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# Efficient dynamic mechanisms with interdependent valuations ☆

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This paper considers a dynamic environment with interdependent valuations and evolving private information. Under the assumption of "independent types", we construct an efficient, incentive-compatible mechanism that is also budget-balanced in every period of the game. Our mechanism works in environments where in each period, each agent observes her own realized outcome-decision payoff from the previous period. This extends the insight of Mezzetti (2004) to the dynamic setting.

## 1. Introduction

An important strand of mechanism design theory is concerned with the design of efficient mechanisms. The mechanism designer would like to allocate the good to the bidder with the highest valuation,[1] provide the public good if and only if the sum of the agents' valuations is greater than the cost, and facilitate trading if and only if the buyer's valuation is higher than the seller's valuation, etc.[2]

The renowned Vickrey–Clarke–Groves (VCG) mechanism established the existence of an efficient, incentive-compatible mechanism for a general class of static mechanism design problems with private values and quasilinear preferences; see Clarke (1971), Groves (1973) and Vickrey (1961). Subsequently, a pair of classic papers, Arrow (1979) and d'Aspremont and Gérard-Varet (1979) (AGV), constructed an efficient, incentive-compatible mechanism in which the transfers were also budget-balanced, using the solution concept of Bayesian–Nash equilibrium, under the additional assumption that private information is independent across agents.

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[1] A leading rationale for the widespread privatization of state-owned assets is to enhance efficiency; see Dasgupta and Maskin (2000). For example, the U. S. Congress explicitly mandated the Federal Communications Commission to promote efficiency in its auctions of frequency bands for telecommunications.

[2] The problem of implementing socially efficient outcomes has also been extensively studied in the dynamic setting; see, for example, Bergemann and Välimäki (2010), Athey and Segal (2013), and Guo and Hörner (2015). Pavan et al. (2014) provide a general treatment of the dynamic mechanism design problem in the independent private-value setting (see also references therein).

In dynamic mechanism design problems with private values, Bergemann and Välimäki (2010) and Athey and Segal (2013) have successfully addressed this question, by means of dynamic extensions of the VCG and AGV mechanisms. However, it is well known that VCG and AGV mechanisms no longer work in settings with interdependent valuations. Indeed, Maskin (1992), Dasgupta and Maskin (2000) and Jehiel and Moldovanu (2001) have demonstrated, in increasing generality, that if information signals are statistically independent, multidimensional (or, if they are single dimensional, but a single crossing condition is violated), and interdependent, then the implementation of efficient mechanisms is generically impossible.

In this paper, we study efficient mechanism design in a dynamic environment with interdependent valuations and evolving private information. Our aim is to construct an efficient, incentive-compatible dynamic mechanism that is also budget-balanced in every period of the game. As in the AGV mechanism and Athey and Segal (2013), we place emphasis on budget balance.

As discussed above, implementation of efficient mechanisms with interdependent valuations runs into difficulties even in the static setting. To overcome such difficulties, we extend the following insight from Mezzetti (2004) to the dynamic setting. In a static mechanism design problem, Mezzetti (2004) constructs a novel and elegant "generalized (or two-stage) Groves mechanism" that bypasses the above difficulties, with the assumption that each agent observes her own realized outcome-decision payoff after the final outcome decision, but before final transfers, are made.[3] While Mezzetti (2004) resolves incentive compatibility, requiring agents to be able to observe their own payoffs before the mechanism ends is a strong assumption in the static setting from an applied perspective. In the dynamic setting, it may seem natural to assume that in each period, each agent could observe her own realized outcome-decision payoff from the previous period.

This assumption is related to the literature on contingent payments; see Hansen (1985), Crémer (1987), Samuelson (1987) and more recently, DeMarzo et al. (2005) and Che and Kim (2010) among others.[4] In this paper, we do not require that the realized outcome-decision payoffs be observable to the mechanism designer, but we rely instead on the agents' reports of their own realized payoffs.

This paper places emphasis on budget balance in every period of the game.[5] Indeed, the construction of an efficient, incentive-compatible mechanism is straightforward. In each period, the mechanism designer makes a transfer to each agent that is an adjusted amount of the sum of the other agents' outcome-decision payoffs from the previous period. This suffices to make each agent the residual claimant of the social surplus and provide the agents with the incentive to be truthful as long as the mechanism prescribes an efficient decision rule. Under the assumption of independent types, we show that dynamic efficiency can be achieved with balanced budget. As in the AGV mechanism and Athey and Segal (2013), our construction of the budget-balanced mechanism requires all the other agents to pitch in to pay each agent's incentive term. This ensures that the budget is balanced in every period of the game. The key difference between our mechanism and the "balanced team mechanism" in Athey and Segal (2013) is as follows. In their paper, only the transfers of the most recent two periods are relevant for each agent's incentive in the current period, since the expectation of the transfers afterwards is zero.[6] However, in our mechanism, all the future transfers could influence the incentive of the current period.

Another approach that studies efficient mechanism design exploits the correlation of private information; see the seminal contribution of Crémer and McLean (1988) in the static setting. More recently, Liu (2014) and Noda (2015) extend the insight of Crémer and McLean (1988) to the dynamic setting and construct efficient and incentive-compatible mechanisms respectively. These results leverage on the inter-temporal correlation of private information and do not apply in our setting. Hörner et al. (2015) apply a similar technique to dynamic Bayesian games.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 constructs the efficient, incentive-compatible and budget-balanced mechanism and Section 4 concludes.

## 2. Model

### 2.1. Setup

**Notation.** We consider a dynamic mechanism design environment with interdependent valuations in a discrete-time, infinite-horizon model. There is a finite set $\mathcal{I} = \{1, 2, \ldots, I\}$ of risk neutral agents. Time is discrete, indexed by $t \in \mathbb{N} = \{0, 1, 2, \ldots\}$. The state of the world $\theta_t^i$ for agent $i$ is a general Markov process on the state space $\Theta^i$. The aggregate state is given by the vector $\theta_t = (\theta_t^1, \theta_t^2, \ldots, \theta_t^I)$ with $\Theta = \prod_{i \in \mathcal{I}} \Theta^i$. Write $\theta_t^{-i} \in \Theta^{-i} = \prod_{j \neq i} \Theta^j$ for the state of all agents except agent $i$. The outcome space is a measurable set $X$ endowed with the $\sigma$-algebra $\mathcal{X}$. The initial state $\theta_0 \in \Theta$ is assumed to be publicly known. The current state $\theta_t \in \Theta$ and current decision $x_t \in X$ define a probability distribution for state variable $\theta_{t+1}$ on $\Theta$ by the law of motion $Q(\cdot|x_t, \theta_t)$.

**Timing.** We consider mechanisms in which, following a publicly observed initial state $\theta_0 \in \Theta$, a decision $x_0 \in X$ is made. Then in each period $t \geq 1$, each agent privately observes her type $\theta_t^i \in \Theta^i$. Agents make reports simultaneously and a public

[3] Two-stage mechanisms can also be used to achieve goals other than efficiency (e.g., surplus extraction); see Mezzetti (2007).

[4] Lehrer (1992) and Tomala (1999) have also adopted similar assumptions of observable payoff in the environment of repeated games.

[5] Nath et al. (2015) study efficient and incentive compatible mechanisms in dynamic environments with interdependent valuations and observable payoffs, but the issue of budget balance is not addressed therein.

[6] See the proof of Proposition 2 in Athey and Segal (2013).

decision $x_t \in X$ is made at the end of each period. Each agent $i$ also receives a transfer $y_t^i \in \mathbb{R}$. We assume that the past reports of each agent and the public decision are observable to all agents. All agents discount the future with a common discount factor $\delta \in (0, 1)$.

**Interdependent valuations.** We allow agents to have interdependent valuations in the sense that agent $i$'s payoff could depend on the signals of all the other agents for each $i \in \mathcal{I}$. If a sequence of types $\{\theta_t\}_{t \geq 0}$ is realized, a sequence of public decisions $\{x_t\}_{t \geq 0}$ and transfers $\{y_t\}_{t \geq 0}$ are determined, then the discounted payoff of agent $i$ is

$$\sum_{t \geq 0} \delta^t [u_i(x_t, \theta_t) + y_t^i],$$

where $u_i \colon X \times \Theta \to \mathbb{R}$ is assumed to be measurable and bounded. We will refer to $u_i$ as the outcome-decision payoff of agent $i$.

**Independent types.** Throughout this paper, we shall assume independent types. That is, conditional on decisions, the private information of agent $i$ does not have any direct effect on the distribution of the current and future types of other agents (we still allow one agent's reports to affect the future types of other agents through the implemented decisions). More formally,

**Definition 1.** Agents have *independent types* if given any $x_t \in X$ and $\theta_t = (\theta_t^1, \theta_t^2, \dots, \theta_t^I) \in \Theta$, the transition probability $Q(\cdot|x_t, \theta_t) = \otimes_{i \in \mathcal{I}} Q_i(\cdot|x_t, \theta_t^i)$, where $Q_i(\cdot|x_t, \theta_t^i)$ is a transition probability from $X \times \Theta^i$ to $\triangle(\Theta^i)$.

**Equilibrium notion.** The truthtelling strategy of agent $i$ always reports her state $\theta_t^i$ in every period $t \geq 1$ truthfully, regardless of the observed past (in particular, regardless of whether she has lied in the past). We will consider perfect Bayesian equilibrium (PBE) in truthtelling strategies, with beliefs that assign probability 1 to the other agents' latest reports being truthful.

### 2.2. *Efficiency*

A social policy is a measurable function $\chi : \Theta \to X$, where $\chi(\theta)$ represents the decision made when the realized state in this period is $\theta$. Starting from an initial type $\theta_0 \in \Theta$, a social policy $\chi$ together with the transition probability $Q$ uniquely determine a probability measure over the sequence of states $(\theta_t)_{t \geq 0} \in \Theta^{\mathbb{N}}$.

In period $t$, efficiency can be obtained at type $\theta_t$ by maximizing the discounted expected surplus:

$$\sup_{\{x_s\}_{s \geq t}} \mathbb{E}\left[\sum_{s \geq t} \delta^{s-t} \sum_{i \in \mathcal{I}} u_i(x_s, \theta_s)\right].$$

We characterize the efficient social policy $\chi^* \colon \Theta \to X$ and the associated social value function $V : \Theta \to \mathbb{R}$ by the following recursion using the principle of dynamic programming:

$$\begin{aligned} V(\theta) &= v(\theta) + \delta \int_{\Theta} V(\tilde{\theta}) Q(d\tilde{\theta}|\chi^*(\theta), \theta) \\ &= \sup_{x \in X} \left[\sum_{i \in \mathcal{I}} u_i(x, \theta) + \delta \int_{\Theta} V(\tilde{\theta}) Q(d\tilde{\theta}|x, \theta)\right], \end{aligned}$$

where $v(\theta) = \sum_{i \in \mathcal{I}} u_i(\chi^*(\theta), \theta)$.[7]

## 3. Mechanism

In this section, we construct an efficient and budget-balanced dynamic mechanism such that truthtelling strategies form a perfect Bayesian equilibrium. As discussed in the introduction, we assume that in each period, each agent observes her own realized outcome-decision payoff from the previous period.

**Assumption.** In each period $t+1$ ($t \geq 1$) and for any $x_t \in X$, each agent $i$ observes her own realized outcome-decision payoff from period $t$.

For simplicity of exposition, in what follows, we go one step further and assume that in each period, the realized outcome-decision payoffs from the previous period are observable to the mechanism designer. The mechanism we construct

[7] Throughout this paper, we assume that the efficient social policy exists.

still works under the original assumption. Indeed, in each period $t+1$, the mechanism designer could require the agents to report their realized outcome-decision payoffs from period $t$. Since for each agent $i$, the report of her own outcome-decision payoff does not affect her utility, we can assume that agent $i$ truthfully reports her outcome-decision payoff from the previous period; see Mezzetti (2004) for further discussions.

In period $t \geq 1$, given reported types $r_{t-1}$ and $r_t$ in periods $t-1$ and $t$ respectively,[8] let

$$\Phi_i(r_{t-1}, r_t^i) = \int\limits_{\Theta_{-i}} V(r_t^i, \tilde{\theta}_t^{-i}) Q_{-i}(d\tilde{\theta}_t^{-i} | \chi^*(r_{t-1}), r_{t-1}^{-i}) - \int\limits_{\Theta} V(\tilde{\theta}_t) Q(d\tilde{\theta}_t | \chi^*(r_{t-1}), r_{t-1}) \text{ and}$$

$$\Psi_i(r_{t-1}, r_t) = \Phi_i(r_{t-1}, r_t^i) - \frac{1}{I-1} \sum_{j \neq i} \Phi_j(r_{t-1}, r_t^j)$$

for each $i \in \mathcal{I}$.

From the mechanism designer's perspective, $\Phi_i(r_{t-1}, r_t^i)$ characterizes the change in the expected social value if agent $i$ reports $r_t^i$ in period $t$, given the report $r_{t-1}$ in period $t-1$.

Construct the following mechanism $(\chi^*, y)$:

1. The socially efficient policy $\chi^*$ is implemented in every period; that is, in period $t$, the allocation is $\chi^*(r_t)$ based on the reports $r_t$.
2. For each $i \in \mathcal{I}$, the transfer to agent $i$ in period $t+1$ (for $t \geq 1$) is

$$y_{t+1}^i = \frac{1}{\delta} \sum_{j \neq i} w_t^j - \frac{I-1}{I\delta} \left[ v(r_t) - \Psi_i(r_{t-1}, r_t) \right],$$

   where $w_t^j = u_j(\chi^*(r_t), \theta_t)$ is the realized outcome-decision payoff of agent $j \in \mathcal{I}$ in period $t$, and $v(r_t) = \sum_{i \in \mathcal{I}} u_i(\chi^*(r_t), r_t)$.[9]

As in the AGV mechanism and Athey and Segal (2013), our mechanism requires all the other agents to pitch in to pay each agent's incentive term, which ensures that the budget is balanced on the equilibrium path.

**Theorem 1.** *Truthtelling strategies form a perfect Bayesian equilibrium in the mechanism $(\chi^*, y)$. Furthermore, on the equilibrium path, the mechanism $(\chi^*, y)$ is budget-balanced in every period of the game.*

**Proof.** The logic of the proof is summarized as follows. Step 1 begins by considering a simpler mechanism $(\chi^*, z)$ where the transfer $z_t^i$ to agent $i$ is an adjusted amount of the sum of the realized outcome-decision payoffs of all the other agents in period $t-1$. We show that truthtelling strategies form a PBE in this mechanism. The idea, as in the standard VCG mechanism, is to make each agent the residual claimant of the full surplus. Step 2 proves that the expected present value of agent $i$'s gain from deviating in the mechanism $(\chi^*, y)$ is the same as in the simple mechanism $(\chi^*, z)$. Therefore, truthtelling strategies still form a perfect Bayesian equilibrium in the mechanism $(\chi^*, y)$. Lastly, Step 3 verifies that on the equilibrium path, the mechanism $(\chi^*, y)$ is budget-balanced in every period of the game.

**Step 1.** We consider a simpler mechanism $(\chi^*, z)$ where the allocation rule is still the efficient social policy $\chi^*$, but the transfer agent $i$ receives in period $t \geq 2$ is $z_t^i = \frac{1}{\delta} \sum_{j \neq i} w_{t-1}^j$, where $(w_{t-1}^1, w_{t-1}^2, \ldots, w_{t-1}^I)$ are the realized outcome-decision payoffs in period $t-1$.[10] By the one-stage deviation principle, to verify PBE it suffices to show that a one-stage deviation of any agent $i \in \mathcal{I}$ to reporting any $r_t^i \in \Theta^i$ instead of her true type $\theta_t^i \in \Theta^i$ in period $t$ is unprofitable. If all agents choose the truthtelling strategy, then the expected discounted payoff of agent $i$ in period $t$ is

$$u_i(\chi^*(\theta_t), \theta_t) + z_t^i + \mathbb{E}\left[ \sum_{k \geq 1} \delta^k (u_i(\chi^*(\theta_{t+k}), \theta_{t+k}) + z_{t+k}^i) | \chi^*(\theta_t), \theta_t \right]$$

$$= u_i(\chi^*(\theta_t), \theta_t) + z_t^i + \mathbb{E}\left[ \sum_{k \geq 1} \delta^k (u_i(\chi^*(\theta_{t+k}), \theta_{t+k}) + \frac{1}{\delta} \sum_{j \neq i} u_j(\chi^*(\theta_{t+k-1}), \theta_{t+k-1})) | \chi^*(\theta_t), \theta_t \right]$$

$$= \sum_{j \in \mathcal{I}} u_j(\chi^*(\theta_t), \theta_t) + z_t^i + \mathbb{E}\left[ \sum_{k \geq 1} \delta^k \sum_{j \in \mathcal{I}} u_j(\chi^*(\theta_{t+k}), \theta_{t+k}) | \chi^*(\theta_t), \theta_t \right]$$

[8] Since the initial state $\theta_0 \in \Theta$ is publicly known, we assume $r_0 \equiv \theta_0$.

[9] We let $y_0^i = y_1^i \equiv 0$ for each agent $i \in \mathcal{I}$.

[10] We let $z_0^i = z_1^i \equiv 0$ for each agent $i \in \mathcal{I}$.

$$= V(\theta_t) + z_t^i.$$

Suppose that agent $i$ reports $r_t^i$ instead. Let $x = \chi^*(r_t^i, \theta_t^{-i})$. Then her expected discounted payoff in period $t$ is

$$u_i(x, \theta_t) + z_t^i + \mathbb{E}\left[\sum_{k\geq 1} \delta^k (u_i(\chi^*(\theta_{t+k}), \theta_{t+k}) + z_{t+k}^i)|x, \theta_t\right]$$
$$= u_i(x, \theta_t) + z_t^i + \mathbb{E}\Bigg[\sum_{k\geq 2} \delta^k (u_i(\chi^*(\theta_{t+k}), \theta_{t+k}) + \frac{1}{\delta}\sum_{j\neq i} u_j(\chi^*(\theta_{t+k-1}), \theta_{t+k-1}))$$
$$+ \delta(u_i(\chi^*(\theta_{t+1}), \theta_{t+1}) + \frac{1}{\delta}\sum_{j\neq i} u_j(x, \theta_t))|x, \theta_t\Bigg]$$
$$= \sum_{j\in\mathcal{I}} u_j(x, \theta_t) + z_t^i + \mathbb{E}\left[\sum_{k\geq 1} \delta^k \sum_{j\in\mathcal{I}} u_j(\chi^*(\theta_{t+k}), \theta_{t+k})|x, \theta_t\right]$$
$$= \sum_{j\in\mathcal{I}} u_j(x, \theta_t) + z_t^i + \delta \int_\Theta V(\tilde{\theta}) Q(\mathrm{d}\tilde{\theta}|x, \theta_t).$$

Since $V$ is the social value function when the decision policy is $\chi^*$, we have

$$V(\theta_t) = \sum_{j\in\mathcal{I}} u_j(\chi^*(\theta_t), \theta_t) + \delta \int_\Theta V(\tilde{\theta}) Q(d\tilde{\theta}|\chi^*(\theta_t), \theta_t)$$
$$\geq \sum_{j\in\mathcal{I}} u_j(x, \theta_t) + \delta \int_\Theta V(\tilde{\theta}) Q(d\tilde{\theta}|x, \theta_t).$$

Thus, a one-stage deviation of any agent $i \in \mathcal{I}$ to reporting any $r_t^i \in \Theta^i$ instead of her true type $\theta_t^i \in \Theta^i$ in period $t$ is unprofitable. Truthtelling strategies form a perfect Bayesian equilibrium.

**Step 2.** We prove that the expected present value of agent $i$'s gain from deviating in the mechanism $(\chi^*, y)$ is the same as in the simple mechanism $(\chi^*, z)$.

In period $t-1$, consider the case where the true type profile is $\theta_{t-1}$ and the reported type profile is $(r_{t-1}^{-j}, \theta_{t-1}^j)$ for some $j \in \mathcal{I}$. That is, agent $j$ truthfully reports $\theta_{t-1}^j \in \Theta^j$ while the other agents arbitrarily report $r_{t-1}^{-j} \in \Theta^{-j}$. We have

$$\int_{\Theta_j} \Phi_j(r_{t-1}^{-j}, \theta_{t-1}^j, \tilde{\theta}_t^j) Q_j(d\tilde{\theta}_t^j|\chi^*(r_{t-1}^{-j}, \theta_{t-1}^j), \theta_{t-1}^j)$$
$$= \int_{\Theta_j}\int_{\Theta_{-j}} V(\tilde{\theta}_t^j, \tilde{\theta}_t^{-j}) Q_{-j}(d\tilde{\theta}_t^{-j}|\chi^*(r_{t-1}^{-j}, \theta_{t-1}^j), r_{t-1}^{-j}) Q_j(d\tilde{\theta}_t^j|\chi^*(r_{t-1}^{-j}, \theta_{t-1}^j), \theta_{t-1}^j)$$
$$- \int_\Theta V(\tilde{\theta}_t) Q(d\tilde{\theta}_t|\chi^*(r_{t-1}^{-j}, \theta_{t-1}^j), r_{t-1}^{-j}, \theta_{t-1}^j)$$
$$= 0,$$

where the first equality follows from the definition of $\Phi_i$.

Thus, for each agent $i \in \mathcal{I}$, if all the other agents truthfully report their types, then the expectation of the term $\sum_{j\neq i} \Phi_j(r_{t-1}, r_t^j)$ in $\Psi_i(r_{t-1}, r_t)$ is 0 (from agent $i$'s perspective) regardless of her own report. In other words, if agent $i$ assigns probability 1 to the event that all the other agents truthfully report their types, then the term $\sum_{j\neq i} \Phi_j(r_{t-1}, r_t^j)$ in the transfer $y_{t+1}^i$ cannot distort her incentive.

Next we consider other terms $v(r_t) - \Phi_i(r_{t-1}, r_t^i)$ in the transfer $y_{t+1}^i$ that could potentially distort agent $i$'s incentives. Suppose that all the other agents adopt the truthtelling strategy; that is, $r_{t-1}^{-i} = \theta_{t-1}^{-i}$ in period $t-1$. As for agent $i$, her past types are payoff-irrelevant since (1) the past types do not enter into her future outcome-decision payoff functions and transfers; and (2) her belief about the opponents' current types depends on her report, but not the true type, in the previous period. As a result, we can assume that agent $i$ truthfully reports in period $t-1$. We focus on the case that the true type of agent $i$ is $\theta_t^i$ but she reports $r_t^i$ in period $t$.

In what follows, we consider the summation of the expectation of $v(r_{t+k}) - \Phi_i(r_{t+k-1}, r^i_{t+k})$ for $k \geq 0$ (from agent $i$'s perspective). If agent $i$ deviates from $\theta^i_t$ to $r^i_t$, we have

$$\int_{\Theta_{-i}} v(r^i_t, \tilde{\theta}^{-i}_t) Q_{-i}(d\tilde{\theta}^{-i}_t | \chi^*(\theta_{t-1}), \theta^{-i}_{t-1}) - \Phi_i(\theta_{t-1}, r^i_t)$$

$$+ \sum_{k\geq 1} \delta^k \int_{\Theta_{-i}} \mathbb{E}\left( v(\tilde{\theta}_{t+k}) | \chi^*(r^i_t, \tilde{\theta}^{-i}_t), \theta^i_t, \tilde{\theta}^{-i}_t \right) Q_{-i}(d\tilde{\theta}^{-i}_t | \chi^*(\theta_{t-1}), \theta^{-i}_{t-1})$$

$$- \int_{\Theta_{-i}} \mathbb{E}\left( \Phi_i(r^i_t, \tilde{\theta}^{-i}_t, \tilde{\theta}^i_{t+1}) | \chi^*(r^i_t, \tilde{\theta}^{-i}_t), r^i_t, \tilde{\theta}^{-i}_t) \right) Q_{-i}(d\tilde{\theta}^{-i}_t | \chi^*(\theta_{t-1}), \theta^{-i}_{t-1})$$

$$- \sum_{k\geq 2} \delta^k \int_{\Theta_{-i}} \mathbb{E}\left( \Phi_i(\tilde{\theta}_{t+k}, \tilde{\theta}^i_{t+k}) | \chi^*(r^i_t, \tilde{\theta}^{-i}_t), r^i_t, \tilde{\theta}^{-i}_t) \right) Q_{-i}(d\tilde{\theta}^{-i}_t | \chi^*(\theta_{t-1}), \theta^{-i}_{t-1})$$

$$= \int_{\Theta_{-i}} v(r^i_t, \tilde{\theta}^{-i}_t) Q_{-i}(d\tilde{\theta}^{-i}_t | \chi^*(\theta_{t-1}), \theta^{-i}_{t-1}) \tag{1}$$

$$- \int_{\Theta_{-i}} V(r^i_t, \tilde{\theta}^{-i}_t) Q_{-i}(d\tilde{\theta}^{-i}_t | \chi^*(\theta_{t-1}), \theta^{-i}_{t-1}) \tag{2}$$

$$+ \int_{\Theta} V(\tilde{\theta}_t) Q(d\tilde{\theta}_t | \chi^*(\theta_{t-1}), \theta_{t-1})$$

$$+ \sum_{k\geq 1} \delta^k \int_{\Theta_{-i}} \mathbb{E}\left( v(\tilde{\theta}_{t+k}) | \chi^*(r^i_t, \tilde{\theta}^{-i}_t), \theta^i_t, \tilde{\theta}^{-i}_t \right) Q_{-i}(d\tilde{\theta}^{-i}_t | \chi^*(\theta_{t-1}), \theta^{-i}_{t-1}) \tag{3}$$

$$- \sum_{k\geq 1} \delta^k \int_{\Theta_{-i}} \mathbb{E}\left( V(\tilde{\theta}_{t+k}) | \chi^*(r^i_t, \tilde{\theta}^{-i}_t), \theta^i_t, \tilde{\theta}^{-i}_t) \right) Q_{-i}(d\tilde{\theta}^{-i}_t | \chi^*(\theta_{t-1}), \theta^{-i}_{t-1}) \tag{4}$$

$$+ \delta \int_{\Theta_{-i}} \mathbb{E}\left( V(\tilde{\theta}_{t+1}) | \chi^*(r^i_t, \tilde{\theta}^{-i}_t), r^i_t, \tilde{\theta}^{-i}_t) \right) Q_{-i}(d\tilde{\theta}^{-i}_t | \chi^*(\theta_{t-1}), \theta^{-i}_{t-1}) \tag{5}$$

$$+ \sum_{k\geq 2} \delta^k \int_{\Theta_{-i}} \mathbb{E}\left( V(\tilde{\theta}_{t+k}) | \chi^*(r^i_t, \tilde{\theta}^{-i}_t), \theta^i_t, \tilde{\theta}^{-i}_t) \right) Q_{-i}(d\tilde{\theta}^{-i}_t | \chi^*(\theta_{t-1}), \theta^{-i}_{t-1}) \tag{6}$$

$$= \int_{\Theta_{-i}} \left[ v(r^i_t, \tilde{\theta}^{-i}_t) - V(r^i_t, \tilde{\theta}^{-i}_t) + \delta \mathbb{E}\left( V(\tilde{\theta}_{t+1}) | \chi^*(r^i_t, \tilde{\theta}^{-i}_t), r^i_t, \tilde{\theta}^{-i}_t) \right) \right] Q_{-i}(d\tilde{\theta}^{-i}_t | \chi^*(\theta_{t-1}), \theta^{-i}_{t-1}) \tag{7}$$

$$+ \int_{\Theta} V(\hat{\theta}_t) Q(d\tilde{\theta}_t | \chi^*(\theta_{t-1}), \theta_{t-1})$$

$$+ \sum_{k\geq 1} \delta^k \int_{\Theta_{-i}} \mathbb{E}\left[ v(\tilde{\theta}_{t+k}) - V(\tilde{\theta}_{t+k}) + \delta V(\tilde{\theta}_{t+k+1}) | \chi^*(r^i_t, \tilde{\theta}^{-i}_t), \theta^i_t, \tilde{\theta}^{-i}_t) \right] Q_{-i}(d\tilde{\theta}_{t(-i)} | \chi^*(\theta_{t-1}), \theta^{-i}_{t-1}) \tag{8}$$

$$= \int_{\Theta} V(\tilde{\theta}_t) Q(d\tilde{\theta}_t | \chi^*(\theta_{t-1}), \theta_{t-1}). \tag{9}$$

The first equality follows from the definition of $\Phi_i$. Terms (1), (2), (5) aggregate to term (7) and terms (3), (4), (6) aggregate to term (8) respectively. It is easy to see that both terms (7) and (8) are equal to zero. Finally, (9) does not depend on agent $i$'s report.

Therefore, the transfer scheme $y$ together with $\chi^*$ provides each agent the same expected gain from deviating as the simple mechanism $(\chi^*, z)$. Since truthtelling strategies form a PBE in the latter mechanism, truthtelling strategies also form a PBE if the mechanism $(\chi^*, y)$ is adopted.

**Step 3.** We show that in the mechanism $(\chi^*, y)$, the transfers $y^i_{t+1}$ balance the budget on the equilibrium path; that is, $\sum_{i\in\mathcal{I}} y^i_{t+1} = 0$. On the equilibrium path, agents truthfully report their types, $r_t = \theta_t$ and $w^i_t = u_i(\chi^*(\theta_t), \theta_t)$. We have

$$
\begin{aligned}
&\sum_{i\in\mathcal{I}} y^i_{t+1}\\
&=\sum_{i\in\mathcal{I}}\left[\frac{1}{\delta}\sum_{j\neq i} w^j_t-\frac{I-1}{I\delta}\left[v(\theta_t)-\Psi_i(\theta_{t-1},\theta_t)\right]\right]\\
&=\frac{1}{\delta}\left[\sum_{i\in\mathcal{I}}\sum_{j\neq i} w^j_t-\sum_{i\in\mathcal{I}}\frac{I-1}{I}\left[v(\theta_t)-\Psi_i(\theta_{t-1},\theta_t)\right]\right]\\
&=\frac{1}{\delta}\left[\sum_{i\in\mathcal{I}}\sum_{j\neq i} w^j_t-\sum_{i\in\mathcal{I}}\frac{I-1}{I}v(\theta_t)\right]\\
&=\frac{1}{\delta}\left[\sum_{i\in\mathcal{I}}\sum_{j\neq i} u_j(\chi^*(\theta_t),\theta_t)-(I-1)\sum_{i\in\mathcal{I}}u_i(\chi^*(\theta_t),\theta_t)\right]\\
&=0,
\end{aligned}
$$

where the third equality is due to the following:

$$
\sum_{i\in\mathcal{I}}\Psi_i(\theta_{t-1},\theta_t)=\sum_{i\in\mathcal{I}}\Phi_i(\theta_{t-1},\theta^i_t)-\frac{1}{I-1}\sum_{i\in\mathcal{I}}\sum_{j\neq i}\Phi_j(\theta_{t-1},\theta^j_t)=0. \qquad \square
$$

**Remark 1.** Consider the case that every state is absorbing; that is, $Q(\theta|x,\theta)=1$ for each $x\in X$ and $\theta\in\Theta$.[11] In this case, the efficient social policy in our mechanism is simply the efficient policy in the static setting, and the corresponding transfer is an adjusted amount of the transfer in the static environment (see Mezzetti, 2004, page 1623), where the adjustment is due to the discount factor. In particular, in our mechanism $(\chi^*, y)$, (1) the transfer $y$ is adjusted by the discount factor $\delta$, and (2) the construction of $y$ depends on the continuation value $V$, which equals $\frac{v}{1-\delta}$ when every state is absorbing.

## 4. Conclusion

In a dynamic environment with interdependent valuations and evolving private information, we construct an efficient, incentive-compatible dynamic mechanism that is also budget-balanced in every period of the game. To overcome the difficulties with interdependent valuations, we assume that in each period, each agent observes her own realized outcome-decision payoff from the previous period. This extends the insight of Mezzetti (2004) to the dynamic setting.

We conclude with several observations. Firstly, our result can be generalized to the case where each agent only observes her own realized outcome-decision payoff after any finite number of periods. Secondly, we see no difficulties in extending our result to the case of time-dependent payoffs. This allows us to cover finite-horizon environments and in particular, Mezzetti (2004). Finally, in dynamic mechanism design problems with private values, the assumption that each agent observes her own realized outcome-decision payoff is trivially satisfied. Therefore, our result can also be viewed as a construction of an efficient, incentive-compatible and budget-balanced mechanism in this setting.

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