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Modelling Inoperability Propagation Mechanism in Interdependent Systems

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***Abstract*—This communication considers the issue of deriving a model to describ how the inoperating level of a particular system (a production unit, a transportation system, an energy supply plant, etc.) of interconnected or interdependent systems (neworking and interdependency are interchangeable in this communication) will impact the operating level of other systems for the purpose of analysis, simulation, prediction, risk assessment, etc. The mechanism of such impacting process may be very complex; for instance to impact the operating level of a system the inoperability of another system may need to reach a certain level (threshold), to combine (synergy) with other events or situations; there may exist some preemptivity condition (that is to destabilize a given system some particular conditions must be satisfied). The main purpose of this communication is therefore to establish a model of inoperability propagation in a networked systems when taking into account as much as possible phenomena such as thresholding, synergy, resilience, etc. Necessity of synergy appeals for a synergetic aggregation operator; to this end, we propose to consider using Choquet integral associeted with a weighted cardinal fuzzy meaure (wcfm) as the appropriate aggregation operator. Furthermore this association leads to a straightforward formula to compute the integral.**

***Keywords*—Inoperability Propagation, Networked Systems, Risk Assessment and Management, Choquet Integral, WCFM, IIM.**

I. INTRODUCTION

Capabilities of many systems (transportation infrastructures, energy and water supply infrastructures, communication infrastructures, production infrastructures, financial infrastructures, etc.) that facilitate modern life in many ways result from their highly interconnection. But, as interconnection increase capability of each individual component system, it increases by the way its vulnerability due to its dependence on the operability level of other systems. Indeed, operations of water supply systems, financial systems, communication systems, for instance rely heavily on the operability level of energy supply system and so on. Modeling adequately interactions of these systems is therefore a fundamental issue for analysts in order to dispose of tools that permit them to describe, to analyze, to predict, to control, etc., as accurate as possible the behavior of these systems. But, a sound modeling process must rely on adequate and most representative scientific tools. First approaches to address interactions modeling of large scale systems come probably from works done by economists among which input-output model (IOM) derived by Wassily Leontief, see [4] [5] [6], to analyze American economy is the most popular one. Indeed, the modeling capacities of this model lead to its adaptation in other domains such as risk assessment and management and dependability analysis, in order to study how local perturbation propagate through a large scale interconnected systems and estimating the resulting operability level using the so called inoperability input-output model (IIM), see [2] [7] [9].

Though models built upon IIM approach are widely used in literature for modeling interactions between component systems of many systems and do give good results, for complex behavior and large scale systems, they may reveal some modeling weakness due among other to how input information are combined (aggregation) to produce output at each component system. Some drawbacks of these approaches for interconnected systems modeling and analysis are related to lack of *synergy* and *preemptivity* consideration of some input signals that are described below.

- There are no real interactions between inputs in the transfer function (transformation of inputs to output at each system) as just a weighted mean is realized whereas sometime some synergy is needed between inputs to produce an output.
- For some systems, their disturbance can not be triggered if the global and/or individual effects of their inputs do not exceed some given thresholds; this is not taken into account in IIM approach nor in some similar modeling tools such as fuzzy cognitive maps (FCM), see [3] [13].

To attempt to overcome drawbacks enumerated previously, and building on IIM approach we propose to aggregate input signals using Choquet integral [1], an aggregation operator that allows taking into account relationships between elements to aggregate.

The remainder of this communication is organized as following: the second section is devoted to presenting the main problem and the modelling process as well as assumptions on which the modelling process rely; section three presents an example to show how the model can be used in real world application situation and the fourth sections concludes the communication.

II. STATEMENT OF THE PROBLEM AND MODELLING

We consider a system of n interconnected component systems or entities (to be understood in a very large way where a component system may be a physical infrastructure asset such electrical energy plant, an economical sector such as agriculture, a social sector such as education, or even a component within a system such as battery in a car, etc.) which structure is described by a directed graph that is a couple (N, E) where N is the set of nodes, each node representing a component system and $E \subseteq N \times N$ is the set of directed arcs; an arc $(i,\, j) \in E$ means that inoperability of node i will impact the inoperability of node j. Without loss of generality, we consider the state of a component system i to be determined by the parameter or indicator $x_i(t) \in [\ 0, \quad 1\ ]$ that represents its inoperability level at time instant t meaning that the system is operating at $1 - x_i(t)$ of its capacity at time instant t; $x_i(t) = 0$ means that this system is fully operational at time t and $x_i(t) = 1$ signifies that the system is out of service. The state of this almost system of systems (some component systems may not be fully independent from others) at time instant t is therefore determined by the vector $\mathbf{x}(t)$ defined by equation (1)

$$\mathbf{x}(t) = \left[\ x_1(t) \quad x_2(t) \quad \ldots \quad x_n(t)\ \right]^T. \tag{1}$$

From now the main purpose is to determine the evolution of inoperability level $\mathbf{x}(t)$ of the system given actual conditions; this process is obtained in two steps, obtaining the overall destabilizing effect at the input of each node i and deriving its dynamics that is how $x_i(t+1)$ will be obtained from $x_i(t)$ and the overall destabilizing effect due to inoperability of its input nodes. We consider that the overall effect of inoperability conditions of other nodes on the actual node i is obtained by aggregating their inoperability level. The aggregation process may constitute a complex process depending on how the global effect is obtained from individual inoperability levels $x_k(t)$. In the case of destabilization, one can reasonably consider that these effects behave in synergy that is, two destabilizing events that happen simultaneously will have more effect than the some of their effect if they happen separately; a well known aggregation operator to take into account such behavior is the so called Choquet integral associated to a synergetic fuzzy measure or capacity. But, in practice, most of the time the inoperability of an input may not have effect on the destabilized node until it exceeds some thresholds: a phenomenon we refer to as trigging process. The modeling process will therefore consists in three steps namely modeling trigging process, modeling aggregation process and finally modeling dynamics of the behavior; these processes will be carried up in the following paragraphs; assumptions upon which models are derived will be embended in the text because of lack of space.

A. Trigging process

In practical situation, trigging process is certainly a very complex process; it may consist in global destabilizing effect reaching some threshold or individual effect going beyond a certain level. In a very simply way, let us denote by $\theta_i\left(\mathbf{x}_{-i}(t)\right)$ the trigging indicator of the inoperability process of node i at time t defined by the following: $\theta_i\left(\mathbf{x}_{-i}(t)\right) = 1$ if inoperability of i triggered at time t and 0 if not where $\mathbf{x}_{-i}(t)$ stands for components of vector $\mathbf{x}(t)$ without $x_i(t)$. As expressed in introduction section, different phenomena may influence how effects combine at the input of a node; in this communication we consider two main such interactions properties, namely thresholding and synergy. Thresholding process take place at the input of nodes and is related to the fact that for an input to exerts its effect, its activity level must be beyond some threshold; let us define these thresholds as follows, $x^0_{j,i}$ is the minimum inoperability level of node j required at the input of i to trigger its inoperability process; we define by $\delta_{j,i}(x_j(t))$, the indicator of trigging defined as: $\delta_{j,i}(x_j(t)) = 1$ if $x_j(t) \geq x^0_{j,i}$ and 0 otherwise. From this, we consider two extreme trigging processes: independency for trigging at the input of a node and simultaneity trigging procedure as defined in the following items.

- Independent case: here the overall trigging do not depend on simultaneous trigging of each input, that is the trigging indicator is given by equation (2)

$$\theta_i\left(\mathbf{x}_{-i}(t)\right) = 1 \tag{2}$$

  and therefore the vector $\mathbf{x}_{-i}(t)$ at the input of node i is given by equation (3)

$$[\mathbf{x}_{-i}(t)]_k = \delta_{k,i}(x_k(t))x_k(t) \tag{3}$$

  which ensures that the impact of a no trigged input at a node is null.

- Simultaneous trigging case: in this case for the inoperability process of a node to trigger each of its input must trigger; the trigging indicator $\theta_i\left(\mathbf{x}_{-i}(t)\right)$ will be given in this case by equation (4)

$$\theta_i\left(\mathbf{x}_{-i}(t)\right) = \left(\prod_{j, j\neq i} \delta_{j,i}(x_j(t))\right) \tag{4}$$

Of course many complex trigging processes may exist in practice; once the trigging is operated, the aggregation that is how effect combine at the input of a node can begin; this is the purpose of the next paragraph.

B. Aggregation of the input impacts

We make an a priori assumption that when confronted to destabilizing effect, the principle of the whole is more than the sum apply so that the aggregation of effects at the input of any node must be done in synergetic way. In this situation the global effect at the input of a node does not consist in just cumulating effect but much more in the sense that the combine effect is beyond the sum of individual effect. In this condition, aggregation process must take into account this synergetic behavior of input effect. Among aggregation operator that respect this issue, Choquet integral associated to a synergetic or super additive fuzzy measure or capacity is well suited here. In this paragraph this kind of aggregation operators will be introduced with a particular emphasis on Choquet integral associated to a weighted cardinal fuzzy measure (wcfm for short) developed by the author and that leads to two improvements:

- a straightforward formula to compute the integral;
- a practical intuitive way to determine the associated fuzzy measure as opposed to difficulties encountered for this purpose in the literature.

When a set $N = \{1,\, 2,\, ...,\, n\}$ of attributes with numerical measures vector $x = \left[\ x_1 \quad x_2 \quad \ldots \quad x_n\ \right]$ must be aggregated using Choquet integral, the first and primary thing to do is to define a fuzzy measure or a capacity over the set N; definition of such fuzzy measure or capacity is given below.

Let $N = \{1, 2, ..., n\}$ be a set of n elements. A capacity or fuzzy measure over N is a set function $\mu : 2^N \to [0, 1]$ verifying $\mu(\emptyset) = 0$, $\mu(N) = 1$, and $\mu(A) \leq \mu(B)$ whenever $A \subseteq B$.

From a capacity μ over N one can determine the interaction indices I_{ij} (that can be helpful in practice to derive such fuzzy measure) between two elements i and j, see [11], with the following meanings.

- $I_{ij} > 0$ means that elements i and j considered individually are not important whereas when considered together they become important; thus there is synergy or complementarity between them.
- $I_{ij} < 0$ means that elements i and j are individually important but taken together the importance does not increase much more that is these attributes are substitutable, there is redundancy.
- $I_{ij} = 0$ means that elements i and j are independent.

Another interesting index associated with a fuzzy measure and that measures the importance or power of a given element i is the so called Shapley index ϕ_i, see [11].

Once an appropriate fuzzy measure is defined, one can consider Choquet integral (defined below, see [1]) as an aggregation operator.

Let μ be a capacity or fuzzy measure over N and x the numerical values vector of elements of N. The Choquet integral $C_\mu(x)$ of x with regard to μ is given by equation (5)

$$C_\mu(x) = \sum_{i=1}^{n} \left(x_{\sigma(i)} - x_{\sigma(i-1)}\right) \mu(A_i); \tag{5}$$

where σ is a permutation over N such that the order of equation (6) is respected

$$x_{\sigma(1)} \leq x_{\sigma(2)} \leq ... \leq x_{\sigma(n)}; x_{\sigma(0)} = 0; \tag{6}$$

and the subset A_i is given by equation (7)

$$A_i = \{\sigma(i), \sigma(i+1), ..., \sigma(n)\}. \tag{7}$$

One can see immediately that difficulty of using Choquet integral as an aggregation operator in practice comes from the necessity to define a fuzzy measure that necessitates specifying $2^{|N|} - 2$ coefficients representing the measure of subsets of N other than $\emptyset$ and N. Thus, if the interaction nature (synergy, redundancy or independence) between elements to aggregate and/or their importance in terms of Shapley index or their interaction indices for instance, are known, this can guide fuzzy measure definition. Rightly, in systems dynamics next state of a systems will results from synergetic aggregation of the state of systems influencing its along with other possible phenomena. Positioning this communication in this framework we propose to use Choquet integral associated to a WCFM, see [11], that is briefly recalled in the following paragraph.

1) Choquet integral associated to a WCFM: Generically, let us consider as known a relative importance weight ω_i (with the condition $\sum_{i \in N} \omega_i = 1$) for each element i of the set N of elements to aggregate that we compactly represent by vector ω with $\omega(i) = \omega_i$; a wcfm μ_ω associated to vector ω is given by equation (8), see [11].

$$\mu_\omega(A) = \frac{|A|}{|N|} \left(\sum_{i \in A} \omega_i\right), \forall A \subseteq N. \tag{8}$$

It is shown in [11] that a weighted cardinal fuzzy measure (WCFM) over N leads to interactions indices given by $I_{ij} = \frac{\omega_i + \omega_j}{|N|}$ so that instead of relative weighting vector if the interaction matrix $\mathbf{I}$ is given then the associated wcfm denoted $\mu_\mathbf{I}$, is defined by the following equation (9)

$$\mu_\mathbf{I}(A) = \frac{1}{2} \left(\sum_{i \in A} \sum_{j \in A} I_{ij}\right), \forall A \subseteq N. \tag{9}$$

The interactions indices associated to this measure must verify the following conditions:

- interactions indices associated to fuzzy measure defined by equation (8) are symmetric, that is $I_{ij} = I_{ji}$;
- they must satisfy $\sum_{i \in N} \sum_{j \in N} I_{ij} = 2$ where N is the universe of elements to aggregate.

In the same way Shapley indices are given by $\phi_i = \frac{\omega_i}{2} + \frac{1}{2|N|}$ (see [11]), so that if these indices satisfying $\phi_i \geq \frac{1}{2|N|}$ $\forall$ i are known (supplied by experts for instance), one can deduce the corresponding relative importance degree as $\omega_i = 2\phi_i - \frac{1}{|N|}$ and so one can defined the corresponding wcfm as given by equation (10)

$$\mu_\phi(A) = \frac{|A|}{|N|} \left(\sum_{i \in A} \left(2\phi_i - \frac{1}{|N|}\right)\right) \tag{10}$$

where ϕ is the Shapley indices vector.

This is an interesting thing as in some situations, it may be more convenient to experts to estimate the synergy of interaction of elements at the input of a node. These parameters can be obtained using, for instance, analytic hierarchy process (AHP) approach, see [8], as sketched in the following. The Choquet integral $C_\omega^{wcfm}(x)$ (when relative importance degrees vector ω is known), or $C_\mathbf{I}^{wcfm}(x)$ (in the case where the matrix of interaction indices $\mathbf{I}$ is available), or $C_\phi^{wcfm}(x)$ (in the case where the Shapley indices vector is given) of numerical vector x associated to the corresponding WCFM is therefore given by equations, (11)-(13)

$$C_\omega^{wcfm}(x) = \sum_{k=1}^{n} \left\{\mu_\omega(A_k) \left(x_{\sigma(k)} - x_{\sigma(k-1)}\right)\right\}; \tag{11}$$

$$C_\mathbf{I}^{wcfm}(x) = \sum_{k=1}^{n} \left\{\{\mu_\mathbf{I}(A_k)\} \left(x_{\sigma(k)} - x_{\sigma(k-1)}\right)\right\}; \tag{12}$$

$$C_\phi^{wcfm}(x) = \sum_{k=1}^{n} \left\{\{\mu_\phi(A_k)\} \left(x_{\sigma(k)} - x_{\sigma(k-1)}\right)\right\}; \tag{13}$$

where σ and A_k are defined as in equations (6) and (7) respectively; and $\mu_\omega(A_k)$, $\mu_{\mathbf{I}}(A_k)$, and $\mu_\phi(A_k)$ are given by equations (8), (9), and (10) respectively.

In practical view point, among previous indices, namely, relative importance weights ω_i, interaction indices I_{ij}, and Shapley indices ϕ_i; relative importance degrees ω_i and interaction indices I_{ij} are certainly those that can be easily estimated by experts using techniques such as analytic hierarchy process (AHP), see [8]. The task of estimating relative importance degrees ω_i is a classical AHP problem; in the following we give an AHP based approach for estimating interaction indices I_{ij}.

a) Obtaining interaction indices I_{jk} from AHP: As stated in previous section, experts may be more at their ease to supply interaction index of two inputs at the input of a given node using methods such as AHP, see [8]. Let us denote by I_{jk} the interaction index of input signals j and k; using AHP analysis, this index can be found as follows: for j, k, l, let us define $\Phi_j(k,l)$ to be a degree measuring the extent to which k is in synergy with j compared to l; this measure can be obtained from AHP standard table [8] and then check for consistent; the relative synergy importance degree υ_{jk} of k with regards to j is therefore given by equation (14)

$$\upsilon_{jk} = \frac{1}{|N|}\sum_{k\in N}\left(\frac{\Phi_j(k,l)}{\sum_{l\in N}(\Phi_j(k,l))}\right); \qquad (14)$$

alternatively one may choose a pivot p and compare all other elements to it to get coefficients $\Phi_j(k,p)$ for any $k \in N$ by answering the question "how important is element k in synergy with element j compared to element p ?" and obtain the $\Phi_j(k,l)$ by equations (15)-(16)

$$\Phi_j(p,p) = 1; \Phi_j(p,k) = \frac{1}{\Phi_j(k,p)} \qquad (15)$$

$$\Phi_j(k,l) = \Phi_j(k,p)\Phi_j^i(p,l) = \frac{\Phi_j(k,p)}{\Phi_j(l,p)}. \qquad (16)$$

The interaction index I_{jk} that fulfills requirements of previous paragraph is then obtained from (14) as given by equation (17)

$$I_{jk} = \frac{\upsilon_{jk} + \upsilon_{kj}}{|N|}. \qquad (17)$$

2) Aggregated destabilizing effect: Let us define by $\omega_i(t)$ the relative importance vector or eventually by $\mathbf{I}_i(t)$ the interaction matrix associated with $\mathbf{x}(t)$ at the input of node i at time instant t, then the global destabilizing effect $d_i(\mathbf{x}_{-i}(t))$ at the input of node i at time t is given by one of expressions of equations (18) and (19)

$$d_i(\mathbf{x}_{-i}(t)) = C^{wcfm}_{\mathbf{I}_i(t)}(\theta_i(\mathbf{x}_{-i}(t))\mathbf{x}_{-i}(t)) \qquad (18)$$

$$d_i(\mathbf{x}_{-i}(t)) = C^{wcfm}_{\omega_i(t)}(\theta_i(\mathbf{x}_{-i}(t))\mathbf{x}_{-i}(t)) \qquad (19)$$

How input disturbance may affect the dynamics of the corresponding component systems or entity may constitute a complex mechanism; in this communication we make assumption that the dynamics of the corresponding node let say node i, in the network is a sort of combination of its actual inoperability level $x_i(t)$ and its actual input global destabilizing effect $d_i(\mathbf{x}_{-i}(t))$. How to obtain this combination is the purpose of the forthcoming paragraph.

C. Dynamics model

Dynamics describe how actual operating situations will influence future ones; basically, for a component system i, how to determine its future inoperability level $x_i(t+1)$ from its actual inoperability level $x_i(t)$ and the overall impact $d_i(\mathbf{x}_{-i}(t))$ of the inoperability of its influencing nodes; basically $x_i(t+1)$ will be given by equation such as (20)

$$x_i(t+1) = f_i(x_i(t), d_i(\mathbf{x}_{-i}(t)) \qquad (20)$$

where f_i is a function that should have some appropriate properties that depend on the capacity of system i to support disturbance or its weakness with regards to inoperability of its influencing systems. In order to represent a realistic time behavior of the inoperability of a system i, function f_i must fulfills some properties. It is obvious that, the operability conditions of a system are affected by other systems, the destabilization of one of them will increase its inoperability level that will affect the systems which operation conditions rely on its and so on; thus for long term all the network may be completely destabilized (that is $x_i(t) \to 1$ when $t \to \infty \ \forall \ i$) or operating at a certain steady state level different from its nominal operating level if corrective actions are not taken; the function f_i is therefore a non decreasing time function; if global disturbance at the input of node i is zero, this system inoperability dynamics will depend only on its internal condition that we evaluate through its internal vulnerability degree α_i. The function f_i is a transfert function of a combination of internal effect and external disturbance as given by equation (21)

$$f_i(x_i(t), d_i(\mathbf{x}_{-i}(t)) = T_i((1+\alpha_i)x_i(t) + \beta_i d_i(\mathbf{x}_{-i}(t))) \qquad (21)$$

where $0 \le \beta_i \le 1$ measures the extent to which external global inoperability will impact the inoperability of system i and T_i is an appropriate transfert function. There exists many transfert functions and the choice of the apropriate one may be application dependent and expertise may be needed to choose it; for instance $T_i(x) = \min(1, x)$ will lead to possible complete destabilization whereas $T_i(x) = \tanh(x)$ may result in systems reaching steady states.

Parameters α_i and β_i will play a great role on the behavior of the system so that a sensitivity analysis with regards to these parameters may be very interesting. How to obtain them in a practical situation is also worth of investigation; risk assessment approaches can be used for this purpose, see for instance [12] and [10]. From the dynamic model described by equations (20) and (21), many scenarios can be analyzed to support sound decision making. For instance, given a shift in the operation conditions of a particular systems, one may consider evaluating how long it will take for another system to become completely inoperable or its inoperability to be increased by a certain amount ?

III. Application

The developed model can be used in almost any socioeconomic domain when there is a need to analyze the effect of interdependency between many systems, component systems, sectors, components, etc. such as infrastructures, economic sectors, social services, etc. for the purpose of monitoring, policy evaluation, prediction, etc.. One particular domain where this model can be helpful is the domain of risk assessment and management in terms of analyzing risk propagation because of the interdependency. Large scale, social, economical, and technical, complex systems such as that of interdependent strategic infrastructures or economic sectors systems need to be addressed in systems engineering perspectives in order to dispose of tools to aid policy and public decision makers make sound and adequate decisions. Thanks to computers performance nowadays in terms of storage capacity and processing rapidity, the possibility to study complex interconnected systems to support sound and adequate decision process in many domains becomes a reality. Furthermore, dynamics, in interconnected systems are very important as future behavior of such systems depend on the actual states of systems with which they are in relations on network basis. For risk management of strategic infrastructures purpose, decision makers may be interested to evaluate how the disruption of an infrastructure will affect the operations results of other infrastructures; see [9] for a study on interdependent economic sectors for instance. Modeling how disruptions of different infrastructures combine to affect the operating level of other infrastructures is a challenging tasks; nevertheless, one can consider in first approximation that effects combine in a synergetic way. To do so, let us consider a network of 5 infrastructures or systems (at a country or a large region level), namely: gas and oil infrastructure (S_1); petroleum and coal infrastructure (S_2); electricity supply infrastructure (S_3); water supply infrastructure (S_4); and road transport infrastructure (S_5).

This application is extracted from [7] for illustrative purpose; in the original study of [7], 7 infrastructures were considered including banking and communication besides of that considered here. The main purpose in that study was to estimate the ripple impact risk (measured in terms of a certain amount of financial loss by solving a static equation) on the considered infrastructure of a scenario of attack on petroleum and coal infrastructure with a certain amount of loss. The estimated "vulnerability coefficients" was done using a public data from Australia; coefficients corresponding to our selected 5 infrastructures are given on the following Table 1.

	S_1	S_2	S_3	S_4	S_5
S_1	0.0181	0.4571	0.0328	0	0.0001
S_2	0.0051	0.0235	0.0132	0.0041	0.1274
S_3	0.0036	0.0017	0.1700	0.0105	0.0030
S_4	0.0001	0.0014	0.0065	0.0360	0.0202
S_5	0.0012	0.0020	0.0049	0.0014	0.0472

Table 1: raw data from [7]

Here, our intention is to develop a dynamic model to simulate scenarios such as "how long it will take before the whole system or some sub-systems will become completely inoperating or operating at a steady state different from their nominal operating levels given that one particular sub-system operating capacity is reduced by a certain percentage?" From the matrix of Table 1, we interpret, for our study, the coefficient on the diagonal as the one step internal vulnerability coefficient α_i of the the corresponding infrastructure and the sum of the rest of coefficients of the corresponding line as its one step external vulnerability coefficient β_i, so that they are given by following equations (22) and (23)

$$\alpha = \begin{bmatrix} 0.0181 & 0.0235 & 0.1700 & 0.0360 & 0.0472 \end{bmatrix} \quad (22)$$

$$\beta = \begin{bmatrix} 0.4900 & 0.1499 & 0.0188 & 0.0282 & 0.0095 \end{bmatrix} \quad (23)$$

To obtain the relative external influence matrix $\omega = [\omega_{ij}]$, we normalize data of previous table when disregarding coefficients on the diagonal to obtain the matrix of following equation (24)

$$\omega = \begin{bmatrix} 0 & 0.9328 & 0.0669 & 0 & 0.0003 \\ 0.0343 & 0 & 0.0883 & 0.0275 & 0.8499 \\ 0.1910 & 0.0894 & 0 & 0.5577 & 0.1620 \\ 0.0039 & 0.0507 & 0.2312 & 0 & 0.7142 \\ 0.1285 & 0.2064 & 0.5176 & 0.1476 & 0 \end{bmatrix} \quad (24)$$

where ω_{ij} is the relative importance degree of j on i; meaning that:

- The external inoperability that will affect the inoperabilities of gas and oil infrastructure (S_1) are petroleum and coal infrastructure (S_2), electricity supply infrastructure (S_3), and the road transportation in the proportion 93.28%, 6.69%, and 0.03% respectively; whereas water supply (S_4) does not have influence on gas and oil infrastructure.
- Petroleum and coal infrastructure (S_2) will be mostly influenced by road transportation (S_5) for 84.99%, followed by electricity supply infrastructure (S_3) for 8.83%, gas and oil infrastructure for 3.43% and finally by water supply for 2.75%.
- The inoperability of electricity supply infrastructure (S_3) will be increased by the inoperability of water supply infrastructure (S_4), gas and oil infrastructure (S_1), road transportation infrastructure (S_5), and petroleum and coal infrastructure (S_2), with the relative importance of 55; 77%, 19.10%, 16.20%; and 8.94% respectively.
- Water supply infrastructure (S_4) functioning conditions will be degraded by inoperability of road transportation infrastructure (S_5, 71.42%), electricity supply infrastructure (S_3, 23.12%), petroleum and coal infrastructure (S_2, 5.07%), and gas and oil infrastructure (S_1, 0.39%).
- The inoperability of road transportation infrastructure (S_5) will be affected by the inoperability of electricity supply infrastructure (S_3, 51.76%), petroleum and coal infrastructure (S_2, 20.64%), water supply (S_4, 14.76%), and gas and oil infrastructure (S_1, 12.85%).

From dynamic model (21), many scenarios can be tested; for instance how disruption on petroleum and coal infrastructure (S_2) will affect the operating level of other infrastructure: let us suppose that at $t = 0$, petroleum and coal supply is cut by 20% of its nominal level, following Figure 1 shows how the other infrastructures inoperability will behave.

One can analyze results shown on Figure 1 as following, two infrastructures namely gas and oil infrastructure (S_1) and electricity supply infrastructure (S_3) are very sensitive to operating condition of petroleum and coal infrastructure;

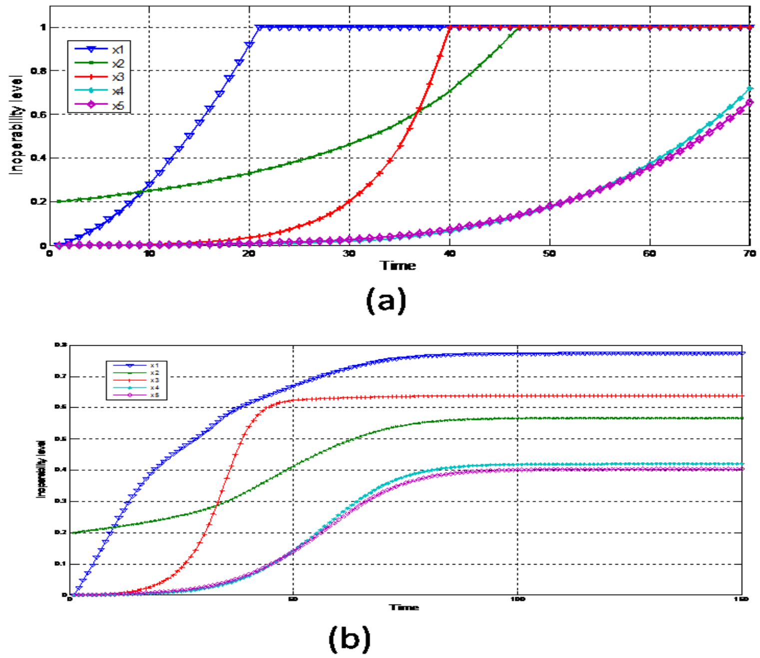

Fig. 1. Inoperabilities behavior for 20% cut of petroleum supply with transfer functions: (a) $T_i(x) = \min(1, x)$ and (b) $T_i(x) = \tanh(x)$.

indeed 20% cut of operability of this one leads to gas and oil infrastructure becoming inoperating after approximately 20 time units and electricity infrastructure fails after 40 time units or reaching a very high steady state inoperability level compared to the initial disturbed infrastructure. Water supply and road transportation operating conditions seem to be less sensitive to the reduction of the operating level of petroleum and coal infrastructure. One should not consider this study as a validation of the presented inoperability propagation model as data as well as conditions and assumptions are probably completely different. But results show that the model presented so far can be able to predict (at least qualitatively in terms of tendancies) sound behavior of networked systems if one dispose of good data to derive different parameters and a good analysis of interconnection relationships mainly to confirm synergetic relationship assumption that underlies the model.

IV. Conclusion

The problem of analyzing and modeling the mechanism of inoperability propagation within a networked or interdependent systems has been considered in this communication. The communication has developed some tools that can be used to assess the strength of the impacts of interdependency mainly in the case of synergetic behavior of those impacts to reach a dynamic model of how inoperability propagate in a such networked systems. Another issue raised in this communication relate to the possibility of an impact to be irrelevant for the destabilization of the impacted system if it does not reached some thresholds, a phenomenon referred to as the trigging process. The developed model is the beginning of a research projects that should not only improve the modeling process, but also verify or confirm the validity of assumptions made, develop parameters estimation models and validate the results on real world problem.

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