expected revenue could be 2.80%–52.68% below the first-best revenue on BK and 1.54%–63.59% on LHB. It is noteworthy that, although M1 displays decent performance based on several metrics in Table 4, it consistently performs worst here. This relates to one of the theoretical shortcomings of M1 discussed in Section 3.1. As $\alpha \to 1$, unlike other models, M1 assumes that retailers order (sometimes significantly high) nonzero quantities. As a consequence, this model results in overpricing by the supplier (see Table 5, M1 results in highest α^*) leading to significantly lower demand, which in turn results in lower revenue compared to other models.

5. Implications and Conclusions

Designing effective supply chain processes and inventory systems requires that the underlying models represent the observed newsvendor behavior reasonably well. Extant models either do not predict the pull-to-center effect or predict only the PTC effect, and hence *fail* to capture the behavior of a significant share of the population. This paper has discussed several prospect theory-based models that differ in their reference point assumptions but can all theoretically predict heterogeneity in the newsvendor context. We rigorously evaluated these models based on several theoretical and empirical criteria. As evident from Table 4, the model with mean demand as the stochastic reference point consistently outperforms the competing models. Using this model, we also derived some prescriptive insights for both retailers and suppliers operating in emerging markets.

The analysis in this paper has important implications for newsvendor modeling. A common thread that binds all the PT models discussed is the important role that loss aversion plays in predicting heterogeneity. The formal models in OM literature frequently account for risk aversion, but not for loss aversion. However, as we argued at the beginning of this paper, heterogeneity in risk aversion has only limited predictive ability, whereas heterogeneity in loss aversion can overcome these limitations. Although loss aversion has received some attention in the OM literature (e.g., Becker-Peth et al. 2013, Herweg 2013, Baron et al. 2015), this paper highlights that while applying loss aversion, one should be mindful of its intimate relationship with reference dependence. A reference point determines the structure of the gains and losses experienced by the decision maker and thereby the manner in which loss aversion plays its role. However, there is still no concrete theory on how reference points are formed and updated (Barberis 2013).

Some settings feature natural candidates for the reference points. For a consumer, the price that she last observed could be her reference point for the next purchase decision. A monthly sales target could be the reference point for a sales agent deciding how much effort to exert in the current month. In a procurement auction, the reservation price could be a reference point for auction participants. In a service system, the service quality experienced in the past could be a reference point for determining whether or not a consumer will return. In several other settings, the reference point could simply be the status quo. The analysis of such settings could be enriched by accounting for the respective reference points – and thereby for loss aversion.

However, our theoretical analysis has revealed that some of these natural candidates may be inconsistent with observed behavior. In such cases one could take a "bottom-up" approach, as used in this paper, to identify the appropriate model. First, analyze the observed data to identify some systematic patterns, both at the individual and aggregate level. Second, depending on the information structure faced by the decision makers in the setting at hand, list the plausible reference points; the analysis in this paper shows how different types of information can be modeled as reference points. Third, discard any model that is inconsistent with the observed data. Fourth, evaluate the remaining models – based on their fit to the data – and thus identify the reference point most likely to be adopted by decision makers; then use this reference point model for further analysis. We hope that this approach will help OM researchers formally incorporate reference dependence and loss aversion in their models.

Future studies could apply the behavioral models discussed in our paper in broader settings where the newsvendor model is used as a building block, such as supply chain contracting and newsvendor competition, to generate new (behaviorally grounded) insights (as in Kirshner and Ovchinnikov 2016), and more testable hypotheses (much in the spirit of Sections 3.3 and 4.3). One could then test these hypotheses either using the existing data or through new experiments, and thereby provide additional evidence either in favor of or against these behavioral models. Although we investigated only prospect theory-based models in this paper, there could be other classes of models (such as bounded rationality models enriched with behavioral features such as risk aversion and loss aversion) that can also predict heterogeneity in newsvendor behavior. The future research could compare such models with the models analyzed in our paper.

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Appendix A: Discussion of Conditions in Proposition 1

Here we discuss the necessity of several conditions used in the definitions of $\mathcal{G}_{\mathrm{P}}$ and $\mathcal{G}_{\mathrm{N}}$ in Proposition 1. As we mentioned in the paper, although q^*_∞ is always below both $\tilde{q}_{\mathrm{EP}}$ and μ, to ensure that q^*_0 is above both $\tilde{q}_{\mathrm{EP}}$ and μ, we need δ to be greater than a threshold $\hat{\delta}$. This is because the lower the value of δ, the flatter the objective function, and hence the narrower the range of its optimum, as λ is varied. To ensure that the range of optimum spans beyond expected profit-maximizing quantity and mean demand (which is necessary for the model to accommodate both PTC and non-PTC behavior), we need the objective function to be not overly flat, and hence δ to be above a threshold.[7] In addition, we need a regularity condition $\beta \le 1-2/(rD)$ to ensure the existence and uniqueness of the threshold $\hat{\delta}$. For this condition to be feasible, we need the assumption $rD > 2$ on the environmental parameters. Yet this assumption is not that restrictive because in the existing studies rD is much higher than two (e.g., $rD = 3600$ in Schweitzer and Cachon (2000), and $rD = 1200$ in Bolton and Katok (2008)). The regularity conditions $\beta \le \alpha/(1-\alpha)$ and $\eta \le (1-\beta)^2/[2\alpha-1]^+$ are also needed to ensure that the threshold $\hat{\delta}$ lies between zero and one.

Then the set $\mathcal{G}_{\mathrm{P}}$ constructed as $\{(\eta,\beta,\lambda,\delta) \in \mathcal{G} : \lambda \in \Lambda(\eta,\beta,\delta,\alpha,r,D), \delta \ge \hat{\delta}(\eta,\beta,\alpha,r,D), \beta \le \min\big(1-2/(rD), \alpha/(1-\alpha)\big), \eta \le (1-\beta)^2/[2\alpha-1]^+\}$, therefore constitutes individuals who exhibit pull-to-center effect, whereas those in the set $\mathcal{G}_{\mathrm{N}} = \{(\eta,\beta,\lambda,\delta) \in \mathcal{G} : \lambda \notin \Lambda(\eta,\beta,\delta,\alpha,r,D), \delta \ge \hat{\delta}(\eta,\beta,\alpha,r,D), \beta \le \min\big(1-2/(rD), \alpha/(1-\alpha)\big), \eta \le (1-\beta)^2/[2\alpha-1]^+\}$ do not exhibit that effect. The nonemptiness of sets $\mathcal{G}_{\mathrm{P}}$ and $\mathcal{G}_{\mathrm{N}}$ shows that the model satisfies (H1). The remaining individuals are in the set $\mathcal{G}_{\mathrm{R}} = \mathcal{G} - (\mathcal{G}_{\mathrm{P}} \cup \mathcal{G}_{\mathrm{N}})$, and they satisfy either $\delta < \hat{\delta}$ or $\beta > \min\big(1-2/(rD), \alpha/(1-\alpha)\big)$ or $\eta > (1-\beta)^2/[2\alpha-1]^+$. Under these conditions, q^*_0 could lie either between or below $\tilde{q}_{\mathrm{EP}}$ and μ, in which case the interval Λ may not be well defined. $\mathcal{G}_{\mathrm{R}}$ constitutes individuals who exhibit PTC effect, and also those who do not; however, partitioning it between these two types is analytically cumbersome, and, more importantly, does not affect the already proven fact that the model can accommodate both types of individuals.

It is worth noting that, unlike Propositions 1 and 2, we do not require any assumption on the magnitude of rD in Propositions 4–6. This is because in PRP models, the consumption utility component is independent of δ, whereas the gain-loss utility component depends on δ. Under such asymmetry (which exists by definition), we need the condition on r and D to ensure the existence and uniqueness of $\hat{\delta}$.

[7] This relationship between the magnitude of δ and the flatness of the objective function is applicable to all the models discussed in this paper. Therefore, we see a threshold condition on δ in all the propositions in the paper. Furthermore, this threshold $\hat{\delta}$ decreases under high-margin conditions ($\alpha < 0.5$), and increases under low-margin conditions ($\alpha > 0.5$), i.e., it is U-shaped in α for all the models. This is because the value of α determines the width of the pull-to-center zone. The larger the width of the PTC zone, the larger should be the range of the optimum for the model to predict quantities outside the PTC zone, and hence the larger the corresponding threshold on δ. Since the width of the PTC zone is largest when $\alpha = 0$ or $\alpha = 1$ and shrinks as α approaches 0.5, the threshold on δ is also largest at the extreme values of α and decreases as $\alpha \to 0.5$.

Appendix B: Population-Level Estimates of Model Parameters

For the competing models listed in Table 1, here we report the means and the standard deviations (in parentheses) of the sampling distributions of the parameters, estimated based on population-level analysis described in Section 3.2.1.

	$\hat{\eta}_{\mathrm{M1}}$	$\hat{\beta}_{\mathrm{M1}}$	$\hat{\sigma}_{\mathrm{M1}}$	$\hat{a}_{\mathrm{M1}}$	$\hat{b}_{\mathrm{M1}}$
BK	1.21	0.24	14.27	0.91	0.07
	(0.94)	(0.05)	(0.83)	(0.39)	(0.05)
LHB	0.64	0.14	16.82	1.59	0.04
	(0.48)	(0.08)	(0.41)	(0.48)	(0.05)

Table 6 **Population level estimates under M1.**

	$\hat{\eta}_{\mathrm{M2}}$	$\hat{\sigma}_{\mathrm{M2}}$	$\hat{a}_{\mathrm{M2}}$	$\hat{b}_{\mathrm{M2}}$
BK	0.77	13.92	3.34	2.81
	(0.53)	(0.84)	(7.51)	(7.52)
LHB	0.34	16.77	8.54	8.27
	(0.14)	(0.39)	(12.89)	(12.73)

Table 7 **Population level estimates under M2.**

	$\hat{\eta}_{\mathrm{M3}}$	$\hat{\sigma}_{\mathrm{M3}}$	$\hat{a}_{\mathrm{M3}}$	$\hat{b}_{\mathrm{M3}}$
BK	0.18	14.27	0.52	0.18
	(0.47)	(0.87)	(0.17)	(0.08)
LHB	0.05	16.93	0.37	0.08
	(0.07)	(0.44)	(0.06)	(0.02)

Table 8 **Population level estimates under M3.**

	$\hat{\eta}_{\mathrm{M4}}$	$\hat{\sigma}_{\mathrm{M4}}$	$\hat{a}_{\mathrm{M4}}$	$\hat{b}_{\mathrm{M4}}$
BK	0.18	14.37	0.46	0.16
	(0.42)	(0.93)	(0.14)	(0.07)
LHB	0.01	17.18	0.35	0.10
	(0.01)	(0.40)	(0.03)	(0.01)

Table 9 **Population level estimates under M4.**

	$\hat{\eta}_{\mathrm{M5}}$	$\hat{\sigma}_{\mathrm{M5}}$	$\hat{a}_{\mathrm{M5}}$	$\hat{b}_{\mathrm{M5}}$
BK	1.10	14.11	1.55	0.66
	(1.65)	(0.86)	(0.96)	(0.52)
LHB	0.15	16.81	0.94	0.36
	(0.74)	(0.46)	(0.21)	(0.09)

Table 10 **Population level estimates under M5.**

Appendix C: Robustness Checks for Population-Level Analysis

We found in Section 3.2.1 that M5 outperforms all the other models, in terms of out-of-sample log-likelihood, on both BK and LHB. In this section, we test the robustness of this result with other goodness of fit measures.

C.1. First-Order Stochastic Dominance

Using the log-likelihood values for the models over 1,000 test sets, we compute the cumulative distribution functions (CDFs) of the sampling distributions of the log-likelihood. We test for whether the first-order stochastic dominance ordering of these CDFs is same as the ordering discussed in Section 3.2.1 based on mean log-likelihoods. For a statistical test of the stochastic dominance ordering between a model pair, we use a one-sided paired Wilcoxon rank-sum test (a nonparametric test) with null hypothesis that the sampled CDFs of the competing models come from the same population against the alternative hypothesis that the CDF of one model tends to have smaller values than that of the other. We perform this test for all possible model pairs (again with Holm–Bonferroni correction for multiple testing) and again find a similar pattern as earlier: all the adjusted p-values are less than 0.001, except for the comparisons M1 $\succ$ M3 and M2 $\succ$ M4 on BK, which now result in adjusted p-values of 0.17 and 0.02 respectively. These results suggest that the log-likelihoods – as random variables – are stochastically ordered in the same manner as the mean log-likelihoods of these models.

C.2. Root-Mean-Square Error

Even though likelihood is a measure commonly used to test the fit of competing models, we next evaluate these models using a more intuitive measure that is based on the distance between the observed and predicted order quantities. For each subject i in the test set, we measure the expected squared distance (or squared error) between the mean order quantity $\hat{q}_i$ placed by that subject and the mean order quantity predicted by model $\mathcal{m}$. We compute the average of the squared errors across all the subjects in the test set and then take its square root; this way, the resultant measure has the same units as order quantities. This measure is known as the *root-mean-square error*, which for model $\mathcal{m}$ is given by

$$\mathrm{RMSE}_{\mathcal{m}} = \left\{ \frac{1}{m_t} \sum_{i=1}^{m_t} \mathbb{E}\left[(\hat{q}_i - q^*_{\mathcal{m}}(\lambda \mid \alpha_i, \mathbb{P}_{\mathcal{m}}))^2 \right] \right\}^{1/2};$$

here m_t is the number of subjects in the test set, and the expectation is taken with respect to the distribution of λ – whose parameters are estimated using training and cross-validation sets.

On each of the 1,000 randomly selected test sets, we compute the RMSE for all competing models using the maximum likelihood parameters estimated from the *corresponding* training and cross-validation sets. Table 11 reports the means and standard deviations of RMSE (across 1,000 test sets) for all the models on BK and LHB data sets.

	$\mathrm{RMSE}_{\mathrm{M1}}$	$\mathrm{RMSE}_{\mathrm{M2}}$	$\mathrm{RMSE}_{\mathrm{M3}}$	$\mathrm{RMSE}_{\mathrm{M4}}$	$\mathrm{RMSE}_{\mathrm{M5}}$
BK	17.23	23.74	21.22	21.04	16.42
	(1.83)	(3.89)	(2.46)	(2.57)	(1.58)
LHB	22.01	25.69	26.24	25.65	20.93
	(1.20)	(1.34)	(1.33)	(1.77)	(1.36)

Table 11 Means and standard deviations (in parentheses) of RMSE for the competing models.

Consistently with our likelihood analysis, we find that M5 predicts the experimental data better (i.e., has lower RMSE) than the other competing models. Its mean RMSE is significantly lower than the mean RMSE of other models with adjusted p-values – which are based on one-sided paired t-test, with Holm–Bonferroni correction for multiple testing – less than 0.001. We can see that the percentage improvement in the prediction error of M5 with respect to other models is 5%–31% on BK and 5%–20% on LHB.

C.3. Separate Analysis for Low- and High-Margin Conditions

Recall that in Section 3.2.1, we pooled the data from low- and high-margin conditions to fit the models. In this section, we present the results from the population-level analysis when the models are fit separately to the data from low- and high-margin conditions. In total, we have four conditions corresponding to two data sets and two experimental conditions in each of these data sets. Table 12 shows how the subjects are split across training, cross-validation, and test sets in these four conditions.

To estimate the model parameters, we employ the same method as in Section 3.2.1 for each of the above four conditions, with the only difference that, instead of 1,000 random splits of the data set, we use only 500 random splits (because the size of training, validation, and test sets is also halved). Table 13 presents the log-likelihood values for the competing models in these four cases. We find that the log-likelihood of M5 is significantly greater than that of the other models in all the four cases, except in the high-margin settings

	Experimental condition	Training set	Validation set	Test set
BK	LM	12	4	4
	HM	11	4	3
LHB	LM	56	19	19
	HM	48	16	16

Table 12 Sizes of training, cross-validation, and test sets across data sets and experimental conditions.

where its log-likelihood is not significantly different from (although greater than) that of M3. Accordingly, we make two conclusions: (i) consistently with the analysis in Section 3.2.1, M5's fit to the data is better than (or as good as) that of the competing models, and (ii) M3 fits the data particularly well in the high-margin settings.

		$\log \mathcal{L}_{\mathrm{M1}}$	$\log \mathcal{L}_{\mathrm{M2}}$	$\log \mathcal{L}_{\mathrm{M3}}$	$\log \mathcal{L}_{\mathrm{M4}}$	$\log \mathcal{L}_{\mathrm{M5}}$
BK	LM	−1613.41	−1613.44	−1613.35	−1613.11	−1608.82
		(87.34)	(87.84)	(87.74)	(82.56)	(88.56)
	HM	−1263.88	−1265.75	−1261.61	−1272.89	−1260.99
		(81.91)	(88.99)	(79.06)	(77.27)	(79.62)
LHB	LM	−1649.49	−1647.47	−1648.79	−1645.95	−1642.82
		(33.87)	(31.91)	(31.99)	(30.12)	(31.86)
	HM	−1373.47	−1373.76	−1371.46	−1383.61	−1371.16
		(25.92)	(25.11)	(25.27)	(23.57)	(25.30)

Table 13 Means and standard deviations (in parentheses) of the sampling distributions of log-likelihood for the competing models when the models are estimated separately for low- and high-margin conditions.

Appendix D: Individual-Level Estimation

In Section 3.2.1, we assumed that the decision making process of all subjects is represented by the same prospect theory model. Under this assumption, we have shown that M5 is most consistent with the data. However, we can go a level deeper and reanalyze the data by assuming that subjects may be heterogeneous in terms of the PT model that best represents their decision making process. Here our aim is to determine if there is a dominant model, one that explains the behavior of a significant proportion of subjects. To that end, we now estimate the model parameters at the individual level.

The two main challenges for this task are: (i) the experimental data available per subject is too thin to obtain reliable estimates, and (ii) it is well known that the loss aversion parameter, when estimated at the individual level, tends to be very volatile (Wakker 2010, p. 265). We resolve the first issue by generating more data per subject using the bootstrap method. To resolve the second issue, instead of obtaining a point estimate of loss aversion parameter, we incorporate uncertainty around that parameter and estimate its distribution. In other words, we allow the distribution of loss aversion to vary across the subjects (in contrast to assuming one distribution for all the subjects as in Section 3.2.1), and we estimate these distributions' parameters from the bootstrapped data.

Specifically, for subject i (who placed n orders in the experiment), we create 100 samples (each of size n) by bootstrapping from that subject's order quantities. These samples can be considered as the order quantities placed by the clones of subject i who have the same characteristics as the original subject.[8] We then write the likelihood function for these bootstrap samples of subject i under each model $\mathcal{M}$ following the method in Section 3.2.1: at the population level, parameter estimation was based on the data from different subjects; now those different subjects are replaced by the clones of the same subject while estimating the parameters for that subject. The parameter set $\mathbb{P}_{i,\mathcal{M}} = \{\eta_{i,\mathcal{M}}, \sigma_{i,\mathcal{M}}, a_{i,\mathcal{M}}, b_{i,\mathcal{M}}\}$ for $\mathcal{M} \in \{\mathrm{M2}, \mathrm{M3}, \mathrm{M4}, \mathrm{M5}\}$, and $\mathbb{P}_{i,\mathrm{M1}} = \{\eta_{i,\mathrm{M1}}, \beta_{i,\mathrm{M1}}, \sigma_{i,\mathrm{M1}}, a_{i,\mathrm{M1}}, b_{i,\mathrm{M1}}\}$. As in the previous section, we use the method of cross-validation to estimate the model parameters. Out of the 100 bootstrap samples generated per subject, we use the orders from 60 samples as the training set, 20 samples as the validation set, and the remaining 20 samples as the test set. The log-likelihood estimated on the test set is used for the purpose of model comparison.

We classify the subject i to be compatible with reference point model $\mathcal{M}$ (i.e., she is of $\mathcal{M}$-type) if that subject's log-likelihood on test set under model $\mathcal{M}$ is higher than that under the other models. Using this classification criterion, Figure 5 reports the proportion of subjects classified as $\mathcal{M}$-type for all $\mathcal{M} \in \{\mathrm{M1}, \mathrm{M2}, \mathrm{M3}, \mathrm{M4}, \mathrm{M5}\}$. We see that – although the subject pool is heterogeneous in terms of reference points – a significant majority is compatible with M5. Specifically, we find that on BK, 19 out of 38 subjects (i.e., 50%) are classified as M5-type, whereas only 6 (16%), 6 (16%), 5 (13%), and 2 (5%) subjects are classified as M1-type, M2-type, M3-type, and M4-type respectively. A similar pattern is observed on LHB as well: out of 174 subjects, 56% are classified as M5-type, 16% as M1-type, 15% as M2-type, 6% as M3-type, and 7% as M4-type in that data set. Thus, as in the population-level analysis, M5 dominates other models in the individual-level analysis as well.

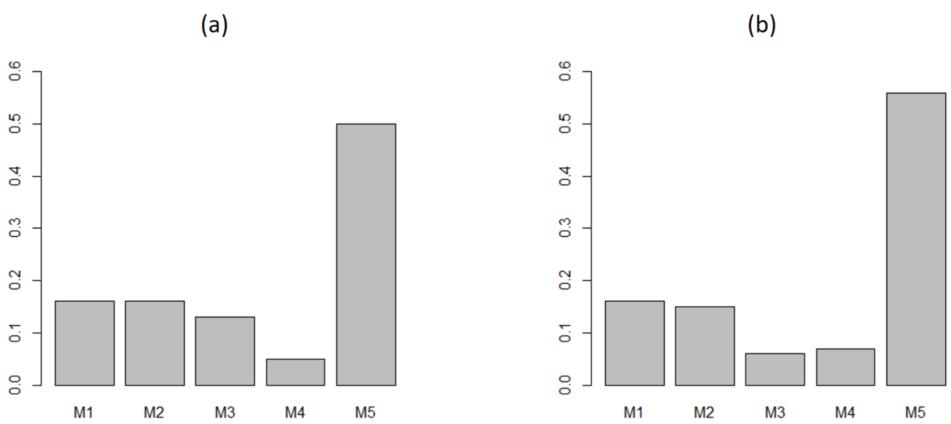

Figure 5 Proportion of subjects classified as compatible with model $\mathcal{M}$, reported for all the competing models using (a) BK, and (b) LHB data sets.

[8] Bootstrapping is a generally employed method to obtain statistically similar samples (Efron and Tibshirani 1994). Using Kolmogorov–Smirnov test, we find that the generated bootstrap samples are statistically similar to the corresponding original samples.

Appendix E: A Note on Parameter Specification

The estimation method employed in Sections 3.2.1 and 3.2.2 fits the candidate reference point models *separately* to the data. This method implicitly assumes that loss aversion could vary with reference point. We use this method because it allows each model to fit the data as best as it can without any externally imposed constraints. Moreover, this method is consistent with the estimation procedure used in several papers which fit decision theoretic models separately to the data even when these models share some behavioral parameters such as loss aversion (e.g., Hardie et al. 1993, Hey and Orme 1994, Bleichrodt et al. 2007, Gaechter et al. 2007, Merkle 2017).

To the best of our knowledge, the existing literature does not offer a conclusive answer to whether or not the parameters such as loss aversion are dependent on the reference point. There is some evidence that the loss aversion of an individual is affected by the operating environment and framing of the problem (Gaechter et al. 2007, Simmons and Novemsky 2008, Wakker 2010, Beauchamp et al. 2012). Since these factors in turn could determine the individual's reference point, this evidence also provides some support for the employed estimation method.

However, to make sure that the observed superiority of M5 is not driven by the implicit assumption that loss aversion could vary across the models, we also performed the analysis where loss aversion and other behavioral parameters (i.e., η and β) are specified at the individual level independent of the reference point. Note that – when the behavioral parameters are exogenously specified to be the same across the models – there is no parameter estimation involved. For a given parameter specification (η,β,λ), we simply need to compare models based on a metric of interest under that specification. We choose this metric to be the squared difference between the mean order quantity $\hat{q}_i$ of subject i, and the order quantity $q^*_{\mathcal{M}}(\eta,\beta,\lambda \mid \alpha_i)$ predicted by model $\mathcal{M}$ under parameter specification (η,β,λ) and experimental condition α_i. We choose this metric instead of log-likelihood because (i) it is intuitive, (ii) it is structurally similar to log-likelihood under normal error assumption, and (iii) it avoids exogenously specifying one more parameter σ (which is external to the models).

We choose the parameters as follows: $\eta \in \mathcal{E} = \{0, 0.1, \ldots, 4.9, 5\}$, $\beta \in \mathcal{B} = \{0, 0.05, \ldots, 0.95, 1\}$, and $\lambda \in \mathcal{L} = \{0, 0.25, \ldots, 49.75, 50\}$. Since we are interested in the ordering of the models in terms of squared difference, for each parameter specification (η,β,λ) in $\mathcal{E}\times\mathcal{B}\times\mathcal{L}$, we obtain $\text{Rank}_{\mathcal{M}}(\eta,\beta,\lambda)$ which is the rank of model $\mathcal{M}$ based on squared difference under that specification. The model with lowest (resp., highest) squared difference is given rank 1 (resp., 5). We evaluate ranks of all the models for all possible specifications in $\mathcal{E}\times\mathcal{B}\times\mathcal{L}$, and then use the following metric to classify individuals across models:

$$\text{MeanRank}_{\mathcal{M}} = \frac{1}{|\mathcal{E}|\cdot|\mathcal{B}|\cdot|\mathcal{L}|} \sum_{(\eta,\beta,\lambda)\in\mathcal{E}\times\mathcal{B}\times\mathcal{L}} \text{Rank}_{\mathcal{M}}(\eta,\beta,\lambda).$$

An individual is classified as $\mathcal{M}$-type if her MeanRank under model $\mathcal{M}$ is lower than that under the other models. With this classification criterion, we find that 79% of the subjects are classified at M5-type under BK, and 84% as M5-type under LHB. Therefore, we can conclude that the observed superiority of M5 based on likelihood maximization methods in Sections 3.2.1 and 3.2.2 is not driven by the assumption that loss aversion could vary across models; in fact, constraining the behavioral parameters to be specified at the individual level (independent of reference point) only hurts other models in favor of M5.

Appendix F: Proofs

Proof of Proposition 1. (i) Proposition 3 of Long and Nasiry (2015) shows that $V(q)$ is unimodal. Using the notation in this paper and from the analysis in the proof of Long and Nasiry's Proposition 3, we have

$$\frac{\partial V}{\partial q} = r\theta\beta^{1+\delta}(q-d_{\min})^{\delta}\left[\frac{\eta(\bar{F}(q)-\alpha)}{\theta\beta^{1+\delta}(q-d_{\min})^{\delta}} - \left(\frac{1-\beta}{\beta}\right)^{\delta} + \frac{\delta(1-\beta)^{\delta}(d_{\max}-q)}{\beta^{1+\delta}(q-d_{\min})} - \lambda\right],$$

where $\theta = r^{\delta-1}/D$. Since $\partial V/\partial q$ is decreasing in λ, it follows that V is submodular in (q,λ).

(ii) Using $\bar{F}(\tilde{q}_{\mathrm{EP}}) = \alpha$ and $\bar{F}(\mu) = 1/2$, we obtain

$$\left.\frac{\partial V}{\partial q}\right|_{q=\tilde{q}_{\mathrm{EP}}} = r\theta\beta^{1+\delta}(\tilde{q}_{\mathrm{EP}}-d_{\min})^{\delta}(\hat{\lambda}_{\mathrm{EP}}-\lambda) \quad \text{for } \hat{\lambda}_{\mathrm{EP}} = \left(\frac{1-\beta}{\beta}\right)^{\delta}\left(\frac{\delta\alpha}{\beta(1-\alpha)}-1\right);$$

$$\left.\frac{\partial V}{\partial q}\right|_{q=\mu} = r\theta\beta^{1+\delta}(\mu-d_{\min})^{\delta}(\hat{\lambda}_{\mu}-\lambda) \quad \text{for } \hat{\lambda}_{\mu} = \frac{2^{\delta}}{(rD)^{\delta}\beta^{1+\delta}}\left[(1/2-\alpha)rD\eta + (\delta-\beta)\left(\frac{rD(1-\beta)}{2}\right)^{\delta}\right].$$

The model predicts both underordering and overordering with respect to (w.r.t.) $\tilde{q}_{\mathrm{EP}}$ if and only if (iff) $\hat{\lambda}_{\mathrm{EP}}$ is positive, which is equivalent to $\delta \geq \beta(1-\alpha)/\alpha$. (The condition $\beta \leq \alpha/(1-\alpha)$ used in the definitions of $\mathcal{G}_{\mathrm{P}}$ and $\mathcal{G}_{\mathrm{N}}$ ensures that this threshold on δ is less than one.) Then for a given margin α, we have $q^* < \tilde{q}_{\mathrm{EP}}$ if $\lambda > \hat{\lambda}_{\mathrm{EP}}$ or $q^* \geq \tilde{q}_{\mathrm{EP}}$ if $\lambda \leq \hat{\lambda}_{\mathrm{EP}}$.

The model similarly predicts optimal order quantities both above and below the mean demand iff $\hat{\lambda}_{\mu}$ is positive. Assume $\delta \geq \beta$. If $\alpha \leq 1/2$, then $\hat{\lambda}_{\mu}$ is trivially positive. So consider the case when $\alpha > 1/2$, and define function $k(\delta) = -(\alpha-1/2)rD\eta + (\delta-\beta)(rD(1-\beta)/2)^{\delta}$ over the domain $[\beta,1]$. The sign of $\hat{\lambda}_{\mu}$ depends on the sign of $k(\delta)$. We see that $k(\delta)$ is increasing in δ because both $(\delta-\beta)$ and $(rD(1-\beta)/2)^{\delta}$ are increasing in δ (the latter follows from the assumption $rD > 2$ and the condition $\beta < 1-2/(rD)$ used in the definitions of $\mathcal{G}_{\mathrm{P}}$ and $\mathcal{G}_{\mathrm{N}}$). Moreover, $k(\beta) = -(\alpha-1/2)rD\eta < 0$ and $k(1) = -(\alpha-1/2)rD\eta + (1-\beta)^2 rD/2 \geq 0$, which follows from the condition $\eta \leq (1-\beta)^2/[2\alpha-1]^+$ specified in the definitions of $\mathcal{G}_{\mathrm{P}}$ and $\mathcal{G}_{\mathrm{N}}$. So there exists a threshold $\hat{\delta}_0 \in [\beta,1]$ such that if $\delta \geq \hat{\delta}_0$ (resp., $\delta < \hat{\delta}_0$), then $k(\delta)$ is positive (resp., negative). Therefore, if we set δ to be higher than $\hat{\delta}_0$, then $\hat{\lambda}_{\mu}$ is positive.

To summarize, the threshold $\hat{\delta}$ used in the definition of $\mathcal{G}_{\mathrm{P}}$ and $\mathcal{G}_{\mathrm{N}}$ is given by

$$\hat{\delta}(\eta,\beta,\alpha,r,D) = \mathbb{1}\{\alpha < 1/2\}\beta(1-\alpha)/\alpha + (1-\mathbb{1}\{\alpha < 1/2\})\hat{\delta}_0, \tag{10}$$

where $\hat{\delta}_0$ is the root of the equation $k(\delta) = 0$, and $\mathbb{1}\{\mathrm{E}\}$ is the indicator variable for the event E. By noting that $\tilde{q}_{\mathrm{EP}} < \mu$ under low-margin, $\tilde{q}_{\mathrm{EP}} > \mu$ under high-margin, and q^* is decreasing in λ, we see that the model predicts the PTC effect when λ is in the interval Λ, which is given by

$$\Lambda(\eta,\beta,\delta,\alpha,r,D) = \mathbb{1}\{\alpha < 1/2\}\left[\hat{\lambda}_{\mathrm{EP}},\hat{\lambda}_{\mu}\right] + (1-\mathbb{1}\{\alpha < 1/2\})\left[\hat{\lambda}_{\mu},\hat{\lambda}_{\mathrm{EP}}\right]. \tag{11}$$

Note that the bounds of these intervals $\hat{\lambda}_{\mathrm{EP}}$ and $\hat{\lambda}_{\mu}$ are also functions of the parameters η, β, δ, α, r and D. The optimal order quantity lies outside the PTC zone when λ is outside the interval Λ. It is easy to see that $\hat{\delta}$ lies between zero and one (because of the conditions $\beta \leq \alpha/(1-\alpha)$ and $\eta \leq (1-\beta)^2/[2\alpha-1]^+$), and Λ is nonempty. Therefore, the sets $\mathcal{G}_{\mathrm{P}}$ and $\mathcal{G}_{\mathrm{N}}$ are nonempty; the former constitutes individuals who exhibit PTC effect, whereas the latter constitutes the ones who do not exhibit the effect.

(iii) For given α_{LM} and α_{HM}, the interval $\Lambda(\alpha_{\mathrm{LM}}) \cap \Lambda(\alpha_{\mathrm{HM}})$ is nonempty because $\alpha_{\mathrm{HM}} < \alpha_{\mathrm{LM}}$, $\hat{\lambda}_{\mu}$ is decreasing in α, and $\hat{\lambda}_{\mathrm{EP}}$ is increasing in α (which can be verified easily). Therefore, the set $\mathcal{G}_{\mathrm{P}}(\alpha_{\mathrm{LM}}) \cap \mathcal{G}_{\mathrm{P}}(\alpha_{\mathrm{HM}}) = \{(\eta,\beta,\lambda,\delta) \in \mathcal{G} : \lambda \in \Lambda(\alpha_{\mathrm{LM}}) \cap \Lambda(\alpha_{\mathrm{HM}}), \delta \geq \max(\hat{\delta}(\alpha_{\mathrm{LM}}),\hat{\delta}(\alpha_{\mathrm{HM}})), \beta \leq \min\left(1-2/(rD), \alpha_{\mathrm{HM}}/(1-\alpha_{\mathrm{HM}})\right), \eta \leq (1-\beta)^2/(2\alpha_{\mathrm{LM}}-1)\}$ is also nonempty. $\square$

Proof of Proposition 2. (i) Put $l(q) = (\Pi(q) + cq)/r$; then (5) yields $l(d_{\min}) = d_{\min}$, $l(d_{\max}) = \mu$, and $l'(q) = \bar{F}(q)$. Now we can write

$$\begin{aligned}\frac{\partial V}{\partial q} &= \eta r(\bar{F}(q) - \alpha) + \frac{r^\delta}{D}\big[-\lambda \bar{F}(q)(l(q) - d_{\min})^\delta - \bar{F}(q)(q - l(q))^\delta + \delta D F(q)\bar{F}(q)(q - l(q))^{\delta-1}\big] \\ &= r\bar{F}(q)(l(q) - d_{\min})^\delta \left\{ \frac{\eta(1 - \alpha/\bar{F}(q))}{(l(q) - d_{\min})^\delta} + \frac{r^{\delta-1}}{D}\left[-\lambda - \frac{(q - l(q))^\delta}{(l(q) - d_{\min})^\delta} + \frac{\delta D F(q)(q - l(q))^{\delta-1}}{(l(q) - d_{\min})^\delta}\right]\right\}.\end{aligned}$$

Since $\partial V/\partial q$ is decreasing in λ, it follows that V is submodular in (q, λ). It is easy to see that $\lim_{q \to d_{\min}} \partial V/\partial q = \eta(r - c) > 0$ and $\lim_{q \to d_{\max}} \partial V/\partial q = -\eta c < 0$. Let

$$h_1(q) = \frac{1 - \alpha/\bar{F}(q)}{(l(q) - d_{\min})^\delta} \quad \text{and} \quad h_2(q) = -\frac{(q - l(q))^\delta}{(l(q) - d_{\min})^\delta} + \frac{\delta D F(q)(q - l(q))^{\delta-1}}{(l(q) - d_{\min})^\delta}.$$

If both $h_1(q)$ and $h_2(q)$ are decreasing in q, then $\partial V/\partial q$ crosses the horizontal axis only once from above and so $V(q)$ is unimodal. Next we show that h_1 and h_2 are decreasing.

The derivative of h_1 w.r.t. q is $m(q)/(l(q) - d_{\min})^{1+\delta}$, where

$$m(q) = \alpha\left[-\frac{f(q)}{\bar{F}(q)^2}(l(q) - d_{\min}) + \delta\right] - \delta \bar{F}(q).$$

Consider q such that $-f(q)(l(q) - d_{\min})/\bar{F}(q)^2 + \delta < 0$. Since $\alpha > 0$, we trivially have $m(q) < 0$. Now consider q such that $-f(q)(l(q) - d_{\min})/\bar{F}(q)^2 + \delta \geq 0$. Then

$$\begin{aligned} m(q) &< -\frac{f(q)}{\bar{F}(q)^2}(l(q) - d_{\min}) + \delta F(q) \\ &< -\frac{f(q)}{\bar{F}(q)^2}(l(q) - d_{\min}) + F(q) = \frac{f(q)}{\bar{F}(q)^2}\left[-(l(q) - d_{\min}) + \frac{F(q)\bar{F}(q)^2}{f(q)}\right] < 0.\end{aligned}$$

Here the first inequality follows from $\alpha < 1$; the second follows from $\delta < 1$; and the third follows because $-(l(q) - d_{\min}) + F(q)\bar{F}(q)^2/f(q) = -(l(q) - d_{\min}) + \bar{F}(q)^2(q - d_{\min})$ is negative since (a) its derivative w.r.t. q is $-2\bar{F}(q)f(q)(q - d_{\min}) - \bar{F}(q)F(q) < 0$ and (b) its value at $d_{\min}$ is zero. Since $m(q) < 0$ for all q, we have that $h_1(q)$ decreases with q.

The derivative of $h_2(q)$ w.r.t. q is $n(q)\delta(q - l(q))^{\delta-2}/(l(q) - d_{\min})^{1+\delta}$, where

$$\begin{aligned} n(q) &= \delta(q - d_{\min})\big[F(q)(l(q) - d_{\min}) - \bar{F}(q)(q - l(q))\big] + (q - l(q))(l(q) - d_{\min})\bar{F}(q) \\ &\quad - (l(q) - d_{\min})(q - d_{\min})F(q) + \bar{F}(q)(q - l(q))^2.\end{aligned}$$

Now observe that the term in brackets is always positive because its derivative w.r.t. q is $f(q)(q - d_{\min}) > 0$ and its value at $d_{\min}$ is zero. So we can now use $\delta < 1$, and write

$$\begin{aligned} n(q) &< (q - d_{\min})[F(q)(l(q) - d_{\min}) - \bar{F}(q)(q - l(q))] + (q - l(q))(l(q) - d_{\min})\bar{F}(q) \\ &\quad - (l(q) - d_{\min})(q - d_{\min})F(q) + \bar{F}(q)(q - l(q))^2 = 0.\end{aligned}$$

Therefore, h_2 is also decreasing in q.

(ii) It is easy to verify that $\tilde{q}_{\text{EP}} - l(\tilde{q}_{\text{EP}}) = (1 - \alpha)^2 D/2$, $l(\tilde{q}_{\text{EP}}) - d_{\min} = (1 - \alpha^2)D/2$, $\mu - l(\mu) = D/8$, and $l(\mu) - d_{\min} = 3D/8$. We can use these equalities to obtain

$$\left.\frac{\partial V}{\partial q}\right|_{q=\tilde{q}_{\text{EP}}} = \frac{\alpha}{D} r^\delta (l(\tilde{q}_{\text{EP}}) - d_{\min})^\delta (\hat{\lambda}_{\text{EP}} - \lambda) \quad \text{and} \quad \left.\frac{\partial V}{\partial q}\right|_{q=\mu} = \frac{r^\delta}{2D}(l(\mu) - d_{\min})^\delta (\hat{\lambda}_\mu - \lambda),$$

where

$$\hat{\lambda}_{\mathrm{EP}} = \left(\frac{1-\alpha}{1+\alpha}\right)^{\delta}\left(\frac{2\delta}{1-\alpha}-1\right) \quad \text{and} \quad \hat{\lambda}_{\mu} = \left(\frac{8}{3rD}\right)^{\delta}\left[(1-2\alpha)rD\eta + (4\delta-1)\left(\frac{rD}{8}\right)^{\delta}\right].$$

Thus the model predicts optimal order quantities both above and below $\tilde{q}_{\mathrm{EP}}$ iff $\hat{\lambda}_{\mathrm{EP}}$ is positive, which is equivalent to $\delta \geq (1-\alpha)/2$. (This threshold on δ is always less than one because $(1-\alpha) < 2$.) Then for a given margin α, we have $q^* < \tilde{q}_{\mathrm{EP}}$ if $\lambda > \hat{\lambda}_{\mathrm{EP}}$ or $q^* \geq \tilde{q}_{\mathrm{EP}}$ if $\lambda \leq \hat{\lambda}_{\mathrm{EP}}$.

In a similar manner, optimal order quantities are predicted both above and below the mean demand iff $\hat{\lambda}_{\mu}$ is positive. Assume $\delta \geq 1/4$. If $\alpha \leq 1/2$, then $\hat{\lambda}_{\mu}$ is trivially positive. So consider the case when $\alpha > 1/2$, and define function $k(\delta) = -(2\alpha-1)rD\eta + (4\delta-1)(rD/8)^{\delta}$ over the domain $[1/4, 1]$. The sign of $\hat{\lambda}_{\mu}$ depends on the sign of $k(\delta)$. We note that $k(\delta)$ is increasing in δ because both $(4\delta-1)$ and $(rD/8)^{\delta}$ are increasing in δ (the latter follows from the assumption $rD > 8$). Moreover, $k(1/4) = -(2\alpha-1)rD\eta < 0$ and $k(1) = -(2\alpha-1)rD\eta + 3rD/8 \geq 0$, which follows from the condition $\eta \leq 3/(8[2\alpha-1]^+)$ specified in the definitions of $\mathcal{G}_{\mathrm{P}}$ and $\mathcal{G}_{\mathrm{N}}$. So there exists a threshold $\hat{\delta}_0 \in [1/4, 1]$ such that if $\delta \geq \hat{\delta}_0$ (resp., $\delta < \hat{\delta}_0$), then $k(\delta)$ is positive (resp., negative). Therefore, if we set δ to be higher than $\hat{\delta}_0$, then $\hat{\lambda}_{\mu}$ is positive.

Hence, the threshold $\hat{\delta}$ used in the definition of $\mathcal{G}_{\mathrm{P}}$ and $\mathcal{G}_{\mathrm{N}}$ is given by

$$\hat{\delta}(\eta, \beta, \alpha, r, D) = \mathbb{1}\{\alpha < 1/2\}(1-\alpha)/2 + (1 - \mathbb{1}\{\alpha < 1/2\})\hat{\delta}_0, \tag{12}$$

where $\hat{\delta}_0$ is the root of the equation $k(\delta) = 0$. If we note that $\tilde{q}_{\mathrm{EP}} < \mu$ under low-margin, $\tilde{q}_{\mathrm{EP}} > \mu$ under high-margin, and q^* is decreasing in λ, then it follows that the model predicts the PTC effect when λ is in the interval Λ, which is given by

$$\Lambda(\eta, \delta, \alpha, r, D) = \mathbb{1}\{\alpha < 1/2\}\left[\hat{\lambda}_{\mathrm{EP}}, \hat{\lambda}_{\mu}\right] + (1 - \mathbb{1}\{\alpha < 1/2\})\left[\hat{\lambda}_{\mu}, \hat{\lambda}_{\mathrm{EP}}\right]. \tag{13}$$

The optimal order quantity lies outside the PTC zone when λ is outside the interval Λ. Since $\hat{\delta}$ lies between zero and one (because of the condition $\eta \leq 3/(8[2\alpha-1]^+)$), and Λ is nonempty, the sets $\mathcal{G}_{\mathrm{P}}$ and $\mathcal{G}_{\mathrm{N}}$ are nonempty; the former set constitutes individuals who exhibit PTC effect, whereas the latter set constitutes the ones who do not exhibit that effect.

(iii) The proof follows the same logic used in proving Proposition 1(iii). □

Proof of Proposition 3. (i) When $\pi = rd_{\min} - cd_{\max}$, any payoff is perceived as gain. Thus the model behaves similarly to the traditional expected utility model with a concave utility function. The result then follows from Eeckhoudt et al. (1995).

(ii) When $\pi = (r-c)d_{\max}$, the value function is given by

$$V(q) = -\lambda\left\{\int_{d_{\min}}^{q} (\pi - (rx - cq))^{\delta}\, dF(x) + \int_{q}^{d_{\max}} (\pi - (r-c)q)^{\delta}\, dF(x)\right\} \tag{14}$$

for all $q \in [d_{\min}, d_{\max}]$. The desired result is obtained if we show that $\partial V/\partial q > 0$ for all $q \leq \tilde{q}_{\mathrm{EP}}$, from which it follows that the maximum of $V(q)$ cannot be less than $\tilde{q}_{\mathrm{EP}}$. For $q \leq \tilde{q}_{\mathrm{EP}}$, we can use (14) to obtain

$$\begin{aligned}
\frac{\partial V}{\partial q} &= \lambda\delta r\left[-\alpha\int_{d_{\min}}^{q} (\pi - rx + cq)^{\delta-1}\, dF(x) + (1-\alpha)(\pi - (r-c)q)^{\delta-1}\bar{F}(q)\right] \qquad (15)\\
&\geq \lambda\delta r\alpha\left[-\int_{d_{\min}}^{q} (\pi - rx + cq)^{\delta-1}\, dF(x) + (1-\alpha)(\pi - (r-c)q)^{\delta-1}\right]\\
&\geq \lambda\delta r\alpha(\pi - (r-c)q)^{\delta-1}\left[-F(q) + (1-\alpha)\right] \geq 0.
\end{aligned}$$

Here the first inequality follows because $\bar{F}(q) \geq \alpha$ for $q \leq \tilde{q}_{\rm EP}$, the second inequality from $(\pi - rx + cq)^{\delta-1} \leq (\pi - (r-c)q)^{\delta-1}$ for $x \leq q$, and the last inequality is again due to $q \leq \tilde{q}_{\rm EP}$. $\square$

Proof of Proposition 4. (i) When $\pi = (r-c)d_{\min}$, the value function is given by

$$V(q) = -\lambda \int_{d_{\min}}^{q_\alpha} r^\delta (q_\alpha - x)^\delta \, dF(x) + \int_{q_\alpha}^{q} r^\delta (x - q_\alpha)^\delta \, dF(x) + \int_{q}^{d_{\max}} (r-c)^\delta (q - d_{\min})^\delta \, dF(x),$$

where $q_\alpha = (1-\alpha)d_{\min} + \alpha q$. Then

$$\frac{\partial V}{\partial q} = \delta r^\delta (q - d_{\min})^{\delta-1} \left\{ -\alpha (q - d_{\min})^{1-\delta} h(q) + (1-\alpha)^\delta \bar{F}(q) \right\} \tag{16}$$

for

$$h(q) = \lambda \int_{d_{\min}}^{q_\alpha} (q_\alpha - x)^{\delta-1} \, dF(x) + \int_{q_\alpha}^{q} (x - q_\alpha)^{\delta-1} \, dF(x),$$

which is increasing in q because its derivative w.r.t. q is $\lambda\alpha^\delta (q-d_{\min})^{\delta-1}/D + (1-\alpha)^\delta (q-d_{\min})^{\delta-1}/D \geq 0$. Denote the term in braces in (16) as $n(q)$. Since $h(q)$ is increasing and $\bar{F}(q)$ is decreasing, it follows that $n(q)$ is decreasing in q with $n(d_{\min}) = (1-\alpha)^\delta > 0$ and $n(d_{\max}) = -\alpha D^{1-\delta} h(d_{\max}) < 0$. Hence $\partial V/\partial q$ crosses the horizontal axis only once from above and $V(q)$ is unimodal. Moreover, $\partial V/\partial q$ is decreasing in λ and so V is submodular in (q, λ).

(ii) Simplifying $n(q)$ yields

$$n(q) = \frac{(q - d_{\min})\alpha^{1+\delta}}{D\delta} \left[-\lambda - \left(\frac{1-\alpha}{\alpha} \right)^\delta + \frac{\delta (1-\alpha)^\delta (d_{\max} - q)}{\alpha^{1+\delta}(q - d_{\min})} \right].$$

Now using $F(\tilde{q}_{\rm EP}) = 1 - \alpha$ and $F(\mu) = 1/2$, we obtain

$$n(\tilde{q}_{\rm EP}) = \frac{(\tilde{q}_{\rm EP} - d_{\min})\alpha^{1+\delta}}{D\delta} \left[-\lambda + \left(\frac{1-\alpha}{\alpha} \right)^\delta \left(\frac{\delta}{1-\alpha} - 1 \right) \right],$$

$$n(\mu) = \frac{(\mu - d_{\min})\alpha^{1+\delta}}{D\delta} \left[-\lambda + \left(\frac{1-\alpha}{\alpha} \right)^\delta \left(\frac{\delta}{\alpha} - 1 \right) \right].$$

The model predicts both underordering and overordering w.r.t. $\tilde{q}_{\rm EP}$ iff $\delta \geq 1 - \alpha$. Specifically, for a given margin α we have $q^* < (\geq)\, \tilde{q}_{\rm EP}$ if $\lambda > (\leq)\, \hat{\lambda}_{\rm EP}$; here the threshold $\hat{\lambda}_{\rm EP} = (1-\alpha)^\delta (\delta/(1-\alpha) - 1)/\alpha^\delta$. Similarly, the model predicts optimal order quantities both above and below the mean demand iff $\delta \geq \alpha$. Then for a given α, we have $q^* < (\geq)\, \mu$ if $\lambda > (\leq)\, \hat{\lambda}_\mu$; in this case, the threshold $\hat{\lambda}_\mu = (1-\alpha)^\delta (\delta/\alpha - 1)/\alpha^\delta$.

In summary, the threshold $\hat{\delta}$ used in the definition of $\mathcal{G}_{\rm P}$ and $\mathcal{G}_{\rm N}$ is given by

$$\hat{\delta}(\alpha, r, D) = \max\{\alpha, 1-\alpha\}. \tag{17}$$

Because $\tilde{q}_{\rm EP} < \mu$ under low-margin, $\tilde{q}_{\rm EP} > \mu$ under high-margin, and because q^* is decreasing in λ, the model predicts the PTC effect when λ is in the interval Λ, which is given by

$$\Lambda(\delta, \alpha, r, D) = \mathbb{1}\{\alpha < 1/2\} \left[\hat{\lambda}_{\rm EP}, \hat{\lambda}_\mu \right] + (1 - \mathbb{1}\{\alpha < 1/2\}) \left[\hat{\lambda}_\mu, \hat{\lambda}_{\rm EP} \right]. \tag{18}$$

When λ is outside the interval Λ, the optimal order quantity lies outside the PTC zone. It is easy to see that $\hat{\delta}$ lies between zero and one, and Λ is nonempty. Therefore, the sets $\mathcal{G}_{\rm P}$ and $\mathcal{G}_{\rm N}$ are nonempty; the former constitutes individuals who exhibit PTC effect, whereas the latter constitutes the ones who do not exhibit the effect.

(iii) The proof follows the same logic used to prove Proposition 1(iii). $\square$

Proof of Proposition 5. (i) Put $q_{\bar{\alpha}} = (1-\alpha)q + \alpha d_{\max}$. Then we can use (6) to write

$$\frac{\partial V}{\partial q} = \delta r^{\delta}(d_{\max}-q)^{\delta-1}\left\{-\alpha^{\delta}F(q) + (1-\alpha)(d_{\max}-q)^{1-\delta}g(q)\right\}; \tag{19}$$

here

$$g(q) = \int_{q}^{q_{\bar{\alpha}}}(q_{\bar{\alpha}}-x)^{\delta-1}\,dF(x) + \lambda\int_{q_{\bar{\alpha}}}^{d_{\max}}(x-q_{\bar{\alpha}})^{\delta-1}\,dF(x),$$

which is decreasing in q because its derivative w.r.t. q is $-\alpha^{\delta}(d_{\max}-q)^{\delta-1}/D - \lambda(1-\alpha)^{\delta}(d_{\max}-q)^{\delta-1}/D \le 0$. Let the term in braces in (19) be denoted $m(q)$. Since $g(q)$ is decreasing and $F(q)$ is increasing, it follows that $m(q)$ is decreasing in q with $m(d_{\min}) = (1-\alpha)D^{1-\delta}g(d_{\min}) > 0$ and $m(d_{\max}) = -\alpha^{\delta} < 0$. Therefore, $\partial V/\partial q$ crosses the horizontal axis only once from above and $V(q)$ is unimodal. We see that $\partial V/\partial q$ is increasing in λ and hence V is supermodular in (q,λ).

(ii) Upon simplifying $m(q)$, we obtain

$$m(q) = \frac{(d_{\max}-q)(1-\alpha)^{1+\delta}}{D\delta}\left[\lambda + \left(\frac{\alpha}{1-\alpha}\right)^{\delta} - \frac{\delta\alpha^{\delta}(q-d_{\min})}{(1-\alpha)^{1+\delta}(d_{\max}-q)}\right].$$

Using $F(\tilde{q}_{\mathrm{EP}}) = 1-\alpha$ and $F(\mu) = 1/2$, we obtain

$$m(\tilde{q}_{\mathrm{EP}}) = \frac{(d_{\max}-\tilde{q}_{\mathrm{EP}})(1-\alpha)^{1+\delta}}{D\delta}\left[\lambda - \left(\frac{\alpha}{1-\alpha}\right)^{\delta}\left(\frac{\delta}{\alpha}-1\right)\right],$$

$$m(\mu) = \frac{(d_{\max}-\mu)(1-\alpha)^{1+\delta}}{D\delta}\left[\lambda - \left(\frac{\alpha}{1-\alpha}\right)^{\delta}\left(\frac{\delta}{1-\alpha}-1\right)\right].$$

The model predicts both underordering and overordering w.r.t. $\tilde{q}_{\mathrm{EP}}$ iff $\delta \ge \alpha$. So for margin α, we have $q^* < (\ge)\, \tilde{q}_{\mathrm{EP}}$ if $\lambda < (\ge)\, \hat{\lambda}_{\mathrm{EP}}$ where the threshold $\hat{\lambda}_{\mathrm{EP}} = \alpha^{\delta}(\delta/\alpha - 1)/(1-\alpha)^{\delta}$. Similarly, the model predicts optimal order quantities both above and below the mean demand iff $\delta \ge 1-\alpha$. Then, for a given α, we have $q^* < (\ge)\, \mu$ if $\lambda < (\ge)\, \hat{\lambda}_{\mu}$, where the threshold $\hat{\lambda}_{\mu} = \alpha^{\delta}(\delta/(1-\alpha) - 1)/(1-\alpha)^{\delta}$.

Therefore, the threshold $\hat{\delta}$ used in the definition of $\mathcal{G}_{\mathrm{P}}$ and $\mathcal{G}_{\mathrm{N}}$ is given by

$$\hat{\delta}(\alpha, r, D) = \max\{\alpha, 1-\alpha\}. \tag{20}$$

Because $\tilde{q}_{\mathrm{EP}} < \mu$ under low-margin, $\tilde{q}_{\mathrm{EP}} > \mu$ under high-margin, and because q^* is increasing in λ, the model predicts the PTC effect when λ is in the interval Λ, which is given by

$$\Lambda(\delta,\alpha,r,D) = \mathbb{1}\{\alpha < 1/2\}\left[\hat{\lambda}_{\mu}, \hat{\lambda}_{\mathrm{EP}}\right] + (1 - \mathbb{1}\{\alpha < 1/2\})\left[\hat{\lambda}_{\mathrm{EP}}, \hat{\lambda}_{\mu}\right]. \tag{21}$$

The optimal order quantity lies outside the PTC zone when λ is outside the interval Λ. It is easy to see that $\hat{\delta}$ is between zero and one, and Λ is nonempty. Therefore, the sets $\mathcal{G}_{\mathrm{P}}$ and $\mathcal{G}_{\mathrm{N}}$ are nonempty; the former constitutes individuals who exhibit PTC effect, whereas the latter constitutes the ones who do not exhibit the effect.

(iii) For given α_{LM} and α_{HM}, the interval $\Lambda(\alpha_{\mathrm{LM}}) \cap \Lambda(\alpha_{\mathrm{HM}})$ is nonempty because $\alpha_{\mathrm{HM}} < \alpha_{\mathrm{LM}}$, $\hat{\lambda}_{\mu}$ is increasing in α, and $\hat{\lambda}_{\mathrm{EP}}$ is decreasing in α (which can be verified easily). Therefore, the set $\mathcal{G}_{\mathrm{P}}(\alpha_{\mathrm{LM}}) \cap \mathcal{G}_{\mathrm{P}}(\alpha_{\mathrm{HM}}) = \{(\lambda,\delta) \in \mathcal{G} : \lambda \in \Lambda(\alpha_{\mathrm{LM}}) \cap \Lambda(\alpha_{\mathrm{HM}}), \delta \ge \max(\hat{\delta}(\alpha_{\mathrm{LM}}), \hat{\delta}(\alpha_{\mathrm{HM}}))\}$ is also nonempty. $\square$

Proof of Proposition 6. (i) From (7), we obtain

$$\frac{\partial V_{<\mu}}{\partial q} = \delta r^{\delta}(\mu-q)^{\delta-1}\left\{-\alpha^{\delta}F(q)+\lambda(1-\alpha)^{\delta}\bar{F}(\mu)+(1-\alpha)(\mu-q)^{1-\delta}g(q)\right\}; \tag{22}$$

here

$$g(q) = \int_{q}^{q_{\bar{\alpha}}}(q_{\bar{\alpha}}-x)^{\delta-1}\,dF(x)+\lambda\int_{q_{\bar{\alpha}}}^{\mu}(x-q_{\bar{\alpha}})^{\delta-1}\,dF(x),$$

which is decreasing in q because $g'(q) = -\alpha^{\delta}(\mu-q)^{\delta-1}/D-\lambda(1-\alpha)^{\delta}(\mu-q)^{\delta-1}/D \le 0$. Let the term in braces in (22) be called $m(q)$. Since $g(q)$ is decreasing and $F(q)$ is increasing, it follows that $m(q)$ is decreasing in q with $m(d_{\min}) = \lambda(1-\alpha)^{\delta}\bar{F}(\mu)+(1-\alpha)(\mu-d_{\min})^{1-\delta}g(d_{\min}) > 0$. Because the domain of $m(q)$ is $[d_{\min},\mu)$, the sign of $m(\mu) = -\alpha^{\delta}F(\mu)+\lambda(1-\alpha)^{\delta}\bar{F}(\mu)$ determines whether or not it has a zero. If $m(\mu) < 0$, which is equivalent to

$$\lambda < \left(\frac{\alpha}{1-\alpha}\right)^{\delta} \equiv \hat{\lambda}_{<\mu}, \tag{23}$$

then $\partial V_{<\mu}/\partial q$ crosses the horizontal axis once and $V_{<\mu}$ is unimodal; otherwise, $\partial V_{<\mu}/\partial q$ is completely positive and $V_{<\mu}$ is increasing. Furthermore, $\partial V_{<\mu}/\partial q$ is increasing in λ and so $V_{<\mu}$ is supermodular in (q,λ).

(ii) Using (8), we obtain

$$\frac{\partial V_{>\mu}}{\partial q} = \delta r^{\delta}(q-\mu)^{\delta-1}\left\{(1-\alpha)^{\delta}\bar{F}(q)-\lambda\alpha^{\delta}F(\mu)-\alpha(q-\mu)^{1-\delta}h(q)\right\}, \tag{24}$$

where

$$h(q) = \lambda\int_{\mu}^{q_{\alpha}}(q_{\alpha}-x)^{\delta-1}dF(x)+\int_{q_{\alpha}}^{q}(x-q_{\alpha})^{\delta-1}dF(x),$$

which is increasing in q because $h'(q) = \lambda\alpha^{\delta}(q-\mu)^{\delta-1}/D+(1-\alpha)^{\delta}(q-\mu)^{\delta-1}/D \ge 0$. Let the term in braces in (24) be called $n(q)$. Then $n(q)$ is decreasing in q because $h(q)$ is increasing and $\bar{F}(q)$ is decreasing. We have $n(d_{\max}) = -\lambda\alpha^{\delta}F(\mu)-\alpha(d_{\max}-\mu)^{1-\delta}h(d_{\max}) < 0$. The domain of $n(q)$ is $(\mu,d_{\max}]$, and $n(\mu) = (1-\alpha)^{\delta}\bar{F}(\mu)-\lambda\alpha^{\delta}F(\mu)$. If $n(\mu) > 0$, which is equivalent to

$$\lambda < \left(\frac{1-\alpha}{\alpha}\right)^{\delta} \equiv \hat{\lambda}_{>\mu}, \tag{25}$$

then $\partial V_{>\mu}/\partial q$ crosses the horizontal axis once and $V_{>\mu}$ is unimodal; otherwise, $\partial V_{>\mu}/\partial q$ is completely negative and $V_{>\mu}$ is decreasing. Also, $\partial V_{>\mu}/\partial q$ is decreasing in λ. Therefore, $V_{>\mu}$ is submodular in (q,λ).

(iii) Let the modes of $V_{<\mu}$ and $V_{>\mu}$ be (respectively) $q^*_{<\mu}$ and $q^*_{>\mu}$. Consider a low-margin setting, and let us put $\alpha = \alpha_{\text{LM}}$ for notational convenience. Simplifying the $m(q)$ from part (i) yields

$$m(q) = -\frac{\alpha^{\delta}}{D}\left[(q-d_{\min})-\frac{(1-\alpha)(\mu-q)}{\delta}\right]+\lambda\left[\frac{(1-\alpha)^{\delta}}{2}+\frac{(\mu-q)(1-\alpha)^{1+\delta}}{\delta D}\right].$$

When product margin is low, $\tilde{q}_{\text{LM}}$ is in the domain of $m(q)$. Using the equality $F(\tilde{q}_{\text{LM}}) = 1-\alpha$, we obtain

$$m(\tilde{q}_{\text{LM}}) = -\alpha^{\delta}(1-\alpha)\left[1-\frac{(\alpha-1/2)}{\delta}\right]+\lambda(1-\alpha)^{\delta}\left[\frac{1}{2}+\frac{1}{\delta}(\alpha-1/2)(1-\alpha)\right]. \tag{26}$$

If $\delta < (\alpha-1/2)$, then $m(\tilde{q}_{\text{LM}}) > 0$ always and $q^*_{<\mu} > \tilde{q}_{\text{LM}}$ – that is, the model predicts only overordering. To accommodate heterogeneity, we assume that $\delta \ge (\alpha-1/2)$.

Now define the following thresholds:

$$\hat{\lambda}_{\text{N}} = \frac{\alpha^{\delta}(1-\alpha)^{1-\delta}\left(\delta-(\alpha-1/2)\right)}{\delta/2+(\alpha-1/2)(1-\alpha)}, \qquad \hat{\lambda}_{1} = \min\left(\hat{\lambda}_{<\mu},\hat{\lambda}_{>\mu}\right) = \left(\frac{1-\alpha}{\alpha}\right)^{\delta}, \tag{27}$$

where the latter equality follows from the definitions of $\hat{\lambda}_{<\mu}$ and $\hat{\lambda}_{<\mu}$ in (23) and (25) respectively, and from the low margin condition under consideration. Given our prior assumption on δ and that $\alpha > 1/2$, $\hat{\lambda}_{\mathrm{N}}$ is always positive. It follows from (26) that $q^*_{<\mu} \leq (>)\, \tilde{q}_{\mathrm{LM}}$ for $\lambda \leq (>)\, \hat{\lambda}_{\mathrm{N}}$. From the analysis in part (i) we know that $q^*_{>\mu}$ exists if $\lambda < \hat{\lambda}_{>\mu}$; otherwise, $V_{>\mu}$ is a decreasing function. Now we define another threshold:

$$\hat{\lambda}_{\mathrm{P}} = \max\left(\hat{\lambda}_{\mathrm{N}}, \hat{\lambda}_1\right). \tag{28}$$

We consider two cases as follows. First, $\lambda \leq \hat{\lambda}_{\mathrm{N}}$: then $q^*_{<\mu}$ lies to the left of the PTC zone and $q^*_{>\mu}$ (when it exists) lies to the right of the PTC zone. The optimal order quantity is therefore predicted to be outside the PTC zone. Second, $\lambda \geq \hat{\lambda}_{\mathrm{P}}$: then $q^*_{<\mu}$ is inside the PTC zone because $\hat{\lambda}_{\mathrm{P}} \geq \hat{\lambda}_{\mathrm{N}}$, and $V_{>\mu}$ is decreasing because $\hat{\lambda}_{\mathrm{P}} \geq \hat{\lambda}_1 = \hat{\lambda}_{>\mu}$. Hence, the optimal order quantity is predicted to be inside the PTC zone.

Following a similar logic as above for a high-margin α, we see that by assuming $\delta \geq 1/2 - \alpha$, the threshold $\hat{\lambda}_{\mathrm{N}}$ – now redefined as the value of λ that sets $n(\tilde{q}_{\mathrm{HM}}) = 0$ (where $n(q)$ is as defined in part (ii)) – is always positive. Moreover, with the thresholds $\hat{\lambda}_1$ and $\hat{\lambda}_{\mathrm{P}}$ as defined in (27) and (28) respectively, the model predicts order quantities outside (resp., inside) the PTC zone when $\lambda \leq \hat{\lambda}_{\mathrm{N}}$ (resp., $\lambda \geq \hat{\lambda}_{\mathrm{P}}$).

To summarize, the threshold $\hat{\delta}$ used in the definition of $\mathcal{G}_{\mathrm{P}}$ and $\mathcal{G}_{\mathrm{N}}$ is given by

$$\hat{\delta}(\alpha, r, D) = \max\{\alpha - 1/2, 1/2 - \alpha\}, \tag{29}$$

the threshold $\hat{\lambda}_{\mathrm{N}}(\delta, \alpha, r, D)$ is the value of λ that sets $n(\tilde{q}_{\mathrm{EP}}(\alpha)) = 0$ for $\alpha < 1/2$ and $m(\tilde{q}_{\mathrm{EP}}(\alpha)) = 0$ for $\alpha \geq 1/2$, and the threshold $\hat{\lambda}_{\mathrm{P}}(\delta, \alpha, r, D)$ is as defined in (28). These thresholds on λ are positive if $\delta \geq \hat{\delta}$, and since $\hat{\delta}$ lies between zero and one, the sets $\mathcal{G}_{\mathrm{P}}$ and $\mathcal{G}_{\mathrm{N}}$ are nonempty. The former set constitutes individuals who exhibit PTC effect, whereas the latter set constitutes the ones who do not exhibit PTC effect.

(iv) For given α_{LM} and α_{HM}, the set $\mathcal{G}_{\mathrm{P}}(\alpha_{\mathrm{LM}}) \cap \mathcal{G}_{\mathrm{P}}(\alpha_{\mathrm{HM}}) = \{(\lambda, \delta) \in \mathcal{G} : \lambda \geq \max(\hat{\lambda}_{\mathrm{P}}(\alpha_{\mathrm{LM}}), \hat{\lambda}_{\mathrm{P}}(\alpha_{\mathrm{HM}})), \delta \geq \max(\hat{\delta}(\alpha_{\mathrm{LM}}), \hat{\delta}(\alpha_{\mathrm{HM}}))\}$ is trivially nonempty. $\square$