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Contents

Corn, Soybeans or Fallow: Dynamic Farmland Allocation under Uncertainty

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ABSTRACT

This chapter develops a theoretical basis for understanding the trade-offs facing a farmer for allocating his farmland among several crops over multiple growing seasons. Specifically, we focus on the farmland allocation among two cash crops (corn and soybeans) and letting the farmland lay fallow to rejuvenate the soil and increase the revenue for the crop grown on this farmland in the subsequent seasons. In each growing period, the farmer chooses the allocation in the presence of revenue uncertainty for each cash crop, and crop rotation benefits across periods, where revenue is stochastically larger and farming cost is lower when a cash crop is grown on a rotated farmland (where the same crop was not grown in the previous period). We solve for the optimal dynamic allocation policy.

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1

Motivation and Description of the Problem

This chapter develops a theoretical basis for understanding the tradeoffs facing a farmer for allocating his farmland among several crops over multiple growing seasons. The results in this chapter are originated from our companion paper (Boyabatlı *et al.*, 2018).

Specifically, we focus on the farmland allocation among two cash crops (corn and soybeans) and letting the farmland lay fallow to rejuvenate the soil and increase the revenue for the crop grown on this farmland in the subsequent seasons. Corn and soybeans are the two most planted crops in the U.S. which account for 55.5% of total acres harvested in 2014 (USDA, 2015a) with an estimated total market value of $92 billion in the same year (USDA, 2015b). Corn is an input for a large range of food and industrial products (such as animal feed and ethanol), whereas soybeans are the world's primary source of animal feed and second largest source of vegetable oil. In the U.S., both crops are grown within the same time period (between late March and June), and thus, they compete for the allocation of farmland.

There are unique characteristics of corn and soybeans that present challenges for the farmland allocation decision. First, there exists *revenue uncertainty in each growing season.* The revenue uncertainty of each crop is driven by a variety of factors including the uncertainty in its harvest volume and the uncertainty in its sale price at the end of the growing season. The harvest volume is uncertain due to unfavorable weather conditions, and infestation of pests and diseases during the growing season (Kazaz and Webster, 2011). The sale price is uncertain because it is tied up to the prevailing price at the regional exchange (spot) markets (Goel and Tanrisever, 2017). As empirically documented in Livingston *et al.* (2015), the revenues for corn and soybeans show considerable variability over time. Second, there exist *crop rotation benefits across growing seasons*. In particular, growing a crop on rotated farmland (where the other crop was grown in the previous season) is more profitable than growing on nonrotated farmland (where the same crop was grown in the previous season). As highlighted by Hennessy (2006), rotating crops can increase the crop revenue by improving the soil structure and breaking the pest cycles; and decrease the farming cost by reducing the need for fertilizers (due to improved soil structure) and pesticides (due to lower pest populations). For example, rotating soybeans with corn increases the corn revenue because soybeans fix the nitrogen content of soil which is beneficial for corn growth, and reduces the farming cost because growing corn requires lower fertilizer (nitrogen) volume.

To delineate the main trade-offs let us focus on the simple case where the farmer always fully allocates the farmland between the two cash crops; that is, no farmland is laid fallow. One may argue that this farmer, to reap the crop rotation benefits, should follow a rotation-based farmland allocation policy; that is, grow each crop on rotated farmland every growing season. However, as intuition suggests, deviating from this allocation policy can be profitable when the revenue of a particular crop is expected to increase in a significant manner in comparison with the revenue of the other crop. In this case, the farmer may choose to grow additional volume of that crop on non-rotated farmland, a phenomenon commonly observed in practice (Meyer, 2012). But then, *when* is the revenue increase significant enough to induce the farmer to deviate? Once it is profitable to deviate, *how much* should the farmer deviate; that is, what is the additional volume grown on the non-rotated farmland? These are the central questions that should be answered by an optimally crafted dynamic farmland allocation policy. The dynamic farmland allocation policy should also consider the farmer's choice of letting part of the farmland lay fallow to increase the revenue for the cash crops grown on this farmland in the subsequent seasons.

Our paper's main contribution is to the literature on farm planning, specifically, to the stream of papers studying farmland allocation problem. The farmland allocation problem has received considerable attention from researchers in the operations management and agricultural economics fields. The majority of papers in this stream considers either single-period models under uncertainty (where crop rotation benefits are irrelevant) or multi-period models under certainty. Yet farmers face significant uncertainty for their crop revenues in a growing period and crop rotation benefits across growing periods. This is the first paper that characterizes the optimal *dynamic* allocation policy under *uncertainty* in the presence of crop rotation benefits.

The papers that focus on single-period models study the interplay between the farmland allocation decision and other operational features, including the penalties associated with cash flow variability (Collender and Zilberman, 1985), government price support for crops (Chavas and Holt, 1990), rainfall uncertainty (Maatman *et al.*, 2002) and irrigation management (Huh and Lall, 2013). As also highlighted by Lowe and Preckel (2004), consideration of crop rotation benefits across growing periods is central to the farmland allocation problem. Only a few papers in the literature consider these benefits and study the dynamic farmland allocation problem under uncertainty. As reviewed in Livingston *et al.* (2015), the focus of these papers is to suggest heuristic allocation policies and numerically evaluate their performance. Among these papers, Taylor and Burt (1984) provide a stochastic dynamic programming formulation for a farmer's decision of when to grow wheat or lay the farmland fallow. Based on their formulation, they develop a heuristic allocation policy and numerically study its performance using a calibration that represents a typical wheat farmer in Montana. Cai *et al.* (2013) propose a stochastic dynamic programming formulation for a farmer's allocation decision between corn and soybeans. They numerically compare the performance of different heuristic allocation policies and show that growing corn in the entire farmland provides the best performance.

Closest to our work, Livingston *et al.* (2015) study the farmland allocation between corn and soybeans in a multi-period framework. They formulate an infinite horizon stochastic dynamic program where in each period, the farmer chooses which crop to grow and the subsequent amount of fertilizer application facing uncertainties in fertilizer cost and revenue for each crop in the presence of crop rotation benefits across periods. They do not provide a theoretical characterization of the optimal solution but numerically analyze the optimal actions of the farmer. Using a calibration based on a typical farmer in Iowa, they suggest that the farmer should implement a rotation-based allocation policy. Because our focus is on farmland allocation, we do not consider fertilizer application decision or the farming cost uncertainty. However, we extend their model to consider growing more than one crop in the same period, a future research direction suggested in their paper, and letting the farmland lay fallow.

2

Modelling Approach and Methodology

We consider a farmer who allocates a single acre of farmland among two cash crops (corn and soybeans) and a fallow crop in each growing season to maximize the expected total profit over a finite number of growing seasons. The farmland allocated to the fallow crop in a growing season in our model corresponds to the farmland laid fallow (where no crop is grown) in that season in practice. We use superscript c, s, and f to denote the corn, soybeans, and the fallow crop related parameters, respectively.

Decision variables. Let $\alpha_t^j \in [0,1]$ for $j \in \{c,s,f\}$ denote the proportion of farmland allocated to corn in time period (growing season) t on which crop j was grown in period $t-1$. For notational convenience, we denote the total proportion of farmland allocated to corn in period t as $\alpha_t \doteq \sum_{j\in\{c,s,f\}} \alpha_t^j$, and corn allocations in period t as $\boldsymbol{\alpha}_t \doteq \left(\alpha_t^c, \alpha_t^s, \alpha_t^f\right)$. Let $\beta_t^j \in [0,1]$ for $j \in \{c,s,f\}$ denote the proportion of farmland allocated to the fallow crop in period t on which crop j was grown in period $t-1$. We denote the total proportion of farmland allocated to the fallow crop in period t as $\beta_t \doteq \sum_{j\in\{c,s,f\}} \beta_t^j$, and the fallow crop allocations in period t as $\boldsymbol{\beta}_t \doteq \left(\beta_t^c, \beta_t^s, \beta_t^f\right)$. The remaining proportion of farmland, $1-\alpha_t-\beta_t$, is allocated to soybeans in period t. Within this allocation it is important to determine the proportion of farmland on which crop $j \in \{c, s, f\}$ was grown in period $t-1$. As we will discuss shortly, this can be determined without defining another three decision variables (which denote the proportion of farmland allocated to soybeans in period t on which crop $j \in \{c, s, f\}$ was grown in period $t-1$).

Revenue uncertainty. Let $\tilde{r}_t^c$ and $\tilde{r}_t^s$ denote the uncertain corn and soybeans revenue per acre in period t, respectively. Because the farmland allocated to the fallow crop in each growing season in our model corresponds to the farmland that is laid fallow (where no crop is grown) in that growing season in practice, the fallow crop revenue per acre is assumed to be zero in each period. We assume that $\tilde{\boldsymbol{r}}_t = (\tilde{r}_t^c, \tilde{r}_t^s)$ follow correlated stochastic processes with Markovian property; that is, the current revenue realizations are sufficient to characterize the distribution of the future revenues.

Crop rotation benefits. To capture the revenue-enhancing crop rotation benefits, we assume that the uncertain revenue per acre of (cash) crop $j \in \{c, s\}$ in period t is $\tilde{r}_t^j$ if it is grown on nonrotated farmland, $(1+b_1^j)\tilde{r}_t^j$ if it is grown on rotated farmland where the other cash crop was grown in the previous period, and $(1+b_2^j)\tilde{r}_t^j$ if it is grown on rotated farmland where the fallow crop was grown in the previous period. We assume $b_2^j \geq b_1^j \geq 0$, i.e., revenue-enhancing crop rotation benefits for cash crop $j \in \{c, s\}$ are (stochastically) larger on the rotated farmland where the fallow crop was grown in the previous season than on the rotated farmland where the other cash crop was grown in the previous season. To capture the cost-reducing crop rotation benefits, we assume that the unit farming cost of cash crop $j \in \{c, s\}$ is ω^j if it is grown on non-rotated farmland, $(1-\gamma_1^j)\omega^j$ if it is grown on rotated farmland where the other cash crop was grown in the previous period, and $(1-\gamma_2^j)\omega^j$ if it is grown on rotated farmland where the fallow crop was grown in the previous period. Similarly, we assume $\gamma_2^j \geq \gamma_1^j \geq 0$. We assume that there is no farming cost associated with the fallow crop in each period.

Formulation. We formulate the farmer's problem as a finite horizon stochastic dynamic program. In each period $t \in [1, T]$, the sequence of events is as follows:

(i) At the beginning of period t, the farmer observes the total corn allocation α_{t-1}, the total fallow crop allocation β_{t-1}, and corn and soybeans revenues $\boldsymbol{r}_{t-1} = (r_{t-1}^c, r_{t-1}^s)$ from period $t-1$. The farmer then chooses the corn allocations $\boldsymbol{\alpha}_t = (\alpha_t^c, \alpha_t^s, \alpha_t^f)$ and the fallow crop allocations $\boldsymbol{\beta}_t = (\beta_t^c, \beta_t^s, \beta_t^f)$ constrained by the available farmland where crop $j \in \{c, s, f\}$ was grown in period $t-1$, i.e., $\alpha_t^c + \beta_t^c \leq \alpha_{t-1}, \alpha_t^f + \beta_t^f \leq \beta_{t-1}, \alpha_t^s + \beta_t^s \leq 1 - \alpha_{t-1} - \beta_{t-1}$. For example, the first constraint ensures that the sum of the proportion of farmland allocated to corn and the fallow crop in this period where corn was grown in the previous period cannot be larger than the proportion of farmland where corn was grown in the previous period.

(ii) At the end of period t, the corn and soybeans revenues $\tilde{\boldsymbol{r}}_t = (\tilde{r}_t^c, \tilde{r}_t^s)$ are realized and the farmer collects the revenues from the crop sales.

The farmer's immediate payoff in period $t \in [1, T]$ is given by

$$
\begin{aligned}
& L(\boldsymbol{\alpha}_t, \boldsymbol{\beta}_t \mid \alpha_{t-1}, \beta_{t-1}, \boldsymbol{r}_{t-1}) && (2.1) \\
\doteq\ & \alpha_t^f \mathbb{E}_t \left[(1+b_2^c)\tilde{r}_t^c - (1-\gamma_2^c)\omega^c\right] + \alpha_t^s \mathbb{E}_t \left[(1+b_1^c)\tilde{r}_t^c - (1-\gamma_1^c)\omega^c\right] + \alpha_t^c \mathbb{E}_t \left[\tilde{r}_t^c - \omega^c\right] \\
+\ & \min\left(1-\alpha_t-\beta_t, \beta_{t-1}-\alpha_t^f-\beta_t^f\right) \mathbb{E}_t \left[(1+b_2^s)\tilde{r}_t^s - (1-\gamma_2^s)\omega^s\right] && (2.2) \\
+\ & \min\left((1-\alpha_t-\beta_t-(\beta_{t-1}-\alpha_t^f-\beta_t^f))^+, \alpha_{t-1}-\alpha_t^c-\beta_t^c\right) \mathbb{E}_t \left[(1+b_1^s)\tilde{r}_t^s - (1-\gamma_1^s)\omega^s\right] \\
+\ & \left((1-\alpha_t-\beta_t-(\beta_{t-1}-\alpha_t^f-\beta_t^f))^+ - (\alpha_{t-1}-\alpha_t^c-\beta_t^c)\right)^+ \mathbb{E}_t \left[\tilde{r}_t^s - \omega^s\right],
\end{aligned}
$$

where $\mathbb{E}_t[.]$ denotes the expectation operator conditional on the available information at time t; that is, $\mathbb{E}_t[.] = \mathbb{E}[.|\boldsymbol{r}_{t-1}]$. In (2.1), the first line corresponds to the total expected profit from growing corn in period t. It is the sum of expected profit from growing corn on three different farmlands: rotated farmland where the fallow crop was grown in the previous period, rotated farmland where soybeans were grown in the previous period, and nonrotated farmland. The remaining three lines in (2.1) denote the total expected profit from growing soybeans in period t. For $1 - \alpha_t - \beta_t$ proportion of the farmland that is allocated to soybeans, to leverage crop rotation benefits, the farmer starts planting soybeans from the rotated farmland where the fallow crop was grown in the previous period which remains (if any) from the corn and the fallow crop allocation in this period (that is given by $\beta_{t-1} - \alpha_t^f - \beta_t^f$). Therefore, rotation benefits for soybeans plantation on the fallow farmland in the previous period are only relevant for $\min\left(1-\alpha_t-\beta_t, \beta_{t-1}-\alpha_t^f-\beta_t^f\right)$ proportion of the farmland. For the remaining $(1-\alpha_t-\beta_t-(\beta_{t-1}-\alpha_t^f-\beta_t^f))^+$ proportion of the farmland allocated to soybeans, again to leverage crop rotation benefits, the farmer starts planting soybeans from the rotated farmland where corn was grown in the previous period which remains (if any) from the corn and the fallow crop allocation in this period (that is given by $\alpha_{t-1}-\alpha_t^c-\beta_t^c$). Therefore, rotation benefits for soybeans on corn farmland in the previous period are only relevant for $\min\left((1-\alpha_t-\beta_t-(\beta_{t-1}-\alpha_t^f-\beta_t^f))^+, \alpha_{t-1}-\alpha_t^c-\beta_t^c\right)$ proportion of the farmland. The remaining proportion of the farmland allocated to soybeans is from the nonrotated farmland which has no rotation benefit.

Let $V_t(\alpha_{t-1}, \beta_{t-1}, \boldsymbol{r}_{t-1})$ for $t \in [1, T]$ denote the optimal value function from period t onward given α_{t-1}, β_{t-1}, and $\boldsymbol{r}_{t-1}$, which equals to

$$\begin{aligned}
\max_{\boldsymbol{\alpha}_t, \boldsymbol{\beta}_t} \quad & \left\{ L(\boldsymbol{\alpha}_t, \boldsymbol{\beta}_t \mid \alpha_{t-1}, \beta_{t-1}, \boldsymbol{r}_{t-1}) + \mathbb{E}_t \left[V_{t+1}(\alpha_t = \sum_{j \in \{c,s,f\}} \alpha_t^j, \beta_t = \sum_{j \in \{c,s,f\}} \beta_t^j, \tilde{\boldsymbol{r}}_t) \right] \right\} \\
\text{s.t.} \quad & \alpha_t^c + \beta_t^c \le \alpha_{t-1}, \alpha_t^f + \beta_t^f \le \beta_{t-1}, \alpha_t^s + \beta_t^s \le 1 - \alpha_{t-1} - \beta_{t-1}, \qquad (2.3) \\
& 0 \le \alpha_t^j \le 1, 0 \le \beta_t^j \le 1 \text{ for } j \in \{c, s, f\},
\end{aligned}$$

with a boundary condition $V_{T+1}(\cdot) = 0$. The farmer's optimal total expected profit over the entire planning horizon is given by $V_1(\alpha_0, \beta_0, \boldsymbol{r}_0)$, where α_0, β_0, and $\boldsymbol{r}_0$ denote the observed corn allocation, the fallow crop allocation and crop revenues at the beginning of the planning horizon, respectively.

3

Results and Insights

We now solve for the farmer's optimization problem stated in (2.3) and characterize the optimal allocation decision and the optimal value function in period $t \in [1, T]$. For this purpose, we first define the following recursive operators:

$$\begin{aligned} K_t^c(\boldsymbol{r}_{t-1}) &= \max\left\{C_t^{(0)}, S_t^{(1)}, F_t\right\}, \\ K_t^s(\boldsymbol{r}_{t-1}) &= \max\left\{C_t^{(1)}, S_t^{(0)}, F_t\right\}, \\ K_t^f(\boldsymbol{r}_{t-1}) &= \max\left\{C_t^{(2)}, S_t^{(2)}, F_t\right\}, \end{aligned} \tag{3.1}$$

where

$$\begin{aligned} C_t^{(0)} &\doteq -\omega^c + \mathbb{E}_t\left[\tilde{r}_t^c + K_{t+1}^c(\tilde{\boldsymbol{r}}_t)\right], \\ C_t^{(1)} &\doteq -(1-\gamma_1^c)\omega^c + \mathbb{E}_t\left[(1+b_1^c)\tilde{r}_t^c + K_{t+1}^c(\tilde{\boldsymbol{r}}_t)\right], \\ C_t^{(2)} &\doteq -(1-\gamma_2^c)\omega^c + \mathbb{E}_t\left[(1+b_2^c)\tilde{r}_t^c + K_{t+1}^c(\tilde{\boldsymbol{r}}_t)\right], \\ S_t^{(0)} &\doteq -\omega^s + \mathbb{E}_t\left[\tilde{r}_t^s + K_{t+1}^s(\tilde{\boldsymbol{r}}_t)\right], \\ S_t^{(1)} &\doteq -(1-\gamma_1^s)\omega^s + \mathbb{E}_t\left[(1+b_1^s)\tilde{r}_t^s + K_{t+1}^s(\tilde{\boldsymbol{r}}_t)\right], \\ S_t^{(2)} &\doteq -(1-\gamma_2^s)\omega^s + \mathbb{E}_t\left[(1+b_2^s)\tilde{r}_t^s + K_{t+1}^s(\tilde{\boldsymbol{r}}_t)\right], \\ F_t &\doteq \mathbb{E}_t\left[K_{t+1}^f(\tilde{\boldsymbol{r}}_t)\right], \end{aligned}$$

with $K^j_{T+1}(\boldsymbol{r}_T) = 0$ for $j \in \{c, s, f\}$. It is easy to establish that $C^{(0)}_t \leq C^{(1)}_t \leq C^{(2)}_t$ and $S^{(0)}_t \leq S^{(1)}_t \leq S^{(2)}_t$ hold by definition.

In (3.1), $K^j_t(\boldsymbol{r}_{t-1})$ denotes the expected marginal profit of farmland in the remaining planning horizon (from period t onward) where crop j was grown in period $t-1$. Consider, for example, $K^c_t(\boldsymbol{r}_{t-1})$. It is given by the maximum profit from three options available to the farmer: (i) growing corn in period t and optimally using the farmland in the remaining periods (which yields the expected marginal profit $\mathbb{E}_t\left[K^c_{t+1}(\tilde{\boldsymbol{r}}_t)\right]$)—which is denoted by $C^{(0)}_t$,—(ii) growing soybeans in period t and optimally using the farmland in the remaining periods (which yields the expected marginal profit $\mathbb{E}_t\left[K^s_{t+1}(\tilde{\boldsymbol{r}}_t)\right]$)—which is denoted by $S^{(1)}_t$, and (iii) growing the fallow crop in period t (which brings zero expected profit in this period) and optimally using the farmland in the remaining periods (which yields the expected marginal profit $\mathbb{E}_t\left[K^f_{t+1}(\tilde{\boldsymbol{r}}_t)\right]$)—which is denoted by F_t.

The farmer's optimization problem stated in (2.3) has a piecewise linear structure–that is, the objective function is piecewise linear in the decision variables α^j_t and β^j_t for $j \in \{c, s, f\}$. Therefore, the optimal allocation decisions can be presented in terms of two other variables: the total proportion of farmland allocated to corn in period t, i.e., $\alpha_t = \sum_{j\in\{c,s,f\}} \alpha^j_t$, and the total proportion of farmland allocated to the fallow crop in period t, i.e., $\beta_t = \sum_{j\in\{c,s,f\}} \beta^j_t$. As we will discuss shortly, there is a one-to-one correspondence between these two variables and the decision variables α^j_t and β^j_t for $j \in \{c, s, f\}$.

Proposition 3.1. In period $t \in [1, T]$, the optimal corn allocation α^*_t and the optimal fallow crop allocation β^*_t are given by $(\alpha^*_t, \beta^*_t) =$

$$\begin{cases}
(0, 1) & \text{if } K^f_t(\boldsymbol{r}_{t-1}) = F_t, \\
(1, 0) & \text{if } K^f_t(\boldsymbol{r}_{t-1}) = C^{(2)}_t \ \& \ K^c_t(\boldsymbol{r}_{t-1}) = C^{(0)}_t \\
(1-\beta_{t-1}, 0) & \text{if } K^f_t(\boldsymbol{r}_{t-1}) = S^{(2)}_t \ \& \ K^c_t(\boldsymbol{r}_{t-1}) = C^{(0)}_t \\
(1-\alpha_{t-1}, 0) & \text{if } K^f_t(\boldsymbol{r}_{t-1}) = C^{(2)}_t \ \& \ K^s_t(\boldsymbol{r}_{t-1}) = C^{(1)}_t \ \& \ K^c_t(\boldsymbol{r}_{t-1}) = S^{(1)}_t \\
(1-\alpha_{t-1}-\beta_{t-1}, 0) & \text{if } K^f_t(\boldsymbol{r}_{t-1}) = S^{(2)}_t \ \& \ K^s_t(\boldsymbol{r}_{t-1}) = C^{(1)}_t \ \& \ K^c_t(\boldsymbol{r}_{t-1}) = S^{(1)}_t \\
(\beta_{t-1}, 1-\alpha_{t-1}-\beta_{t-1}) & \text{if } K^f_t(\boldsymbol{r}_{t-1}) = C^{(2)}_t \ \& \ K^s_t(\boldsymbol{r}_{t-1}) = F_t \ \& \ K^c_t(\boldsymbol{r}_{t-1}) = S^{(1)}_t \\
(0, 1-\alpha_{t-1}-\beta_{t-1}) & \text{if } K^f_t(\boldsymbol{r}_{t-1}) = S^{(2)}_t \ \& \ K^s_t(\boldsymbol{r}_{t-1}) = F_t \ \& \ K^c_t(\boldsymbol{r}_{t-1}) = S^{(1)}_t \\
(\beta_{t-1}, 0) & \text{if } K^f_t(\boldsymbol{r}_{t-1}) = C^{(2)}_t \ \& \ K^s_t(\boldsymbol{r}_{t-1}) = S^{(0)}_t \\
(0, 0) & \text{if } K^f_t(\boldsymbol{r}_{t-1}) = S^{(2)}_t \ \& \ K^s_t(\boldsymbol{r}_{t-1}) = S^{(0)}_t \\
(1-\alpha_{t-1}, \alpha_{t-1}) & \text{if } K^f_t(\boldsymbol{r}_{t-1}) = C^{(2)}_t \ \& \ K^s_t(\boldsymbol{r}_{t-1}) = C^{(1)}_t \ \& \ K^c_t(\boldsymbol{r}_{t-1}) = F_t \\
(1-\alpha_{t-1}-\beta_{t-1}, \alpha_{t-1}) & \text{if } K^f_t(\boldsymbol{r}_{t-1}) = S^{(2)}_t \ \& \ K^s_t(\boldsymbol{r}_{t-1}) = C^{(1)}_t \ \& \ K^c_t(\boldsymbol{r}_{t-1}) = F_t \\
(\beta_{t-1}, 1-\beta_{t-1}) & \text{if } K^f_t(\boldsymbol{r}_{t-1}) = C^{(2)}_t \ \& \ K^s_t(\boldsymbol{r}_{t-1}) = F_t \ \& \ K^c_t(\boldsymbol{r}_{t-1}) = F_t \\
(0, 1-\beta_{t-1}) & \text{if } K^f_t(\boldsymbol{r}_{t-1}) = S^{(2)}_t \ \& \ K^s_t(\boldsymbol{r}_{t-1}) = F_t \ \& \ K^c_t(\boldsymbol{r}_{t-1}) = F_t.
\end{cases}$$

The optimal allocation decisions are characterized based on which of the three options, growing (i) corn, (ii) soybeans or (iii) the fallow crop in period t (and optimally using the farmland in the remaining periods), is the most profitable on the farmland where crop $j \in \{c, s, f\}$ was grown in the previous period, as captured by the recursive operators $K_t^j(\boldsymbol{r}_{t-1})$ given in (3.1). For example, consider the first case presented in Proposition 3.1. When $K_t^f(\boldsymbol{r}_{t-1}) = F_t$, because $C_t^{(0)} \leq C_t^{(1)} \leq C_t^{(2)}$ and $S_t^{(0)} \leq S_t^{(1)} \leq S_t^{(2)}$ by definition, we have $K_t^c(\boldsymbol{r}_{t-1}) = F_t$ and $K_t^s(\boldsymbol{r}_{t-1}) = F_t$. In other words, growing the fallow crop is the most profitable option regardless of which crop was grown in the previous period. Therefore, the whole farmland is optimally allocated to the fallow crop, i.e., $\beta_t^* = 1$ (and thus $\alpha_t^* = 0$). Consider another example (seventh case presented in Proposition 3.1): what is the optimal allocation when $K_t^s(\boldsymbol{r}_{t-1}) = F_t$, $K_t^f(\boldsymbol{r}_{t-1}) = S_t^{(2)}$, and $K_t^c(\boldsymbol{r}_{t-1}) = S_t^{(1)}$? In this case growing the fallow crop is the most profitable option on the farmland where soybeans were grown in the previous period while growing soybeans is the most profitable option on the farmland where the fallow crop or corn was grown in the previous period. Therefore, $\beta_t^* = 1 - \alpha_{t-1} - \beta_{t-1}$ and $1 - \alpha_t^* - \beta_t^* = \beta_{t-1} + \alpha_{t-1}$; in other words, no farmland is allocated to corn, i.e., $\alpha_t^* = 0$. The other cases are characterized in a similar fashion.

The optimal levels for the original decision variables $\alpha_t^{j^*}$ and $\beta_t^{j^*}$ for $j \in \{c, s, f\}$ in the farmer's optimization problem stated in (2.3) can be obtained from α_t^* and β_t^* characterizations, respectively using the previous period's allocation for each crop $(\alpha_{t-1}, \beta_{t-1}, 1 - \alpha_{t-1} - \beta_{t-1})$. For example, when $\alpha_t^* = 1 - \alpha_{t-1}$, because $1 - \alpha_{t-1} = (\beta_{t-1}) + (1 - \alpha_{t-1} - \beta_{t-1})$, it follows that $\alpha_t^{f^*} = \beta_{t-1}$, $\alpha_t^{s*} = 1 - \alpha_{t-1} - \beta_{t-1}$, and $\alpha_t^{c*} = 0$. Consider another example, $\beta_t^* = 1 - \beta_{t-1}$. In this case, because $1 - \beta_{t-1} = (\alpha_{t-1}) + (1 - \alpha_{t-1} - \beta_{t-1})$, we obtain $\beta_t^{c*} = \alpha_{t-1}$, $\beta_t^{s*} = 1 - \alpha_{t-1} - \beta_{t-1}$, and $\beta_t^{f^*} = 0$.

Referring to our research questions posed in Section 1, Proposition 3.1 characterizes i) the specific conditions under which the revenue increase for a particular crop is significant enough to induce the farmer to deviate from a rotation-based allocation policy, and ii) the specific crop volume related to this deviation; that is, the additional volume of each crop grown on non-rotated farmland. In our companion paper

Boyabatlı *et al.* (2018), we use the characterization of the optimal allocations decisions in Proposition 3.1 to propose a simple heuristic allocation policy, which we characterize in closed form. Using a model calibration based on a farmer growing corn and soybeans in Iowa, we show that the proposed policy not only outperforms the commonly suggested heuristic policies in the literature, but also provides a near-optimal performance.

We note that Proposition 3.1 identifies three strategies that emerge as a part of the optimal allocation policy. In particular, there is *monoculture* strategy where only one of the crops is grown on the entire farmland—this strategy corresponds to cases i, ii, and ix in Proposition 3.1. There is *rotate* strategy where each crop is only grown on rotated farmland—this corresponds to cases iv, v, vi, vii, x, xi, xii, and $xiii$. Finally, there is *mixed* strategy (the remaining cases in Proposition 3.1) where one of the cash crops is grown on rotated farmland where the fallow crop was grown in the previous period, and the other cash crop is grown both on rotated farmland where the other cash crop was grown in the previous period and on non-rotated farmland.

We close this section with an important observation. Once the farmer optimally follows a monoculture allocation policy in period t—that is, the whole farmland is only allocated to a single crop—the farmer also optimally follows a monoculture policy in the subsequent periods. The following corollary formalizes this observation.

Corollary 3.1. i) When the whole farmland is allocated to corn in period $t-1 \in [0, T-1]$, i.e., $\alpha_{t-1} = 1$ and $\beta_{t-1} = 0$, the optimal corn and the fallow crop allocation (α_t^*, β_t^*) in period t are given by

$$(\alpha_t^*, \beta_t^*) = \begin{cases} (1,0) & \text{if } K_t^c(\boldsymbol{r}_{t-1}) = C_t^{(0)}, \\ (0,0) & \text{if } K_t^c(\boldsymbol{r}_{t-1}) = S_t^{(1)}, \\ (0,1) & \text{if } K_t^c(\boldsymbol{r}_{t-1}) = F_t. \end{cases}$$

ii) When the whole farmland is allocated to the fallow crop in period $t-1$, i.e., $\alpha_{t-1} = 0$ and $\beta_{t-1} = 1$, the optimal corn and the fallow crop allocation (α_t^*, β_t^*) in period t are given by

$$(\alpha_t^*, \beta_t^*) = \begin{cases} (1,0) & \text{if } K_t^f(\boldsymbol{r}_{t-1}) = C_t^{(2)}, \\ (0,0) & \text{if } K_t^f(\boldsymbol{r}_{t-1}) = S_t^{(2)}, \\ (0,1) & \text{if } K_t^f(\boldsymbol{r}_{t-1}) = F_t. \end{cases}$$

iii) When the whole farmland is allocated to soybeans in period $t-1$, i.e., $\alpha_{t-1} = 0$ and $\beta_{t-1} = 0$, the optimal corn and the fallow crop allocation (α_t^*, β_t^*) in period t are given by

$$(\alpha_t^*, \beta_t^*) = \begin{cases} (1,0) & \text{if } K_t^s(\boldsymbol{r}_{t-1}) = C_t^{(1)}, \\ (0,0) & \text{if } K_t^s(\boldsymbol{r}_{t-1}) = S_t^{(0)}, \\ (0,1) & \text{if } K_t^s(\boldsymbol{r}_{t-1}) = F_t. \end{cases}$$

4

Future research

In this paper, we restrict our attention to crop planning decision. As discussed in Lowe and Preckel (2004), crop production, however, involves subsequent operational decisions during cultivation (e.g., fertilizer and pesticides application, irrigation planning) and harvesting (e.g., harvest timing). Those operational decisions have an impact on crop revenues which we assume uncertain but exogenous in our model. Combining the crop planning decision with those other operational decisions in crop production should prove to be an interesting avenue for future research.

Our model (implicitly) assumes that the farmland allocation decision has no impact on crop revenues. This is a reasonable assumption for *commodity* crops, such as corn and soybeans as considered in this paper, where the production volume of an individual farmer is insignificant in comparison with the aggregate production volume that are traded in the exchange (spot) markets. However, this is not a reasonable assumption for a crop where the production volume of an individual farmer constitutes to a significant portion of the aggregate production volume. In this case, the farmer's allocation decision has an impact on the crop revenue because it alters the crop's availability in the market. Studying the farmland allocation decision in this setting requires a general equilibrium model that formalizes the interplay between the crop availability and the crop revenue, and should prove to be an interesting avenue for future research.

Finally, relaxing the assumptions made on the crop features gives rise to a number of interesting areas for future research. First, we assume that the crop rotation benefit is valid for one period. This is a reasonable assumption for corn and soybeans, as empirically documented in Hennessy (2006), but is a limitation for other crops. Second, there can be constraints on the farmland allocation of each crop due to, for example, limited availability of crop-specific resources (such as seeds and fertilizers) or government regulations. Generalizing our model to consider these issues is an interesting avenue for future research.

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